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ON SOLVING AN UNBALANCED TRANSPORTATION PROBLEM

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ABSTRACT

In this paper a modification is suggested for formulating an unbalanced transportation problem. An example has been presented and solved to illustrate the drawback of the conventional formulation and the advantage of using the suggested modification in the formulation of an unbalanced transportation problem.

INTRODUCTION

The problem of transporting a product from several factories (supply origins) to a number of warehouses (demand destinations) is generally considered as a very good application area for linear programming technique. The standard formulation of a single product transportation problem is as follows:

Let m = number of supply origins;

n = number of demand destinations;

a_i = amount available at supply origin 'i' (i=1, 2, ..., m);

b_j = amount required at demand destination 'j' (j=1, 2, ..., n);

C_{ij} = cost of transporting one item from supply origin 'i' to demand destination 'j';

X_{ij} = number of units transported from supply origin 'i' to demand destination 'j' and

Z = total cost of transporting the item.

The objective is to

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \; X_{ij} \tag{1}$$

Subject to the following conditions

$$\left.\text{all } X_{ij} \geq 0 \qquad \text{for } \begin{array}{l} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, n \end{array}\right] \tag{2}$$

and

$$\left.\begin{array}{ll} \sum_{i=1}^{m} X_{ij} \geq b_j & \text{for } j = 1, 2, \ldots, n \\ \sum_{j=1}^{n} X_{ij} \leq a_i & \text{for } i = 1, 2, \ldots, m \end{array}\right] \tag{3}$$

Three situations may arise depending on the total number of units available for transport and the total number of units required at the demand destinations.

<u>Case I</u>: $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

This is the case when the total number of units available at the supply origins is equal to the total number of items available at the demand destinations. A transportation problem pertaining to this is termed as the balanced transportation problem. In a balanced transportation the set of conditions represented by (3) are set in the form of equations.

Standard methods are available for solving the balanced transportation problem.

Case II: $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

This case represents an unbalanced transportation problem. We obtain a balanced transportation problem by introducing a dummy demand destination (n+1) with requirement of

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

and $C_{i(n+1)} = 0$ for all 'i'.

We now have a balanced transportation problem involving m supply origins and (n+1) demand destinations.

Case III: $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

This also represents an unbalanced transportation problem. In order to obtain the optimal solution and to be able to apply the techniques available for solving the balanced transportation problem, we create a dummy supply origin (m+1) with availability equal to

$$a_{m+1} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$$

and the cost of transporting one item from i = m+1 to the jth demand destination is assumed to be

$$C_{(m+1)j} = 0 \text{ for } j = 1, 2, \ldots, n.$$

The implication of this assumption is that there is no penalty cost associated with unfulfilled demand at a destination, see Budnick et al.[2] This mathematical formulation of the unbalanced transportation problem, in the case of the total demand exceeding the total supply, is the most common available in the Management Science literature, see Davis et al.,[3] Lee et al.,[4] Taha[6] and Anderson et al.[1] However, in practical situations the problem may have to be reformulated specifying from where the extra demand is to come and at what cost or alternatively what penalty costs will be incurred for not meeting the demand.

SOLUTION OF THE TRANSPORTATION PROBLEMS

Before a solution of the transportation problem is attempted, the first step is to make sure that we have a balanced transportation problem. If the problem is unbalanced, then it must be balanced by creating a dummy supply original or demand destination.

The optimal solution of a balanced transportation problem is achieved in two stages. At first a good initial solution is obtained and then the initial solution is improved till the optimal solution is achieved. The number of iterations required to obtain the optimal solution from the initial solution very often depends on the initial solution. If it is possible to obtain a very good initial solution then the optimal solution is obtained in very few iterations.

There are two standard methods for obtaining a good initial solution of the transportation problem. These are:

1. Least Cost Method (LCM)
2. Vogel's Approximation Method (VAM)

According to LCM, we allocate maximum number of units to the cell having the least transport cost. On the other hand in VAM, see Reinfeld & Vogel,[5] maximum number of units are allocated to the least cost cell in the row or column for which the highest unit penalty cost is obtained.

We will concentrate our attention to the problem of determining good initial solution of the unbalanced transportation problem. We will illustrate the technique with the help of an example. At first the example will be solved using the existing techniques. The drawback of the current technique for solving the unbalanced transportation problem will be highlighted and a suggestion will be made to overcome the drawback. The example will be solved with the suggested approach for solving the unbalanced transportation problem. Finally computational results are given to illustrate the advantage of using the suggested approach.

EXAMPLE

The following data is available for an unbalanced transportation problem involving three supply origins and three demand destinations.

Demand Destination 'j'

Supply Origin 'i'	1	2	3	Available a_i
	6	19	15	40
	12	10	21	50
	15	14	17	50

C_{ij} Table

Required b_j	30	40	50

SOLUTION

The resulting balanced transportation problem after introducing a dummy demand destination is as follows:

'j'

'i'		1	2	3	4 (Dummy)	a_i
	1	6	19	15	0	40
	2	12	10	21	0	50
	3	15	14	17	0	50
b_j		30	40	50	20	

Stage I: The initial solution is obtained using LCM and VAM. As there is a choice of allocating among the three zeros in the dummy column, so there are three possible initial solutions to the transportation problem using LCM. The solutions and the total cost associated with each solution are given below:

Solution by LCM			Solution by VAM
Solution I	Solution II	Solution III	
X_{11} = 20	X_{11} = 30	X_{11} = 30	X_{11} = 30
X_{14} = 20*	X_{13} = 10	X_{13} = 10	X_{13} = 10
X_{21} = 10	X_{22} = 30	X_{22} = 40	X_{22} = 40
X_{22} = 40	X_{24} = 20*	X_{23} = 10	X_{23} = 10
X_{33} = 50	X_{32} = 10	X_{33} = 30	X_{33} = 30
Z = 1490	X_{33} = 40	X_{34} = 20*	X_{34} = 20*
	Z = 1450	Z = 1450	Z = 1450

* Allocation in the dummy column.

Stage II: The optimal solution was then obtained and the minimum cost for this problem is found to be 1410.

DRAWBACKS OF THE CURRENT METHOD FOR SOLVING UNBALANCED PROBLEMS

The generally accepted formulation of the unbalanced transportation problems assumes zero as the cost of transporting one item from a supply origin to the dummy demand destination (or from the dummy supply origin to any demand destination). Due to this reason when we apply LCM or VAM for obtaining the initial solution of the transportation problem, the allocation is first made to the zero value cells in the dummy supply origin or demand destination.

As a matter of fact no transport to or from the dummy actually takes place. Therefore, the allocation of items to the dummy column or row should be made last. In order to achieve this, the unit cost of transportation to a cell in the dummy row/column is taken to be larger than the largest C_{ij} value.

A SUGGESTED METHOD FOR UNBALANCED PROBLEMS

In order to ensure that the allocation of units to the cells in the dummy row/column is made last, the unit of cost of transportation to a cell in a the dummy row/column is assumed as

$$C_{i,dummy} \text{ or } C_{dummy,j} = 1 + \text{Max}\ (C_{ij})$$

As a result of this modification in the C_{ij} value in the dummy row/column, the units to the cells in the dummy row/column are allocated last.

An initial solution is obtained in Stage I and then the optimal solution is derived in Stage II.

As no actual transport of items takes place to or from the dummy, so in determining the actual transport cost, we ignore the cost of transporting to or from dummy.

SOLUTION OF THE EXAMPLE USING THE SUGGESTED METHOD

The largest C_{ij} is 21, so all C_{i4} values (cost of transporting for a supply origin to the dummy destination (j=4) will be assumed to be 22.

The format of the table for stage I is as follows:

		'j' 1	2	3	4	a_i
	1	6	19	15	22	40
'i'	2	12	10	21	22	50
	3	15	14	17	22	50
b_j		30	40	50	20	

The initial solution obtained on applying LCM or VAM is as follows:

$$X_{11} = 30$$
$$X_{13} = 10$$
$$X_{22} = 40$$
$$X_{24} = 10^*$$
$$X_{33} = 40$$
$$X_{34} = 10^*$$

* Allocation in the dummy column.

The total cost of this solution is 1410. Note that the cost of transporting to cells in the dummy column is ignored in determining the total transport cost.

Incidentally this is the optimal solution as revealed in our earlier attempt to solve the problem by the conventional methods.

The following table gives the results of computations on 80 imbalanced transportation problems.

Problem Size mxn	No. of Problems	*Percentage reduction in cost of the initial solution = y
3 x 4	20	2.69
6 x 8	20	3.42
9 x 12	20	2.95
12 x 15	20	1.87

$$* \; y = 100 \times \frac{(Z' - Z)}{Z'}$$

Where Z' = Total transportation cost of the lowest cost initial solution using the conventional formulation (C_{ij} involving a dummy = 0).

Z = Total transportation cost of the initial solution using the formulation suggested in this paper.

CONCLUDING REMARKS

The author of this paper has solved a large number of unbalanced transportation problems using the suggested modification in definining the unit cost of transport involving the dummy either as a supply origin or the demand destination. The initial solutions obtained as a result of using the suggested modification had equal or very often lower total cost. Due to this the optimal solutions were frequently obtained in Stage I or required only a few iterations in Stage II.

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