Journal of Agricultural and Resource Economics 46(3):464–478 ISSN: 1068-5502 (Print); 2327-8285 (Online)
Copyright 2021 the authors doi: 10.22004/ag.econ.304776

Single-Commodity versus Joint Hedging in Cattle Feeding Cycle: Is Joint Hedging Always Essential?

Chengcheng J. Fei, Dmitry V. Vedenov, Reid B. Stevens, and David P. Anderson

The paper analyzes the effectiveness of joint- versus single-commodity hedging for inputs and outputs of the cattle feeding cycle using the second-order lower partial moment (LPM_2) as the risk measure. Joint hedging always results in higher hedging effectiveness than the single-commodity hedging, but the difference is often small. The difference in performance is found to be explained by the commodity price dependence measures (Kendall's τ). Ranges of τ leading to substantial improvement in risk reduction due to joint hedging are identified. The joint hedging strategy is worth implementing when the observed price dependence measures fall within the identified ranges.

Key words: copulas, downside risk, multicommodity hedging, risk management

Introduction

Cattle feedlot operators are subject to price risk on both the input (feedstuff) and the output (cattle) sides. Since the feeding cycle takes up to 4 months, the unfavorable price movements between the purchase of the feed and the sale of the finished livestock are of particular concern. Hedging with futures is commonly used to reduce this risk.

The main task in setting up a hedging strategy with futures is to determine the hedge ratios that minimize a measure of risk of concern to the hedger. The hedge ratios can be determined either by hedging each commodity individually (single-commodity hedging) or by hedging both commodities simultaneously (joint hedging). The latter accounts for the cross-dependence between the input and output prices but is much more complex to implement than the single-commodity hedging.

The difference between the single-commodity hedging and joint hedging has been examined under the mean-variance (MV) criterion in several multicommodity hedging cases (e.g., Peterson and Leuthold, 1987; Fackler and McNew, 1993). The analysis finds that the optimal hedge ratios implied by the joint hedging strategy are usually smaller than those suggested by single-commodity hedging.

In recent years, minimization of downside risk measures, such as the lower partial moment (LPM_2), has been increasingly used as a criterion in determining the optimal hedge ratios. These measures may better capture the objective of the hedger who are more concerned with losses than gains (Mattos, Garcia, and Nelson, 2008; Power and Vedenov, 2010). In particular, Power and Vedenov (2010) apply the LPM_2 criterion to the cattlemen's joint hedging problem and find that the LPM_2 hedge ratios are typically lower than the MV hedge ratios. However, their analysis is

---

Chengcheng J. Fei (corresponding author) is a post doc research associate, Dmitry V. Vedenov is an associate professor, Reid B. Stevens is a former assistant professor, and David P. Anderson is a professor, all in the Department of Agricultural Economics at Texas A&M University.

Fei acknowledges partial support from the National Science Foundation under Grant Addressing Decision Support for Water Stressed FEW Nexus Decisions Numbered 1739977. Vedenov acknowledges partial support from Hatch project TEX0-2-9258.

Review coordinated by Darren Hudson.

focused only on joint hedging and does not compare the performance of single-commodity hedging versus joint hedging. Given that joint hedging accounts for the cross-dependence among commodity prices, it should always perform better than the single-commodity hedging. However, the LPM_2 minimization is a complex methodology that may be costly to implement.

We investigate how much improvement could be obtained by joint hedging in feedlot operations, what determines the degree of improvement, and under what conditions it is worth implementing joint hedging. To address these questions, we build upon Power and Vedenov's (2010) theoretical framework but use a longer dataset (2001–2016), which incorporates periods of recent volatility in both live cattle and corn prices. We evaluate the hedging effectiveness of the joint and single-commodity hedging strategies in the live cattle and corn markets under the LPM_2 criterion and measure the difference in hedging performance between the two strategies over the period from January 2001 through December 2016. To the best of our knowledge, this is the first attempt to compare the performance of joint and single-commodity hedging strategies using the downside risk criterion over such an extensive timeframe.

The primary contributions of the paper are in (i) comparing the performance of joint and single-commodity hedging strategies using the LPM_2 criterion; (ii) attempting to identify the factors that lead to differences in hedging effectiveness between joint and single-commodity hedging; and (iii) finding the conditions under which implementation of joint hedging leads to sufficiently improvement in risk reduction to justify the computational effort.

Literature Review

Cattle feedlot operation is a multicommodity activity, with risk inherent both in the commodities' own prices and in the interaction between prices of different commodities. The benefits of multicommodity hedging strategies accounting for this interaction have been reported in the literature (e.g., Elam and Donnell, 1992; Haigh and Holt, 2002).

In terms of risk criteria, earlier studies primarily use the traditional mean-variance model (e.g., Peterson and Leuthold, 1987; Noussinov and Leuthold, 1999; Collins, 2000). However, variance minimization equally penalizes upside and downside deviations from the mean, even though these are not equally undesirable by commodity hedgers. One-sided risk measures may better reflect real risk objective of the commodity hedgers than the mean-variance criterion (Lien and Tse, 2002). In particular, the lower partial moment (LPM) family (Fishburn, 1977) has been increasingly used in recent hedging literature.

The nth-order lower partial moment of a random variable x relative to a desired threshold $\bar{x}$ is defined as

$$LPM_n(x) = \int_{-\infty}^{\bar{x}} [\bar{x} - x]^n dF(x), \qquad (1)$$

where $F(x)$ is the cumulative distribution function of the random variable x. LPMn minimization is consistent with $(n+1)$th-order stochastic dominance (Ingersoll, 1987) and is suitable for representation of a wide class of utility functions in the expected utility maximization model (Levy, 2015). Experimental observations also suggest that the LPM family is more suitable for representing the utility of agricultural producers and commodity hedgers (Unser, 2000).

Empirically, the second-order LPM criterion ($n = 2$) has been extensively used in the commodity hedging analysis (Eftekhari, 1998; Demirer and Lien, 2003; Cotter and Hanly, 2006; Mattos, Garcia, and Nelson, 2008; Power and Vedenov, 2010; Liu, Vedenov, and Power, 2017). The expected return without hedging is commonly selected as the threshold of the second-order LPM criterion (e.g., Demirer, Lien, and Shaffer, 2005; Power and Vedenov, 2010; Liu, Vedenov, and Power, 2017).

Power and Vedenov (2010) introduce the LPM_2 criterion to joint hedging and compare the optimal joint hedge ratios under the LPM_2 and the MV criteria in a study of a typical cattle feedlot operation. They find that the LPM_2 hedge ratios can be much smaller than the MV hedge ratios, and even observe instances of negative optimal hedge ratios for corn (so-called "Texas hedge"). Power and Vedenov also suggest that the LPM_2 hedge ratios may be driven by a more general measure of dependence among the commodity prices, rather than the traditional linear correlation.

Liu, Vedenov, and Power (2017) consider the multicommodity hedging problem of a crude oil refinery and indicate that the hedging effectiveness under the LPM_2 criterion is higher than that under the MV criterion. However, both Power and Vedenov Power and Vedenov (2010) and Liu, Vedenov, and Power only consider joint hedging under different criteria rather than compare performance of single-commodity and joint hedging. Further, the most relevant work, Power and Vedenov, only tests the performance of joint hedging over a single time period.

Problem Setup and Methodology

Problem Setup

Our model is based on the cattle operating cycle framework discussed in detail in Power and Vedenov (2010) and a white paper from CME Group (2014). At the beginning of each operating cycle, the feedlot operator purchases feeder cattle (around 800 lb/head) and sufficient feed from the spot market. After 17 weeks of feeding, the operator sells the mature cattle (around 1,200 lb/head) on the spot market. Corn is the major feed component with 50 bushels of corn consumed, on average, by one head of cattle over the feeding period. Though soybean meal is a commonly used protein source, it is only a small part of the input and is usually not included in the hedging models of cattle feedlot (Power and Vedenov, 2010; CME Group, 2014).[1]

Given the limitations of storage capacity and the uncertainty in the spot market of corn, without loss of generality, we assume that the feedlot operator purchases the first half of the required corn (25 bu/head) at the beginning of the feeding period and purchases the second half of the required corn in the middle of feeding period (at the end of the 8th week). We also assume that a representative feedlot operator raises100 heads of cattle.

The operator hedges the future purchase of corn and sale of live cattle in the futures market at the beginning of the operating period. The operator offsets the corn contract when corn is purchased on the spot market and closes the live cattle futures contract at the end of the feeding period. We assume the feedlot operators choose hedge ratios to minimize the downside risk they face.

The net cash flow from corn with hedging is

$$CF^C\left(h^C\right) = -p_0^C Q_0^C - p_1^C Q_1^C + h^C Q_1^C\left(f_1^C - f_0^C\right), \tag{2}$$

where h^C is the hedge ratio of corn; $\{Q_0^C, Q_1^C\}$ are the quantities of corn purchased at the beginning of the feeding period and the end of the 8th week, respectively; $\{p_0^C, f_0^C\}$ are the initial (observed) spot and futures prices; and $\{p_1^C, f_1^C\}$ are the spot and futures prices of corn at the end of the 8th week, respectively. Note that the term $-p_0^C Q_0^C$ is known and has no effect on the hedging decision.

The net cash flow from live cattle with hedging is

$$CF^{LC}\left(h^{LC}\right) = +p_1^{LC} Q^{LC} + h^{LC} Q^{LC}\left(f_0^{LC} - f_1^{LC}\right), \tag{3}$$

where h^{LC} is the hedge ratio of live cattle, Q^{LC} is the quantity of live cattle sold in the 17th week, f_0^{LC} is the initial futures price observed, and $\{p_1^{LC}, f_1^{LC}\}$ are the spot and futures prices of live cattle at the end of the 17th week, respectively.

[1] Soybean meal and other ingredients such as dried distillers' grains (DDGs) are important inputs in the cattle feeding operation. However, the proportion of these components in the total cost is quite small and generally has little impact on the net cash flow.

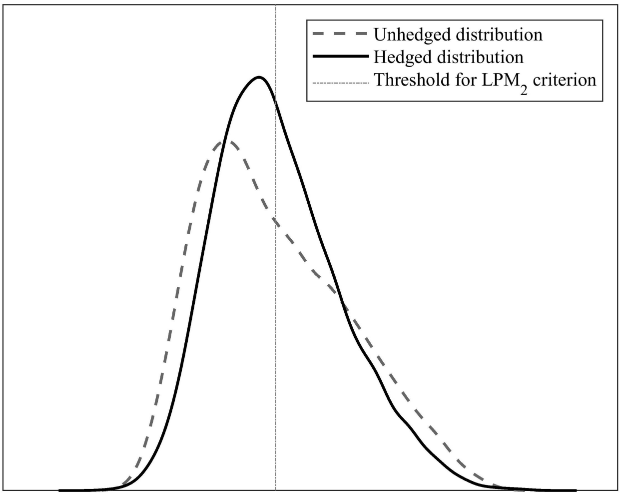

Figure 1. Effect of Hedging on the Distribution of Net Cash Flows under the LPM$_2$ Criterion

The total net cash flow from the feedlot operation is then

$$\begin{aligned} CF\left(h^{C},\, h^{LC}\right) &= CF^{LC}\left(h^{LC}\right) + CF^{C}\left(h^{C}\right) \\ &= -p_0^C Q_0^C - p_1^C Q_1^C + p_1^{LC} Q^{LC} + h^C Q_1^C\left(f_1^C - f_0^C\right) + h^{LC} Q^{LC}\left(f_0^{LC} - f_1^{LC}\right). \end{aligned} \tag{4}$$

We assume that the feedlot operator attempts to minimize the risk of shortfall relative to the target level of net cash flow and therefore chooses the hedge ratios for corn, h^{C*}, and live cattle, h^{LC*}, based on minimization of the second-order lower partial moment (LPM$_2$).

The LPM$_2$ of a random variable x relative to a desired threshold $\bar{x}$ is defined as

$$LPM_2(x) = \int_{-\infty}^{\bar{x}} [\bar{x} - x]^2\, dF(x), \tag{5}$$

where $F(x)$ is the cumulative distribution function of x. In our case, x is an appropriate cash flow (equations 2–4), and the threshold $\bar{x}$ is set to its expected value without hedging. Intuitively, the LPM$_2$ criterion is a measure of the area under the probability density function (pdf) of x (cash flow) to the left of the threshold. Hedging changes the shape of the distribution and possibly shifts it to the right. The optimal hedge ratios are then the ones that result in the highest shift of the distribution mass to the right of the threshold (Figure 1).

For the single-commodity hedging, the optimal hedge ratios for corn, h^{C*}, and live cattle, h^{LC*}, are determined independently by minimizing the LPM$_2$ criteria over the cash flow from corn (equation 2) and the cash flow from live cattle (equation 3) separately. Specifically, we determine the optimal single-commodity hedge ratio of corn (h^{C*}) as

$$h^{C*} = \arg\min_{h^C} LPM_2\left(\overline{CF}^C,\, h^C\right) = \arg\min_{h^C} \int_{-\infty}^{\overline{CF}^C} \left[\overline{CF}^C - CF^C\left(h^C\right)\right]^2 dF\left(CF^C\left(h^C\right)\right) \tag{6}$$

and the optimal single-commodity hedge ratio of live cattle (h^{LC*}) as

$$(7)\quad \begin{aligned} h^{LC*} &= \arg\min_{h^{LC}} LPM_2\left(\overline{CF}^{LC},\ h^{LC}\right) \\ &= \arg\min_{h^{LC}} \int_{-\infty}^{\overline{CF}^{LC}} \left[\overline{CF}^{LC} - CF^{LC}\left(h^{LC}\right)\right]^2 dF\left(CF^{LC}\left(h^{LC}\right)\right), \end{aligned}$$

where $F\left(CF^C\left(h^C\right)\right)$ and $F\left(CF^{LC}\left(h^{LC}\right)\right)$ are the cumulative distribution functions of cash flows from corn and live cattle, respectively, and $\overline{CF}^C = E\left(CF^C(0)\right)$ and $\overline{CF}^{LC} = E\left(CF^{LC}(0)\right)$ are the respective reference levels set equal to the corresponding expected net cash flow without hedging.

For joint hedging, the feedlot operator chooses both optimal hedge ratios simultaneously by minimizing the LPM_2 criterion over the net cash flow from the entire operation (equation 4). Specifically, we determine the vector of optimal hedge ratios, $\vec{\boldsymbol{h}}^* = \begin{bmatrix} h^{C*} \\ h^{LC*} \end{bmatrix}$, as

$$(8)\quad \vec{\boldsymbol{h}}^* = \begin{bmatrix} h^{C*} \\ h^{LC*} \end{bmatrix} = \arg\min_{\vec{\boldsymbol{h}}} LPM_2\left(\overline{\boldsymbol{CF}},\ \vec{\boldsymbol{h}}\right) = \arg\min_{\vec{\boldsymbol{h}}} \int_{-\infty}^{\overline{CF}} \left[\overline{\boldsymbol{CF}} - CF\left(\vec{\boldsymbol{h}}\right)\right]^2 dF\left(CF\left(\vec{\boldsymbol{h}}\right)\right),$$

where $\overline{\boldsymbol{CF}} = E\left[CF\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)\right]$ is the reference level equal to the expected total net cash flow without hedging and $F\left(CF\left(\vec{\boldsymbol{h}}\right)\right)$ is the cumulative distribution function of the total net cash, which implicitly accounts for cross-dependence between the corn and live cattle prices.

Under our assumptions, the hedge ratios are not restricted to be positive in either case. An operator that decides not to hedge commodity j would have $h^j = 0$, while $h^j < 0$ would imply speculation in corresponding commodity.[2] We also assume that $Q^{LC} = 120{,}000$ lb and $Q^C = 2{,}500$ lb for the feedlot with 100 head.

Calculating Optimal Hedge Ratios

Since there is no closed-form solution to the LPM_2 minimization problem, the optimal hedge ratios can only be calculated numerically. Distributions of cash flows are constructed from the joint distribution of corresponding spot and futures prices, which are derived from the historical data using a copula approach. We then use the Monte Carlo method to generate draws from the constructed distributions. Finally, we use the Nelder–Mead derivative-free method (Miranda and Fackler, 2002) to minimize the LPM_2 criterion numerically.

We employ the rolling window approach to efficiently use the available data. The weekly optimal hedge ratios are computed using a 100-week wide rolling window. Specifically, for the tth week ($t > 100$), the spot and future prices observed from week $t100$ to week $t-1$ are treated as the historical data, and the observation in the tth week is treated as the initial price. For example, the first window uses observations 1, 2, ..., 100 as historical data and observation 101 as the initial price; the second window uses observations 2, 3, ..., 101 as historical data and observation 102 as the initial price; and so on. With weekly observations, a 100-observation window means that the hedgers use the trading data from roughly the past 2 years to make the hedging decisions.

To implement the Monte Carlo procedure, we first construct the series of spot and futures shock series for corn and live cattle, $\left\{\varepsilon_t^C, \eta_t^C,\ \varepsilon_t^{LC},\ \eta_t^{LC}\right\}$, using the log-differences

[2] Note that, in practical application, speculative positions taken by a hedger may involve unfavorable regulatory and financial implications. For computational reasons only, we do not impose constraints on the sign of hedge ratios.

of historical prices$\{p_t^C, f_t^C, p_t^{LC}, f_t^{LC}\}$, where $\varepsilon_t^C = \ln p_t^C - \ln p_{t-8}^C$, $\eta_t^C = \ln f_t^C - \ln f_{t-8}^C$, $\varepsilon_t^{LC} = \ln p_t^{LC} - \ln p_{t-17}^{LC}$, and $\eta_t^{LC} = \ln f_t^{LC} - \ln f_{t-17}^{LC}$. The price shock series are then used to estimate the corresponding marginal probability density functions using the kernel density approach (Wand and Jones, 1994) as well as the copula densities of the joint distributions using the mirror image kernel approach (Charpentier, Fermanian, and Scaillet, 2007). For joint hedging, we estimate the four-dimensional copula density, $c(\cdot,\cdot,\cdot,\cdot)$, using all series of shocks, $\{\varepsilon_t^C, \eta_t^C, \varepsilon_t^{LC}, \eta_t^{LC}\}$, while for the single-commodity hedging, the copula densities of corn shocks, $c^C(\cdot,\cdot)$, and live cattle shocks, $c^{LC}(\cdot,\cdot)$, are estimated separately.

The joint distributions of shocks are then constructed by combining the marginal probability density functions and the corresponding copula densities. For example, the joint distribution of all four shocks could be written as

$$g\left(\varepsilon_t^C, \eta_t^C, \varepsilon_t^{LC}, \eta_t^{LC}\right) = g_1\left(\varepsilon_t^C\right) g_2\left(\eta_t^C\right) g_3\left(\varepsilon_t^{LC}\right) g_4\left(\eta_t^{LC}\right) \times c\left(G_1\left(\varepsilon_t^C\right), G_2\left(\eta_t^C\right), G_3\left(\varepsilon_t^{LC}\right), G_4\left(\eta_t^{LC}\right)\right), \quad (9)$$

where $g_i(\cdot)$ are the marginal probability density functions, $G_i(\cdot)$ are the marginal cumulative distribution function corresponding to g_i, and $c(\cdot)$ is the corresponding copula density function.

We then generate 10,000 Monte Carlo draws, $\{u_1^i, u_2^i, u_3^i, u_4^i\}_{i=1}^{10,000}$, from the joint distribution, $g\left(\varepsilon_t^C, \eta_t^C, \varepsilon_t^{LC}, \eta_t^{LC}\right)$, following Cherubini, Luciano, and Vecchiato (2004). Finally, the realizations of spot and futures prices on the closing dates are generated by multiplying the exponentiation of the simulated shocks by the initial price (first observation outside of the window) (i.e., $p_{1,i}^C = e^{u_1^i} p_{t+1}^C$, where p_{t+1}^C is the initial corn spot price, etc.). The simulated series and the initial prices at time t are used to calculate the net cash flows in equations (2)–(4). The Nelder–Mead derivative-free method (Miranda and Fackler, 2002) is then applied to calculate the optimal hedge ratios.[3]

Hedging Effectiveness

We use the hedging effectiveness measure (Ederington, 1979) to compare the performance of different hedging strategies. Hedging effectiveness is generally defined as the percentage of risk offset by hedging relative to the risk without hedging. Specifically, we calculate hedging effectiveness of LPM_2 as

$$HE_{LPM_2} = 1 - \frac{LPM_2\left(CF\left(\boldsymbol{h}^*\right)\right)}{LPM_2\left(CF\left(0\right)\right)}, \quad (10)$$

where $\boldsymbol{h}^*$ are the optimal hedge ratios determined in equations (6)–(7) or equation (8), respectively.

Data

To implement the methodology described previously, the corn futures price from the Chicago Board of Trade (CBT, CME group) and the live cattle futures price from the Chicago Mercantile Exchange (CME) are used. The prices of Corn Number 2 Yellow in Central Illinois (underlying asset of corn futures) and the U.S. Department of Agriculture five-area weighted average price of live cattle (underlying asset of live cattle futures) are used as spot prices.[4,5]

[3] Additional details of the procedure can be found in Power and Vedenov (2010). Note that our results are robust to the choice of data window length and the number of Monte Carlo draws.

[4] The five areas of the live cattle price index are specified by the USDA and include Texas/Oklahoma/New Mexico, Kansas, Nebraska, Colorado, and Iowa/Minnesota.

[5] While Central Illinois is geographically far from the feedlot regions and may not necessarily represent the local prices at the feedlot, this is the only spot price series in Datastream that is long enough for our purposes. Thus, we essentially assume that the basis between the cash prices in Central Illinois and feedlot regions is stable over time and therefore does not affect hedging decisions.

Table 1. Summary Statistic of Corn and Live Cattle Spot and Futures Prices, 2001–2016

	Mean	Standard Deviation	Min.	Max.
Corn spot price ($/bu)	3.7454	1.7231	1.6450	8.4800
Corn futures prices ($/bu)	3.9206	1.6367	1.9575	8.3475
Live cattle spot price ($/lb)	1.0182	0.2636	0.6203	1.7138
Live cattle futures price ($/lb)	1.0206	0.2545	0.6068	1.7020

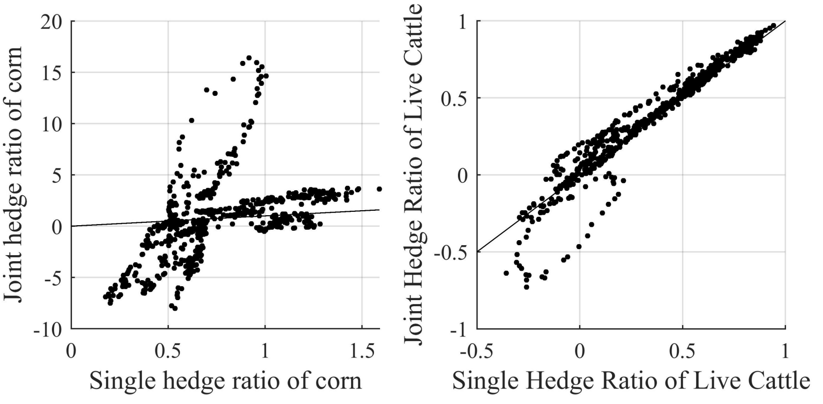

Figure 2. Corn and Live Cattle Hedge Ratios Relative to the 45° Line

All data are collected from Thomson Reuters (2018) for the period from January 2001 to December 2016. Since the live cattle spot price is only available at a weekly frequency, while daily data are available for the other commodities, we convert corn prices and live cattle futures prices to weekly frequency by using the prices on Wednesday of each week.[6] Table 1 reports the summary statistics for the raw data.

Since futures and spot prices are known to have trends and be autocorrelated (Tsay, 2005; Enders, 2008), we convert all raw price series to log-differences (or shocks), as described in the methodology section. Further, to minimize the effect of contract rollover on our results, we build the continuous time series at the level of log-differences rather than raw prices. In other words, we first construct the series of log-differences for each contract month and then, on the day when a particular month stops trading, we switch to the series of log-differences for the next available contract.

Results

Hedge Ratios

We calculated optimal hedge ratios for both single-commodity and joint hedging strategies for each 100-week window in the dataset (718 windows total). Figure 2 shows the scatterplot of single-commodity versus joint hedge ratios for corn and live cattle.

If the single-commodity and joint hedge ratios were similar, then the dots should be close to the 45° line. Most of the live cattle hedge ratios are indeed located around this line and are in the $[-1,1]$ range. However, the corn hedge ratios are scattered much more widely around the 45° line, implying substantially different corn hedge ratios for single-commodity versus joint hedging. Under a single-commodity hedging strategy, the corn hedge ratios are concentrated in the [0,1] interval

[6] Wednesday is selected because it is less likely to be holiday on which markets are closed. If the data on Wednesday are not available, the data on the next trading day are used. The results are robust to using the prices from other days of the week.

Table 2. Summary Statistics of Hedging Effectiveness (*N* = 718)

	Single-Commodity Hedging (%)	Joint Hedging (%)
Min.	−3.8	0.0
Mean	22.8	28.1
Max.	78.5	78.8
St. dev.	20.2	19.2
% positive	90.4	100.0

Notes: Negative hedging effectiveness means increase in risk, compared with no hedging.

with fairly low variability. Under the joint hedging strategy, however, the corn ratios spread across the $[-10, 20]$ range.[7]

The high variability of corn hedge ratios under the joint hedging can be explained by the fact that the total cost of corn is only 15% of the revenue of live cattle. Since the proportion of corn in the overall cash flow is small, the LPM_2 value is much less sensitive to changes in corn prices, than to changes in live cattle prices. As seen in equations (2)–(4), the hedge ratios essentially serve as scaling factors in determining the total cash flows. This leads to large changes in the values of corn hedge ratio in response to much smaller changes in the live cattle hedge ratio when the commodities are hedged jointly.

Similar to Power and Vedenov's (2010) results, 283 out of 718 observation windows result in negative corn hedge ratios when using the joint hedging strategy, which implies that speculation in corn is an optimal strategy. However, we find a larger variance of corn hedge ratios than that found in Power and Vedenov. We also find speculation in live cattle being an optimal strategy in 103 windows, unlike Power and Vedenov, who found no negative hedge ratios for live cattle for the period they considered.

Hedging Effectiveness

The hedge ratios alone do not tell the whole story. It is important to consider how well the hedge reduces risk. We use the hedging effectiveness criterion (equation 10) to measure hedging performance. Table 2 presents the summary statistics of hedging effectiveness under the single-commodity and joint hedging strategies.

We find that joint hedging reduces the total risk faced by the feedlot operator in 100% cases, while single-commodity hedging does so in only 90.4% cases. In other words, single-commodity hedging actually increases the overall downside risk in 62 out of the 718 observations. Further, the average risk-reduction of joint-hedging is higher than that of single-commodity hedging (28.1% vs. 22.8%)

We then calculate the difference between hedging effectiveness for joint and single-commodity hedging strategies (denoted as $\Delta_{HE} = HE_{joint} - HE_{single}$) for each window we simulate. This measure confirms that the joint hedging strategy performs no worse than the single-commodity hedging strategy in any window, and performs better on average, with Δ_{HE} as high as 34.7% in some cases (Table 3).

To explain the difference in hedging effectiveness between the joint and single-commodity hedging strategies, we use measures of dependence between spot and futures price shocks across commodities (e.g., Liu, Vedenov, and Power, 2017). Traditionally, Pearson correlation is used to measure the dependence. However, Pearson correlation only reflects the linear association between the variables, which is more suitable for the variance minimization criterion. For the LPM_2 criterion, we are concerned with the dependence over the entire domain of the distribution and therefore use

[7] Note that the axes in Figure 2a have different scales in order to better illustrate the range of the hedge ratios.

Table 3. Summary Statistics of Difference in Hedging Effectiveness between Joint- and Single-Commodity Strategies

	Δ_{HE}
Min.	0.00002
Mean	0.0905
Median	0.0295
Max.	0.3470

Table 4. Summary Statistics of Pairwise Kendall's τ for Spot (S) and Futures (F) prices of Corn (C) and Live Cattle (LC)

	Min.	Max.	Mean
$\tau(C_S, C_F)$	0.6488	0.9103	0.8110
$\tau(C_S, LC_S)$	-0.3972	0.2558	-0.0294
$\tau(C_S, LC_F)$	-0.3766	0.2764	-0.0732
$\tau(C_F, LC_S)$	-0.3790	0.2566	-0.0405
$\tau(C_F, LC_F)$	-0.4154	0.2885	-0.0713
$\tau(LC_S, LC_F)$	0.2638	0.6416	0.4755

Kendall's τ as a measure of dependence. Kendall's τ is calculated as

$$\tau = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j>i} sgn(X_i - X_j)(Y_i - Y_j), \tag{11}$$

where $(X_1, Y_1),\ (X_2, Y_2), \ldots,\ (X_n, Y_n)$ is a set of joint observations of the random variables X and Y, respectively, and n is the total number of observations. Kendall's τ of two random variables is high when the variables are concordant (i.e., $X_1 > X_2$ whenever $Y_1 > Y_2$, and $X_1 < X_2$ whenever $Y_1 < Y_2$) and low otherwise. If the ranking of the X variables is exactly the same as (or perfectly reversed with) the ranking of the Y variables, then Kendall's τ equals 1 (or -1). If X and Y are independent, the coefficient would be approximately 0. Kendall's τ (τ in the rest of the paper) measures the degree of the joint dependence between two series over the entire distribution, is less likely to be distorted by the outliers than Pearson correlation, and performs better in capturing the dependence of nonelliptical distributions (Embrechts, Lindskog, and McNeil, 2001; Liu, Vedenov, and Power, 2017). We calculate τ based on the shocks (i.e., log-differences) of the spot and futures prices to be consistent with our approach to modeling the joint distributions.

Given four series of price shocks, we can calculate six pairwise τ (Table 4). The data suggest that corn spot and corn futures price shocks are highly concordant. The dependence between live cattle spot and live cattle futures price shocks is weaker than corn, but still positive. The average cross-dependence between the two commodities is near 0, but with significant variability on both sides.

The scatterplots of differences in hedging effectiveness (Δ_{HE}) versus each τ are shown in Figure 3. Conforming to the results in Table 3, Δ_{HE} is less than 10% in 620 out of 718 observed windows (dots in Figure 3). In these situations, the small gain in hedging effectiveness may not be worth the cost of implementing joint hedging. However, in the remaining 98 windows (13.6% of all observations), joint hedging leads to larger improvements in hedging effectiveness, reaching as high as 34.7%.

Visually, the plots in Figure 3 suggest a parabolic dependence, with higher corresponding to both very high and very low values of τ. To confirm this conjecture and statistically test the impact of the six τ on hedging effectiveness, Δ_{HE} is regressed on a quadratic function of pairwise τ. Table 5 reports the results.

We find that Δ_{HE} is primarily explained by the cross-commodity price shocks dependence, with the corresponding quadratic terms and cross-products significant at 0.1% confidence level. The

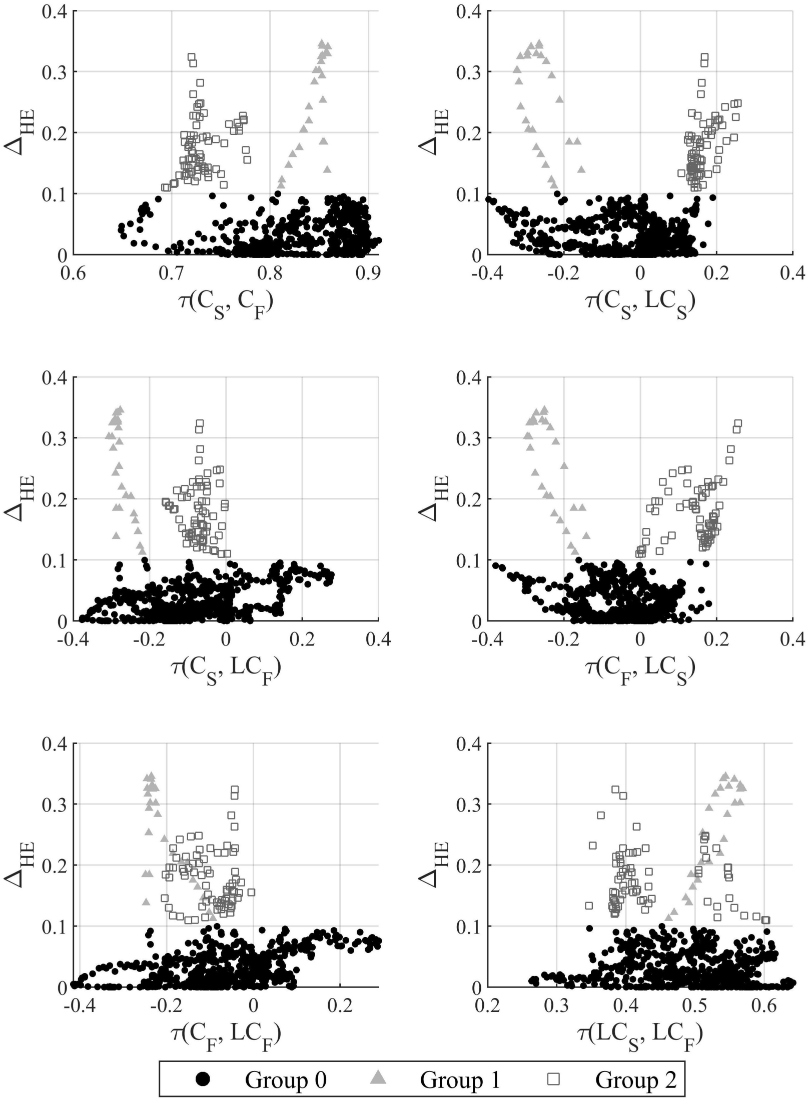

Figure 3. Differences in Hedging Effectiveness vs. Kendall's τ

interactions of cross-commodity τ have the largest marginal effects on the difference in hedging effectiveness. The τ of corn spot/corn futures price shocks explains much of the difference in hedging effectiveness in terms of its quadratic form, cross-products, and itself. The dependence of live cattle spot and live cattle futures price shocks has the smallest effect on Δ_{HE}.

The observations with high Δ_{HE} can be naturally separated into two groups corresponding to very high and very low τ of corn spot/corn futures price shocks. These groups are marked separately in the six subplots of Figure 3. Group 1 (marked by Δ in Figure 3) contains the observations with τ of corn spot/corn futures shocks greater than 0.8. For Group 1, joint hedging can lead to an average 23.8% improvement in the downside risk reduction relative to the single-commodity hedging strategy (Table 6). Group 1 corresponds to the range of τ listed in Table 7. All of the four cross-commodity τ of corn and live cattle price shocks in this group tend to have large negative values. In other words, Group 1 corresponds to the market conditions characterized by a discordance (negative dependence) between the corn and live cattle prices.

Table 5. Regression of Difference in Hedging Effectiveness between Joint- and Single-Commodity Strategies on Kendall's τ

Δ_{HE}	Coefficients
$\tau(C_S, C_F)$	1.215***
$\tau(C_S, LC_S)$	9.180***
$\tau(C_S, LC_F)$	−3.327**
$\tau(C_F, LC_S)$	−4.088***
$\tau(C_F, LC_F)$	2.899**
$\tau(LC_S, LC_F)$	−2.635***
$\tau(C_S, C_F)^2$	−1.484***
$\tau(C_S, LC_S)^2$	−9.296***
$\tau(C_S, LC_F)^2$	−11.776***
$\tau(C_F, LC_S)^2$	−0.141
$\tau(C_F, LC_F)^2$	−9.667***
$\tau(LC_S, LC_F)^2$	0.147
$\tau(C_S, C_F)\,\tau(C_S, LC_S)$	−9.774***
$\tau(C_S, C_F)\,\tau(C_S, LC_F)$	3.728**
$\tau(C_S, C_F)\,\tau(C_F, LC_S)$	5.043***
$\tau(C_S, C_F)\,\tau(C_F, LC_F)$	−3.465**
$\tau(C_S, C_F)\,\tau(LC_S, LC_F)$	3.112***
$\tau(C_S, LC_S)\,\tau(C_S, LC_F)$	24.812***
$\tau(C_S, LC_S)\,\tau(C_F, LC_S)$	10.882**
$\tau(C_S, LC_S)\,\tau(C_F, LC_F)$	−19.867***
$\tau(C_S, LC_S)\,\tau(LC_S, LC_F)$	0.150
$\tau(C_S, LC_F)\,\tau(C_F, LC_S)$	−14.389***
$\tau(C_S, LC_F)\,\tau(C_F, LC_F)$	22.219***
$\tau(C_S, LC_F)\,\tau(LC_S, LC_F)$	−1.104
$\tau(C_F, LC_S)\,\tau(C_F, LC_F)$	9.854**
$\tau(C_F, LC_S)\,\tau(LC_S, LC_F)$	−1.783**
$\tau(C_F, LC_F)\,\tau(LC_S, LC_F)$	1.635*
Adjusted R-Squared	0.887

Notes: Single, double, and triple asterisks (*, **, ***) indicate statistical significance at the 10%, 5%, and 1% level, respectively.

Table 6. Summary Statistics of Differences in Hedging Effectiveness for Group 1 (N = 27)

	Δ_{HE}
Min.	0.1017
Mean	0.2377
Median	0.2481
Max.	0.3470

Group 2 corresponds to low values of corn spot/corn futures τ (less than 0.8) and is marked by the □ in Figure 3. The joint hedging in this group can reduce as much as 32% of the downside risk over the single-commodity hedging strategy, with an average reduction of 18.9% (Table 8). The ranges of the dependence measures (Kendall's τ) in Group 2 are much different than those in Group 1. The corn spot/corn futures τ and live cattle spot/live cattle futures τ are close to the smallest observed values (Table 9), while corn spot/live cattle spot τ and corn futures/live cattle spot τ have large positive values. In other words, Group 2 corresponds to the market conditions characterized by lower dependence between each commodity's own spot and futures prices but strong positive cross-commodity dependence.

Table 7. Ranges of Kendall's τ Defining Group 1 (*N* = 27)

	Min.	Max.
$\tau(C_S, C_F)$	**0.8112**	**0.8589**
$\tau(C_S, LC_S)$	**−0.3231**	**−0.1327**
$\tau(C_S, LC_F)$	−0.3069	−0.2192
$\tau(C_F, LC_S)$	−0.2958	−0.1184
$\tau(C_F, LC_F)$	**−0.2485**	**−0.0933**
$\tau(LC_S, LC_F)$	0.4618	0.5689

Notes: Bold values are bounds of the right-hand three-dimensional region in Figure 4.

Table 8. Summary Statistics of Differences in Hedging Effectiveness for Group 2 (*N* = 71)

	Δ_{HE}
Min.	0.1001
Mean	0.1890
Median	0.1763
Max.	0.3201

Table 9. Ranges of Kendall's τ Defining Group 2 (*N* = 71)

	Min.	Max.
$\tau(C_S, C_F)$	**0.6932**	**0.7770**
$\tau(C_S, LC_S)$	**0.1075**	**0.2558**
$\tau(C_S, LC_F)$	−0.1584	0.0032
$\tau(C_F, LC_S)$	−0.0026	0.2566
$\tau(C_F, LC_F)$	**−0.2043**	**−0.0049**
$\tau(LC_S, LC_F)$	0.3467	0.6032

Notes: Bold values are bounds of the left-hand three-dimensional region in Figure 4.

The corn futures/live cattle futures τ and the live cattle spot/live cattle futures τ do not seem to contribute to the separation of the regions. This is consistent with the estimated model (Table 5), which shows that the corresponding coefficients are not significant.

To confirm the ability of the identified ranges of Kendall's τ to discriminate among the instances of high and low differences in hedging effectiveness, we calculate the number of "false positives" (i.e., observations with low difference in hedging effectiveness (low Δ_{HE}) that are located within the ranges of τ identified as Group 1 (Table 7) and Group 2 (Table 9)). Out of 620 observations with a difference in hedging effectiveness of less than 10%, only seven fall into either of the regions associated with Group 1 or Group 2.

The calculation of six Kendall's τ may seem to be too complex for practical applications. However, it turns out that if we only use three τ to predict the difference in hedging effectiveness, we could still arrive at a highly accurate grouping. The selected τ are the ones capturing dependence between the corn spot/corn futures, corn spot/live cattle spot, and corn futures/live cattle futures price shocks. Figure 4 shows the three-dimensional regions corresponding to Group 1 and Group 2.

The right-hand box (Region 1) is bounded by the maximum and minimum values of the selected three τ of Group 1 in Table 7 (in bold). The left-hand box (Region 2) is bounded by the maximum and minimum values of the selected three τ of Group 2 in Table 9 (in bold). Even using this simplified grouping, only 7 out of the 620 observations with Δ_{HE}—less than 10%—are misclassified, with 4 observations wrongly assigned to Group 1 and the other 3 wrongly assigned to Group 2. The overall prediction accuracy is 99.03%. Therefore, we can conclude that if the values of the selected three

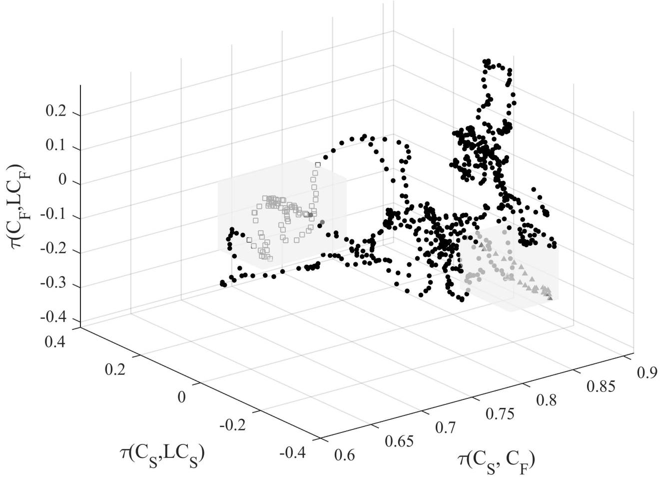

Figure 4. Three-Dimensional Regions of Kendall's τ Corresponding to High Differences in Hedging Effectiveness

Kendall's τ fall into either the Region 1 or Region 2, joint hedging would be much more effective than the single-commodity hedging.

Over time, the market may move in and out of a particular region. However, our results suggest that cattle operators could always determine the current market condition by tracking the three identified measures of dependence. In particular, τ of corn spot/corn futures, corn spot/live cattle spot, and corn futures/live cattle futures price shocks can be calculated based on the historical experience over the previous 100 weeks. When the values of all three τ are found to be located in one of the three-dimensional regions defined in Table 7 or Table 9 (bold values), the cattle operators could substantially reduce the downside risk of the net cash flow by using a joint hedging strategy. In all other cases, the cattle feedlot operator could simply use the single-commodity hedging strategy, since the additional reduction of downside risk by joint hedging is less than 10%, and likely would not offset the added difficulty of implementing the much more complex joint hedging model.

Conclusion

We compare the effectiveness of joint and single-commodity hedging strategies of a feedlot operator. The optimal hedge ratios are based on the LPM_2 minimization, which captures the feedlot operators concerns with downside risk rather than the upside return's deviation. Since the LPM_2 optimization problem does not have a closed-form solution, we use a Monte Carlo approach and numerical optimization methods to calculate the optimal hedge ratios and hedging effectiveness. We apply a 100-week rolling window to estimate a series of hedge ratios and hedging effectiveness measures from January 2001 through December 2016.

We then calculate the difference in hedging effectiveness Δ_{HE}between the joint- and single-commodity hedging strategies. Over the entire sample, joint hedging has higher hedging effectiveness than the single-commodity hedging, but the difference between the two varies.

To explain the variability of Δ_{HE}, the latter is regressed on the quadratic form of the Kendall's τ (i.e., measures of dependence between different price pairs). The results of the regression show that Δ_{HE} is driven primarily by the cross-dependence between the corn and live cattle prices.

Further, we identify two ranges (three-dimensional regions) of Kendall's τ for corn spot/corn futures, corn spot/live cattle spot, and corn futures/live cattle futures price shocks which correspond to a 10% or higher improvement in hedging effectiveness from the joint hedging relative to the single-commodity hedging. One of the regions reflects the market conditions characterized by the strong negative dependence between the corn and live cattle prices. The other region seems to correspond to situations of weak dependence between each commodities' own spot/futures price shocks but high positive cross-commodity dependence.

A practical recommendation arising from our results is that the feedlot operators could monitor the values of the three Kendall's τ when deciding on the single-commodity versus joint hedging. When the values fall into one of the two identified regions, joint hedging should be used to increase hedging effectiveness. Otherwise, single-commodity hedging would be sufficient to reduce the downside risk faced by the cattle feedlot operators.

[First submitted May 2020; accepted for publication August 2020.]

References

Charpentier, A., J.-D. Fermanian, and O. Scaillet. "The Estimation of Copulas: Theory and Practice." In J. Rank, ed., *Copulas: From Theory to Application in Finance,* London, UK: Risk Books, 2007, 35–64.

Cherubini, U., E. Luciano, and W. Vecchiato. *Copula Methods in Finance*. Chichester, UK: Wiley, 2004.

CME Group. "An Introduction to Cattle Feeding Spreads." 2014. Available online as https://www.cmegroup.com/trading/agricultural/files/AC-378_CattleFeedingWhitePaper_r2.pdf.

Collins, R. A. "The Risk Management Effectiveness of Multivariate Hedging Models in the U.S. Soy Complex." *Journal of Futures Markets* 20(2000):189–204. doi: 10.1002/(SICI)1096-9934(200002)20:2<189::AID-FUT5>3.0.CO;2-V.

Cotter, J., and J. Hanly. "Reevaluating Hedging Performance." *Journal of Futures Markets* 26(2006):677–702. doi: 10.1002/fut.20212.

Demirer, R., and D. Lien. "Downside Risk for Short and Long Hedgers." *International Review of Economics & Finance* 12(2003):25–44. doi: 10.1016/S1059-0560(02)00144-2.

Demirer, R., D. Lien, and D. R. Shaffer. "Comparisons of Short and Long Hedge Performance: The Case of Taiwan." *Journal of Multinational Financial Management* 15(2005):51–66. doi: 10.1016/ j.mulfin.2003.12.001.

Ederington, L. H. "The Hedging Performance of the New Futures Markets." *Journal of Finance* 34(1979):157–170. doi: 10.2307/2327150.

Eftekhari, B. "Lower Partial Moment Hedge Ratios." *Applied Financial Economics* 8(1998): 645–652. doi: 10.1080/096031098332682.

Elam, E., and W. Donnell. "Cross Hedging Cattle Rations Using Corn Futures." *Texas Journal of Agriculture and Natural Resources* 5(1992):13–20.

Embrechts, P., F. Lindskog, and E. McNeil. "Modelling Dependence with Copulas." Rapport Technique, Département de Mathématiques, Institut Fédéral de Technologie de Zurich, Zurich, Switzerland, 2001.

Enders, W. *Applied Econometric Time Series*. Hoboken, NJ: Wiley, 2008, 3rd ed.

Fackler, P. L., and K. P. McNew. "Multiproduct Hedging: Theory, Estimation, and an Application." *Review of Agricultural Economics* 15(1993):521–535. doi: 10.2307/1349486.

Fishburn, P. C. "Mean-Risk Analysis with Risk Associated with Below-Target Returns." *American Economic Review* 67(1977):116–126.

Haigh, M. S., and M. T. Holt. "Crack Spread Hedging: Accounting for Time-Varying Volatility Spillovers in the Energy Futures Markets." *Journal of Applied Econometrics* 17(2002):269–289. doi: 10.1002/jae.628.

Ingersoll, J. E. *Theory of Financial Decision Making*. Totowa, N.J: Rowman & Littlefield, 1987.

Levy, H. *Stochastic Dominance: Investment Decision Making under Uncertainty*. Cham, Switzerland: Springer International Publishing, 2015, 3rd ed.

Lien, D., and Y. K. Tse. "Some Recent Developments in Futures Hedging." *Journal of Economic Surveys* 16(2002):357–396. doi: 10.1111/1467-6419.00172.

Liu, P., D. Vedenov, and G. J. Power. "Is Hedging the Crack Spread No Longer All It's Cracked Up to Be?" *Energy Economics* 63(2017):31–40. doi: 10.1016/j.eneco.2017.01.020.

Mattos, F., P. Garcia, and C. Nelson. "Relaxing Standard Hedging Assumptions in the Presence of Downside Risk." *Quarterly Review of Economics and Finance* 48(2008):78–93. doi: 10.1016/j.qref.2006.01.003.

Miranda, M. J., and P. L. Fackler. *Applied Computational Economics and Finance*. Cambridge, MA: MIT Press, 2002.

Noussinov, M. A., and R. M. Leuthold. "Optimal Hedging Strategies For The US Cattle Feeder." *Journal of Agribusiness* 17(1999):1–19. doi: 10.22004/ag.econ.14679.

Peterson, P. E., and R. M. Leuthold. "A Portfolio Approach to Optimal Hedging for a Commercial Cattle Feedlot." *Journal of Futures Markets* 7(1987):443–457. doi: 10.1002/fut.3990070407.

Power, G. J., and D. Vedenov. "Dealing with Downside Risk in a Multi-Commodity Setting: A Case for a 'Texas Hedge'?" *Journal of Futures Markets* 30(2010):290–304. doi: 10.1002/fut.20411.

Thomson Reuters. "Datastream." 2018. URL https://www.refinitiv.com/en.

Tsay, R. S. *Analysis of Financial Time Series*. Hoboken, NJ: Wiley, 2005, 2nd ed.

Unser, M. "Lower Partial Moments as Measures of Perceived Risk: An Experimental Study." *Journal of Economic Psychology* 21(2000):253–280. doi: 10.1016/S0167-4870(00)00004-0.

Wand, M. P., and M. C. Jones. *Kernel Smoothing*. Boca Raton, FL: CRC Press, 1994.