THE BAYESIAN DECISION MODEL WITH MORE THAN ONE PREDICTOR– AN APPLICATION TO THE STOCKING RATE PROBLEM

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The variables a manager faces in making decisions may be divided into two broad categories–those which are determined by the manager and those which are outside of his control. Agricultural economists have made many efforts to develop expectation models for one or more of the uncontrollable variables facing farmers and have suggested procedures for utilizing the resulting expectations. Recent developments in statistical decision theory provide a logically consistent framework for incorporating the predictions of expectation models [4, pp. 192-196]. Applications of Bayesian analysis utilizing predictions of one uncontrollable variable have been reported in the literature [1, 3]. However, many decision problems logically require expectations of two uncontrollable variables (such as price and yield) or more. This article illustrates a method of including predictors for more than one uncontrollable variable in the Bayesian framework, and reports some empirical results of an application to a stocking rate problem.

ANALYSIS

The analysis is based on fourteen years (1950-1963) of experimental grazing data for loam soils at Heavener in Eastern Oklahoma. The pasture is in common Bermuda grass overseeded with annual clovers and fertilized with one hundred pounds of 0-40-20, annually. Comparable grazing yield data were not available for other fertilization rates. Thus, only one level of fertilizer application is considered in the analysis.

Choice grade stocker steers, weighing 500 pounds on the first of April, are to be pastured on the Bermuda grass from April to the end of October. A producer's net return per acre for the i^{th} stocking rate is given by equation (1):

$$NR_{ai} = SR_i \left[P_N W_{gi} + P_{N\text{-}M} W_b - PC_i\right] \quad (1)$$

where

NR_{ai} is net return per acre for the i^{th} stocking rate,

SR_i is the i^{th} stocking rate in steers per acre,

P_N is the price per cwt. of 500-800 lb. choice steers during the first week of November,

W_{gi} is the gain per steer in cwt. from March to November for the i^{th} stocking rate,

$P_{N\text{-}M}$ is the margin between buying and selling price in dollars per cwt.,

W_b is the buying weight per steer in cwt., and

PC_i is production cost (other than purchase cost) per steer for the i^{th} stocking rate.

The net return per steer, the term in brackets in equation (1), is multiplied by steers per acre or stocking rate to obtain net return per acre. The last term, production costs per steer, varies by stocking rate due to differences in minerals required, veterinary expense, and costs of maintaining the Bermuda grass pasture.

Assuming the purchase weight per stocker (W_b), as well as the production costs per head, for a given stocking rate (PC_i) are constants, the expected return per acre for the i^{th} stocking rate is:

$$\overline{NR}_{ai} = SR_i\left[\overline{P}_N(\overline{W}_{gi}) + \overline{P}_{N\text{-}M}(W_b) - PC_i\right] \quad (2)$$

where the bar (–) over a term indicates an expected or average value.

The variance of net return per acre can be derived

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from the following equation:

$$\text{Var}(NR_{ai}) = E(NR_{ai} - \overline{NR}_{ai})^2 \qquad (3)$$

Substituting (1) and (2) into (3), expanding the square and gathering terms results in:

$$\text{Var}(NR_{ai}) = SR_i^2\, E[(P_N-\overline{P}_N)^2 (W_{gi}-\overline{W}_{gi})^2 + \overline{P}_N^{\,2}(W_{gi}-\overline{W}_{gi})^2 + \overline{W}_{gi}^{\,2}(P_N-\overline{P}_N)^2 + W_b^{\,2}(P_{N\text{-}M}-\overline{P}_{N\text{-}M})^2 + 2W_b(P_N \times W_{gi} - \overline{P}_N \times \overline{W}_{gi})(P_{N\text{-}M}-\overline{P}_{N\text{-}M})]. \qquad (4)$$

If the form of the net return distribution can be assumed to be normal or lognormal, the expected net return per acre and its variance (estimated from equations 2 and 4) can be used to estimate the probability of obtaining a specified level of returns for each stocking rate. The following section presents information on the net returns distribution for three alternative stocking rates at Heavener, Oklahoma, assuming the producer uses no estimate of potential beef production per acre or prices. Then, a prediction of the pounds of beef produced per acre is developed and the net returns distribution is presented for producers using this information in selecting the stocking rate. The final section develops a price prediction model, and illustrates how both the production and price predictors may be used in decision making.

SELECTING THE ANNUAL STOCKING RATE

Evaluation of the Bermuda grass production, and its response to weather conditions at Heavener, indicates it is reasonable to consider three alternative stocking rates as the actions available to the farmer. The light stocking rate selected is .825 head per acre, while the average and heavy stocking rates are 1.0 and 1.325 head per acre, respectively. Cost and return estimates in the analysis are based on a unit 40 acres in size. This is large enough to take advantage of certain economies in transporting livestock, but small enough to represent typical livestock producers in the area. The average production costs for each of the three stocking rates are shown in Table 1.

The "No Data" Solution

Perhaps the simplest method of analysis assumes that the producer has no information indicating the probable production or price level for the coming year. This "no data" analysis utilizes only a prior distribution of uncontrollable variables. If one is willing to accept the production and price variation

TABLE 1. ESTIMATED COSTS AND RETURNS FROM SPRING BUY-FALL SELL STOCKER OPERATION (40 ACRES)

Number of Steers	33	40	53
PRODUCTION COSTS			
Weight per head (lbs) 500			
Price $/cwt. $25.50			
Cost of steers	$4,207.50	$5,100.00	$6,757.50
Mineral, vet., and med.[a]	29.04	35.20	46.64
Buying and trucking[b]	105.00	121.00	143.00
Trucking and marketing[c]	211.28	246.00	303.31
Interest on capital	128.33	155.74	206.11
Labor[a]	31.68	38.40	50.88
Miscellaneous[a]	10.56	12.80	16.96
Costs of Bermuda grass	429.20	429.20	429.20
TOTAL SPECIFIED COSTS	$5,152.59	$6,138.34	$7,953.60

[a]Data interpreted from [5].

[b]Includes $1 per head charge for contract buying.

[c]Includes charges per head of $.12 for delivery, $1 for yardage, $1 for feed, $.005 for insurance, $.25 for auction, and $1.30 for commission.

experienced in the past as the estimate of future variation, then historic data can be used to estimate the appropriate net return distributions. Data for the 14-year period, 1950 to 1963, were used to estimate the mean, variance and covariance terms shown in Table 2.

The average net return per acre, and its variance, could be calculated using the values in Table 2 with equations (2) and (4) above. Instead, the net returns were calculated using (1) and the actual price and gain values that occurred for each stocking rate during each of the 14 years. This resulted in a series of 14 net return values for each stocking rate that were used to test the form of the distribution.[1] The Kolmogorov-Smirnov one-sample test was used to test if the net returns distribution for each stocking rate differed significantly from a normal distribution. The distributions of net returns for the light, average and heavy stocking rates do not differ from the normal distribution at the .15, .10 and .15 levels, respectively. The expected value, the variance, and the assumption of a normal distribution were used to develop the distribution of net returns shown in Table 3.

The expected net return for the average stocking rate (1.0 head per acre) is $20.27 per acre. The figures presented are net returns at points on the probability distribution for the stocking rate indicated. Thus, the first figure under the 1.0 stocking rate indicates the probability of receiving returns less than –$22.99 per acre is .05. Other figures in the table are interpreted in a similar manner.

A producer selecting the strategy based on maximum expected returns would select the .825 stocking rate. Producers wishing to maximize expected returns subject to a small probability of low or negative returns would also select the .825 stocking rate. Only those producers willing to sacrifice some expected income for the opportunity of larger gains in favorable years would select the 1.0 or 1.325 stocking rate. Even though the dollar values have not been converted to utility values, it appears the low stocking rate would be the optimal strategy to follow in each year for producers that are not using either production or price predictors.

The Solution with a Production Predictor

Producers have some information indicating the probable level of forage production which can be used to predict potential beef production. The weather variables used to predict beef production include precipitation, maximum average temperatures and wind for the November-March period. The amount of precipitation for this period largely determines the amount of moisture available at the beginning of the growing season. Average maximum temperature is directly associated with Bermuda grass growth in the early spring. The wind in the November-March period may not be much of a preconditioning variable, but it is closely related to the amount of wind that occurs during the growing season—correlation between the two variables is .63.

The equation used to predict beef production is:

$$B_p = -143.901 - \underset{(.848)}{4.174}\, Y_1 + \underset{(2.211)}{7.314}\, Y_2 + \underset{(1.701)}{15.297}\, Y_3 \qquad (5)$$

where the values in parentheses are the standard errors,

B_p is the estimated pounds of gain per acre for the April-October grazing period,

Y_1 is the miles of wind for the November-March period,

Y_2 is the monthly average maximum temperature for the November-March period, and,

Y_3 is the number of inches of precipitation for the November-March period.

The estimate of beef production is made at the time of the stocking-rate decision. The observations are placed into one of three intervals, less than 210, from 210 to 300, and greater than 300, respectively. These three levels were selected because they have approximately equal probability of occurrence and because the grazing produced for the three levels appears to satisfy the needs of the three stocking rates, respectively.

Deviations between the actual production in the 14-year period and the amount predicted by (5) were used to estimate the variance of weight gain and the covariance terms presented in column (e) of Table 2. These coefficients were substituted into equations (2) and (4) to estimate the expected net return and variance for each of the three stocking rates for each

[1]The net returns equation, and the estimates of the coefficients in Table 2, could have been used with a Monte Carlo simulation approach to generate a net returns distribution. The K-S test could then be used to determine if the resulting distribution differed significantly from the normal. However, the procedure used is preferable on two counts. First it is less involved computationally. Second, it does not require one to assume a distribution for the price and gain variables in the Monte Carlo simulation model.

TABLE 2. MEANS, VARIANCES AND COVARIANCES FOR THE ANALYSIS

(a) Variable	(b) Unit	(c) Mean	(d) No Data Variance	(e) Production Prediction Variance	(f) Production and Price Prediction Variance
Price in November	$/cwt.	23.80	22.670	22.670	2.372
March-November Price Margin	$/cwt.	−1.70	12.137	12.137	2.372
Weight Gain w/Light Stocking Rate	cwt./head	1.9415	0.0874		
Weight Gain w/Average Stocking Rate	cwt./head	2.3600	0.0936		
Weight Gain w/Heavy Stocking Rate	cwt./head	2.7128	0.1664		
			Covariance	Covariance	Covariance
March-November Margin and Value added with Light Stocking Rate	$/cwt.		18.674	−16.16	4.192
March-November Margin and Value added with Average Stocking Rate	$/cwt.		21.150	20.10	8.406
March-November Margin and Value added with Heavy Stocking Rate	$/cwt.		18.536	29.40	7.427

		Mean Gain by Stocking Rate			Variance	Variance
		.825	1.0	1.325		
Weight Gain w/Prediction of Low Beef Production	cwt./head	2.40	2.00	1.50	.0851	.0851
Weight Gain w/Prediction of Average Beef Production	cwt./head	2.70	2.40	1.80	.0283	.0283
Weight Gain w/Prediction of High Beef Production	cwt./head	3.00	2.70	2.40	.0198	.0198

TABLE 3. NET RETURNS IN DOLLARS PER ACRE BY STOCKING RATE, USING NO PREDICTIORS

Probability of Obtaining Smaller Returns	Stocking Rate in Head Per Acre		
	.825	1.0	1.325
.05	−14.99	−22.99	−38.32
.10	− 7.09	−13.50	−25.96
.20	2.56	− 1.89	−10.84
.50	20.99	20.27	18.01
.80	39.42	42.43	46.86
.90	49.07	54.04	61.98
.95	56.97	63.53	74.34
Expected Value	20.99	20.27	18.01

of three predicted levels of beef production per acre. These nine pairs of expected value-variance estimates and the assumption of a normal distribution were used to develop Table 4.

A producer selecting the stocking rate strategy, based on maximum expected net returns per acre, would select the light stocking rate when the predicted level of beef production was below 210, the average stocking rate for predicted beef production from 210 to 300, and the heavy stocking rate for predicted beef production of more than 300 pounds per acre. The reader should notice that producers willing to sacrifice some expected income for a smaller probability of a loss may prefer the light to the average stocking rate for predicted beef production from 210 to 300 pounds and either the light or medium to the heavy rate when predicted beef production is over 300 pounds. It is also conceivable that some producers may prefer the opportunity for larger gains with the heavy stocking rate when predicted beef production is between 210 and 300 pounds. However, the average and heavy stocking rates are clearly inferior to the light rate when the predicted production level is below 210.

The Solution with a Price and a Production Predictor

The high variation in returns per acre with the relatively accurate predictor of production suggests that a price forecast model may also be useful in selecting the stocking rate. A multiple linear regression equation utilizing three variables readily observable by producers is used as the price forecast model. The explanatory variables included are the price of steers during the last week of March to reflect yearly price conditions, the inventory of calves in Oklahoma on January 1 to reflect supply conditions, and the previous year's U.S. per capita disposable income to indicate demand conditions. The equation was fitted to two cycles of cattle data (beginning in 1950).[2]

The prediction equation, using the above variables, is:

$$P_N = 1.244 + \underset{(0.096)}{0.422\, X_1} - \underset{(0.482)}{0.846\, X_2} + \underset{(0.003)}{0.010\, X_3} \qquad (6)$$

where the values in the parentheses are the standard errors,

P_N is the price per cwt. of 500-800 lb. choice stocker-feeder steers in Oklahoma City the first week in November,

X_1 is the price per cwt. of 500-800 lb. choice stocker-feeder steers in Oklahoma City the last week in March,

X_2 is the number of beef calves in Oklahoma in 100,000 head on January 1, and,

[2]A trend variable accounting for the affect of the Korean Conflict was also included in the equation fitted. The value of the coefficient was 8.944 and was significant at the .01 level. Although this variable was instrumental in determining the equation, it is not necessary to include it in the forecast model.

TABLE 4. NET RETURNS IN DOLLARS PER ACRE BY STOCKING RATE AND PREDICTED LEVEL OF BEEF PRODUCTION

Probability of Obtaining Smaller Returns	Stocking Rate in Head Per Acre		
	.825	1.0	1.325
	Predicted Beef Production of Less than 210 Pounds per Acre		
.05	– 7.81	–14.43	–27.73
.10	– 2.48	– 8.38	–20.29
.20	4.04	– 0.98	–11.21
.50	16.49	13.14	6.14
.80	28.94	27.26	23.46
.90	35.46	34.66	32.57
.95	40.79	40.71	40.01
Expected Value	16.49	13.14	6.14
	Predicted Beef Production of 210-300 Pounds per Acre		
.05	–13.06	–19.21	–37.39
.10	– 5.28	–10.02	–25.39
.20	4.23	1.21	–11.54
.50	22.38	22.66	15.60
.80	40.53	44.11	42.74
.90	50.05	55.34	56.96
.95	57.82	64.53	68.59
Expected Value	22.38	22.66	15.60
	Predicted Beef Production of More Than 300 Pounds per Acre		
.05	–12.62	–18.69	–29.74
.10	3.64	8.05	–15.63
.20	7.33	4.96	1.61
.50	28.27	29.80	34.52
.80	49.21	54.64	67.43
.90	60.18	67.65	84.67
.95	69.16	78.29	98.78
Expected Value	28.27	29.80	34.52

X_3 is the per capita disposable income in the United States in the previous year.

Coefficients of all variables were significant at the .05 level. Ninety percent of the variation in the data is explained by this equation. The deviations between November prices predicted with (6) and the actual November prices occurring were used to estimate the variance of November prices, the variance of the March-November price margin and the covariance terms presented in column (f) of Table 2. These coefficients were used with equations (2) and (4) to develop the expected return and variance estimates used to construct Table 5.

The values in Table 5 show the distribution of net returns per acre for each of the three stocking rates for alternative predicted November prices and alternative margins *when the predicted beef production per acre is below 210 pounds.* The comparable tables, constructed for other predicted levels of beef production, are not presented due to the space limitation. A

TABLE 5. NET RETURNS IN DOLLARS PER ACRE BY STOCKING RATE, UNDER ALTERNATIVE PRICES AND MARGINS, WHEN PREDICTED BEEF PRODUCTION IS BELOW 210 LBS.

Probability of Obtaining Smaller Returns	Predicted November Price 28.00			30.00			32.00			34.00		
	Predicted March-November Price Margin											
	–4.00	–2.00	0.00	–4.00	–2.00	0.00	–4.00	–2.00	0.00	–4.00	–2.00	0.00
	Stocking Rate of .825 Head Per Acre											
.05	– 2.93	5.32	13.57	0.54	8.79	17.04	4.79	12.24	20.49	7.42	15.67	23.92
.10	1.07	9.32	17.57	4.65	12.90	21.15	8.21	16.46	24.71	11.76	20.01	28.26
.20	5.97	14.22	22.47	9.68	17.93	26.18	13.38	21.63	29.88	17.07	25.32	33.57
.50	15.31	23.56	31.81	19.27	27.52	35.77	23.23	31.48	39.73	27.19	35.44	43.69
.80	24.65	32.90	41.15	28.86	37.11	45.36	33.08	41.33	49.58	37.31	45.56	53.81
.90	29.55	37.80	46.05	33.89	42.14	50.39	38.25	46.50	54.75	42.62	50.87	59.12
.95	33.55	41.80	50.05	38.00	46.25	54.50	42.47	50.72	58.97	46.96	55.21	63.46
Expected Value	15.31	23.56	31.81	19.27	27.52	35.77	23.23	31.48	39.73	27.19	35.44	43.69
	Stocking Rate of 1.0 Head per Acre											
.05	–11.82	–1.82	8.18	–13.34	–3.34	6.66	–5.04	4.96	14.96	–1.69	8.31	18.31
.10	– 7.02	2.98	12.98	– 7.33	2.67	12.67	0.03	10.03	20.03	3.52	13.52	23.52
.20	– 1.16	8.84	18.84	0.02	10.02	20.02	6.22	16.22	26.22	9.89	19.89	29.89
.50	10.04	20.04	30.04	14.04	24.04	34.04	18.04	28.04	38.04	22.04	32.04	42.04
.80	21.64	31.64	41.64	28.06	38.06	48.06	29.86	39.86	49.86	34.19	44.19	54.19
.90	27.10	37.10	47.10	35.41	45.41	55.41	36.05	46.05	56.05	40.56	50.56	60.56
.95	31.90	41.90	51.90	41.42	51.42	61.42	41.12	51.12	61.12	45.77	55.77	65.77
Expected Value	10.04	20.04	30.04	14.04	24.04	34.04	18.04	28.04	38.04	22.04	32.04	42.04
	Stocking Rate of 1.325 Head per Acre											
.05	–22.36	–9.11	4.14	–18.99	–5.74	7.51	–15.65	–2.40	10.85	–12.32	0.39	14.18
.10	–17.62	–4.37	8.88	–14.11	–0.86	12.39	–10.63	2.62	15.87	– 7.16	6.09	19.34
.20	–11.82	–1.43	14.68	– 3.53	9.72	22.97	– 4.51	8.74	21.99	– 0.86	12.39	25.64
.50	– 0.76	12.49	25.74	3.22	16.47	29.72	7.19	20.44	33.69	11.17	29.42	37.67
.80	10.30	23.55	36.80	14.59	27.84	41.09	18.89	32.14	45.39	23.20	36.45	49.70
.90	16.10	29.35	42.60	20.55	33.80	47.05	25.01	38.26	51.51	29.50	42.75	56.00
.95	20.84	34.09	47.34	25.43	38.68	51.93	30.03	43.28	56.33	34.66	47.91	61.16
Expected Value	– 0.76	12.49	25.74	3.22	16.47	29.72	7.19	20.44	33.69	11.17	29.42	37.67

producer using this information would use the two prediction equations to estimate potential beef production per acre, the November selling price, and the March-November price margin. Assume the three predicted values are below 210 pounds, $28 and –$2, respectively. This producer would compare the distributions in Table 5 for the three stocking rates with a price of $28. and a margin of –$2. The expected net returns per acre for the light, medium, and heavy stocking rate are $23.56, $20.04 and $12.49, respectively. Considering the expected value, and the distributions, indicates the light stocking rate would be chosen. Although the light stocking rate is optimum for all price and margin conditions presented in Table 5, the data indicate that the advantage of the light stocking rate decreases as the predicted price and the predicted margin increase. This indicates that the medium and heavy stocking rate will become optimal as the table is expanded to include higher price and margin levels.

CONCLUSIONS

A more complete analysis of this stocking rate problem should consider alternative fertilization rates, alternative starting weights for the cattle, and a larger number of (predicted) beef production categories per acre. The methodology is adequate to consider these additional alternatives when the experimental data become available.

The application suggests that farm management research workers can provide farmers an opportunity to apply statistical decision theory to a variety of enterprise problems. Researchers must develop relatively accurate prediction models for the important uncontrollable variables and the relevant table(s) of net return distributions. Such tables would provide the information farmers need to make enterprise decisions based on their preference function.

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