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The Vanishing U.S. Cattle Cycle: A Stochastic Cycle Approach

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We re-examine the existence of cattle cycles based on U.S. beef cow inventories from 1979 to 2019. Our analysis begins with a basic first-order stochastic cycle model and finds a cattle cycle of 16.54 years, significantly longer than the presumed 10- to 12-year cycle. Typically cycles become longer before they disappear. Upon further investigation, we re-estimate the length of the cycle by applying a second-order stochastic cycle model which improves goodness of fit. Surprisingly, the cycle length is estimated to be infinitely long, implying the complete disappearance of the beef cow cycle.

Key words: cattle dynamics, cattle inventory

Introduction

The dynamics of beef cattle supply and the existence of cattle cycles have been widely researched topics in the last 4 decades. Jarvis (1974) was first to treat beef cows in the context of capital goods and recognized that increasing beef prices can actually lead to reduced slaughter in the short run. This study influenced empirical approaches to modeling the beef cattle herd such as formulated by Rucker, Burt, and LaFrance (1984) and stimulated a theoretical treatment of the dynamics of livestock production by Rosen (1987). Rosen, Murphy, and Scheinkman (1994) specifically addressed the existence of cattle cycles. More recently Aadland (2004) constructed a model to describe the putative 10-year cattle cycle by assuming that producers maximize a discounted stream of future profits subject to biological constraints and market forces.

Under a fully rational expectations theory, forward-looking ranchers should anticipate future cattle cycles, which were known to be approximately 10 years, and take actions to mitigate them. Of course, under the animal spirits paradigm (where emotions affect economic decisions), their actions could endogenously generate or realize another 10-year cycle in the future. There are also quite a few studies that question the appropriateness of fully rational expectations in the cattle industry (Nerlove and Fornari, 1998; Baak, 1999; Chavas, 2000) as most cattle market participants are found to be not fully rational (i.e., quasirational or boundedly rational). However, Aadland (2004) showed that individual optimizing behavior endogenously generated approximate 10-year cycles in cattle supply even under a less restrictive assumption that ranchers are only boundedly rational (i.e., ranchers have limited access to future information and form expectations of future prices based on only past or current information). As will be seen, the extraordinary change in the beef cow cycle suggests that previous studies of the cattle cycle may need to be re-evaluated.

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Since it is difficult to visualize a persistent cycle from the recent semiannual or annual data, we turn to the business cycle literature because periodic patterns in business cycles are typically not easily discerned. In 1927, Eugen Slutzky published a paper (subsequently available in English in 1937), which was the first to introduce the notion of a stochastic cycle. He posited that under certain circumstances the summation of random causes may be the source of a cyclical process. He also noted that cycles may transition over time, making it difficult to detect a regular pattern. Slutzky (1937) was able to closely approximate a nineteenth-century English business cycle using his stochastic model. Beveridge and Nelson (1981) reviewed business cycle analyses, and their study was one of the first to recognize and account for the nonstationarity of macroeconomic data. Harvey (1985) developed a stochastic cycle model that allowed the amplitude and phase of the cycle to vary over time. Harvey divided his datasets of macroeconomic series into two periods. He found that before 1948, a 6- to 9-year cycle was evident in gross national product. For more recent data, Harvey was less confident that cyclical components could be identified. A highly influential paper by Howitt and McAfee (1992) addressed the idea that the business cycle results from exogenous shocks that lead to economic fluctuations in an otherwise stable system. They constructed a rational expectations–animal spirits cycle. The notion of animal spirits relates to random waves of optimism or pessimism that are not necessarily based on economic conditions. Howitt and McAfee proposed,

> When spirits are high, firms expect a high level of employment, and hence a high level of aggregate demand. . . This encourages firms to hire more aggressively, thus validating the original expectation (p. 494).

They showed that the animal spirits cycle may be stable under Bayesian learning:

> If people start with diffuse priors, there is a positive probability that accidental correlations during the early stages of learning could lead them forever into the self-fulfilling beliefs of the animal spirits cycle (p. 505).

Nevertheless, they did conclude that the probability of a perpetual cycle seemed remote. Hodrick and Prescott (1997) introduced a low-pass filter to de-trend economic time series. If properly de-trended, the cycle then is the residual of the series. Harvey and Trimbur (2003) reconsidered the use of model-based time series filters to extract cyclical components. They extended the original trend plus cycle model of Harvey (1985) to allow for higher-ordered cycles. They showed that a second-order stochastic cycle model tends to fit the data better and can provide a smoother (i.e., less noisy) cyclical representation. Based on their experience the second-order model should also yield more plausible values for the cycle length. To the best of our knowledge, this is the first study that applies business cycle models to depict the cyclical pattern in the U.S. beef cattle inventory.

Data Description

The data used in this study are sourced from the USDA National Agricultural Statistics Service (see https://quickstats.nass.usda.gov/), which reports the U.S. beef cow inventory twice a year, in January and July. While the January beef cow inventory is based on survey data, the midyear numbers are derived from marketing information and a smaller survey. Figure 1 plots annual beef cow inventory numbers corrected for mean and trend for the 62-year period from 1920 to 1981.[1] It is evident that generally numbers peaked mid-decade and tended to bottom out around the beginning of each decade—suggesting a 10-year cattle cycle. In fact, when a trigonometric regression with $\sin(\lambda t)$ and $\cos(\lambda t)$ along with a polynomial in time is performed over this period, λ is estimated to be 0.6235, which implies a cycle length of 10.08 years.

However, Figure 1 stands in contrast to the trend and seasonally adjusted semiannual beef cow inventories for the recent 40 years (1979–2019) depicted in Figure 2.[2] Since 1995, the number

[1] A linear trend has been removed from the original series.
[2] A quadratic trend and a semiannual dummy variable are included in the regression.

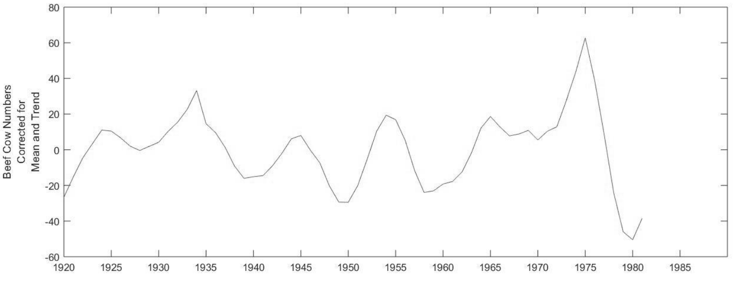

Figure 1. U.S. Annual Beef Cow Numbers, 1920–1981, Corrected for Mean and Trend

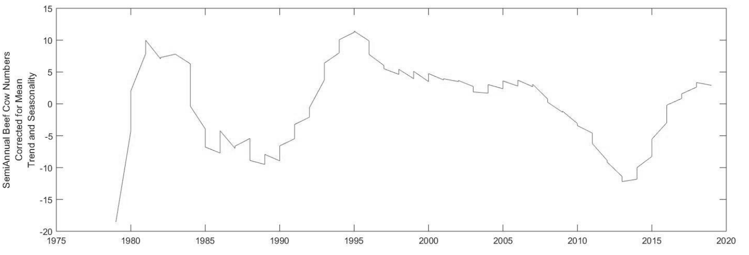

Figure 2. U.S. Semiannual Beef Cow Numbers, 1979–2018, Corrected for Mean, Trend, and Seasonality

of beef cows (trend and seasonally adjusted) appears to have little cyclicality. We do need to mention that there are two missing values for this series: midyear 2013 and 2016. These values were calculated by estimating an autoregressive model of order four with constant, seasonal dummy, and trend and performing a grid search over the missing values such that the model's sum of squared errors was minimized. The resulting estimates of beef cow numbers were 29.9 million for 2013 and 31.4 million for 2016. The little cyclical pattern observed in the more recent beef cow cycle—whether it is significantly longer than 10 years or has disappeared—calls for the need to re-examine the cattle cycle.

The Basic Stochastic Cycle Model

We begin by analyzing the semiannual beef cow inventory data using the Harvey and Trimbur (2003) approach, which we refer to in this study as the basic stochastic cycle model, to attempt to estimate the length of the beef cow cycle. The irregular patterns evidenced in Figure 2 suggest that the amplitudes and phases of this series have been evolving over time. To address these issues, we investigate a random walk plus cycle model that allows for (i) analysis of nonstationary data, (ii) direct estimation of cycle length, and (iii) shifting phase and changing amplitudes.

The model is specified to be a random walk (with drift) with a stochastic cycle (Harvey, 1989) for the stochastic series y_t:

(1) $$y_t = \mu_t + \psi_t + x_t\delta + \varepsilon_t;$$

(2) $$\mu_t = \mu_{t-1} + \beta + \eta_t;$$

(3) $$\psi_t = \rho\left\{\cos(\lambda)\,\psi_{t-1} + \sin(\lambda)\,\psi^*_{t-1}\right\} + \kappa_t;$$

(4) $$\psi^*_t = \rho\left\{-\sin(\lambda)\,\psi_{t-1} + \cos(\lambda)\,\psi^*_{t-1}\right\} + \nu_t;$$

where μ_t and ψ_t represent dynamic unobservables associated with a random walk (with drift β) and a cyclical process (with frequency λ), respectively; x_t is coded as 1 if the inventory number is reported in January, and 0 otherwise; the coefficient ρ is termed the damping factor, and $\rho = 1$ if the cyclical process is nonstationary; and the error processes ε_t, η_t, κ_t, and v_t are assumed to be *i.i.d.* normal with variance–covariance matrix

$$\begin{bmatrix} \sigma_\varepsilon^2 & 0 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 & 0 \\ 0 & 0 & \sigma_\kappa^2 & 0 \\ 0 & 0 & 0 & \sigma_v^2 \end{bmatrix}.$$

The initial conditions ψ_0 and ψ_0^* determine the initial amplitude and phase shift of the series and μ_0 denotes the initial level of the series, which can be estimated in lieu of fitting a constant to the model. Parker and Shonkwiler (2014) showed that the reduced form of the model can be written in terms of the observable process y_t, the unknown parameters, and the error processes:

$$\begin{aligned} \Delta y_t = {} & 2\rho\cos\lambda\Delta y_{t-1} - \rho^2\Delta y_{t-2} + \beta^* + \eta_t - 2\rho\cos\lambda\,\eta_{t-1} + \rho^2\eta_{t-2} + \Delta\kappa_t \\ & - \rho\cos\lambda\Delta\kappa_{t-1} + \rho\sin\lambda\Delta v_{t-1} + \Delta\xi_t - 2\rho\cos\lambda\Delta\xi_{t-1} + \rho^2\Delta\xi_{t-2}, \end{aligned} \tag{5}$$

where Δ denotes the (first) difference operator, β^* represents the sum of deterministic terms, and $\xi_t = x_t\delta + \varepsilon_t$. In time series parlance, the series is a type of ARIMA(2,1,2) process. This representation is valid when y_t has a random walk component (i.e., $\sigma_\eta^2 > 0$) and consequently the series must be first differenced to achieve stationarity. If this is not the case, the model with $\sigma_\eta^2 = 0$ simplifies to

$$y_t = 2\rho\cos\lambda\,\xi_{t-1} + \rho^2\xi_{t-2}, \tag{6}$$

or a type of ARMA(2,2) process with constant trend. An additional simplification of the dynamic process occurs when all the noise in the system is due to the stochastic cycle. In this case $\sigma_\varepsilon^2 = 0$, and then

$$\begin{aligned} y_t = {} & 2\rho\cos\lambda\, y_{t-1} - \rho^2 y_{t-2} + \beta t + \kappa_t - \rho\cos\lambda\,\kappa_{t-1} \\ & + \rho\sin\lambda\, v_{t-1} - 2\rho\cos\lambda\, x_{(t-1)}\delta + \rho^2 x_{(t-2)}\delta. \end{aligned} \tag{7}$$

Estimation Results Model I

A MATLAB program using double-sided numerical first and second derivatives of the likelihood function implied by the state-space model was used to obtain parameter estimates (Harvey, 1989, p. 126). Estimation results for the basic stochastic cycle model are reported in Table 1, where μ_0, Ψ_0, and Ψ_0^* represent the initial values for μ_t, Ψ_t, and Ψ_t^*, respectively; δ is the coefficient on the semiannual dummy variable; and the other parameters are defined as in equations (1)–(4) with the dependent variable being semiannual beef cow inventory from 1979 to 2019. Imposing the customary restriction (Harvey, 1989, p. 39) that $\sigma_\kappa^2 = \sigma_v^2$ for the cow herd size, we found that σ_ε^2 converged to 0 under maximum likelihood estimation of the stochastic cycle model. We adopted a logarithmic time trend as it resulted in a marginally better fit than a linear counterpart;[3] its coefficient (β) is estimated to be positive and significant at the 5% level. The highly significant coefficient on the semiannual dummy variable (δ) shows that the cow herd tends to be smaller by almost 700,000 (herd

[3] The logarithmic time trend is sensitive to the value of the beginning time. We use $t = 1$ to date the first observation that was observed in January 1979.

Table 1. First-Order Cycle Estimation Results

	First-Order Cycle I		First-Order Cycle II	
Parameter	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**
μ_0	323.800	18.030	337.810	15.338
β	10.310	5.194	5.872	4.030
δ	−6.750	0.347	−9.828	1.156
σ_ε	0		0	
λ	0.190	0.023	0.190	0.019
ρ	0.943	0.015	0.948	0.014
$\sigma_\kappa = \sigma_\nu$	1.677	0.327	1.665	0.298
σ_η	2.568	0.377	2.341	0.347
$\Psi_0 = \Psi_0^*$	51.203	16.490	43.580	13.528
Δy_{t-3}			0.234	0.091
Log-likelihood	−206.980		−202.260	
LBQ p-value	0.030		0.258	
R^2	0.983		0.985	

Notes: Standard errors are robust standard errors. The dependent variable is U.S. semiannual beef cow inventory from 1979 to 2019. μ_0, Ψ_0, and Ψ_0^* represent the initial values for μ_t,Ψ_t, and Ψ_t^*, respectively. β is the drift term within the random walk component, μ_t. δ is the coefficient on the semiannual dummy variable. λ represents cycle frequency, and ρ is the damping factor of the cycle. σ_ε, σ_η, σ_κ, and σ_ν are standard deviations for disturbance terms ε, η, κ, and ν, respectively. Δy_{t-3} is the first difference between y_{t-3} and y_{t-4}.

size is expressed in units of 100,000) on January 1 than on July 1. The estimate of λ implies a cattle cycle of $\frac{2\pi}{\lambda}$, about 33.1 periods or 16.57 years. Using the delta method, we find an approximate asymptotic 95% confidence interval for the cycle length of 12.68 to 20.46 years. Clearly this is significantly longer than the presumed 10-year cycle. Further the estimated amplitude, $\hat{\rho} = 0.94$, suggests that the cycle does not decay much from period to period.

The statistically significant estimate of σ_η indicates that there is a nonstationary component to beef cow numbers. Therefore the reduced form of the model follows equation (5), with $\xi_t = x_t\delta$ and $\delta = [\beta_t \beta_{DV}]$. Here, we also impose the restriction $\psi_0 = \psi_0^*$; whereas it is often customary to set $\psi_0 = \psi_0^* = 0$. Clearly, the initial condition for the cyclical component is significantly different than 0.

We calculate robust standard errors (White, 1982) to ensure that inference is robust to distributional misspecification such as nonnormality or heteroskedasticity; however, the standard errors may be affected by autocorrelated errors. Therefore, we calculate the Ljung–Box (1978) Q test (LBQ) for serially correlated residuals based on $m = 20$ lagged residual correlations and adjusted for degrees of freedom determined by m less the number of parameters estimated excluding the constant, μ_0, and the variance terms. The p-value of the test under the hypothesis of temporally independent errors is shown to be 0.030 for Model I. Inspection of the correlogram then led to the inclusion of the stationary variable Δy_{t-3}.

Estimation Results Model II

The first-order stochastic cycle model augmented with Δy_{t-3} has estimated parameters that are largely unchanged except for the coefficient on the logarithmic trend, which becomes statistically insignificant. Interestingly, λ is estimated with more precision and the model now suggests a beef cow cycle of 16.54 years with a 95% confidence interval of 13.35–19.74 years. The variance of the cycle is $\sigma_\psi^2 = \sigma_\kappa^2 / \left(1 - \rho^2\right) = 27.46$, which can be compared to the variance of the random walk, which is $t\sigma_\eta^2$, or 438.46 at the 80th observation. We cannot reject the hypothesis of independent errors at conventional levels based on the LBQ test calculated as above. In addition, the estimated amplitude ($\hat{\rho} = 0.95$) does not vary much compared with that in Model I.

Were we to stop the analysis at this point, we would conclude that the augmented random walk model with stochastic trend demonstrates a substantial lengthening of the beef cattle cycle. The 95% interval for the cycle length does not include the 10- to 12-year cycle that has been historically accepted. For this reason, we extend the analysis to a second-order stochastic cycle representation to determine whether it offers an improvement in fit and corroborating evidence of a lengthening cattle cycle.

The Higher-Order Stochastic Cycle Model

Generalized Stochastic Cycles

Instead of the basic model, which incorporates a first-order stochastic cycle, described previously. we apply a higher-order stochastic model to explore the periodic dynamics of the beef cow inventories. Periodic behavior can be better identified by generalized cycles as they have more power concentrated near the spectral peak (Trimbur, 2006). Examples in Harvey and Trimbur (2003) showed that smoother cycles in economic series can be extracted by generalized cyclical processes. Allowing for variation in the amplitude and phase over time, as the first-order stochastic cycle does, the higher-order cycles also allow the disturbances themselves to be periodic, thus producing a more flexible description of periodic behavior in time series data. Additionally, the interpretation of parameters remains straightforward and as such the cyclical pattern can be studied directly. Extensions to multivariable models are also possible.

The unobserved component, $\psi_t^{(m)}$, is defined as an mth-order stochastic cycle if

(8) $$\begin{bmatrix} \psi_t^{(1)} \\ \psi_t^{(1)*} \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(1)} \\ \psi_{t-1}^{(1)*} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \ t = 1,2,\dots T;$$

(9) $$\begin{bmatrix} \psi_t^{(i)} \\ \psi_t^{(i)*} \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(i)} \\ \psi_{t-1}^{(i)*} \end{bmatrix} + \begin{bmatrix} \psi_{t-1}^{(i-1)} \\ \psi_{t-1}^{(i-1)*} \end{bmatrix}, \ t = 1,2,\dots T, \ i = 2,\dots m;$$

where κ_t and κ_t^* are uncorrelated white-noise processes with 0 mean and common variance σ_κ^2 and ρ is the damping factor, which is bounded between 0 and 1. The cycle is stationary if $\rho < 1$. The stochastic nature of cycles comes from the disturbances $\begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}$. Higher-order stochastic cycles are more flexible than the first-order one because when order increases, the disturbances can impact cycles in different ways, resulting in different dynamic properties. For instance, a second-order stochastic cycle, $\psi_t^{(2)}$, has a first-order stochastic cycle, $\psi_{t-1}^{(1)}$, as a driving component, which allows the shocks to the second-order cycle to be periodic as well.

Although cycles can be reduced to autoregressive moving-average (ARMA) form, they are more often represented in a state-space form, as this is the easiest way in terms of estimation and offers more intuitive parameterization than the reduced form. Therefore, we put an mth-order cycle in a state-space form, with the transition equation specified as

(10) $$\boldsymbol{\psi}_t = \boldsymbol{T}_m \boldsymbol{\psi}_{t-1} + \boldsymbol{c}_m \otimes \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix},$$

where $\boldsymbol{\psi}_t = \left[\psi_t^{(m)}, \ \psi_t^{(m)*}, \psi_t^{(m-1)}, \ \psi_t^{(m-1)*}, \dots, \psi_t^{(1)}, \ \psi_t^{(1)*}\right]'$ is the state vector with $2m$ elements. The last element of the $m1$ vector $\boldsymbol{c}_m$ is 1, with zeros elsewhere. $\boldsymbol{T}_m$ is given by

(11) $$\boldsymbol{T}_m = \boldsymbol{I}_m \otimes \boldsymbol{T} + \boldsymbol{S}_m \otimes \boldsymbol{I}_2,$$

where the transition matrix $\boldsymbol{T}$ is given by $\boldsymbol{T} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix}$, and $\boldsymbol{S}_m$ is a matrix with ones on the diagonal strip to the right of the main diagonal and zeros elsewhere. The variance–covariance matrix, VAR$(\cdot)$, of the disturbance vector $\boldsymbol{c}_m \otimes \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}$ is given by

(12) $$\mathrm{VAR}\left(\boldsymbol{c}_m \otimes \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}\right) = \boldsymbol{c}_m \boldsymbol{c}_m^{'} \otimes \begin{bmatrix} \sigma_\kappa^2 & 0 \\ 0 & \sigma_\kappa^2 \end{bmatrix}.$$

Fitting Annual Beef Cow Inventories using Second-Order Stochastic Cycle Model

We then start analyzing this series with the random walk second-order stochastic cycle model:

(13) $$y_t = \mu_t + \psi_t^{(2)} + \log(t_t)\beta + \varepsilon_t,$$

(14) $$\mu_t = \mu_{t-1} + \eta_t,$$

(15) $$\begin{bmatrix} \psi_t^{(1)} \\ \psi_t^{(1)*} \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(1)} \\ \psi_{t-1}^{(1)*} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix},$$

and

(16) $$\begin{bmatrix} \psi_t^{(2)} \\ \psi_t^{(2)*} \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(2)} \\ \psi_{t-1}^{(2)*} \end{bmatrix} + \begin{bmatrix} \psi_{t-1}^{(1)} \\ \psi_{t-1}^{(1)*} \end{bmatrix},$$

where μ_t is the trend component and $\psi_t^{(2)}$ is the second-order stochastic cycle component. All parameters and disturbances follow the same definition as in equations (1)–(4), with the semiannual dummy represented by xt. We can also write equations (13)–(16) in the state-space form, with the measurement equation and transition equation specified as

(17) $$y_t = \boldsymbol{Z}\boldsymbol{\alpha}_t + \boldsymbol{X}_t\boldsymbol{\beta} + \varepsilon_t \text{ (measurement equation)},$$

(18) $$\boldsymbol{\alpha}_t = \begin{bmatrix} \mu_t \\ \boldsymbol{\psi}_t \end{bmatrix} = \begin{bmatrix} \mu_{t-1} + \eta_t \\ \boldsymbol{T}_2 \boldsymbol{\psi}_{t-1} + \boldsymbol{c}_2 \otimes \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix} \end{bmatrix} \text{ (transition equation)},$$

where $\boldsymbol{Z} = [1\ 1\ 0\ 0\ 0]$, and ε_t is assumed to be *i.i.d.* normally distributed with mean 0 and constant variance σ_ε^2.

The transition equation for the second-order cyclical component is

(19) $$\boldsymbol{T}_2 = \begin{bmatrix} \rho\cos(\lambda) & \rho\sin(\lambda) & 1 & 0 \\ -\rho\sin(\lambda) & \rho\cos(\lambda) & 0 & 1 \\ 0 & 0 & \rho\cos(\lambda) & \rho\sin(\lambda) \\ 0 & 0 & -\rho\sin(\lambda) & \rho\cos(\lambda) \end{bmatrix}, \ c_2 = [0,1]'$$

The system involves the eight parameters μ_0, β, ρ, λ, σ_ε, $\sigma_\kappa = \sigma_{\kappa*}$, σ_η, and the initial condition $\psi_0 = \psi_0^*$. The system is estimated by maximum likelihood, and optimization is achieved by Kalman filter algorithm.

Table 2. Second-Order Cycle Estimation Results

	Second-Order Cycle I		Second-Order Cycle II	
Parameter	**Estimate**	**Std. Err.**	**Estimate**	**Std. Err.**
μ_0	349.260	5.924	358.190	5.218
β	−5.003	4.086	−7.067	3.377
σ_ε	0		0	
λ	0		0	
ρ	0.792	0.041	0.791	0.030
$\sigma_\kappa = \sigma_\nu$	1.978	0.469	2.119	0.585
σ_η	2.153	0.473	1.715	0.447
$\Psi_0 = \Psi_0^*$	28.445	5.788	24.984	4.618
Δy_{t-3}			0.217	0.066
Log-likelihood	−204.540		−198.730	
LBQ p-value	0.076		0.934	
R^2	0.984		0.986	

Notes: Standard errors are robust standard errors. The dependent variable is U.S. annual beef cow inventory from 1979 to 2019. μ_0, Ψ_0, and Ψ_0^* represent the initial values for μ_t,Ψ_t, and Ψ_t^*, respectively. β is the drift term within the random walk component, μ_t. λ represents cycle frequency, and ρ is the damping factor of the cycle. σ_ε, σ_η, σ_κ, and σ_ν are standard deviations for disturbance terms ε, η, κ, and ν, respectively. Δy_{t-3} is the first difference between y_{t-3} and y_{t-4}.

As mentioned previously, given the concern that the results from using semiannual data may partially be an artifact of the different survey used to calculate midyear inventories, we use the second-order cycle model to fit annual beef cow inventory data, which include the same 40 observations used earlier. Table 2 reports the second-order cycle estimation results. All the parameters have the same definitions as in Table 1. As the model iterated to convergence, the estimate of λ became quite small. Because λ must be in the interval between 0 and π, inclusive, it was replaced by its square to maintain nonnegativity. Surprisingly, the estimate of λ^2 converged to 0, signifying an infinitely long cattle cycle. As with the first-order cycle model, the estimate of σ_ε also converged to 0. Because the LBQ statistic for the base model suggests some temporal nonindependence of the error terms, we will focus on the model augmented with Δy_{t-3}.

Comparing the second-order cycle augmented model to the first-order cycle augmented model, we see that the logarithmic time trend has a coefficient that is now negative and statistically significant. The estimate of the damping factor, ρ, shrinks and the amount of the relative variation in the random components switches: The variance of the cycle is $\sigma_\psi^2 = \left(1+\rho^2\right)\sigma_\kappa^2/\left(1-\rho^2\right)^3 = 144.88$, much greater than the variance of the first-order cycle, while the variance of the random walk at the 80th observation is 235.16, much less than that of the first-order cycle model. In choosing between models, we observe that the log-likelihood and implied R^2 of the second-order model are both greater than those of the first-order model, and this is accomplished with one less parameter. Further, the very small value of the LBQ statistic (and consequently large p-value) indicates a much better representation of the data by the second-order augmented model.

Implications of $\lambda = 0$

The fact that λ becomes 0 in the second-order cycle model implies the complete disappearance of the beef cow cycle. It may seem contradictory that a stochastic cycle model would converge to a solution that indicates the absence of a cycle. To better understand the meaning of $\lambda = 0$, consider the autocorrelation functions, $\rho(\tau)$, determined by the two stochastic cycle models. For the first-order model $\rho(\tau) = \rho^\tau \cos(\lambda\tau)$ and for the second-order model, the autocorrelation function is $\rho(\tau) = \rho^\tau \cos(\lambda\tau)\left(1 + \frac{1-\rho^2}{1+\rho^2}\tau\right)$. When $\lambda = 0$, $\cos(\lambda\tau) = 1$ for any τ. We can compare the

Table 3. Autocorrelation Functions

Lag = τ	Observed Cow Numbers	First-Order Cycle	Second-Order Cycle
1	0.931	0.931	0.974
2	0.906	0.835	0.916
3	0.799	0.718	0.841
4	0.744	0.586	0.758
5	0.623	0.446	0.673
6	0.569	0.304	0.591
7	0.450	0.165	0.514
8	0.395	0.034	0.444
9	0.281	−0.085	0.381
10	0.239	−0.189	0.325

autocorrelation functions implied by the two models to that of the observed beef cow inventory series in Table 3, where we see that the second-order stochastic cycle model more closely represents the observed autocorrelation of the beef cow inventory series. In fact, for any $\lambda > 0$, the implied autocorrelation function will become negative at some point. This feature may not be consistent with the data-generating process.

Conclusion and Discussion

We began our analysis on the dynamics of beef cow numbers with a basic random walk (first-order) stochastic cycle model utilizing semiannual data of beef cow inventories over a recent 40-year period. Results from the augmented first-order stochastic model imply a cattle cycle of 16.54 years with an asymptotic standard deviation of 1.63 years. We find that a second-order random walk stochastic cycle model provides a better representation of beef cow numbers both in terms of fit as measured by the log-likelihood and implied R^2 and fit as measured by the Ljung–Box Q statistic. However, this model yields the unexpected result that the beef cattle cycle disappears.

This raises a question: Will there the beef cow herd continue to expand and contract? Our model does not suggest otherwise. Depending on where a starting point is chosen, there will likely be some pattern of beef cow numbers that allows a cyclical interpretation. The value of our stochastic cycle model is to inform analysts that, using recent data, the cyclical movements are not highly predictable and probably do not conform to a putative 9- to 12-year beef cow cycle. Yes, there are patterns in beef cow numbers that reasoned analysts may interpret as cycles, but our models show there is currently no regularity to cattle cycles such as existed in the past.

For example, the Livestock Marketing Information Center (LMIC) has identified three complete cattle cycles since 1979. Using plots of annual data on total cattle inventory, the LMIC has suggested an 11-year cycle beginning in 1979, a 14-year cycle beginning in 1990, and a 10-year cycle beginning in 2004. But unlike earlier patterns in total cattle inventories, the distances from peaks to troughs are muted, making cycle identification somewhat subjective. In fact, a basic stochastic cycle analysis of these data over the last 41 years actually identifies an infinitely long cycle, even though the dynamic properties of the total annual cattle inventory series differ markedly from those of the semiannual beef cow inventory series.

Although the identification of commodity cycles can be viewed as purely a statistical exercise, the presence of cycles has important implications. Beveridge and Nelson (1981, p. 151) have pointed out that the observation of cyclical components in economic time series "has played an important role in shaping our thinking about economic phenomena." In this case, we observe either a significant lengthening of the cattle cycle or its complete disappearance. Typically, cycles become longer before they disappear. While the popular press has already buried the cattle cycle (e.g., Speer, 2014), given the observed patterns in herd size since the 1990s, the second-order stochastic cycle model provides evidence for its disappearance.

This leads to speculation as to why the cycle in recent decades has not followed the approximate 10-year cycle observed over most of the twentieth century. Since 2000, a number of events have been linked to a lengthening cattle cycle—specifically due to a long-term trend of decreasing beef cow numbers. There were a number of droughts in major cow–calf-producing areas from 2000 to 2008 and again in the southern plains from 2010 to 2013 (Petry, 2015). Higher row crop prices and the consequent expansion of crop production may also have led to a reduction in cow–calf operations. The 2012 Census of Agriculture observed a decline of almost 175,000 cow–calf operations over the previous 20 years, with a bulk of these operations having fewer than 50 cows. These exogenous shocks hitting the cattle industry could partly explain the change in patterns of beef cow numbers.

While it is debatable whether rational expectation theory holds in the cattle industry, our stochastic cycle models identify a lengthening or disappearing cattle cycle, which certainly gives more credence to a rational expectations paradigm. It will be a matter of time to determine whether the cycle re-emerges and reverts to a traditional cycle length of approximately 10 years.

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