Commercial Disappearance and Composite Demand for Food with an Application to U.S. Meats

Albert J. Reed, J. William Levedahl, and J. Stephen Clark

When elementary prices move strictly proportionately, aggregation over a group of diverse products is valid, and group demand responses can be decomposed into quality and quantity responses. This study shows that when relative elementary prices and group prices are stochastically independent, a similar decomposition is valid. Empirical results suggest consumers respond to changes in prices and income mostly by altering the quality of meat products. These findings imply that using commercial disappearance as a proxy for food demand can be misleading for policy analysis.

Key words: commodity aggregation, Composite Commodity Theorem, composite demand, Generalized Composite Commodity Theorem, quantity-quality decomposition

Introduction

Data from the Consumer Expenditure Survey (U.S. Department of Labor, Bureau of Labor Statistics) suggest that since the late 1980s, U.S. expenditures for at-home meat and meat products have risen. From 1988 through 2000, per capita meat expenditures grew at an average annual rate of 4.3%, although spending patterns among the different meat categories varied. Over this same period, per capita expenditures for poultry rose at an annual average rate of 4.5%, followed by pork (3.5%), other meat (3.07%), and beef (2.2%). These expenditure trends are illustrated in figure 1.

Empirical analysis of such expenditure patterns can reveal information useful for public policy decisions. For example, growth in meat expenditures attributed to increases in the physical units of meat consumed might have more direct implications for consumer health and nutrition issues than if expenditure growth were attributed to shifts toward a mix of more costly meat products. Resolving such questions depends on accounting for expenditure patterns among elementary products within broad food categories.

The Composite Commodity Theorem (CCT) (Hicks; Leontief) justifies the construction of composite measures of demand that describe, in a consistent way, consumer purchases of diverse elementary products. The validity of the CCT rests on the presumption of strictly proportional price movements, a restriction which is virtually always rejected empirically. Nevertheless, the CCT does imply that expenditures can be decomposed into the product of composite price and composite demand, and that composite demand

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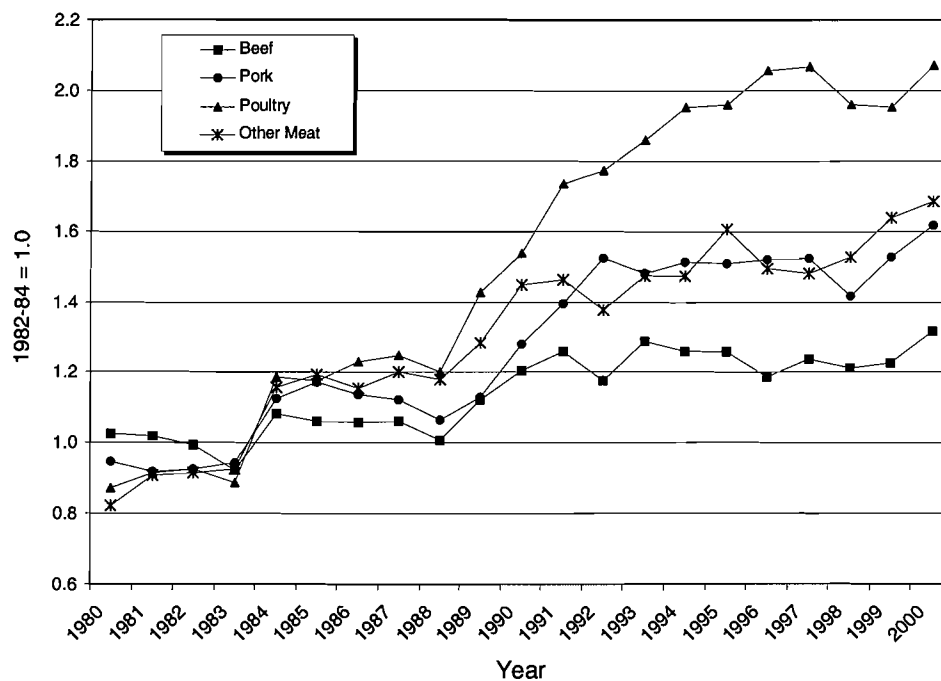
Source: Consumer Expenditure Survey (U.S. Department of Labor, Bureau of Labor Statistics)

Figure 1. Per capita meat expenditures, 1980–2000

can be decomposed into the product of composite physical quantity and Theil's measure of composite quality, which reflects the *mix* of diverse products chosen by consumers (Nelson 1991). This analysis derives virtually the same decompositions of group expenditures and composite demand, and provides the same interpretation of composite quality under the more plausible and refutable restrictions of the Generalized Composite Commodity Theorem (GCCT) (Lewbel).

Increasingly, the GCCT has been used to justify various product aggregation schemes (Eales, Hyde, and Schrader; Asche, Bremnes, and Wessels; Williams and Shumway). Based on tests of the GCCT, this study finds beef, pork, and poultry to be valid meat aggregates. The existence of valid aggregates has the additional implication that composite beef, pork, and poultry can be decomposed into quality and quantity components.

Both the CCT and the GCCT point to the drawback of using commercial disappearance as a proxy for composite food demand in empirical work. Commercial disappearance does not account for the diversity of consumer food products. By definition, the retail-equivalent measure of commercial disappearance is an estimate of the physical quantity of a group of elementary consumer food products. The theory of aggregation over products clearly asserts that only in the case in which consumers view a group of goods as identical can commercial disappearance represent a valid measure of composite demand. When food product diversity is ruled out, the mix of products, and therefore Theil's measure of quality, cannot fluctuate over time.

However, both the CCT and the GCCT suggest disappearance data, as a measure of physical quantity, can be used to enrich composite demand analysis. Using a measure of physical quantity similar to commercial disappearance, Nelson (1990) reports regression estimates of quantity and quality elasticities for various foods in both the Ivory

Coast and the United States. These regressions are valid only under the restrictions imposed by the CCT. An implication of the current study is that the same regressions remain valid under the more plausible and refutable restrictions of the GCCT.

Based on annual expenditure and commercial disappearance data, estimates of demand and quantity regressions for beef, pork, and poultry indicate the price and income elasticities of quality are notably larger than the price and income elasticities of quantity. The results describe consumers responding to changes in prices and income by changing the mix of meat products to a greater extent than they change the quantity of meat products. This description is consistent with the notion that changing consumer preferences are altering the kinds of food products marketed (Kinsey and Senauer).

Theory

This section appeals to the Composite Commodity Theorem (CCT) (Hicks; Leontief) and the Generalized Composite Commodity Theorem (GCCT) (Lewbel) as ways to justify composite beef, pork, and poultry measures of demand and quality. Both theorems imply that changes in the mix of purchases of diverse goods alter composite demand and composite quality.

In an application of the CCT, Nelson (1991) shows that by expressing composite demand and Theil's measure of composite quality in terms of elementary products, consumers not only determine composite measures of demand and quantity, but also choose quality by selecting the mix of products.[1] The usefulness of these results for applied demand analysis is limited by the fact that they depend on strictly proportional movements of elementary prices—a condition unlikely to hold empirically. In this section, virtually the same relationships between composite demand and quality known to hold under the CCT are demonstrated to remain valid under the more plausible restrictions of the GCCT.

The notation used throughout the remainder of the article is as follows. Suppose that over some time interval, consumers purchase a vector of $i = 1, 2, ..., n$ different final elementary products $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ at prices $\mathbf{p} = [p_1, p_2, ..., p_n]^T$, so that total expenditures (income) is $y = \mathbf{p}^T\mathbf{x}$. Because each x_i is considered homogeneous, the x_i and p_i are termed elementary quantities and elementary prices, respectively. Elementary budget shares are defined as $w_i = (p_i x_i)/y$. By a suitable relabeling, denote the last $n - k$ elementary goods, $\mathbf{x}_G = [x_{k+1}, x_{k+2}, ..., x_n]^T$ as the elementary demands of group G, for which consumers face prices $\mathbf{p}_G = [p_{k+1}, p_{k+2}, ..., p_n]^T$, so that group expenditures are $y_G = \mathbf{p}_G^T\mathbf{x}_G \equiv \Sigma_{i \in G} p_i x_i$, where $G = \{i: i = k+1, k+2, ..., n\}$. The average price (index) for group G is P_G, with relative prices denoted as $\rho_G = [p_{k+1}/P_G, p_{k+2}/P_G, ..., p_n/P_G]^T$. If each of the elements of $\mathbf{x}_G$ are measured in a common physical unit, then $q_G \equiv x_{k+1} + x_{k+2} + ... + x_n = \Sigma_{i \in G} x_i$ represents a measure of physical quantity for group G. Conditioned on this measure, the unit value for the group is $V_G = y_G/q_G$, and Theil's composite quality variable, defined as the unit value-to-price ratio, is denoted $v_G = V_G/P_G$.

From the n different final products, the problem is to form $M < n$ groups where $\mathbf{q} = [q_1, q_2, ..., q_G, ..., q_M]^T$; $\mathbf{V}(\mathbf{v}) = [y_1/q_1, y_2/q_2, ..., y_G/q_G, ..., y_M/q_M]^T$; $\mathbf{P} = [P_1, P_2, ..., P_G, ..., P_M]^T$;

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[1] The idea that consumers *choose* composite quality originated with Theil. This view of quality contrasts with the notion that consumers *respond* to quality either within a household production context (Lancaster) or a standard optimization problem (Fisher and Shell; Hanneman).

$\rho = [\rho_1^T, \rho_2^T, ..., \rho_G^T, ..., \rho_M^T]^T$; and $\mathbf{v} = [V_1/P_1, V_2/P_2, ..., V_G/P_G, ..., V_M/P_M]^T$ denote vectors of composite quantities, unit values, price indices, relative prices, and composite qualities, respectively.

Theil presumes that optimizing consumers choose vectors of *composite* quantity and quality by solving $\max_{\mathbf{q},\mathbf{v}} U = f(\mathbf{q}, \mathbf{v})$ subject to $y = \mathbf{V}(\mathbf{v})^T\mathbf{q}$. To incorporate exogenous prices into the budget constraint, the optimization problem is restated as:

(1)
$$\max_{\mathbf{q},\mathbf{v}} U = f(\mathbf{q}, \mathbf{v}),$$

$$\text{s.t.:} \quad y = \sum_{G=1}^{M} P_G v_G q_G.$$

Theil defines composites as groups of elementary goods with prices that are functionally related. Regardless of the nature of these functions, because (1) contains $2M$ "unknowns" and $M+1$ "knowns," (1) may be solved as a discrete-continuous time problem (Hanneman), but it cannot be solved for smooth, interior solutions. In the special case where a group is defined as one in which the elementary prices move proportionately, Nelson (1991) appeals to the CCT to show that Theil's problem involving $2M$ quantities and qualities can be condensed into a problem involving M composite demands.

According to the CCT, proportionate movements in elementary prices within group G imply, for base-period prices, p_i^o, $p_i = P_G p_i^o$ ($\forall i \in G$), so that group expenditures are $y_G = P_G \Sigma_{i \in G} p_i^o x_i$. Multiplying this expression by q_G/q_G and comparing the result to $P_G v_G q_G$ in (1) gives:

(2)
$$Q_G \equiv y_G/P_G = \sum_{i \in G} p_i^o x_i,$$

(3)
$$v_G \equiv y_G/(P_G q_G) \equiv Q_G/q_G = \left[\sum_{i \in G} p_i^o (x_i/q_G) \right].$$

Since it has been shown that the term $\Sigma_{i \in G} p_i^o x_i$ in (2) summarizes and exhausts the behavioral implications of utility theory (Deaton and Muellbauer, chap. 5; Silberberg, chap. 11), Q_G is a measure of composite consumer demand. Therefore, Theil's problem can be restated as $\text{Max}_{\mathbf{Q}} U = f(\mathbf{Q})$ subject to $y = \Sigma_{G=1}^{M} (P_G Q_G)$, where $\mathbf{Q} = [Q_1, Q_2, ..., Q_G, ..., Q_M]^T$ (Nelson 1991).

Equations (2) and (3) define Q_G and v_G in terms of composite variables usually available to analysts. According to (2), composite demand is the ratio of group expenditures to the group price index, and (3) states that Theil's composite quality variable is the ratio of composite demand to composite physical quantity.[2] Moreover, (2) and (3) imply $Q_G \equiv v_G q_G$, so that composite demand can be decomposed into composite quantity and quality. By choosing one attribute to measure physical quantity, the CTT suggests analysts can think of consumers differentiating elementary products on the basis of all other attributes which are summarized by Theil's measure of quality. Equations (2) and (3) state that given observations on group prices and expenditures, composite demand is unique regardless of the physical unit chosen to measure quantity (Nelson 1991).

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[2] Note that composite quality (v_G) is defined by Theil as the unit value-to-price ratio, so $v_G \equiv V_G/P_G \equiv (E_G/q_G)/P_G \equiv (E_G/P_G)/q_G \equiv (Q_G/q_G)$.

Traditionally, proxies for composite food demand in empirical work are often based on a single physical attribute (e.g., Huang 1985, 1993). That is, if each x_i is expressed in terms of a common physical unit (e.g., retail pounds), the sum $q_G \equiv \Sigma_{i \in G} x_i$ is often used as a measure of composite demand. However, if consumers value different goods within a composite differently, q_G cannot represent a valid measure of composite demand because it is not invariant to the physical unit chosen.[3] For example, even though one pound of hamburger equals one pound of lean steak, this equality will almost certainly not hold if the goods are expressed in terms of fat content or calories (or texture, flavor, convenience, etc.) per pound (Nelson 1991). Summing over different physical attributes is likely to result in different preference orderings.

Equations (2) and (3) also link composite variables to elementary consumer products. Equation (2) states that composite demand is a linear combination of the physical units of elementary products purchased, with fixed base-period elementary prices used as weights. Thus, the subjective differences among the elementary goods are reflected in differences among the elementary prices. Further, there are two ways in which composite demand can change: when consumers alter the physical *amount* of elementary goods, or when they alter the *mix* of different elementary goods. Equation (3) states that when consumers alter the *mix* of goods, Theil's measure of composite quality changes. Specifically, composite quality (v_G) rises (falls) as consumers purchase a higher (lower) proportion of relatively high-priced elementary goods. In combination, equations (2) and (3) indicate composite demand variables based on a single physical attribute are valid only when fluctuations in the mix of different consumer products are ruled out.

Unlike the CCT, product aggregation based on separability does not explicitly link quality to the mix of different products or inputs. For example, Chinloy defines quality of labor inputs as the ratio of composite demand to quantity, but cannot explicitly link this measure to a variable mix of elementary labor inputs.

A drawback of the CCT is that it requires prices to move strictly proportionately. This requirement can seriously restrict the potential range of products to be aggregated (Deaton and Muellbauer, pp. 121–22). For example, if one wishes to form a composite dairy demand variable, but cheese prices are more volatile than fluid milk prices, the CCT suggests these two products would not be elements of the same composite group. Under the considerably more relaxed requirements of the GCCT, such aggregation might be possible.

The GCCT provides a stochastic interpretation of Theil's model of quantity and quality which closely follows Nelson's (1991) interpretation. Although the GCCT is developed in the context of budget shares, note that Theil's budget constraint $y = \Sigma_{G=1}^{M}(P_G v_G q_G)$ can be rewritten as $1 = \Sigma_{G=1}^{M}(P_G^* v_G q_G)$, where $P_G^* = P_G / y$. Define $\mathbf{P}^* = [P_1/y, ..., P_M/y]^T$ as the vector of normalized prices, and $\mathbf{p}^* = [p_1/y, ..., p_n/y]^T$ as the vector of elementary normalized prices, so that $\mathbf{W} = [W_1, ..., W_M]^T = [P_1^* v_1 q_1, ..., P_M^* v_M q_M]^T$ represents the M-vector of composite shares, and $\mathbf{w} = [w_1, ..., w_n]^T = [p_1^* x_1, ..., p_n^* x_n]^T$ represents the n-vector of elementary shares. Note also, the ith element of ρ is $\rho_i = p_i/P_G = p_i^*/P_G^* \, \forall \, i \in G$.

According to the GCCT, composite shares are presumed to be functions of composite normalized prices so that $W_G = W_{Gh}(\mathbf{P}^*) + \varepsilon_G$, where $E(\varepsilon_G | \mathbf{P}^*) = 0$, E is a mathematical expectations operator, and the subscript h denotes predicted value. Because composite

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[3] For an eloquent discussion of the topic summarized in this paragraph, see Nelson (1991).

shares are also sums of elementary shares, they can also be represented as $W_G = W_{Gh}^*(\mathbf{p}^*) + e_G$, $E(e_G | \mathbf{p}^*) = 0$. Let $\kappa = E\rho$, with κ_i representing the ith element of κ. As shown in appendix A, if ρ is independent of $\mathbf{P}$, terms in Theil's budget constraint can be written as:

$$(4) \qquad P_G^* v_G q_G = P_G^* \sum_{i \in G} \kappa_i x_i + \varepsilon_G.$$

Equation (4) resembles (2) in that a composite variable (in this case a share) is expressed as a linear combination of quantities of elementary goods. In this case, however, the independence condition required by the GCCT implies that the means of relative prices, rather than base-period prices, are used as weights.

Lewbel derives conditions under which composite budget shares consistently summarize consumer demand. Given the vectors ρ and $\mathbf{P}$ are independent, $E(P_G^* v_G q_G | \mathbf{P}^*)$ satisfy adding-up and homogeneity, and either satisfy or nearly satisfy Slutsky symmetry. That is, given valid (i.e., rational) elementary demands, the stochastic group share functions are integrable (or nearly so) because they are assumed to be derived from a utility-maximization problem. If, in addition to independence between ρ and $\mathbf{P}$, the income gradient of the elementary indirect utility function depends only on *group* prices and income, the utility implied by composite demand represents the best predictor of actual consumer utility (Lewbel, theorem 3).[4] This separability-type restriction becomes tractable for indirect utility functions from which homothetic (corollary 4), PIGLOG (corollary 5), AIDS (corollary 6), and the exactly aggregable translog (corollary 7) demands are derived. It is in this sense that the composite shares defined by (4) exhaust the behavioral implications of utility theory.

Given that composite shares, $P_G^* v_G q_G$, represent a valid measure of demand, equation (4) is central to analyses of consumer expenditures under the relatively mild requirements of the GCCT. Note that since $E(\varepsilon_G | \mathbf{P}^*) = 0$, $E(P_G^* v_G q_G | \mathbf{P}^*) = P_G^* E(v_G q_G | \mathbf{P}^*) = P_G^* E(\Sigma_{i \in G} \kappa_i x_i | \mathbf{P}^*)$,

$$(5) \qquad E(Q_G | \mathbf{P}^*) \equiv E(v_G q_G | \mathbf{P}^*) \equiv E(y_G / P_G | \mathbf{P}^*) = \sum_{i \in G} \kappa_i x_i.$$

Therefore, as with the CCT, quantity demand is a linear combination of physical measures of elementary goods with average relative prices serving as weights. The practice of using composite physical quantity (q) as a proxy for demand in empirical work presumes physical quantity can be described as a function of prices and income. Because (5) *maintains* that composite demand is a function of prices and income, it follows that composite quality is a valid function of prices and income. A stochastic version of this statement is written as:

$$(6) \qquad E(Q_G | \mathbf{P}^*) \equiv E(v_G q_G | \mathbf{P}^*) = E(q_G | \mathbf{P}^*) E(v_G | \mathbf{P}^*)$$

which, from (5), implies:

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[4] The elementary (composite) indirect utility function is the utility function from which the elementary (composite) demand functions are derived.

$$(7) \qquad E(v_G | \mathbf{P^*}) \equiv \sum_{i \in G} \kappa_i \big[x_i / E(q_G | \mathbf{P^*}) \big] . \,{}^{5}$$

As with the CCT, (7) indicates Theil's measure of composite quality summarizes the mix of products consumers purchase under the restrictions of the GCCT. According to equation (7), on average, composite quality rises (falls) as consumers purchase a higher (lower) average proportion of relatively high (low) priced products.

Moreover, since $E(y\varepsilon_G | \mathbf{P^*}) = y(E\varepsilon_G | \mathbf{P^*}) = 0$, equations (4), (5), and (6) imply:

$$(8) \qquad E(y_G | \mathbf{P^*}) \equiv P_G E(Q_G | \mathbf{P^*}) = P_G E(q_G | \mathbf{P^*}) E(v_G | \mathbf{P^*}).$$

Equation (8) states that conditioned on group prices and income, average group expenditures can be decomposed into group price and average group demand, and that average group demand can be further decomposed into average physical quantity and average quality. Hence, under less stringent conditions of the GCCT, (8) provides essentially the same framework for decomposing consumer expenditure data as the CCT.

Both the GCCT and the CCT indicate expenditure data directly reflect consumer behavior. Let $\eta_G^{(y)}$, $\eta_G^{(Q)}$, $\eta_G^{(v)}$, and $\eta_G^{(q)}$ denote the income elasticities of group G expenditures, demand, quality, and quantity, and let $\varepsilon_{GH}^{(y)}$, $\varepsilon_{GH}^{(Q)}$, $\varepsilon_{GH}^{(v)}$, $\varepsilon_{GH}^{(q)}$ denote the Hth price elasticity for group G expenditures, demand, quality, and quantity. Then, if δ_{GH} denotes the Kronecker delta, so that $\delta_{GH} = 1$ when $G = H$, and $\delta_{GH} = 0$ otherwise, it follows from (8) that

$$(9) \qquad \eta_G^{(y)} \equiv \eta_G^{(Q)},$$

$$(10) \qquad \varepsilon_{GH}^{(y)} \equiv \delta_{GH} + \varepsilon_{GH}^{(Q)},$$

$$(11) \qquad \eta_G^{(Q)} \equiv \eta_G^{(v)} + \eta_G^{(q)},$$

$$(12) \qquad \varepsilon_{GH}^{(Q)} \equiv \varepsilon_{GH}^{(v)} + \varepsilon_{GH}^{(q)},$$

which are the same elasticity relationships implied by the CCT (Nelson 1991). Note, (9) and (11) imply $\eta_G^{(y)} = \eta_G^{(v)} + \eta_G^{(q)} \equiv \eta_G^{(Q)}$, or that income elasticities of group expenditures equal income elasticities of group demand. Based on this relationship, when diverse products are consistently aggregated, the composite can be treated as a single good (Cramer). Moreover, from equation (10), cross-price elasticities estimated from group expenditure data equal cross-price elasticities estimated from composite demand data, and the own-price elasticities of group expenditures equal one plus the own-price elasticities of demand. Equations (9)–(12) establish that under both the CCT and the GCCT, expenditure data can be used to make direct inference on consumer behavior.

Both the CCT and the GCCT imply that when consumers alter the mix of products, composite quality changes. To show this, consider the case in which composite quality remains constant and equals θ for all periods t. Composite demand is then expressed as follows:

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[5] Let $f(\mathbf{v}, \mathbf{q}, \mathbf{P})$ represent the probability distribution of quality, quantity, and price. Then $f(\mathbf{v}, \mathbf{q}, \mathbf{P}) = f_1(\mathbf{P}) * [f_2(\mathbf{v}, \mathbf{P})/f_1(\mathbf{P})]$ $* [f(\mathbf{v}, \mathbf{q}, \mathbf{P})/f_2(\mathbf{v}, \mathbf{P})] = f_1(\mathbf{P}) * f_2(\mathbf{v}|\mathbf{P}) * f_3(\mathbf{q}|\mathbf{P}, \mathbf{v})$. The practice of regressing quantity on prices and income suggests the conditional distribution of quantity, $f_3(\mathbf{q}|\mathbf{P}, \mathbf{v}) = f_3(\mathbf{q}|\mathbf{P})$, implying $f(\mathbf{v}, \mathbf{q}, \mathbf{P})/f_1(\mathbf{P}) = f_2(\mathbf{v}|\mathbf{P}) * f_3(\mathbf{q}|\mathbf{P})$, and the result in the text obtains.

$$(13) \qquad Q_{Gt} = \theta q_{Gt} = \theta(x_{k+1} + x_{k+2} + \dots + x_n)_t .$$

Comparing (13) to (2) or (5) implies $p_i^* = \theta$, or $\kappa_i = \theta$, $\forall i \in G$. Consequently, consumers are purchasing elementary goods with identical base or mean relative prices, implying the elementary goods are homogeneous. In this case, changes in the mix of products are ruled out.

The U.S. Department of Agriculture (USDA) computes retail-equivalent measures of commercial disappearance using formulas similar to (13). Commercial disappearance for a commodity is calculated as the residual of total farm production of the commodity minus the sum of net exports, changes in stocks, shipments, and military purchases all expressed in terms of some common physical unit of farm commodities (Putnam and Allshouse). Let F_G denote this farm-based residual for goods making up group G. Then the retail-equivalent measure of commercial disappearance is $\phi_G F_G$, where ϕ_G is a single output-farm input coefficient for the industry producing goods for group G. Because $F_{Gt} \equiv \Sigma_{i \in G} f_{it}$, where f_{it} is the amount of farm product used in the production of the ith retail product, the retail equivalent measure of commercial disappearance is $\phi_G \Sigma_{i \in G} f_{it}$. This equivalent measure takes the form of (13) when $x_{it} = f_{it}$ and $\theta = \phi_G$. In the case where the quantity of elementary products is measured in terms of retail-equivalent weight, (13) shows that $x_{it} = \phi_G f_{it}$, and $\theta = 1$.

The above discussion suggests that because commercial disappearance ignores differences among elementary products, these data are likely to describe only a portion of consumer behavior. The remainder of this article attempts to measure how well commercial disappearance data, in fact, reflect composite beef, pork, and poultry demand.

The GADS Demand-Quantity System

In this study, estimates of composite demand, quantity, and quality elasticities for at-home beef, pork, and poultry are computed from estimates of a system combining Bewley and Young's version of the generalized addilog demand system (GADS) with a system of quantity equations. The GADS demand-quantity system is specified in this section.

The version of GADS introduced by Bewley and Young is a convenient linearization of Theil's highly nonlinear multinomial extension of the linear logit model because it provides an economic interpretation of the estimated coefficients and facilitates inference. In the Bewley and Young version of the addilog system, Slutsky symmetry is required to hold exactly at a given point (and is assumed to hold approximately elsewhere). Let t denote a time index. For an M-system of composite demands with M-composite budget shares $(W_{1t}, W_{2t}, \dots, W_{Mt})$, this version of GADS requires Slutsky symmetry to hold exactly at some predetermined value of the budget shares. Denote this point as $\mathbf{W}^* = (W_1^*, W_2^*, \dots, W_M^*)$. $\mathbf{W}^*$ could be any data point provided $\Sigma_{J=1}^M W_J^* = 1$ (for example, the sample mean). Following Bewley and Young, denote $\ln(W_t^+) = \Sigma_{J=1}^M W_J^* \ln(W_{Jt})$ and $\ln(P_t) = \Sigma_{J=1}^M W_J^* \ln(P_{Jt})$. Let $\varepsilon_{IJ}^{(Q)}$ represent the Marshallian elasticity of the Ith demand with respect to the Jth price, and let $\eta_I^{(Q)}$ represent the Ith income elasticity of demand. If $u_{It}^{(Q)}$ denotes the Ith model error, Bewley and Young's version of GADS for this M-equation system of composite demands is given as:

$$(14) \qquad W_I^* \ln(Q_{It}/W_t^+) = a_I + \sum_{J=1}^M \pi_{IJ}^{(Q)} \ln(P_{Jt}) + \theta_I^{(Q)} \ln(y_t/P_t) + u_{It}^{(Q)}, \qquad I = 1, \dots, M,$$

where

$$(15) \qquad \pi_{IJ}^{(Q)} = W_I^* \varepsilon_{IJ}^{(Q)} + W_I^* W_J^* \eta_I^{(Q)}$$

and

$$(16) \qquad \theta_I^{(Q)} = W_I^* \eta_I^{(Q)}.$$

In this version of the addilog, the price coefficients (i.e., $\pi_{IJ}^{(Q)}$) in each of the M linear equations of (14) are Slutsky parameters defined at point $\mathbf{W}^*$, so that a test of the restrictions $\pi_{IJ}^{(Q)} = \pi_{JI}^{(Q)}$ represents a test of Slutsky symmetry at point $\mathbf{W}^*$.

A convenience of Bewley and Young's version of GADS is that the demand elasticities are linear functions of *only* model parameters and the predetermined point $\mathbf{W}^*$. In particular, $\varepsilon_{IJ}^{*(Q)} = (1/W_I^*)\pi_{IJ}^{(Q)}$, $\eta_I^{(Q)} = (1/W_I^*)\theta_I^{(Q)}$, and $\varepsilon_{IJ}^{(Q)} = (1/W_I^*)\pi_{IJ}^{(Q)} - (W_J^*/W_I^*)\theta_I^{(Q)}$ are the compensated price, income, and Marshallian price elasticities, respectively (at $\mathbf{W}^*$). Because Bewley and Young's version of GADS is a linearization around the fixed point $\mathbf{W}^*$, variances of the estimated elasticities (i.e., $\varepsilon_{IJh}^{*(Q)}$, $\eta_{Ih}^{(Q)}$, and $\varepsilon_{IJh}^{(Q)}$) based on (15) and (16) are:

$$\text{Var}\left[\varepsilon_{IJh}^{*(Q)}\right] = (1/W_I^*)^2 \text{Var}\left[\pi_{IJh}^{(Q)}\right],$$

$$\text{Var}\left[\eta_{Ih}^{(Q)}\right] = (1/W_I^*)^2 \text{Var}\left[\theta_{Ih}^{(Q)}\right], \text{ and}$$

$$\text{Var}\left[\varepsilon_{IJh}^{(Q)}\right] = (1/W_I^*)^2 \text{Var}\left[\pi_{IJh}^{(Q)}\right] + \left(W_J^*/W_I^*\right)^2 \text{Var}\left[\theta_{Ih}^{(Q)}\right] - 2\left(W_J^*/W_I^{*2}\right) \text{Cov}\left[\pi_{IJh}^{(Q)}, \theta_{Ih}^{(Q)}\right],$$

given the estimated variance/covariance of the model coefficients $\pi_{IJh}^{(Q)}$ and $\theta_{Ih}^{(Q)}$.

Because the sum over the M-equations of the left-hand side of equation (14) reduces to $\ln(y_t/P_t)$, least squares automatically imposes $\Sigma_{I=1}^M a_I = 0$, $\Sigma_{I=1}^M \pi_{IJ}^{(Q)} = 0$, $\Sigma_{I=1}^M \theta_I^{(Q)} = 1$, and therefore $\Sigma_{I=1}^M u_{It}^{(Q)} = 0$ (Bewley and Young). Hence, the covariance matrix of residuals of an estimated GADS is singular, and an equation is dropped in estimation. In the case of no serial correlation and no cross-equation restrictions, OLS, maximum likelihood (ML), and seemingly unrelated regression (SUR), parameter estimates and standard errors of these estimates would be invariant to the equation dropped (Berndt and Savin). In the case of symmetry-restricted (or any other cross-equation restriction) estimates, only SUR and ML estimates of coefficients and standard errors would be invariant to the omitted equation (Berndt and Savin).

Conceptually, (11) and (12) state that Marshallian price and income elasticities of demand can be decomposed into quality and quantity components. To decompose estimates of demand elasticities requires estimates of quality or quantity elasticities. Note, if $\varepsilon^{(q)}$ and $\eta^{(q)}$ are price and income elasticities of quantity, and $e^{(q)}$ is a model error term, premultiplying the Ith double-log representation of quantity, $\ln(q_{It}) = c_I + \Sigma_j \varepsilon_{IJ}^q \ln(P_{Jt}) + \eta_I^q \ln(y_t) + e_{It}^q$, by W_I^* yields:

$$(17) \qquad W_I^* \ln(q_{It}) = c_I^* + \sum_{J=1}^{M} \pi_{IJ}^{(q)} \ln(P_{Jt}) + \theta_I^{(q)} \ln(y_t/P_t) + u_{It}^{(q)}, \qquad I = 1, ..., M,$$

where

$$(18) \qquad \pi_{IJ}^{(q)} = W_I^* \varepsilon_{IJ}^{(q)} + W_I^* W_J^* \eta_I^{(q)},$$

$$(19) \qquad \theta_I^{(q)} = W_I^* \eta_I^{(q)},$$

and where $u_{It}^{(q)} = W_I^* e_{It}^{(q)}$. As with the GADS demand specification, the price and income elasticities of quantity that satisfy (18) and (19) are defined at the fixed point $\mathbf{W}^*$. However because theory imposes no adding-up restrictions on the quantity equations, the covariance of estimated quantity residuals is not singular.

The particular demand-quantity system specified by (14) and (17) also provides a decomposition of *compensated* demand elasticities into quality and quantity components at point $\mathbf{W}^*$. Note that GADS-compensated elasticities are $\varepsilon_{IJ}^{*(Q)} = \pi_{IJ}^{(Q)}/W_I^* = \varepsilon_{IJ}^{(Q)} + W_J^* \eta_I^{(Q)}$; therefore, according to (11) and (12), $\varepsilon_{IJ}^{*(Q)} = (\varepsilon_{IJ}^{(q)} + \varepsilon_{IJ}^{(v)}) + W_J^*(\eta_I^{(q)} + \eta_I^{(v)}) \equiv \varepsilon_{IJ}^{*(q)} + \varepsilon_{IJ}^{*(v)}$, where $\varepsilon_{IJ}^{*(q)} \equiv \varepsilon_{IJ}^{(q)} + W_J^* \eta_I^{(q)}$, and $\varepsilon_{IJ}^{*(v)} \equiv \varepsilon_{IJ}^{(v)} + W_J^* \eta_I^{(v)}$. The decomposition of compensated demand elasticities will be computed below.[6]

Empirical Results

This section presents empirical results in support of the construction of composite beef, pork, and poultry demand. The demand variables and commercial disappearance variables are then used to estimate a meat demand-quantity system defined by (14) and (17). As in Nelson (1990), the parameter estimates are used to decompose demand elasticities into quantity and quality components. The present study shows that such a decomposition is valid under the less stringent conditions of the GCCT.

It is worth emphasizing that any particular dichotomy of attributes into quantity and quality is artificial. If one wished to investigate the response of fat content to changes in prices and income, for example, the quantity and quality decomposition of demand elasticities would differ from the estimates presented in this section. In this case, composite quantity would be measured in terms of grams of fat, and quality would represent an index of all the attributes of meat products except fat. Accordingly, the estimated decomposition of demand elasticities would differ from the estimates presented below, but the estimates of composite demand elasticities would remain unchanged.

Evidence of Meat Composites Using the GCCT

If beef, pork, and poultry are valid composites, the GCCT argues the vector of elementary relative prices (ρ) for the three groups would be independent of the vector of the composite price indices $(\mathbf{P})$ and income. Lewbel checks necessary conditions for the GCCT by testing whether each integrated (log) relative price is cointegrated with its (log) nominal or deflated group price. A finding of no cointegration represents evidence in favor of the GCCT. This section follows Lewbel's procedure.

Results of tests of the null of a unit root for 13 (log) relative prices, the three (log) nominal, and three (log) deflated group price indices are reported in table 1. Annual reported Consumer Price Index (CPI) data are used to construct the time-series variables, and the "Food-at-Home" component of the CPI is used to deflate the three group price indices. To provide the greatest number of degrees of freedom, the tests are based on the longest possible annual time series available (in most cases back to 1955), and reported (rather than computed) group price indices are used in the tests. Based on the

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[6] Despite being defined from "Slutsky-like" equations, it is important to note that $\varepsilon_{IJ}^{*(q)}$ and $\varepsilon_{IJ}^{*(v)}$ are not utility-compensated quantity and quality elasticities. Unlike $W_I^* \varepsilon_{IJ}^{*(Q)} = W_J^* \varepsilon_{JI}^{*(Q)}$, this means $W_I^* \varepsilon_{IJ}^{*(q)}$ need not equal $W_J^* \varepsilon_{JI}^{*(q)}$, and $W_I^* \varepsilon_{IJ}^{*(v)}$ need not equal $W_J^* \varepsilon_{JI}^{*(v)}$.

Table 1. Results of Unit Root and Cointegration Tests

Log of Composite Prices for:	NULL HYPOTHESES		Sample Interval
	$I(1)$ Nominal	$I(1)$ Deflated	
Beef	-1.90 (0)	-3.24 (0)*	1955-2000
Pork	-2.43 (2)	-1.59 (6)	1955-2000
Poultry	-3.93 (0)*	-3.51 (0)*	1955-2000

Log of Relative Prices for:	NULL HYPOTHESES			Sample Interval
	$I(1)$	Not Cointegrated with:		
		Nominal R_I	Deflated R_I	
Chuck Roast	-1.35 (6)	-1.61 (6)	NC	1955-2000
Ground Beef	-1.17 (5)	-1.25 (5)	NC	1955-2000
Round Steak	-2.38 (0)	-2.53 (0)	NC	1955-2000
Sirloin Steak	-2.54 (0)	-2.51 (0)	NC	1960-2000
Round Roast	-2.46 (0)	-2.10 (0)	NC	1964-2000
Other Beef	-4.08 (0)*	NC	NC[†]	1978-2000
Bacon	-1.66 (2)	-1.80 (2)	-1.76 (2)	1955-2000
Chops	-0.80 (2)	-0.57 (2)	-1.37 (2)	1955-2000
Ham	-2.83 (0)	-1.66 (0)	-2.01 (0)	1978-2000
Other Pork	-3.67 (2)*	NC	NC	1978-2000
Whole Chicken	-2.66 (4)	NC	NC	1955-2000
Chicken Parts	-2.66 (2)	NC	NC	1978-2000
Other Poultry	-1.91 (2)	NC	NC	1978-2000

Notes: An asterisk (*) denotes rejection of the null hypothesis at the 0.10 level of significance; NC = not computed; NC[†] = cointegration test is not computed, and the Spearman rank correlation test (-0.561) is discussed in the text. $I(1)$ tests that the series is integrated of order 1 against the alternative of trend stationarity. The values reported are augmented Dickey-Fuller (ADF) τ_t-statistics associated with the coefficient of the lagged level variable in the regression of the first-difference variable on a constant, time trend, and a number of lagged first-difference terms (reported in parentheses). The number of lagged terms was chosen as the highest significant lag order from either the autocorrelation or partial autocorrelation function of the first-difference series, using SHAZAM 8.0 (White). The values reported for the cointegration tests are ADF-statistics of the residuals from a regression of log-relative prices on the log-nominal or log-deflated group price index, a constant and time trend, and a number of lagged-difference residuals (reported in parentheses) that are determined by SHAZAM. Finite sample critical values are obtained from MacKinnon.

results from table 1, one cannot reject the null of a unit root at the 0.10 level of significance for 11 of the 13 relative prices, for two of the three nominal group price indices, and for one of the three deflated group price indices.

The results suggest only a limited number of tests for spurious regressions (i.e., cointegration tests) need to be computed to check for valid aggregation. For example, because the unit root tests indicate that both the nominal and the deflated group price indices for poultry may be stationary, neither can be cointegrated with the three integrated relative elementary poultry prices. Further, the findings from table 1 show one cannot reject, at the 0.10 level, the null that any one of the integrated relative elementary beef prices is spuriously related to the integrated nominal group price index for beef. In addition, none of the integrated relative pork prices appear to be cointegrated with either the nominal or deflated group price indices for pork.

The single exception to evidence in favor of aggregation is possible correlation between the relative price of other beef and the deflated group price for beef. Because the test results suggest both series are stationary, a Spearman rank correlation test was performed (Davis, Lin, and Shumway). A value of the test statistic equal to −0.561 rejects the null (at the 0.05 level) of zero correlation between the relative price of other beef and the nominal group price index of beef. With the exception of this test, all the results reported in table 1 justify the formation of demand composites for beef, pork, and poultry.

Davis, Lin, and Shumway argue the foregoing test procedure requiring the data to satisfy every pairwise test independently of every other test would reject the GCCT more often than required by theory. They propose a multi-comparison or familywise procedure that uses pairwise results to test whether the relative prices within a group are jointly independent of their composite price. Given the individual cointegration tests reported in table 1 did not reject the GCCT, these familywise tests would necessarily not reject the GCCT.[7]

Estimates of Demand, Quantity, and Quality Elasticities

Annual U.S. per capita expenditure and commercial disappearance data from 1980 to 2000 (21 observations) are used to estimate the demand-quantity system given by (14) and (17). The analysis is restricted to this relatively short sample interval because expenditure data obtained from the Bureau of Labor Statistics' Consumer Expenditure Survey (CES) are available on a continuous basis only since 1980, but measures of commercial disappearance data (Putnam and Allshouse) are only computed annually. In the previous section, annual time series from 1955–2000 suggest beef, pork, and poultry are valid composites. These composites are assumed valid for the 1980–2000 period.[8] The composite demand variables are constructed as the ratio of per capita consumer expenditures to the associated group price index. Retail-equivalent commercial disappearance data, adjusted for at-home use, are used to measure per capita retail-equivalent physical quantity. Details of the adjustment are given in appendix B.

A six-equation GADS demand-quantity system consisting of three demand and three physical quantity equations was estimated jointly. Serial correlation of the six model errors is ruled out, and the results are based on an estimate of the covariance matrix with no restrictions on the error covariance between the demand and quantity equations. Because the goal is to compare the magnitudes of the demand and quantity equations for beef, pork, and poultry, both the demand and quantity equations for other meat are dropped from estimation. With no serial correlation, the GADS demand subsystem [i.e., (14)] is singular and all estimates are invariant to dropping the other meat demand equation even when estimated jointly with the quantity equations. However, the quantity subsystem [i.e., (17)] is not singular. When the entire system is estimated jointly with cross-equation restrictions imposed on the GADS demand equations, the quantity

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[7] The Davis, Lin, and Shumway critique cannot be applied to the single Spearman rank correlation test discussed above because it is a single test which is not part of a multi-comparison procedure.

[8] Alternatively, unit root and cointegration tests reported in the previous section could be performed using annual price data from 1980–2000 corresponding to the sample period for which CES data are available. These tests would have been based on approximately one-half of the degrees of freedom achieved with the longer time series. Given the results in the previous section, it seems unlikely that tests based on the shorter time series would reject the composites.

parameters are not invariant to the quantity equation dropped. However, the effect of dropping the other meat quantity equation in estimation was found to be small.[9]

Iterated seemingly unrelated regression (ITSUR) was used to estimate the six-equation demand-quantity system for at-home beef, pork, and poultry at the point $\mathbf{W}^* = [0.345, 0.216, 0.172, 0.267]$.[10] Because ITSUR estimates converge asymptotically to maximum-likelihood estimates, adjusted likelihood-ratio (LR) statistics (Laitinin) are used to test the six integrability restrictions (three symmetry plus three homogeneity) and to test for time trends in the system. The adjustments to the LR statistics account for the bias of using the estimated, rather than the actual, covariance matrix (Moschini, Moro, and Green).

The test results suggest time trends should be included in the specification of the demand and quantity system. The adjusted LR statistic associated with the null of integrability is $\chi_{[6]}^2 = 13.199$ when a time trend is included in each of the six equations, and is $\chi_{[6]}^2 = 27.482$ when no time trend is included. These results imply that integrability is rejected at approximately the 0.04 level of significance when time trends are included in the model, but at approximately the 0.0001 level of significance when time trends are excluded.[11]

Moreover, the adjusted LR statistic associated with the null of no time trend in the six equations with integrability imposed is $\chi_{[6]}^2 = 29.628$, which rejects the null at the 0.00005 level. The results are consistent with the notion that changes in consumer tastes and preferences may have followed a linear time trend over the period. The estimates discussed in the remainder of this section are computed with a time trend included in each of the demand-quantity equations, and with homogeneity and symmetry restrictions imposed on the demand subsystem.

Coefficient estimates and standard errors of the six-equation demand-quantity system are used to compute estimates and t-values of Marshallian price and income elasticities of demand plus quantity and quality elasticities for the three meat groups.[12] Elasticity estimates and t-values are reported in table 2. Point estimates of the own-price elasticities of demand for composite beef (-1.425), pork (-0.462), and poultry (-1.106) are negative; however, only the beef and poultry elasticities are statistically different from zero. The three income elasticities of demand are positive and statistically significant, with estimates for pork (1.172) and poultry (1.206) indicating possible luxuries.

The estimated own-price and income elasticities of demand are relatively elastic, owing to the relatively large and mostly significant income and price elasticities of quality. The estimated own-price and income elasticities of quantity are found to be small and statistically insignificant. The income elasticity estimates indicate that as per capita income rises, the per capita physical quantity of the three meat composites remains virtually unchanged, but consumers increase demand by purchasing a more expensive mix of each of the three categories of meat products.

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[9] Seemingly unrelated parameter estimates of the quantity equations with cross-equation restrictions placed on the GADS demand subsystem changed only slightly from the estimates of the unrestricted demand-quantity system. The unrestricted demand-quantity system is invariant to the equation dropped because it collapses to OLS.

[10] These points represent the average budget shares (as a proportion of all at-home meat expenditures) for at-home beef, pork, poultry, and other meat over the sample. Moreover, with only 21 observations, no attempt was made to estimate the system as a set of cointegrated regressions (e.g., Park and Ogaki).

[11] Also, with time trends included in the model, the adjusted LR statistic associated with only the symmetry restriction is $\chi_{[3]}^2 = 8.008$ (p-value = 0.046), and with only the homogeneity restriction is $\chi_{[3]}^2 = 2.415$ (p-value = 0.491).

[12] Parameter estimates are available on request from the authors.

Table 2. Elasticities of Demand, Quantity, and Quality

Description	Demand		Quantity		Quality	
	Elasticity	*t*-Value	Elasticity	*t*-Value	Elasticity	*t*-Value
Beef:						
Beef Price	−1.425	−8.22	−0.103	−0.54	−1.322	−5.16
Pork Price	−0.225	−1.39	0.313	0.94	−0.538	−1.46
Poultry Price	−0.094	−0.88	0.322	3.47	−0.416	−3.04
Other Meat Price	0.836	4.32	−1.317	−7.82	2.153	8.13
Income	0.908	8.59	−0.008	−0.10	0.916	7.52
Pork:						
Beef Price	−0.449	−1.66	0.052	0.21	−0.502	−1.60
Pork Price	−0.462	−1.01	0.032	0.07	−0.494	−0.93
Poultry Price	−0.082	−0.38	−0.974	−7.60	0.891	4.19
Other Meat Price	−0.179	−0.42	0.794	3.64	−0.972	−2.35
Income	1.172	5.97	−0.065	−0.67	1.238	6.50
Poultry:						
Beef Price	−0.292	−1.17	1.020	4.61	−1.311	−4.13
Pork Price	−0.111	−0.39	−1.635	−4.33	1.524	3.26
Poultry Price	−1.106	−3.67	−0.070	−0.70	−1.036	−3.53
Other Meat Price	0.302	0.69	0.494	2.74	−0.191	−0.43
Income	1.206	4.72	0.073	0.93	1.133	4.70

Table 3. Compensated Price Elasticities

Description	Demand		Quantity		Quality	
	Elasticity	*t*-Value	Elasticity	*t*-Value	Elasticity	*t*-Value
Beef:						
Beef Price	−1.112	−6.87	−0.105	−0.57	−1.007	−4.11
Pork Price	−0.029	−0.18	0.311	0.93	−0.340	−0.92
Poultry Price	0.062	0.57	0.320	3.40	−0.258	−1.86
Pork:						
Beef Price	−0.045	−0.18	0.030	0.12	−0.075	−0.25
Pork Price	−0.208	−0.46	0.018	0.04	−0.227	−0.43
Poultry Price	0.120	0.54	−0.985	−7.53	1.104	5.10
Poultry:						
Beef Price	0.124	0.57	1.045	4.89	−0.921	−3.11
Pork Price	0.150	0.54	−1.619	−4.27	1.769	3.81
Poultry Price	−0.899	−2.91	−0.058	−0.57	−0.841	−2.80

Similarly, the relatively large own-price elasticity estimates for beef and poultry can be attributed to a relatively strong and statistically significant impact on the quality of beef and poultry products purchased, and a small and statistically insignificant impact on the per capita quantity purchased. That is, an increase in the beef or poultry price leads to a considerably stronger and more significant reduction in the quality rather than the per capita quantity of elementary products purchased. The own-price elasticity of quality for pork is relatively large but not statistically different from zero. This evidence of a relatively large quality response is consistent with the findings for the United States reported by Nelson (1990).

Table 3 reports estimates of the compensated demand elasticities and their decomposition into quality and quantity components. The point estimates suggest four of the six cross-price elasticities of demand are positive. This result is consistent with the notion that the major meat categories are substitutes in demand (Chalfant, Gray, and White). With 21 observations, however, none of the cross-price elasticities of demand appear to be statistically different from zero (at the 0.05 level). Still, estimates of the compensated own-price elasticities can provide a description of consumers' quantity-quality tradeoff. For example, the decomposition of the compensated own-price elasticity for beef suggests that holding utility constant, a 1% increase in the price of beef results in a decrease of beef demand by 1.112%. The results of the decomposition indicate the compensated response to an increase in beef prices would be achieved by substituting quantity for quality, as consumers purchase a higher proportion of lower-priced beef products. A similar interpretation holds for poultry.

Conclusions

A number of demographic changes, income growth, and technological efficiencies now appear to be driving consumers to spend food dollars differently than in the past (Kinsey and Senauer). These changes seem to imply, for example, that food products which save time or are easy to assemble at home may be more preferred than in the past—and in particular, may be more preferred than less costly, fresh and unassembled products. While a test of such a hypothesis might be possible from an analysis of a vast amount of detailed elementary product data, this study argues such inference might also be gleaned from more readily obtainable composite data. As a first step, empirical evidence is presented which is mostly consistent with the notion that despite the different elementary meat products purchased, the composite demands for beef, pork, and poultry can be treated as demands for single goods.

As noted by Nelson (1991), if elementary product prices move strictly proportionately, the Composite Commodity Theorem could be used to decompose group expenditures and group demand into the physical *amount* and *mix* of elementary products, where the mix of products defines quality as a variable of consumer choice. This study demonstrates that virtually the same convenient interpretations apply under the plausible conditions of the Generalized Composite Commodity Theorem.

The empirical estimates of composite demand elasticities for beef, pork, and poultry reveal notable differences between quantity and quality elasticities when commercial disappearance is chosen as a measure of composite quantity. In general, this means the feature of product diversity cannot be avoided. The results for this specific decomposition suggest the practice of using commercial disappearance as a proxy for composite U.S. meat demand can be seriously misleading because disappearance data rule out variations in the mix of consumer products purchased over time. In particular, the results indicate that in response to rising incomes, U.S. consumers have increased the per capita demand for meat products, while the income response of commercial disappearance has been small. The interpretation of this finding is that consumers have substituted quality for quantity by purchasing the same per capita physical amount of products in the form of a relatively more expensive mix.

Moreover, in response to changes in meat price inflation, consumers appear to have altered the mix more than the per capita amount of elementary meat products. Such

interpretations are based on the presumption that differences among relative prices of elementary products reflect differences in consumers' valuation of the different final products. This same fundamental presumption lies at the heart of mechanisms governing the movement of farm and composite retail food prices (Wohlgenant 1999).

Admittedly, the expenditure-based demand variables used above may also fail to fully account for consumers' demand responses. Exclusion of new products from expenditure data may have led to systematic errors in the variables used in the empirical analysis. Or such omissions may have led to errors in the group-price indices from which composite variables were constructed. At this point it is unclear if more detailed meat expenditure data and quality-adjusted group price indices would make notable changes in the difference between quantity and quality responses. What is clear is that unless one wishes to make the assumption that food products within a group are identical, using the ratio of group expenditures to physical quantity (i.e., the unit value) as a measure of an exogenous group price should be avoided.

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Appendix A:
Derivation of Text Equation (4)

Based on the restrictions implied by the GCCT, this appendix derives equation (4) in the text. The GCCT maintains that composite shares, W_G, can be expressed as the sum of the function, W_{Gh}, which maps normalized composite prices (i.e., $\mathbf{P}^*$) to composite budget shares and a stochastic error term. The subscript h denotes a predicted value, where prediction is based on the minimum mean squared error. Hence, $W_G = W_{Gh}(\mathbf{P}^*) + \varepsilon_G$, where ε_G is a stochastic error term satisfying $E(\varepsilon_G | \mathbf{P}^*) = 0$, and E is the mathematical expectations operator. This defines $W_{Gh} \equiv E(W_G | \mathbf{P}^*) = E(P_G^* v_G q_G | \mathbf{P}^*)$.

Since *elementary* shares are a function of normalized elementary prices (i.e., $\mathbf{p}^*$), and because composite shares are the sum of elementary shares (i.e., $W_G \equiv \Sigma_{i \in G} w_i$), then $W_G = W_{Gh}^*(\mathbf{p}^*) + e$, where $E(e | \mathbf{p}^*) = 0$, and $W_{Gh}^* = E(W_G | \mathbf{p}^*) = E(\Sigma_{i \in G} p_i^* x_i | \mathbf{p}^*)$. Because $\mathbf{P}^* = \mathbf{P}(\mathbf{p}^*)$, the conditioning set used to form W_{Gh} is a subset of the conditioning set used to form W_{Gh}^*, the law of iterated expectations implies $E[E(W_G | \mathbf{p}^*) | \mathbf{P}^*] = E(W_G | \mathbf{P}^*) \equiv W_{Gh}$, which means

$$P_G^* v_G q_G = E\left(\sum_{i \in G} p_i^* x_i \,\middle|\, \mathbf{P}^*\right) + \varepsilon_G = E\left(\sum_{i \in G} (P_G^* \rho_i) x_i \,\middle|\, \mathbf{P}^*\right) + \varepsilon_G.$$

According to the GCCT, a necessary condition for W_G to be a valid demand variable is that ρ is stochastically independent of $\mathbf{P}^*$. Let H denote the cumulative distribution function of ρ, such that $\kappa \equiv E\rho = \int \rho \, dH(\rho)$. If ρ is independent of $\mathbf{P}$, then

$$E\left(\sum_{i \in G} (P_G^* \rho_i) x_i \,|\, \mathbf{P}^*\right) = \sum_{i \in G} \int \left(P_G^* \rho_i \, dH(\rho)\right) x_i = P_G^* \sum_{i \in G} \kappa_i x_i,$$

or

$$P_G^* v_G q_G = P_G^* \sum_{i \in G} \kappa_i x_i + \varepsilon_G,$$

where κ_i is the ith element of κ. This is equation (4) in the text.

Appendix B:
Data Sources

Reported annual CPI index data (1982–84 = 100) (U.S. Department of Labor, Bureau of Labor Statistics) from 1955 through 2000 for beef, pork, and poultry were used as composite price indices. The price index for all other meat was constructed as a Laspeyres index using meat expenditure data. The reported "All Food-at-Home" component of the Consumer Price Index (CPI) was used to deflate the beef, pork, and poultry price indices in the independence tests. The relative elementary prices were constructed as the ratio of an "elementary" CPI component to the corresponding group price index. For example, the relative price of chuck roast was constructed as the CPI component for chuck roast divided by the CPI for beef, and the relative price of bacon is the CPI component for bacon divided by the pork CPI.

Estimation of the GADS demand-quantity system required the construction of demand variables for at-home beef, pork, and poultry. These variables were constructed as U.S. annual expenditure per capita for the meat category divided by the CPI for the group. U.S. expenditures on the three meat categories were calculated from the diary portion of the Consumer Expenditure Survey (CES) for each year 1980–2000. For any year, the CES Diary Survey contains micro-data files detailing the household's weekly food expenditures and a weight representing the household's contribution to the U.S. total. Total annual U.S. expenditures for any particular category is the sum of the weighted household responses. Dividing this result by the U.S. resident plus armed forces overseas (July 1) population estimate yields per capita annual estimates of expenditures. Quantity demand is simply the ratio of this number by the associated group CPI.

Estimates also require the construction of annual at-home per capita quantity variables. Retail-equivalent measures of commercial disappearance for beef, pork, and poultry (Putnam and Allshouse) are adjusted to obtain an at-home estimate. Retail-equivalent measures of commercial disappearance are estimates of the total physical quantity available for consumption. The proportion of each composite meat product consumed at home is estimated for each product using various numbers of restrictions implied by theory (Wohlgenant 1989), and the most reasonable set of estimates was chosen. In this study, the estimated at-home proportions used are: 0.468 (beef), 0.787 (pork), and 0.775 (poultry) (Reed, Elitzak, and Wohlgenant). That is, estimated pounds of beef consumed at home are 0.468 times the retail-equivalent estimate of commercial disappearance of beef. Again, the at-home estimate is divided by the estimate of the U.S. resident plus armed forces overseas (July 1) population number to obtain a per capita quantity estimate.