How Much of Commodity Price Behavior Can a Rational Expectations Storage Model Explain?

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Abstract: A rational expectations competitive storage model is applied to the U.S. corn market

to assess the aptness of this framework in explaining monthly price behavior in an actual

commodity market. Relative to previous models, extensive realism is added to the model in

terms of how production activities and storage costs are specified. By modeling convenience

yield, "backwardation" in prices between crop years does not depend on the unrealistic

assumption of zero ending stocks. Our model produces cash prices that are distributed with

positive skewness and kurtosis, and mean and variance that increase over the storage season,

consistent with the persistence and the occasional spikes observed in commodity prices. Futures

prices are generated as conditional expectations of spot prices at contract maturity, and the

variances of futures prices have realistic time-to-maturity and seasonal patterns. Model

realizations of cash and futures prices over many "years" are used to demonstrate the wide

variety of price behaviors that can be observed in an efficient market with a similar market

structure, implying that marketing strategies based on short, historical samples of prices to

manage price risk can be misleading.

1. Introduction

Models of spot prices of financial assets typically assume that prices observed daily (or at lower frequencies) are not autocorrelated (Hull, 2000, p. 219). In contrast, commodity prices have systematic behavior, even if commodity markets are efficient. A typical commodity price series is variable and autocorrelated with seasonal and perhaps cyclical components. Net of seasonality and cycles, these prices may be mean reverting to some long-run average, associated with trends in the macro-economy, population growth, and technological changes. Occasional spikes are observed when prices jump abruptly and temporarily to a high level relative to their long-run average. Thus, distributions of prices are skewed to the right and often display kurtosis (e.g., Myers, 1994; Deaton and Laroque, 1992).

Many models have been developed to depict the systematic behavior of prices, but few have done so adequately (Tomek and Myers, 1993; Brorsen and Irwin, 1996). Both time-series and structural models have required ad hoc assumptions to capture all the nonlinear features of commodity prices (see Tomek and Peterson, 2001, for a review). Little attention has been paid to the fundamental question of whether or not the primary parameters of price distributions can be recovered from the estimated model parameters. Because of these challenges in modeling, answers to fundamental questions in price risk management remain largely ambiguous. A deeper understanding of agricultural commodity price behavior is essential, given the heightened price risk from increasingly liberalized domestic agricultural and trade policies worldwide.

A major impediment to estimating the model parameters or the parameters of price distributions directly is that the number of observations is limited. Unlike financial markets, commodity markets typically do not have high-quality, high-frequency observations on cash prices. At best, prices are reported in daily ranges for selected location and quality

specifications. Some questions relevant to decision-makers in agriculture can be answered by using monthly or quarterly observations, but since commodity markets undergo frequent structural changes related to changes in farm policies, the number of such observations available from a single structural regime is often small (for an unusual example of the construction of a long sample, see Voituriez, 2001). With a limited number of observations, the estimated parameters are fragile, and it is not possible to draw any conclusions regarding outcomes of economic decisions. The short-sample problem can be overcome if a model can be developed, which can simulate observations from probability distributions that are consistent with those of observable prices.

The objective of this paper is to examine how much of a commodity's price behavior can be understood in terms of a structural model. We develop a model that can produce "realistic" probability distributions of monthly prices, by which we mean that the model should generate price behavior comparable to what has been observed in a recent, finite time period, and also provide plausible realizations of future price behavior within a similar market structure. Our point of departure is the nonlinear rational expectations commodity storage model. This model emphasizes nonlinearity in storage—that aggregate storage cannot be negative (Gustafson, 1958)—and Muth's (1961) rational expectations hypothesis. Williams and Wright (1991) synthesize the modern theory of competitive storage, which appends supply, demand, and market clearing conditions to the intertemporal arbitrage equation of the classic model (Working, 1949). Rational price expectations are endogenous to the model.

This partial equilibrium framework has simulated some of the time-series features of spot prices (e.g., Deaton and Laroque, 1992, 1996; Chambers and Bailey, 1996; Rui and Miranda, 1995; Routledge et al., 2000). Deaton and Laroque (1992), for example, replicate the degree of

skewness and kurtosis in observed price distributions, but fail to account for the degree of autocorrelation.[1] Such studies, however, typically apply a "generic" model to multiple commodities, and ignore commodity-specific characteristics such as the distinction between annual crops with storage and continuously produced, perishable commodities. Applications of the rational expectations storage framework to commodity-specific markets have focused on simulating policy scenarios (Miranda and Glauber, 1993; Miranda and Helmberger, 1988; Gardner and López, 1996; Lence and Hayes, 2000). Moreover, most research specifies annual models, although a few quarterly models exist (Williams and Wright, 1991; Pirrong, 1999). Chambers and Bailey (1996) propose a monthly framework, but assume a monthly harvest for seven commodities, including soybeans whose production is seasonal.[2]

We assess the aptness of a rational expectations framework in explaining monthly price behavior for an actual commodity market. Our work is the first example of a monthly model that permits carryover of an annual crop into the following crop year. Considerable effort is spent on adding realism to the model in terms of the timing of production activities, which we believe critically drives price behavior. Rather than estimating model parameters from a short sample, we examine whether, for parameters consistent with recent history, the model can generate cash

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[1] They used deflated annual prices, which may have introduced autocorrelation that did not exist in the nominal series.

[2] Model validation focuses on its prediction that prices follow a two-regime process depending on whether or not inventories are held (Deaton and Laroque, 1995, 1996; Chambers and Bailey, 1996; Ng, 1996; Michaelides and Ng, 2000). Beck (2001) tests an implication of a nonnegative constraint on storage, i.e., the difference in asymmetry of price distributions between storable and non-storable commodities, and finds no significant difference.

and futures price distributions that are comparable to that of the data.[3] Given estimates of the

distributions, price series similar to those faced by agents in the industry can be simulated,

allowing us to overcome the small sample problem and thereby analyze long-run economic

consequences of (say) risk management strategies.

The U.S. corn (maize) market is modeled at a monthly frequency. Corn is storable and

homogeneous (relative to other commodities), satisfying the basic characteristics of a commodity

for which the storage model was originally conceptualized. Moreover, the volume of trading for

futures contracts for corn is the largest among agricultural commodity futures in the U.S., and the

simulations permit analysis of common rules-of-thumb for marketing seasonally produced

commodities. Thus, the U.S. corn market should be a useful example that is potentially

generalizable to other storable commodities and locations.

The next section describes characteristics of the recent price series as a minimum

requirement that our model should be able to explain. In addition to basic statistics of the sample

prices, seasonal patterns in mean and variance of cash and futures prices are noted. Then, the

conceptual model is developed, the numerical model is specified, and the equilibrium solutions

are reported. The subsequent section compares price behavior implied by the model to the

characteristics of the sample. The distribution of simulated prices is comparable to the sample

price distribution up to the fourth moment, with a similar degree of autocorrelation. Monthly

patterns in mean and variance are replicated. These simulations also demonstrate that a wide

range of outcomes are possible from the same market structure. For example, the means and the

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[3] While individual components of the model are calibrated to the sample means, the overall model is not

guaranteed to reproduce the data in equilibrium. Thus, our procedure is one of many that are more

broadly defined than calibration in the strict sense (Dawkins et al., 2001).

variances of the distributions of the simulated prices increase over the storage season, but this increased variability is consistent with prices spiking in an occasional year (with large returns to storage) and declining in other years (with negative returns to storage). Our results suggest that the rational expectations commodity storage framework is valuable for analyzing consequences of economic decisions in commodity markets. Based on a long sample that can only be obtained from simulations, our analysis casts doubt on the value of some common rules-of-thumb used in marketing commodities, offering a step toward a fuller understanding of the issues involved in managing price risk.

2. Cash and futures price data

Commodity prices at any point in time vary because of differences in location, quality, and delivery terms. We consider a representative monthly cash price series for No.2 yellow corn at a Central Illinois market. Such a series is highly correlated with transaction prices in other Corn Belt locations in the U.S. Given the changes in loan rates (support levels) and in supply management programs for corn in the 1980s, our sample includes monthly observations from crop years 1989/90 through 1997/98, i.e., September 1989 through August 1998. The sample period characterizes a market-oriented situation while embodying most, if not all, standard features of commodity price behavior. There is no obvious trend over time; the prices are variable with positive autocorrelation; and there is a prominent spike in early 1996 (Figure 1). For these 108 observations, the sample mean, standard deviation, skewness, kurtosis, and autocorrelation coefficients are reported in Table 1.

As a description of the price series, a Gamma distribution is fitted to the sample by method-of-moments estimation to obtain monthly distributions allowing for skewness and kurtosis. These distributions for selected months are plotted in Figure 2. The estimates provide

sensible approximations of intra-year price behavior for U.S. corn. The modes increase over the storage period through May reflecting higher storage costs, and revert toward the harvest level during the growing season. The probability masses of prices during the harvest and immediate post-harvest periods are relatively centered. Prices become more dispersed as planting approaches, and this trend continues until August, when it is reversed in anticipation of the new crop. These seasonal patterns need to be explained by any useful model.

A model of commodity futures price behavior must take into account the theoretical relationship between futures and cash prices over time. The current futures contract price equals the expected spot price at contract maturity assuming no risk premium and no basis risk. In other words, futures prices for any individual contract in an efficient market do not mean revert, since they are the expected values of a particular month's price. Previous analyses, which report mean-reversion in futures prices, may suffer from statistical weaknesses (Irwin et al., 1996). Also, analyses of mean-reversion based on time series constructed from a sequence of prices from nearby futures contracts prices (i.e., using the current maturing contract prices until near the maturity date and then switching to the subsequent maturing contract prices) likely draw misleading results, since the series is analogous to cash prices.

The variance of futures prices for an individual contract is influenced mainly by the flow of information and its uncertainty (see Streeter and Tomek, 1992, for other factors). Variability is expected to increase as contract maturity approaches (Samuelson, 1965), and is affected by the seasonality in the information flow in the underlying market. For a December corn futures contract, for example, the time-to-maturity effect implies larger price variability in December than in May, ceteris paribus, which has been confirmed in empirical studies (e.g., Fackler and Tian, 1999; Goodwin et al., 2000). Due to uncertainty about the expected harvest during the

growing season, the variance of futures contract prices will also have a seasonal component, ceteris paribus.

A descriptive analysis confirms these features of futures price behavior in our sample. Monthly futures price observations were constructed by averaging the settlement prices of the first and third Wednesdays (Thursday, if Wednesday was a holiday) of every month at the Chicago Board of Trade. December and May corn contract prices for the 12 months prior to maturity for the years 1990 through 1997 and 1991 through 1998, respectively, were fitted to a Gamma distribution by method-of-moment estimation. In Figures 3 and 4, these distributions in selected months are plotted for each contract. For both contracts, the time-to-maturity effect is clearly observed—the longer the time-to-maturity, the less dispersed the price distribution, and the dispersion increases monotonically over time. Comparing the distributions at maturity, May contract prices have a larger dispersion than December contract prices, consistent with the seasonal differences in the variances of cash price. The monthly modes of December futures prices appear to have a seasonal pattern, which is inconsistent with an efficient market, but is likely due to the short sample.

3. Conceptual model

The model incorporates minimum key features of the corn sector in the U.S. in the 1990s. Namely, the crop is planted in April and harvested from September through November by producers who are assumed to be expected profit maximizers. The planting decision is conditional on a realized supply shock. Between planting and harvest, monthly crop estimates provide information on the expected new crop size, and during months preceding harvest, news arrives regarding how much of the annual crop will be harvested next month. A larger-than-average proportion of the annual crop may be harvested in September of an "early" year, or in

November of a "late" year. Agents adjust their expectations accordingly. Available supply at the beginning of each month is either consumed or stored. Monthly demand is subject to shocks, and risk-neutral arbitrageurs make monthly storage decisions.

Specifically, units of time are indexed by t and m, where t is calendar year and m is month (1 = January). To simplify notation, the index t is omitted whenever the implied indexes are unambiguous. Producers, consumers, and storers form expectations and base their decisions on information available at the beginning of each month, which is represented by a vector of state variables $\boldsymbol{\theta}_m$. Thus, expectations given information at month m are conditional on $\boldsymbol{\theta}_m$, i.e.,

$$E_m[\cdot] \equiv E[\cdot \mid \boldsymbol{\theta}_m].$$

In April, producers decide how large a crop (measured in aggregate bushels of corn expected at harvest) to plant. Due to uncertain growing conditions, the planted crop size is an estimate of crop size at harvest. During the months between planting and harvest, the expected crop size, or crop estimate (H), follows a random walk:

$$H_{m+1} = H_m + \varepsilon_{m+1}, \quad m = 5, \ldots, 10, \quad H_m > 0 \quad \forall m, \tag{1}$$

where ε_{m+1} is a mean-zero independent disturbance. In the model, planting is completed at the end of April, and the May crop estimate equals the planted crop size, i.e., $H_5 = H_4$.

Each producer is a price taker and makes the planting decision based on April's beginning inventory and supply shock, which defines April's state of the world. Supply shocks represent unsystematic changes in costs of production. Production technology described by a convex, increasing cost function is assumed to be fixed. The producer's problem is to maximize expected, discounted profit by choosing crop size to be planted (H_4):

$$\max_{H_4} E_4 \left[\sum_{m=9}^{11} \alpha_m \frac{H_m P_m}{(1+r)^{m-4}} \right] - C_{\theta_4}[H_4, \ y].$$

The first term is expected discounted revenue, computed as the weighted average of crop size

(H) times price (P) in September through November, where the weights are the monthly harvest

shares (α) divided the discount factor for a monthly interest rate of r; $C_{\theta_4}[\cdot]$ is the state-

dependent total cost to plant and maintain a given amount of crop size, where y is an

independently distributed random supply shock. Given that H_m is a random walk,

$E_4[H_m P_m] = H_4 E_4[P_m]$, thus solving the first-order condition yields the supply function that

depends on the expected (discounted) price at harvest and supply shock:

$$H_4 = S_{\theta_4}\left[E_4\left[\sum_{m=9}^{11} \alpha_m \frac{P_m}{(1+r)^{m-4}} \right], \ y \right], \tag{2}$$

where $S_{\theta_4}[\cdot]$ is the state-dependent aggregate supply function.

During September and October, random proportions of the expected crop size are

actually harvested; the remaining crop is harvested in November, revealing the true total crop

size.[4] Thus, the incoming harvest (h) is

$$h_m = \alpha_m H_m, \qquad m = 9, 10, 11 \tag{3}$$

where α_9 and α_{10} are independent random numbers between 0 and $\bar{\alpha}_m < 1$ (m = 9, 10) such that

$\sum_{m=9}^{10} \bar{\alpha}_m H_m \le H_{11}$, and α_{11} is a number between 0 and 1 such that $H_{11} = \sum_{m=9}^{11} h_m$. Expectations of

these random proportions, $\hat{\alpha}_{m-1}$ ($= E_{m-1}[\alpha_m]$), are revealed as news a month prior to their

realizations during August through October.

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[4] According to Garcia et al. (1997, p. 561) "errors [of USDA crop forecasts] are quite small by the time of

November announcements."

Monthly available supply (A) is equal to the carryover (s) from the previous month, except in months when the carryover is augmented by the incoming harvest:

$$A_m = \begin{cases} s_{m-1}, & m = 1, \ldots, 8, 12 \\ s_{m-1} + h_m, & m = 9, 10, 11. \end{cases} \tag{4}$$

Quantity demanded (q) is a state-dependent function of current price and demand shock (u),

$$q_m = D_{\theta_m}[P_m, u_m], \qquad m = 1, \ldots, 12 \tag{5}$$

where $\partial D_{\theta_m}/\partial P_m < 0$ and $\lim\limits_{q_m \to 0} D_{\theta_m}^{-1}[q_m] = \infty$; i.e., consumption must be positive for the market to clear. Since the monthly specification of demand functions accounts for any seasonality in demand, demand shocks are specified as independently distributed, residual disturbance around this seasonality.

Storage is carried out by risk-neutral arbitrageurs, whose profit equals the expected appreciation in price less opportunity and carrying costs associated with storage. Carrying cost consists of the physical cost of storage minus the convenience yield of stocks, which depends on supply and demand for storage (Telser, 1958).[5] Depreciation in quality due to storage is assumed to be unimportant for corn. Assuming a constant monthly discount rate r and denoting the total carrying cost as $K_{\theta_m}[s]$, the storer's problem is:

$$\max_{s_m \geq 0} \frac{\mathrm{E}_m[P_{m+1}]s_m}{(1+r)} - P_m s_m - K_{\theta_m}[s_m], \ m = 1, \ldots, 12,$$

with the first-order conditions:

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[5] Brennan (1958) includes a possible risk premium that is associated with the risk from carrying stocks, such as financial loss from an unexpected fall in prices, but it is very small or nonexistent (Leuthold et al., 1989).

$$\frac{E[P_{m+1} \mid \boldsymbol{\theta}_m]}{(1+r)} = P_m + K'_{\boldsymbol{\theta}_m}[s_m], \quad s_m > 0, \qquad m = 1, \ldots, 7, 11, 12,$$

$$\frac{E[P_{m+1} \mid \boldsymbol{\theta}_m]}{(1+r)} \leq P_m + K'_{\boldsymbol{\theta}_m}[s_m], \quad s_m \geq 0, \qquad m = 8, 9, 10$$

(6)

where $K'_{\boldsymbol{\theta}_m}[\cdot]$ is the state-dependent marginal carrying cost per unit of storage. This specification

allows for risk-averse storage behavior despite the assumption of risk neutrality, since risk

aversion is observationally equivalent to a storage model with a nonlinear storage cost function

such as this (e.g., Just, 1975).[6] These conditions guarantee no arbitrage opportunities in

equilibrium. Imposing positive consumption removes the possibility of stock-outs except in the

months preceding harvest.

Equations (1) through (6) imply an equilibrium characterized by a rational expectations

price function, $P = \Phi[\boldsymbol{\theta}]$, and a planting equation, $H_4 = \Psi[\boldsymbol{\theta}_4]$, which solve the following

discrete-time, continuous-state, functional equations:

$$\Phi[\boldsymbol{\theta}_m] = \begin{cases} \mathrm{E}_m[\Phi[\boldsymbol{\theta}_{m+1}]]/(1+r) - K'_{\boldsymbol{\theta}_m}[s_m], & m = 1, \ldots, 7, 11, 12, \\ \max\{\mathrm{E}_m[\Phi[\boldsymbol{\theta}_{m+1}]]/(1+r) - K'_{\boldsymbol{\theta}_m}[s_m], D^{-1}_{\boldsymbol{\theta}_m}[s_{m-1}]\}, & m = 8, 9, 10 \end{cases}$$

(7a)

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[6] The utility-maximization problem for a risk-averse storer can be expressed in terms of the certainty

equivalent of a risky prospect (Mas-Colell et al., 1995, chapter 6). Denoting the risk premium (the

difference between the mean of the risky profit and its certainty equivalent) as ρ, and including the

opportunity cost and constant marginal physical storage cost k in the profit, the certainty equivalent

maximization problem is:

$$\max_{s_m \geq 0} \quad \frac{\mathrm{E}_m[P_{m+1}]s_m}{(1+r)} - P_m s_m - k s_m - \rho[s_m], \quad m = 1, \ldots, 12 \,.$$

But, this is identical to the model assuming risk neutrality, replacing the carrying cost $K_{\boldsymbol{\theta}}[s]$, with the

sum of the physical storage cost and risk premium.

$$D_{\theta_m}^{-1}\left[A_m - s_m\right] = \Phi\left[\theta_m\right], \qquad \forall m \tag{7b}$$

$$\Psi\left[\theta_4\right] = S_{\theta_4}\left[\hat{P}\left[\Psi\left[\theta_4\right]\right]\right] \tag{7c}$$

$$\hat{P}\left[H_4\right] = E_4\left[\sum_{m=9}^{11}\alpha_m\Phi\left[\theta_m\right]\Big/(1+r)^{m-4}\,\Big|H_4\right] \tag{7d}$$

where $\hat{P}[\cdot]$ represents the expected harvest price given a planted crop size of H_4, and the state

variables in each month are:

$$\theta_m = \begin{cases} \{m,\,A_m,\,u_m,\,y\}, & m = 4, \\ \{m,\,A_m,\,u_m,\,H_m\}, & m = 5, 6, 7, \\ \{m,\,A_m,\,u_m,\,H_m,\,\hat{\alpha}_m\}, & m = 8, 9, 10, \\ \{m,\,A_m,\,u_m\}, & \text{otherwise.} \end{cases}$$

Equation (7a) states that current price must equal the discounted expected future price,

less the marginal carrying cost K', except in months prior to harvest. During August through

October, stock-outs, although improbable, could occur in which case the price equals the level

implied by consuming all the carryover from the previous month, $D_{\theta_m}^{-1}\left[s_{m-1}\right]$. In all months, the

equilibrium price depends on inverse demand evaluated at available supply that was not stored

(equation 7b). Equation (7c) is the planting function; the crop planted is the state-dependent

supply function evaluated at expected harvest price. Expected harvest price, in turn, depends on

planted crop size, because crop size influences price received at harvest (equation 7d). Three

states exist in all months: the month of the year, the available supply level, and the realized

demand shock. In April, a supply shock conditions the planting decision; during May through

October, the expected size of the incoming crop is relevant; and expected proportions of crop

harvested in the subsequent month are revealed during August through October. In November,

the information of the realized crop is incorporated into available supply. Storage and

consumption are endogenous variables that are derivable from price, and hence are functions of

the state variables.

The existence of price functions in a competitive storage model with seasonal production is proved in Chambers and Bailey (1996). The assumption of rational, forward-looking behavior among the agents implies that stochastic dynamic programming or a similar recursive method must be employed to solve the model. The possibility that the nonnegativity constraint on storage may bind adds further complications. Since the functions cannot be solved in closed form, the problem is solved numerically.

4. Numerical model

4.1. Solution method

The recent literature on numerical solution methods uses variations of a functional approximation method to replace the original functional equation problem with a finite-dimensional (discrete) problem (e.g., Williams and Wright, 1991; Rui and Miranda, 1995; Deaton and Laroque, 1992). Miranda (1998) compares the accuracy and efficiency of different numerical strategies for computing approximate solutions to the nonlinear rational expectations commodity market model, and his results support the superiority of the cubic spline function collocation method over the traditional space discretization, linearization, and least-square curve-fitting methods.

The cubic spline collocation method involves three steps. First, the solutions to the problem, price and planting functions, are approximated by finite linear combinations of known cubic b-spline functions. Second, collocation nodes are selected for both functions, and lastly, continuous distributions of random state variables are replaced by discrete distributions. The procedure replaces the original functional equation problem (7a)–(7d) with a finite-dimensional problem, where unknowns are coefficients of the approximating collocation functions, storage levels, planted crop size, and expected harvest price.

The equilibrium conditions have a recursive structure in that the planting equation does not enter the storer's arbitrage equation (7a). From the storer's point of view, harvest is exogenous, and information on expected crop size becomes available only after the crop has been planted and growing conditions become known. Hence, a solution can be obtained in a sequence of independent operations. First, equations (7a) and (7b) are solved to determine the equilibrium market price at all possible states. Second, these price functions are used to determine expected harvest price ($\hat{P}$) from equation (7d), based on transition probabilities that link future states to the choice of planted crop size (H_4). Finally, the relationship between $\hat{P}$ and H_4 can be used to solve for the equilibrium planting equation in (7c).

To solve for price function coefficients and storage levels, Miranda (1998) proposes a two-step function-iteration algorithm, where the solutions to monthly storage are found by holding the guesses for monthly price functions constant, and vice versa. Here, we combine his proposal and backward induction similar to that of dynamic programming, since the equilibrium price conditions in equation (7a) match the optimal conditions from a dynamic programming problem that maximizes social welfare by choosing storage levels, given an exogenous harvest size. The details of the solution procedure are found in the Appendix.

4.2. Parameter specification

The model endogenously determines monthly prices and annual planting as functions of six state variables: the month of the year, available supply, crop estimates, harvest timing, demand shocks, and the supply shock. The dynamics and distributions of the latter four random state variables are parameterized to represent observed market conditions during the 1990s, as are the coefficients of the demand, supply, and convenience yield functions. Available supply evolves according to the endogenous storage and consumption decisions, which depend on

prices, but is restricted to lie within monthly bounds that are specified to include the observed range. Observations of available supply are imputed from quarterly total supply and ending stocks reported by the Economic Research Service (ERS) at the U.S. Department of Agriculture (USDA).

The expected crop size in May (H_5) assumes values between the minimum and maximum annual harvested crop in the sample period (6.3 to 10.1 billion bushels). The values are associated with probabilities assuming a normal distribution with a data-consistent mean and standard deviation. Subsequently, the crop estimate is revised at the beginning of each month from June through November (when, by assumption, the actual crop size is revealed) with Markov transition probabilities. The standard deviations of the differences between the adjacent months' crop estimates in the sample (World Agricultural Outlook Board, USDA) are used as the standard deviations of the error terms in equation (1).

Harvest timing is defined by proportions of crop harvested during September, October, and November (α), which are taken from the USDA National Agricultural Statistics Service's crop progress report. Since 91.7 percent of the U.S. annual crop was harvested during the three months in our sample, the percentages are adjusted so that the average of the three-month sum equals one. The adjusted September, October, and November mean proportions are 0.12, 0.55, and 0.32, respectively. Based on the sample evidence, normality is assumed for the logarithms of September percentages, while October percentages are assumed to be normally distributed; the two distributions are independent. November percentages are determined by the total crop size in November.

Monthly demands are specified as constant elasticity functions with multiplicative demand shocks, with the price elasticities set at –0.25 for all months.[7] The demand functions are calibrated to the modes of the estimated monthly price distributions (Figure 2) and monthly means of total disappearance (quarterly total disappearance reported by the ERS divided by three). Demand shocks (u) are specified as mean one, normal random variables. Standard deviations are imputed from the ratio of consumption and predicted consumption, assuming the constant-elasticity demand functions with calibrated coefficients, and range between 0.083 and 0.124 billion bushels.

The supply equation is also assumed to take a constant-elasticity form with a multiplicative, normal supply shock (y) with the supply elasticity set at 0.2.[8] The function is calibrated to the sample-average harvest price discounted to April ($2.30 per bushel) and the mean of annual production (8.29 billion bushels), with the mean of supply shock set to one.[9]

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[7] Estimates of annual demand elasticities vary from –0.54 to –0.73 (Holt, 1994; Holt and Johnson, 1989; Shonkwiler and Maddala, 1985). Subotnik and Houck (1979) report ranges of quarterly price elasticities of feed, of food, and of exports as –0.15 to –0.22, 0 to –0.034, and –0.71 to –2.0, respectively.

[8] Estimated acreage elasticities with respect to changes in expected price range from 0.05 to 1.04 (Holt, 1994, 1999; Chavas and Holt, 1990, 1996; Tegene et al., 1988; Lee and Helmberger, 1985), with an apparent consensus around 0.2; the most recently published price elasticity of acreage for the 1991-95 period is 0.293 (Lin et al., 2000). Estimated supply (production) elasticities range from 0.28 to 0.39 (Shonkwiler and Maddala, 1985).

[9] Discounted harvest price is calculated for each crop year in the sample as a weighted average of September, October, and November prices, where each month's price is discounted to April and weighted

Because expected harvest prices are not observed, the variance of y is inferred from the sample. The calibrated supply curve is evaluated at the minimum and maximum observed harvest prices and compared to the minimum and maximum observations of planted crop, respectively. The average of squared deviations between predicted and observed planting is used as an estimate of the variance of y, giving a standard deviation of 0.09 billion bushels.

The net carrying charge of storage is specified as the difference between current and expected prices, and consists of storage costs and marginal convenience yield. If the expected price in the next period differs from the current price only by the costs of storage, prices would increase indefinitely, on average, relative to the previous month. Yet, observed prices typically increase from October through May and then decrease. This puzzle of price backwardation is a long-standing issue (Frechette and Fackler, 1999), and a conventional rational expectations storage model uses aggregate stock-outs to account for price backwardation, ignoring convenience yield. But zero stocks of corn have never occurred in recorded history in the U.S. (though they were very small in 1934 and 1936).

Thus, we model marginal convenience yield, despite the fact that it is unobservable, as a source of price backwardation. Observations for marginal convenience yield (CY) are calculated as the remainders of the non-arbitrage equation evaluated at monthly prices $CY_m = P_m + k - P_{m+1}/(1+r)$, where the monthly physical cost of storage (k) and the discount rate (r) are specified as 3 cents per bushel and 0.0083, respectively. The monthly averages of marginal convenience yield were close to zero from October through April and ranged from 9.5 to 21.2 cents per bushel during the remaining months. To be consistent with the observed price and inventory behavior,

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by the percentages of crop harvested in that month ex post. The monthly discount rate is based on an annual rate of 0.1 ($r = 0.0083 = 0.1/12$).

marginal convenience yield is assumed to be a decreasing, convex function of storage level (s), which is analogous to Rui and Miranda's (1995) specification, and a decreasing function of expected crop size (H). Specifically, marginal convenience yield is

$CY_m = \omega_m s_m{}^{\xi_m} \exp[\zeta_m(H_m - \overline{H})]$, where ω_m, ξ_m, and ζ_m are parameters, $\xi_m < 0$, and $\overline{H}$ is the

sample average crop size. Setting the elasticity parameter ξ_m to -1, the constants ω_m are obtained by calibrating these functions through the monthly average convenience yield, storage level, and historical average crop size. The shift parameters ζ_m associated with expected crop size are calibrated so that the shift factors $\exp[\zeta_m(H_m - \overline{H})]$ take on their maximum values when crop estimates are lowest during the sample period. The convenience yield was set to zero for months October through April. This specification effectively eliminates stock-outs.

For improved computational accuracy, quantities and prices are scaled so that one equals 10 billion bushels and $10 per bushel, respectively. Monotonicity is imposed on functional solutions during the solution process. The model is solved for a convergence tolerance level of 10^{-8}. Cubic spline interpolation and Gaussian quadrature methods are calculated by the computer routines developed by Miranda and Fackler (1999); the algorithm for L-U factorization built in MATLAB version 5.3 is used.

The rational expectations model explains observed economic behavior through supply and demand shocks, but the combined effects are difficult to analyze. Thus, the model is solved in a series of nested cases, starting with the case of certainty and exogenous supply, and introducing crop estimates, demand shocks, and harvest timing sequentially. Then, the planting decision is endogenized, and lastly, the uncertainty regarding production costs is added, resulting in what we refer to as the full model. Comparing results across the nested cases allows us to relate aspects of simulated behavior to certain model specifications. Due to the limited space, we

report only the results for the full model below but discuss observations based on the comparisons. The results for all nested cases are available from the authors upon request.

5. Model solutions

5.1. *Equilibrium cash price and planting functions*

The equilibrium price functions are monotone decreasing, convex functions of available supply and crop size estimates, respectively, in all months, with a negative cross derivative. They are increasing in demand shocks (i.e., smaller quantity), and at a faster rate at lower availability levels. Similarly, prices are more responsive to the changes in demand shocks when smaller crops are expected. When a small proportion is expected to be harvested in September (a late harvest), much of the uncertainty regarding total crop size is transferred to the remaining harvest months, causing August prices to increase at a given level of availability, ceteris paribus. The impact is more apparent in the October price function, since November is the terminal harvest month. When a smaller proportion of the crop is harvested in November, the upward shift in the October price function is greater at lower availability levels and with smaller crop estimates, respectively.

Equilibrium planting, a function of available supply in April, is derived from intersections between the supply function and the inverse demand function expected at harvest, which are increasing and decreasing functions of the expected harvest price, respectively. A positive supply shock (i.e., lower cost of production) induces a larger crop to be planted, and smaller stocks in April shift the expected inverse demand outward, ceteris paribus. The planting function is decreasing in available supply in April and increasing in supply shocks.

5.2. Time-series behavior and long-run distributions

Equilibrium price and planting functions alone do not tell us much about the validity of the model, since they have no observable counterparts. Rather, the model is "good" if it generates equilibrium price series that are comparable to those observed. We have used price levels to parameterize demand and supply functions, but higher moments of distributions as well as serial correlation in price series are not guaranteed to be data-consistent.

Using the equilibrium functions, equilibria were sequentially generated for 10,000 "years" by drawing random disturbances consistent with the model. At the beginning of each month, availability is realized as the amount carried over from the previous month plus any incoming harvest, and together with other realized state variables, determines the current price through the price function. Then, the quantity consumed at that price is determined from the demand function, and the carryover to the next month is the difference between availability and consumption.

Table 1 reports basic statistics for the simulated price series based on the full model, along with their sample counterparts. The model successfully simulates the prices at a comparable mean level and variability around the mean. The overall price distribution is positively skewed and more peaked than normal to the extent comparable to the sample. Williams and Wright's (1991) claim that storage induces skewness is confirmed under our specification of random shocks. Deaton and Laroque (1992) note the inability of their model to replicate the degree of autocorrelation (their model results ranging between 0.08 to 0.48), but our model is quite successful regarding this point.

In terms of seasonal patterns, a comparison of the monthly statistics between the sample and simulated prices provides further evidence that the full model generates "realistic" price

distributions (Table 2). Since the sample only contains nine observations for each month, only

the mean and standard deviations are reported. Simulated average price levels are slightly higher

than observed in the sample (6 to 16 cents per bushel), but the spread between the highest (May)

and lowest (October) mean prices of 36 cents corresponds to that observed in the sample. The

seasonal pattern in price mean reflects an equilibrium outcome of planting, storage, and

consumption decisions. Uncertainty regarding crop size during the growing season produces a

monthly pattern in price variability consistent with the sample. Calibrated demand shocks help

align simulated variability with the observed during the months before planting. The seasonal

differences in variability, however, are not as large as in the sample, and the simulated monthly

maximum prices are higher and the minimum prices are lower than those observed.

Table 2 also reports monthly statistics for simulated storage and consumption, which are

derived from prices, and crop estimates (including planted crop size) and incoming harvest,

which depend on planting. Storage declines steadily through the post-harvest season, smoothing

out intertemporal consumption. In the sample, the standard deviation of storage is 10 to 40

percent of the mean, while that of consumption is 8 to 10 percent. In the full model, the standard

deviation of storage ranges between 12 and 35 percent of the mean, and that of consumption

ranges between 9 to 12 percent.

The equilibrium planting function generates planted crop size with similar magnitude and

variability as the sample. The simulated crop estimates show no systematic trend over the

growing season, reflecting the Markov specification. In contrast, the means of the USDA

monthly crop estimates seem to decline over the growing season in our sample, but the literature

suggests little or no bias in longer time series (Sumner and Mueller, 1989; Garcia et al., 1997).

The specification of a supply shock at planting time generated a consistent level of uncertainty in

crop estimates throughout the growing season. Standard deviations of crop harvested were brought in line with the sample by the random realization of harvest timing calibrated to the sample.

The magnitude of the simulated convenience yield and its relationship with storage levels are illustrated in Figure 5 for selected months. In May, the simulated observations of convenience yield and storage are tightly distributed along a decreasing convex locus, because the variability of expected crop size is limited. As the growing season proceeds and the variability of expected crop size increases, convenience yield becomes more dispersed, and its magnitude increases.

The full model is tested for its robustness under alternative specifications of five parameters: the interest rate, storage cost, and elasticity coefficients with respect to demand, supply, and convenience yield. While magnitudes of the moments varied slightly, the seasonal patterns in the moments remain unchanged for all variables. The results of the sensitivity analysis are reported in Peterson and Tomek (2000).

5.3. Futures price functions

The full model is used to obtain December and May futures price functions, dependent on the state in each month starting a year prior to maturity. Available supply, expected crop size, and percentage of crop harvested, which are observable monthly in reality, are discretized within respective ranges, and each combination of discrete state variables is regarded as an initial state. The model is simulated from all initial states to both December and May, 10,000 times. For each initial state, the averages of December and May conditional prices over the 10,000 simulations are computed as prices of December and May futures contracts. The functional relationships

between state variables and futures prices are formalized by the cubic spline collocation method, analogous to the spot price functions.

Both futures price functions are monotone decreasing with respect to available supply and expected crop size; the cross derivatives are negative. With respect to expected percentage of crop arriving next month, futures price functions are increasing at high levels of available supply, but not monotone at low levels of available supply. Statistics of simulated December and May futures prices, respectively, along with their sample counterparts are reported in Table 3. The simulated means, in contrast to the sample means, do not exhibit any trend, except anomalous spikes in September and October. Variability of both futures price series is small in the early months of the contract and increases towards maturity, which pattern is replicated by the simulated prices ignoring September and October.

The likely factor causing the anomalies in the September and October distributions is the discretized specification of states regarding harvest timing. Despite careful calibration, it places a larger probability mass than in reality on those expected harvest proportions that implies high prices. If September and October are ignored, the simulated futures prices are plausible representations of futures prices in an efficient market, exhibiting a time-to-maturity effect and positive skewness indicated by observed futures prices.

5.4. Implied price behavior

Equilibrium prices generated by our model allow for numerous seasonal patterns, which are consistent with the specified market structure. The model permits changes in post-harvest prices of various magnitudes; the largest simulated price increase from November to May in 10,000 simulations is $2.23 per bushel. A lucky farmer could face such highly advantageous price increases. Alternatively, prices do not necessarily increase monotonically, and it is

possible (with a small probability) for prices to decline seasonally after harvest. The largest

simulated seasonal decrease is $2.98 per bushel. This suggests that marketing strategy

recommendations based on short, historical samples can be misleading.

For example, one proposed risk-management strategy is to diversify post-harvest cash

sales, under the assumption that diversification can reduce risk, since cash prices are not

perfectly correlated from one month to the next. In a typical year, cash prices are lowest

immediately following harvest and then increase through the post-harvest season. As expected

prices increase, however, so does the expected variance. Thus, it is not necessarily the case that

diversifying sales will reduce the variance of returns. Indeed, comparing simulated November

prices with those in the following May indicates that when opportunity and storage costs are

taken into account, the mean return is reduced by two cents per bushel while the variability of

returns increases by 20 percent.

Treating hedging decisions as a simple portfolio problem, the hedge ratio that minimizes

the variance of returns is computed from the simulated prices. For post-harvest and pre-harvest

hedges, the ratios were 0.994 and 0.987, respectively.[10] Since the model ignores yield and basis

risk, the optimal hedge ratio should be approximately one, and the estimated ratios help confirm

the internal consistency of the model. Moreover, our results are similar to those in the empirical

literature, providing further evidence that the simulated price behavior is consistent with that

observed (Myers and Thompson, 1989; Baillie and Myers, 1991; McNew and Fackler, 1994).

Another proposed marketing strategy involves establishing positions in current marketing

year futures contracts when prices are high, and rolling them over to new crop futures in an

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[10] A post-harvest hedge assumes that a farmer sells May futures at harvest and offsets in May. A pre-

harvest hedge is assumed to be placed in May using a December futures contract, and is lifted at maturity.

attempt to obtain a higher return for the next crop. The simulations demonstrate that this is a poor strategy. The relationship between the simulated price level of old crop (May) futures and the spread between new (December) and old crop futures prices in April is plotted in Figure 6. This relationship is similar to that depicted by historical data (Lence and Hayenga, 2001, Figure 3). At low levels of May futures prices, the spread is a small in absolute value, negative number. As the price level increases, the spread decreases at an increasing rate until the rate of change becomes unity. Consequently, high prices in May are associated with a large in absolute value spread, implying that a high price for the current crop cannot be maintained for the new crop by rolling over a hedge into the December contract.

6. Concluding remarks

This paper approaches model building from a viewpoint that the parameters of price distributions should be consistent with, and identifiable from, a structural model. A rational expectations commodity storage model of the U.S. corn market, calibrated to the experience of the 1990s, is used to examine the aptness of this framework in explaining price behavior in an actual commodity market. Extensive realism was added to the model in terms of modeling the timing of production activities. Moreover, price backwardation was modeled by including convenience yield as a component of carrying costs. Thus, unlike standard models in the literature that assume a constant carrying cost, backwardation does not depend on stock-outs, which in reality have never occurred for U.S. corn.

Monthly price series, simulated from the models' equilibrium price functions, are distributed with comparable first through fourth moments and autocorrelation coefficients as the prices faced by market participants during the 1990s. The season-high prices occur in May and the season-low in November, while price variability is the smallest in November and highest

during the growing season (May through August), consistent with the sample. Simulated

December and May futures price distributions exhibit a time-to-maturity effect similar to those

estimated from the sample period. Furthermore, the distributions place some (small) probability

mass on events outside the observed range during the reference time period, allowing for various

seasonal patterns, including unlikely ones. The results are robust with respect to changes in

parameter values.

Such a variety of results suggest that the experience of the 1990s is only a tiny proportion

of outcomes that could have occurred under the same market structure. Analyses of a long

sample of simulated prices raise questions about the consequences of using some of the

commonly recommended marketing strategies, such as diversification of cash sales, as a way to

reduce the variability of returns or to enhance average returns. In addition to an improved

understanding of long-run distributions of commodity prices, our model can be used to gain

insights into the possible realizations of prices that may be encountered in different finite time

periods. If, for example, one looks at simulations in terms of 40-year "lifetimes," it is clear that a

corn producer can face a variety of outcomes over different finite periods. A producer could be

"lucky" or "unlucky" in terms of the behavior of prices, and these differences arise

notwithstanding the assumptions of an efficient market with rational decision-makers.

The model includes six state variables, but a model, no matter how complex, is an

abstraction from the actual world. So many factors evidently combine to affect prices in reality

that most of them cannot be disentangled from noise. The model's complexity cannot be easily

enhanced because of the "curse of dimensionality;" computational time increases almost

exponentially as state variables are added. It should also be noted that the rational expectations

framework implies efficient markets, and consequently, the model would need to be modified to consider hypotheses related to market inefficiency.

Nonetheless, our results suggest that a relatively simple model can be developed that produces prices that mimic the complexity of actual commodity price behavior. The virtue of the approach taken in this paper is that both the simulated cash and futures prices are consistent with recent historical experience as well as reflecting an efficient market with rational decision-makers. The simulations, therefore, are useful in generating prices that permit empirical analyses of the long-run impacts of economic decisions.

Appendix: Details of the solution method

The unknown monthly price function $P[\cdot]$ (decomposed into 12 monthly component functions $P_m[\cdot]$, $m = 1, \ldots, 12$), the planting function $\Psi[\cdot]$, and the expected harvest price function $\hat{P}[\cdot]$ are approximated by finite linear combinations of cubic b-spline functions, respectively. Function approximants are required to hold precisely at collocation nodes. The collocation nodes for the price and planting functions are specified as discretized values of available supply at the beginning of the month, A_n^m, $n = 1, \ldots N$, and discrete levels of crop size H_n^4, $n = 1, \ldots N_P$, are specified as those for the expected price function. The collocation nodes are equally spaced within the approximation range, and the same number of cubic-spline functions and collocation nodes is chosen.

Demand and supply shocks assume I_m-by-1 vector $\mathbf{u}_m$ and J-by-1 vector $\mathbf{y}$ with probability vectors $\mathbf{w}_m$ and $\mathbf{x}$, respectively. Actual and expected proportions of crop harvested assume similar G_m-by-1 vectors $\boldsymbol{\alpha}_m$ and $\hat{\boldsymbol{\alpha}}_{m-1}$ with corresponding probability vector $\mathbf{z}_m$. Normal random variables are discretized according to the Gaussian quadrature principles (Miranda and Fackler, 2002).

The expected crop size (H) is discretized to an L-by-1 vector with elements (H_l, $l = 1, \ldots, L$) that are equally spaced. A priori probability vector is associated with it in May. For all adjacent months between May and November, the crop estimate is assumed to follow an L-state Markov chain with Markov transition matrixes with elements:

$$v_{k,l}^m = \Pr\left\{ H^{m+1} = H_k \mid H^m = H_l \right\}, \qquad l, k = 1, \ldots, L; \quad m = 5, \ldots, 10.$$

Following Pirrong (1999), the transition probability is calculated as:

$$v_{k,l}^m = \mathrm{N}[h_1] - \mathrm{N}[h_2]$$

where $h_1 = \left((H_{k+1} + H_k)/2 - H_l\right)/\sigma_m$, $h_2 = \left((H_k + H_{k-1})/2 - H_l\right)/\sigma_m$, σ_m is the standard deviation

of the error term ε_m in equation (1), and $N[\cdot]$ is the standard normal cumulative distribution

function.

Hence, the function approximants in vector notation are:

$$\mathbf{P}_m[\mathbf{A}_m] \approx \boldsymbol{\Phi}_m[\mathbf{A}_m]\mathbf{c}_m, \quad \forall m$$

$$\boldsymbol{\Psi}[\mathbf{A}_4] \approx \boldsymbol{\Phi}_S[\mathbf{A}_4]\mathbf{c}_S$$

$$\mathbf{\hat{P}}[\mathbf{H}_4] \approx \boldsymbol{\Phi}_P[\mathbf{H}_4]\mathbf{c}_P$$

where elements of $\boldsymbol{\Phi}$ and $\mathbf{c}$ matrices are cubic b-spline functions and collocation coefficients,

respectively. $\mathbf{P}_m[\cdot]$ and $\mathbf{c}_m$ are N by Θ_m (the number of discrete combinations of state variables in

a given month excluding the one specified as collocation nodes), $\boldsymbol{\Phi}_m[\cdot]$ and $\boldsymbol{\Phi}_S[\cdot]$ are N by N,

$\boldsymbol{\Psi}[\cdot]$ and $\mathbf{c}_S$ are N by J, $\mathbf{\hat{P}}[\cdot]$ and $\mathbf{c}_P$ are N_P by 1, and $\boldsymbol{\Phi}_P[\cdot]$ is N_P by N_P. The results in the paper

use $I_m = I = 5$, $J = 30$, $G_9 = G_{10} = 3$, $G_{11} = 9$, $L = 10$, $N = 30$, and $N_P = 100$.

Prior to the initial iteration, the monthly collocation nodes and initial values for the

collocation coefficient matrix $\mathbf{c}$ are assigned. The iteration begins by solving equation (7a) for

October carryover levels $\mathbf{s}_{10}$ with the November price coefficients $\mathbf{c}_{11}$ fixed using Newton's

method. Once such storage levels are found, equation (7b) is solved for the (updated) October

price function coefficients $\mathbf{\hat{c}}_{10}$ using L-U factorization. Using these updated October price

coefficients, equation (7a) is solved for September carryover levels $\mathbf{s}_9$, which are used to obtain

new values of the September price coefficients $\mathbf{\hat{c}}_9$, and the procedure is repeated for the

remaining ten months backwards in time. At the end of a 12-month iteration, the norm of the

difference between the old and new November price function coefficients, $\mathbf{c}_{11}$ and $\mathbf{\hat{c}}_{11}$, is

compared to a predetermined convergence level. If it is below the level, the new set of

coefficients becomes a part of the solution and the algorithm stops; otherwise the iteration resumes with the new set of coefficients replacing the old guess.

Using the equilibrium price functions, the expected harvest price function is solved. Prices during the harvest months are obtained from simulating the corn model from various availability levels and planted crop sizes in April through November for 5,000 times. Expected harvest price $(\hat{P})$ is calculated as the average over the 5,000 simulations of September, October, and November prices weighted by the ratio between crop harvested in each month and annual crop size. For each availability level, we can interpolate a functional relationship between crop size and expected harvest price over the range of crop size as:

$$\hat{\mathbf{P}}_n = \mathbf{\Phi}_P[\mathbf{H}_4]\mathbf{c}_{P,n} \qquad \forall A_n, n = 1,\ldots,N$$

by solving for $\mathbf{c}_P$ using L-U factorization, which is equation (7d).

Lastly, analogously to the monthly price functions, Miranda's (1998) two-step algorithm is applied to solve for the planting function defined in equation (7c). First,

$$\mathbf{\Phi}_S[\mathbf{A}_4]\mathbf{c}_S = S_\theta[\mathbf{\Phi}_P[\mathbf{H}_4]\mathbf{c}_P]$$

is solved for planted crop size $\mathbf{H}_4$ by Newton's method with the coefficients $\mathbf{c}_S$ fixed. Once such crop size is found, $\mathbf{c}_S$ is updated according to:

$$\mathbf{H}_4 = \mathbf{\Phi}_S[\mathbf{A}_4]\mathbf{c}_S$$

via L-U factorization. The iteration is repeated until the coefficients converge.

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Table 1 Basic Statistics of Sample and Simulated Monthly Price Series

Parameters:	Sample (Sept 1989 - Aug 1997)[a]	Model
Number of observations	108	119,952 [b]
Mean (US$/bushel)	2.595	2.696
Standard deviation	0.575	0.571
Skewness[c]	2.325	1.310
Kurtosis[c]	5.795	4.333
First-order autocorrelation	1.483	0.862
Second-order autocorrelation	-0.557	0.018

[a] Source: Grain and Feed Market News, Agricultural Marketing Service, U.S. Department of Agriculture, "Cash Prices at Principal Markets," No.2 yellow corn at Central Illinois.

[b] The total of 120,000 observations were simulated, but the first 48 were deleted to eliminate the impact of the initial condition.

[c] The skewness measure is $\mu_3/(\mu_2)^{1.5}$ and the kurtosis measure is $(\mu_4/\mu_2^2)-3$, where μ_r is the rth central moment.

Table 2 Simulated Outcomes of the Full Model[a]

Month	Price (P), $/bu.					Storage (s), mil. bu.					Consumption (q), mil. bu.				
	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January	2.67	2.59	0.51	8.30	1.57	6428	6422	865	9453	3187	739	739	70	1051	462
	(2.56)	(2.49)	(0.44)	(3.53)	(2.07)						(745)	(757)	(60)	(831)	(650)
February	2.72	2.64	0.51	8.34	1.59	5691	5687	836	8684	2617	737	736	71	992	474
	(2.60)	(2.58)	(0.48)	(3.71)	(2.05)	(4740)	(4789)	(631)	(5678)	(3800)	(745)	(757)	(60)	(831)	(650)
March	2.77	2.69	0.52	8.53	1.64	5052	5048	811	7977	2113	639	638	68	910	389
	(2.69)	(2.59)	(0.51)	(3.92)	(2.16)						(644)	(657)	(54)	(727)	(547)
April	2.83	2.75	0.53	8.78	1.66	4412	4401	788	7333	1608	640	638	76	921	315
	(2.74)	(2.50)	(0.67)	(4.47)	(2.23)						(644)	(657)	(54)	(727)	(547)
May	2.87	2.72	0.62	8.31	1.54	3772	3764	772	6658	1137	639	638	83	1039	344
	(2.77)	(2.51)	(0.80)	(4.86)	(2.20)	(2812)	(2843)	(587)	(3709)	(1718)	(644)	(657)	(54)	(727)	(547)
June	2.84	2.73	0.63	9.18	1.54	3251	3241	758	6072	687	522	520	49	745	345
	(2.72)	(2.59)	(0.78)	(4.74)	(2.09)						(527)	(533)	(54)	(619)	(432)
July	2.79	2.67	0.63	9.86	1.45	2732	2718	744	5530	323	519	517	53	721	341
	(2.65)	(2.34)	(0.80)	(4.70)	(2.16)						(527)	(533)	(54)	(619)	(432)
August	2.75	2.63	0.66	9.29	1.44	2215	2203	732	5035	135	517	517	52	700	321
	(2.57)	(2.45)	(0.76)	(4.48)	(1.86)	(1234)	(1308)	(490)	(2113)	(426)	(527)	(533)	(54)	(619)	(432)
September	2.59	2.49	0.64	7.67	1.16	2489	2435	861	6048	84	873	871	100	1321	524
	(2.47)	(2.35)	(0.42)	(3.39)	(2.08)						(876)	(891)	(69)	(952)	(780)
October	2.51	2.41	0.59	6.94	1.22	6206	6198	1412	11125	1894	872	869	100	1261	537
	(2.40)	(2.27)	(0.40)	(3.12)	(1.92)						(876)	(891)	(69)	(952)	(780)
November	2.57	2.49	0.50	8.38	1.49	7907	7902	926	11116	4254	870	870	92	1184	566
	(2.46)	(2.41)	(0.38)	(3.22)	(2.02)	(6972)	(6940)	(724)	(8080)	(5937)	(876)	(891)	(69)	(952)	(780)
December	2.62	2.54	0.50	8.34	1.53	7168	7161	895	10289	3730	739	739	69	1001	484
	(2.51)	(2.42)	(0.40)	(3.36)	(2.06)						(745)	(757)	(60)	(831)	(650)

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 2 (Continued)[a]

Month	Crop Estimate (*H*), million bushels					Harvest (*h*), million bushels				
	Mean	Median	Std. Dev.	Max.	Min.	Mean	Median	Std. Dev.	Max.	Min.
January										
February										
March										
April	8294	8300	632	10685	5562					
May	8294	8300	632	10685	5562					
	(8649)	(8575)	(619)	(9840)	(7850)					
June	8297	8304	674	10831	5403					
	(8543)	(8500)	(634)	(9840)	(7850)					
July	8299	8307	723	11047	5013					
	(8386)	(8275)	(747)	(9700)	(7450)					
August	8297	8297	830	11506	4825					
	(8234)	(8122)	(775)	(9276)	(7348)					
September	8298	8296	846	11487	4875	1147	989	556	3947	385
	(8210)	(8118)	(838)	(9268)	(7229)	(1091)	(852)	(686)	(2824)	(544)
October	8296	8301	876	11928	4731	4589	4605	1219	8546	1565
	(8257)	(8022)	(966)	(9602)	(6962)	(4502)	(4343)	(1241)	(5988)	(2499)
November	8300	8304	922	11827	4771	2571	2492	1303	6923	0
	(8294)	(7934)	(1235)	(10051)	(6338)	(2682)	(2816)	(1385)	(4782)	(483)
December										

[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.

Table 3 Simulated Futures Prices[a]

|	December Futures, $/bushel						May Futures, $/bushel				
Month	Mean	Median	Std. Dev.	Max.	Min.	Month	Mean	Median	Std. Dev.	Max.	Min.
December	2.60	2.59	0.24	3.44	1.88	May	2.86	2.83	0.28	4.25	2.07
	(2.57)	(2.54)	(0.14)	(2.88)	(2.43)		(2.79)	(2.73)	(0.33)	(3.55)	(2.49)
January	2.61	2.59	0.24	3.44	1.86	June	2.85	2.83	0.29	4.93	2.05
	(2.60)	(2.57)	(0.16)	(2.92)	(2.42)		(2.81)	(2.80)	(0.34)	(3.54)	(2.42)
February	2.60	2.59	0.24	3.43	1.85	July	2.86	2.83	0.30	4.65	2.05
	(2.65)	(2.63)	(0.20)	(3.07)	(2.41)		(2.76)	(2.63)	(0.41)	(3.69)	(2.39)
March	2.61	2.59	0.24	3.44	1.86	August	2.85	2.81	0.32	4.96	2.00
	(2.69)	(2.64)	(0.22)	(3.13)	(2.43)		(2.70)	(2.62)	(0.34)	(3.42)	(2.35)
April	2.61	2.60	0.25	3.45	1.87	September	2.95	2.89	0.42	7.19	1.99
	(2.70)	(2.62)	(0.26)	(3.27)	(2.46)		(2.71)	(2.59)	(0.37)	(3.39)	(2.36)
May	2.61	2.56	0.36	4.42	1.65	October	3.50	3.37	0.70	10.66	2.10
	(2.69)	(2.62)	(0.34)	(3.48)	(2.38)		(2.67)	(2.62)	(0.35)	(3.25)	(2.24)
June	2.61	2.56	0.37	5.27	1.63	November	2.85	2.80	0.37	7.57	1.98
	(2.71)	(2.70)	(0.35)	(3.45)	(2.29)		(2.69)	(2.67)	(0.35)	(3.35)	(2.28)
July	2.60	2.56	0.39	4.94	1.62	December	2.85	2.80	0.37	7.60	1.95
	(2.65)	(2.51)	(0.43)	(3.60)	(2.25)		(2.70)	(2.63)	(0.39)	(3.50)	(2.28)
August	2.60	2.54	0.42	5.40	1.57	January	2.85	2.80	0.37	7.42	1.93
	(2.57)	(2.48)	(0.35)	(3.29)	(2.20)		(2.73)	(2.65)	(0.45)	(3.67)	(2.26)
September	2.73	2.64	0.55	8.09	1.57	February	2.85	2.80	0.38	7.31	1.91
	(2.57)	(2.46)	(0.38)	(3.25)	(2.21)		(2.76)	(2.75)	(0.46)	(3.73)	(2.21)
October	3.46	3.29	0.91	12.40	1.69	March	2.85	2.80	0.38	7.21	1.94
	(2.54)	(2.48)	(0.37)	(3.18)	(2.09)		(2.80)	(2.74)	(0.49)	(3.84)	(2.22)
November	2.60	2.53	0.49	8.58	1.56	April	2.86	2.81	0.38	7.33	1.95
	(2.55)	(2.55)	(0.39)	(3.29)	(2.12)		(2.81)	(2.56)	(0.66)	(4.36)	(2.30)
December	2.60	2.53	0.49	8.30	1.49	May	2.85	2.71	0.62	6.90	1.35
	(2.56)	(2.56)	(0.41)	(3.35)	(2.12)		(2.84)	(2.59)	(0.86)	(4.94)	(2.25)

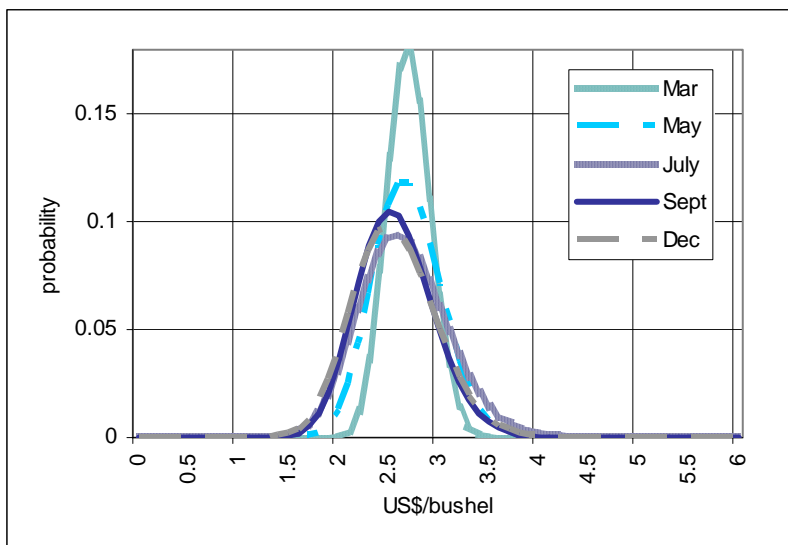[a] Numbers in parentheses are observed values for the period 1989/90 - 1997/98.
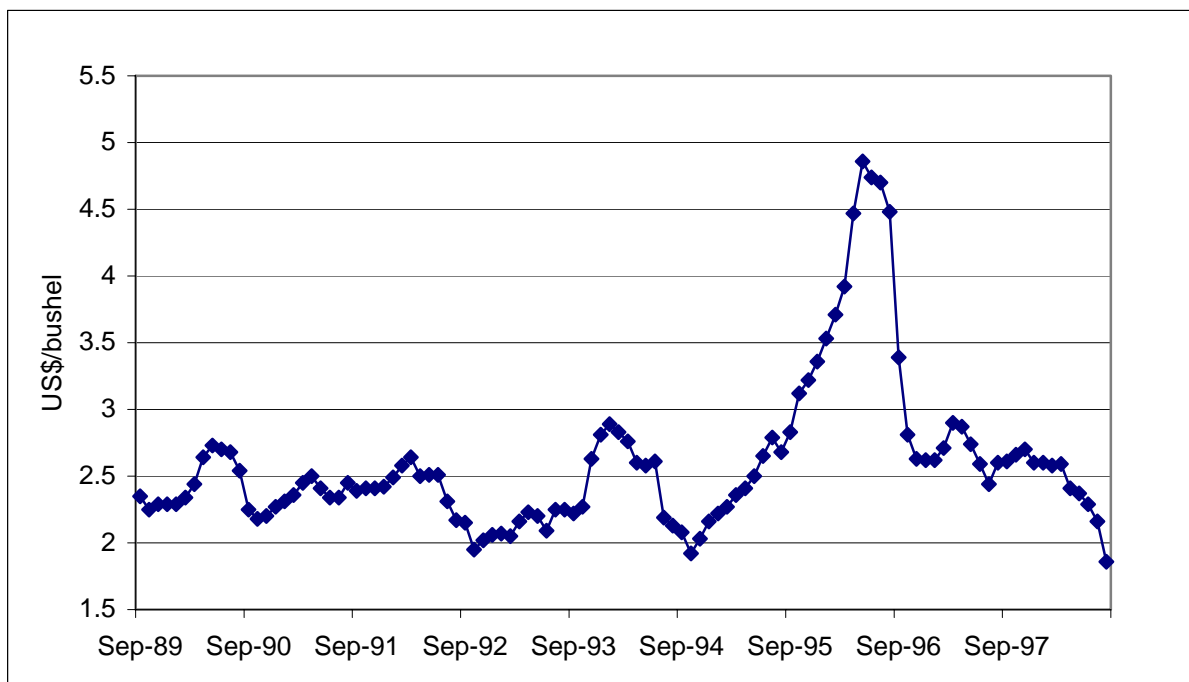
Source: Agricultural Market Service, U.S. Department of Agriculture

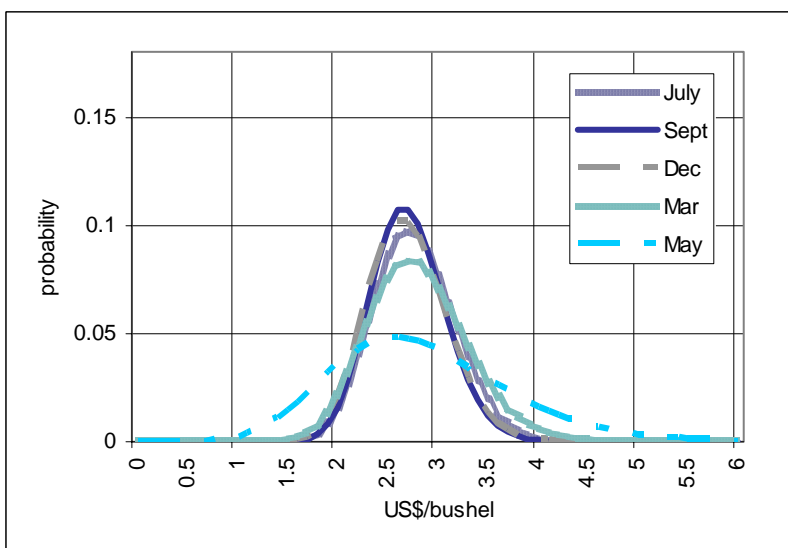Figure 1 Monthly Nominal Cash Corn Prices (No. 2, Yellow) at Central Illinois
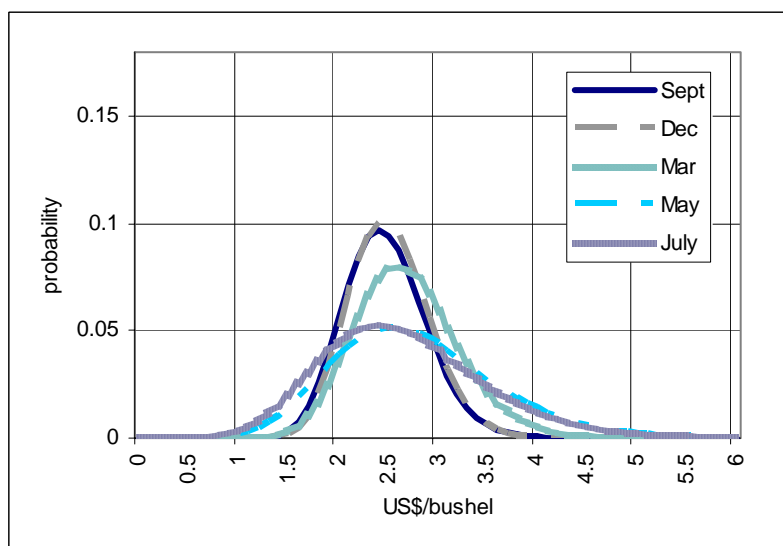
Source: Estimated from 1989/90-1987/88 observations.

Figure 2 Estimated Distributions of Monthly Cash Prices, Selected Months

Source: Estimated from 1989/90-1987/88 observations.

Figure 3 Estimated Distributions of December Futures Contract Price, Selected Months

Source: Estimated from 1989/90-1987/88 observations.

Figure 4 Estimated Distributions of May Futures Contract Price, Selected Months
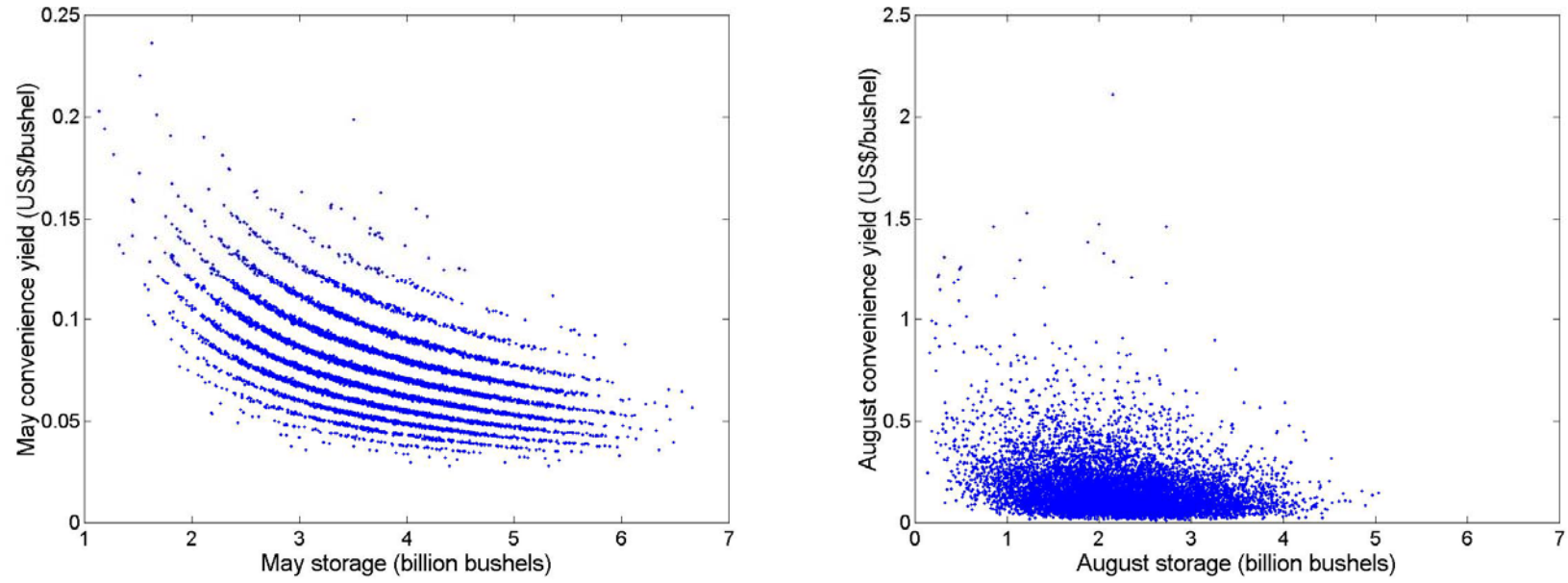
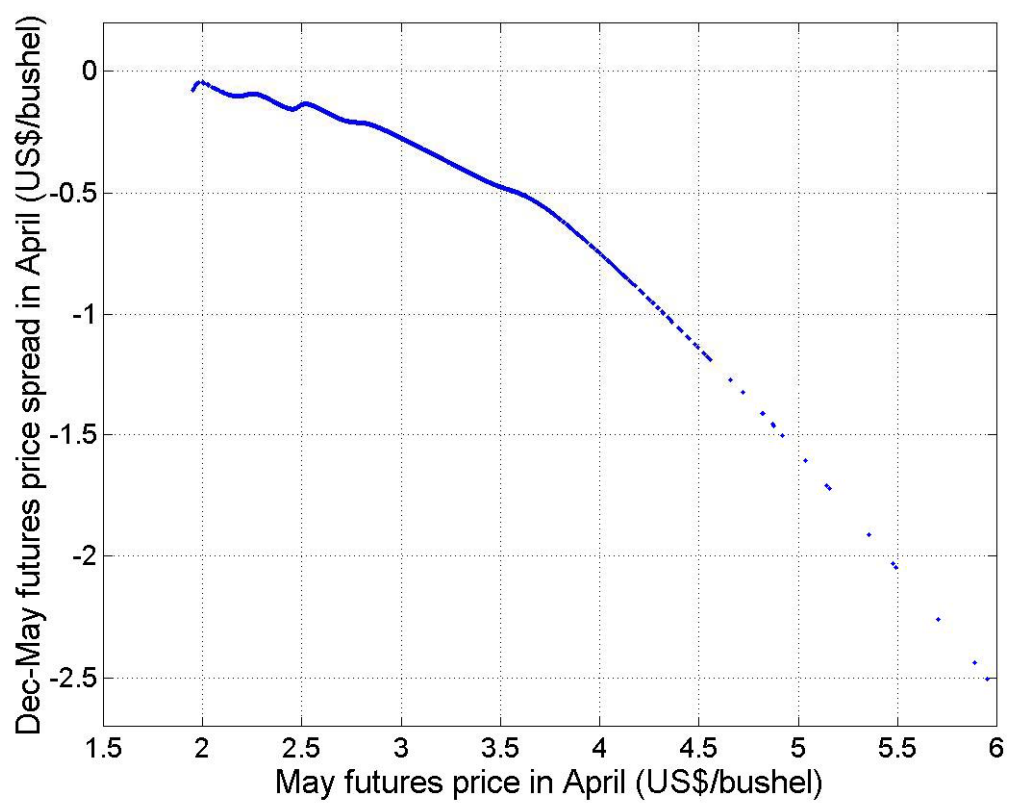Figure 5 Simulated Convenience Yield, Selected Months

Figure 6 Simulated December-May Futures Price Spread versus May Futures Price in April