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Using Grid Soil Sampling to Determine Profit-Maximizing Phosphorus Application Rates in Wheat

Brian E. Mills, B. Wade Brorsen, and D. Brian Arnall

Past research on the profitability of precision phosphorus (P) application has used a small number of fields and a short time frame. Data on grid-sampled fields provided by producers are used to define the distribution of phosphorus within fields. Expected yields and net present value (NPV) are simulated to compare variable and uniform rate P. The highest NPV used a variable rate that changed each year based on yield and predicted carryover. A variable rate using the same rates for 4 years was inferior to simply applying a little extra P at a uniform rate.

Key words: grid sampling, Oklahoma, precision agriculture, profitability, soil sampling, variable rate

Introduction

Insufficient available soil phosphorus (P) can greatly limit crop yields. Therefore, it is important to apply a phosphorus fertilizer. Historically, a uniform rate has been applied across the entire field. This uniform rate was determined by taking samples from locations across the field, mixing them together to form a composite sample, and determining the recommended uniform phosphorus rate from that one sample. When soil P varies across the field, this method can lead to under- or overapplication of phosphorus, which in turn can lead to decreased profits.

One technology to reduce phosphorus misapplication is grid soil sampling, which involves sampling grid cells and obtaining phosphorus recommendations for each one. The most common grid cell sizes are 1 acre, 2.5 acres, and 5 acres. Grid sampling allows a variable rate to be applied instead of a uniform rate. However, grid sampling costs around \$10/acre for a 2.5-acre grid size. This cost includes collecting the sample as well as the chemical analysis to determine nutrient availability at each grid cell location.

Producers need to know when using a variable rate is profitable. Schnitkey, Hopkins, and Tweeten (1996) found that applying a variable rate of phosphorus was profitable in a corn–soybean rotation when grid sampling costs were only \$1.00/acre. However, the benefits of variable-rate fertilizer are less for lower-valued commodities such as wheat (Lowenberg-DeBoer and Swinton, 1997; Weisz et al., 2003; Adamchuk, Morgan, and Lowenberg-Deboer, 2004).

Many studies on precision agriculture profitability have been conducted on a single field (McBratney et al., 2005). Since soil conditions vary from field to field, as Bullock et al. (2009)

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argued, studies across multiple fields may have different results. For example, Lawes and Robertson (2011) found that variable-rate fertilizer applications generated significant returns on one-third of the 20 fields examined.

Further, there is still the question of whether there is a price and yield for wheat at which variable-rate application becomes profitable. Havlin and Heiniger (2009) and Mills, Wade Brorsen, and Arnall (2020) found that higher prices increase the returns to variable-rate fertilization. But since yield directly affects soil P, it is also important to examine how higher yield potential can impact variable-rate profitability. To more accurately determine when or whether variable-rate fertilizer application can be profitable, a large number of fields as well as various grain prices and yield should be examined.

The objective of this paper is to determine the profitability of variable-rate phosphorus application for wheat production. This is accomplished by examining variable-rate profitability across 147 grid sampled fields sent to us voluntarily by producers in Oklahoma and Kansas. All fields were grid sampled at 2.5 acres. This paper is similar to other studies on variable-rate phosphorus, such as Havlin and Heiniger (2009) and Lawes and Robertson (2011). However, we use a much larger number of fields. Variable rate is compared to a uniform rate for two yield potential scenarios, 40 bu/acre and 60 bu/acre. Yield is simulated across a 4-year time frame using a deterministic plateau function. The yield potential is the maximum yield, or plateau, of the yield function. A 40 bu/acre scenario was selected because this is around the average yield in Oklahoma for grain-only wheat production. The 60 bu/acre scenario was to determine whether higher yields would impact the results.

Most studies only looked at a 1- or 2-year timeframe, whereas this study is across a 4-year period. This study also examines both current university recommendations and using a uniform rate above university recommendations.

The Oklahoma State University recommended phosphorus application rates were based on Macnack et al. (2017). Phosphorus recommendations are not based on the wheat yield goal method (Warren et al., 2017). Instead, a sufficiency approach is the common practice, where phosphorus is applied based on the sufficiency level of available phosphorus in the soil.

For a variable rate, phosphorus rate was determined at each grid cell across the field. A uniform rate uses a composite sample. Due to the data in this study being from grid-sampled fields, a composite sample was calculated for each field using the grid sample values. The data are used to get the distribution of phosphorus across space. Wheat yield was simulated using Oklahoma State yield functions (Macnack et al., 2017). The profitability of variable rate fertilization is determined by comparing the net present value (NPV) of variable rate to that of uniform rate across a 4-year period.

We ultimately consider six scenarios: (i) variable rate following university recommendations with the same rate for 4 years, (ii) uniform rate following university recommendations with the same rate for 4 years, (iii) uniform rate with the same rate for 4 years but with optimal additional phosphorus applied above university recommendations, (iv) uniform rate based on a composite sample taken each year, (v) uniform rate based on a composite sample taken each year and optimal additional phosphorus applied above university recommendations, and (vi) variable rate using carryover formulas to predict the rate needed on each grid for each year. The last four scenarios all show improvements over following university recommendations; scenario (vi) has the highest NPV.

Sufficiency Application Approach

With the sufficiency approach, the percentage of available soil P in relation to the critical level corresponds to the percentage of potential wheat yield relative to the plateau. In other words, a soil test of 22.1 parts per million (ppm) will be 80% sufficient, and expected yield will be 80% of the

Table 1. Phosphorus Fertilizer Rates for Wheat Production for Sufficiency Approach Given Bray-1 P Soil Test Levels

Available Soil P (ppm)	Relative Yield	Applied P (lb/acre)
0	0.25	80
4.3	0.45	60
8.5	0.8	40
17.0	0.9	20
27.6	1.0	0

Notes: Available soil P is based on the Bray-1P soil test. Relative yield is yield in relation to the plateau. Applied P is the amount of phosphate (P₂O₅) applied.
 Source: Macnack et al. (2017).

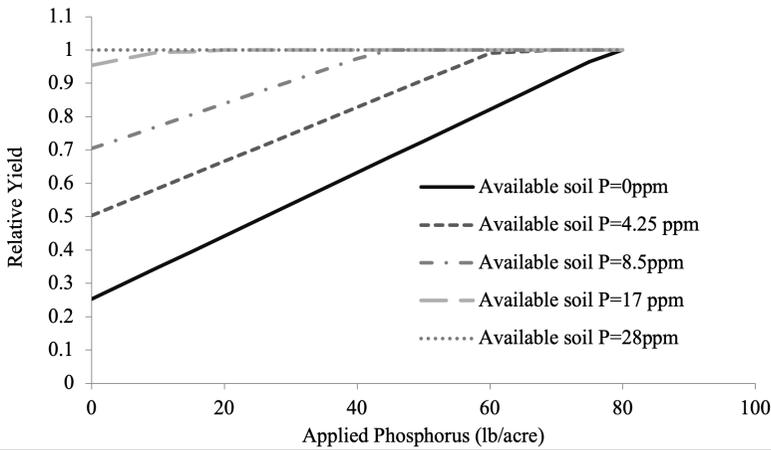


Figure 1. Relative Wheat Yield Response to Applied Phosphorus Given the Bray-1P Soil Test for Available Soil P from Macnack et al. (2017)

Notes: Yield relative to the plateau.

plateau yield. In a field with a yield potential, or plateau, of 40 bu/acre, if soil P were 80% sufficient, then expected yield would be 32 bu/acre.

Yield losses from available soil P below the critical level can be avoided by applying phosphorus. Table 1 shows the yield given the level of available soil P and the phosphorus application rate needed to avoid yield losses. Figure 1 is a graphical representation of the effect of soil P and applied phosphorus on yield.

There are other approaches to determining phosphorus application rates, such as the maintenance, build-up, or build-up and maintenance concepts (Macnack et al., 2017). The maintenance approach is based on applying phosphorus given the amount removed through yield. This approach is not well suited for fields with low soil P, which is why it is often paired with a build-up approach. In the build-up approach, phosphorus is applied at higher amounts, above crop needs, in order to build soil P to the critical level of 27.6 ppm. This is usually done over a period of 4–8 years. The build-up and maintenance approach is to build soil P to the critical level and then replace nutrient removal from harvest. The build-up approaches are usually used in fields where the producer either owns the land or has a long-term lease.

It should also be noted that, in wheat, the sufficiency approach will apply a higher amount of phosphorus than what is removed through crop production, leading to a build-up of available soil P. Which of these approaches is profit maximizing depends on the crop, and there is a need for continued research. The most common method in Oklahoma is the sufficiency approach, which is why we include it here.

The Optimization Problem

To select the phosphorus application strategy, variable or uniform, that maximizes NPV, the following objective function was maximized:

$$\max_{k,v} NPV = \sum_{t=1}^4 \sum_{n=1}^N \left(\frac{Price_t Y_{kntv} - r_k (AppliedP_{kntv})}{(1+i)^t} - s_k \right)$$

(1) $k = \{VR, UR, UOver, URupdate, UOptimal, VROptimal\}$
 $0 \leq v \leq 35,$

where NPV is the discounted expected revenue stream minus the cost of phosphorus application and soil sampling summed across years t and grid cells within a field n ; k is the choice variable for application scenarios where VR is the variable-rate scenario; v is the choice variable for application over university recommendations from 0 to 35 lb of extra phosphorus; UR is the uniform rate scenario, UOver is the uniform rate with application above the university recommendation, URupdate is the uniform rate with yearly composite sampling scenario, UOptimal is the uniform rate with application above university recommendation and a yearly composite sample, and VROptimal is a variable rate using carryover formulas; t is the year; $Price_t$ is the expected wheat price in year t ; Y_{kntv} is the expected yield for location n , year t , amount of overapplication v , and application type k , where either a variable rate or a uniform rate is applied; i is the interest rate, r_k is the phosphorus application cost per pound; $AppliedP_{kntv}$ is the phosphorus application rate for a variable or uniform rate at grid cell n , period t , application scenario k , and overapplication v ; and s_k is the cost of soil sampling for application type k . As defined in equation (1), the data from producers are used to define the distribution of P across space and are thus used in calculating the expectation.

The optimization in equation (1) is carried out subject to a set of constraints. These are (i) yield as a function of soil P and applied P, (ii) the rule used to determine how much phosphorus to apply, and (iii) phosphorus carryover functions. Yield is specified as a function of both soil P and the amount of applied P:

$$(2) \quad Y_{kntv} = Y (SoilP_{kntv}, AppliedP_{kntv}),$$

where yield, Y_{kntv} , for application type k , grid cell n , time t , and overapplication v is a function of the soil P, $SoilP_{kntv}$, and the applied P from fertilizer, $AppliedP_{kntv}$. The university-recommended phosphorus application rates for P₂O₅ lb/acre, from Macnack et al. (2017), are a function of the soil P:

$$AppliedP_{kntv} = (\alpha_0 + \alpha_1 SoilP_{kntv} + \alpha_2 SoilP_{kntv}^2) + v \text{ for } k = VR; SoilP_{kntv} \leq 27.6$$

$$AppliedP_{kntv} = \left(\alpha_0 + \alpha_1 \overline{SoilP}_{kntv} + \alpha_2 \overline{SoilP}_{kntv}^2 \right) + v \text{ for } k = UR; \overline{SoilP}_{kntv} \leq 27.6$$

(3) $AppliedP_{kntv} = v \text{ for } k = VR; SoilP_{kntv} > 27.6$
 $AppliedP_{kntv} = v \text{ for } k = UR; \overline{SoilP}_{kntv} > 27.6$
 $AppliedP_{kntv} \geq 0$
 $v = 0,$

where for a variable rate this varies by n , a uniform rate uses an average of the soil P within a field. For both a variable and uniform rate, phosphorus rates are only dependent on the soil P level found in the first year.

The equation of motion for soil P depends on the previous year's soil P, applied P, and yield. Equations (2) and (3) were developed based on small plot research and so we argue that it is appropriate to treat them as recommendations for grids rather than as whole-field recommendations. Within a small plot, there is less variability of factors such as soil type or soil nutrient levels compared to the field level. As such, small plots may not translate to the results seen at the field level. Therefore, the yield functions and application rates found with small plot research may be more applicable to the grid cells than to the whole field.

One problem with the university recommendations for the uniform rate is that they do not maximize net present value. Net present value could be increased by applying a uniform rate higher than the recommended rate, by yearly composite sampling, or a combination of both. We also consider these three options because they would reduce yield losses due to underapplication. The uniform rate with added phosphorus is found by

$$\begin{aligned}
 \text{Applied}P_{kntv} &= \left(\alpha_0 + \alpha_1 \overline{\text{Soil}P}_{k1v} + \alpha_2 \overline{\text{Soil}P}_{k1v}^2 \right) + v \text{ for } k = URover; \overline{\text{Soil}P}_{k1v} \leq 27.6 \\
 \text{Applied}P_{kntv} &= v \text{ for } k = URover; \overline{\text{Soil}P}_{k1v} > 27.6 \\
 \text{Applied}P_{kntv} &\geq v \\
 0 &\leq v \leq 35,
 \end{aligned}
 \tag{4}$$

the uniform rate with yearly composite sampling is found by

$$\begin{aligned}
 \text{Applied}P_{kntv} &= \left(\alpha_0 + \alpha_1 \overline{\text{Soil}P}_{ktv} + \alpha_2 \overline{\text{Soil}P}_{ktv}^2 \right) + vu \text{ for } k = URupdate; \overline{\text{Soil}P}_{ktv} \leq 27.6 \\
 \text{Applied}P_{kntv} &= v \text{ for } k = URupdate; \overline{\text{Soil}P}_{ktv} > 27.6 \\
 \text{Applied}P_{kntv} &\geq v \\
 v &= 0,
 \end{aligned}
 \tag{5}$$

and the combination of both is

$$\begin{aligned}
 \text{Applied}P_{kntv} &= \left(\alpha_0 + \alpha_1 \overline{\text{Soil}P}_{ktv} + \alpha_2 \overline{\text{Soil}P}_{ktv}^2 \right) + v \\
 &\text{for } k = UROptimal; \overline{\text{Soil}P}_{ktv} \leq 27.6 \\
 \text{Applied}P_{kntv} &= v \text{ for } k = UROptimal; \overline{\text{Soil}P}_{ktv} > 27.6 \\
 \text{Applied}P_{kntv} &\geq v \\
 0 &\leq v \leq 35,
 \end{aligned}
 \tag{6}$$

where the additional phosphorus above university recommendations where profit is maximized, v , is selected using the grid search method. Under the *URupdate* scenario, applied phosphorus is dependent on the mean soil P found in each year; the *UROptimal* scenario both applies phosphorus above university recommendations and is dependent on a yearly composite sample.

Last, the optimal variable-rate scenario was found by modifying equation (3) to

$$\begin{aligned}
 \text{Applied}P_{kntv} &= \alpha_0 + \alpha_1 \text{Soil}P_{kntv} + \alpha_2 \text{Soil}P_{kntv}^2 + v \text{ for } k = VROptimal; v = 0 \\
 \text{Applied}P_{knt} &\geq 0,
 \end{aligned}
 \tag{7}$$

where applied phosphorus is updated yearly.

Methods to Determine Yield, Phosphorus Application, and Soil Phosphorus Levels

Net present values for variable and uniform rates were determined by estimating grain-only wheat yield, soil P levels, and phosphorus application rates over a 4-year period. Phosphorus application rates and yield are based on the sufficiency approach (Macnack et al., 2017), which treats yield as a plateau function in which yield losses occur when available soil P is below the critical level of 27.6 ppm for a Bray-1P test.

Available soil P is found through soil testing, and there are a number of tests available. Oklahoma State University recommendations for phosphorus application (Macnack et al., 2017) are based on Mehlich III soil tests, which is a test for the amount of available P in the soil. But many of the grid soil tests provided use the Bray-1P test for available P. These tests differ in that the soil P found from each is not a one-to-one comparison. Therefore, Mehlich III values were converted to Bray-1P levels by the following approximation from Hergert and Shaver (2009):

$$(8) \quad \text{Bray1P} = 0.85\text{MehlichIII},$$

where a Bray-1P test would be equivalent to 85% of a Mehlich III test.

Yield was simulated using a deterministic plateau function given available soil P and applied P. Using the data in Table 1, the relative yield function was derived as

$$(9) \quad \begin{aligned} Y_{kntv} &= 0.25 + 0.01\text{Applied}P_{kntv} + 0.07\text{Soil}P_{kntv} - 0.001\text{Soil}P_{kntv}^2 \\ &\quad - 0.0003(\text{Soil}P_{kntv}\text{Applied}P_{kntv}) \text{ for } \text{Soil}P_{knt} \leq 27.6 \\ Y_{knt} &= 1 \text{ for } \text{Soil}P_{knt} > 27.6 \end{aligned}$$

where Y_{kntv} is the yield for application type k , grid cell n , and year t ; $\text{Applied}P_{kntv}$ is applied phosphorus; and $\text{Soil}P_{kntv}$ is the soil P level.

The yield response to soil P is quadratic due to the nature of phosphorus. In the soil, not all phosphorus is readily available to the plant. As the amount of total phosphorus in the soil increases, the portion that is available to plants increases as well. Therefore, the effect of available soil P on yield decreases as available soil P increases because there is more total phosphorus in the soil that can become available during the growing season.

The yield response is linear in applied P at a given soil P. At a given soil P, doubling application rate above the recommended rate will not change the response as the plateau will be reached. The excess will not go into plant uptake and instead will go into the soil. Applied P is not immediately tied up by chemical processes and therefore ends up being linearly related. This relationship is based on the level of soil P, so an interaction term is also included.

Yield is examined for two yield potential scenarios of 40 bu/acre and 60 bu/acre. Yield potential is not the same measure as the yield goal method used in nitrogen application, which has been found to be an inaccurate determinant of optimal application rates (Camberato, 2015; Nafziger, 2018). One reason the yield goal method has worked poorly for nitrogen is that it implicitly assumes a stochastic plateau function with an intercept of 0. Optimal nitrogen application is also highly dependent on weather. Phosphorus, unlike nitrogen, is an immobile nutrient, meaning that phosphorus is only removed from the soil through plant uptake or soil runoff. Therefore, the yield response function for phosphorus does not vary as much due to weather and is much easier to quantify.

Phosphorus application rates were estimated using the application rates given Bray-1P levels found in Table 1. The equation used to determine the phosphorus application rates in pounds of

Table 2. Phosphorus Fertilizer Needed to Increase Soil P to a Given Level over 4 Years

Current Soil P (lb/acre)	Phosphorus Rate (lb/acre) Needed to Reach Target Soil P of		
	30 lb/acre	40 lb/acre	45 lb/acre
5	360	448	488
10	256	344	392
15	176	272	312
20	112	200	248
25	56	144	192
30	0	96	136
35	0	48	88
40	0	0	40

Notes: Soil P is amount of available soil P found from Bray-1P soil test. Target soil P is the level of available soil P that will be reached. Phosphorus rate is amount of phosphate (P₂O₅) to apply and does not account for phosphorus losses from yield uptake. Source: Minor, Stecker, and Brown (2019).

P₂O₅ per acre is

$$\begin{aligned}
 &AppliedP_{kntv} = 80 - 5.03SoilP_{kn1v} + 0.008SoilP_{kn1v}^2 \text{ for } k = VR; SoilP_{kn1v} \leq 27.6 \\
 &AppliedP_{kntv} = 80 - 5.03\overline{SoilP}_{kn1v} + 0.008\overline{SoilP}_{kn1v}^2 \text{ for } k = UR; \overline{SoilP}_{k1v} \leq 27.6 \\
 &AppliedP_{kntv} = v \text{ for } k = VR; SoilP_{kn1v} > 27.6 \\
 &AppliedP_{kntv} = v \text{ for } k = UR; \overline{SoilP}_{k1v} > 27.6 \\
 &AppliedP_{knt} \geq v \\
 &v = 0,
 \end{aligned}
 \tag{10}$$

where a variable rate at location *n* and year *t* depends on soil P, and a uniform rate depends on average soil P of the field. Due to the high price of grid sampling, it is only done every 3–5 years. Thus, the variable application rate only depends on the soil P of the first year. In this study, the same variable rate is applied in years 1–4.

Similar to variable rate, the common practice for a uniform rate is to find a composite sample and then base a uniform rate off of that for 3–5 years. This is done despite the relatively low cost of a composite sample. Applying the same phosphorus rate for 3–5 years can lead to over- or underapplication for both variable and uniform rate. By allowing phosphorus rates to change by year, this over- or underapplication would be reduced.

This study looks across multiple years, and it is therefore necessary to define how soil P changes from year to year. Soil P in a given year is found by

$$SoilP_{kntv} = SoilP_{knt-1,v} + \Delta SoilP_{knt-1,v},
 \tag{11}$$

where *SoilP_{kntv}* is the soil P for a given year, *SoilP_{knt-1,v}* is the previous year’s soil P, and $\Delta SoilP_{knt-1,v}$ is the change in soil P.

The change in soil P was determined using phosphorus build rates from Minor, Stecker, and Brown (2019). These build rates can be seen in Table 2. From these build rates, the change in soil P was estimated by the following:

$$\Delta SoilP_{kntv} = 0.072(NetAppliedP_{kntv}) + 0.001(NetAppliedP_{kntv})(SoilP_{kntv}),
 \tag{12}$$

where *NetAppliedP_{kntv}* is the fertilizer that is not used up by the crop. The change in soil P also depends on the soil P level, such that the change in soil P increases as soil P increases.

Phosphorus use is relatively inefficient in wheat production in that a lot of the phosphorus applied does not get used by the crop. Wheat uses 0.5 lb of P for every bushel of wheat produced (Macnack

et al., 2017). Therefore, the net of applied P given wheat yield is

$$(13) \quad \text{NetApplied}P_{kntv} = \text{Applied}P_{kntv} - 0.5Y_{kntv}.$$

If the amount of applied phosphorus is higher than what wheat yield removes, then the soil P level will increase. If it is lower than what wheat yield takes off, then the soil P level will decrease. As has been discussed before, the same phosphorus rate is applied in years 1–4. So if under-application happens in year 1, there can also be under application in years 2–4. This under-application becomes especially important when soil P decreases below 27.6 ppm and yield losses occur. These yield losses will increase each year as soil P decreases more each year. There could also be a situation in which no under-application happens in year 1 but does occur in years 2–4.

Since under application reduces yield, applying phosphorus over the university recommendation should reduce the yield losses under uniform rate application. This was done for a uniform rate by adjusting equation (10) to be

$$(14) \quad \begin{aligned} \text{Applied}P_{kntv} &= \left(80 - 5.03\overline{\text{Soil}P}_{k1v} + 0.008\overline{\text{Soil}P}_{k1v}^2\right) + v \\ &\text{for } k = \text{URover}; \overline{\text{Soil}P}_{k1v} \leq 27.6 \\ \text{Applied}P_{kntv} &= v \text{ for } k = \text{URover}; \overline{\text{Soil}P}_{k1v} > 27.6 \\ \text{Applied}P_{kntv} &\geq v \\ 0 &\leq v \leq 35, \end{aligned}$$

where v is the amount of phosphorus applied above university recommendations. A grid search was used in which v was varied from 0 to 35 lb/acre to determine the optimal amount of application above the university recommendation.

Last, phosphorus rates that varied from year to year were compared to common practice. For a uniform rate, phosphorus rates were determined from a yearly composite sample. A variable rate was assumed to have perfect information. In other words, in the first year, grid sampling would be used to determine variable rates. In the following years, a producer is assumed to use yield, phosphorus rates, and starting soil P values to determine a variable rate each year.

This study uses similar methods and data as Mills, Wade Brorsen, and Arnall (2020), who looked at the profitability of variable-rate lime application in wheat. While both studies examine the net returns to variable-rate application, they differ greatly due to the significant differences in lime and phosphorus management and application. There is also a lack of empirical research into interaction effects between nutrients, especially in wheat. Since these interaction effects are not well defined, the yield response to phosphorus is evaluated separately from lime.

Grid Sampled Soil Data and Costs

The data used came directly from producers. In total, 278 grid sampled fields from Oklahoma, Kansas, Missouri, and Texas were sent in. In order to have the same grid size for all fields, as well as due to missing or incomplete data, 147 fields were used. As a result of these constraints, the fields used were only in Oklahoma and Kansas. Figure 2 shows the location and number of fields by county. Two of the fields did not have county location data available and are not included in the figure. Each field was grid sampled at 2.5 acres.

This study used a price of grid sampling of \$10/acre, which is the price charged by a local firm for 2.5-acre grids. The 278 initial fields are the same that are used in Mills, Wade Brorsen, and Arnall (2020). This study, however, uses 147 fields compared to the 111 used in Mills, Brorsen, and Arnall; due to missing data, only 50 of the fields were used in both studies.

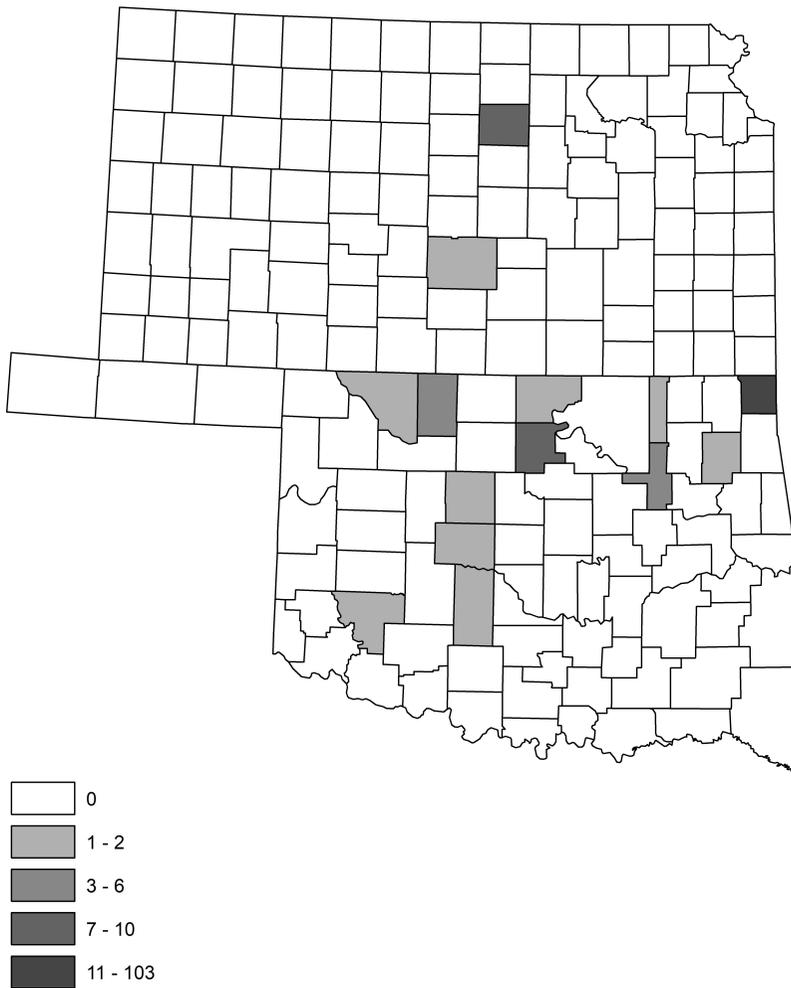


Figure 2. Number of Fields Used by County in Oklahoma and Kansas

There were 4,861 total soil samples across the 147 fields. Each soil sample represents a grid cell within a given field. The average field size was 79.3 acres and ranged from 3.74 acres to 316.72 acres. The average Bray-1 P soil test was 26 ppm, with a low of 0 ppm and a high of 367 ppm. Of the total samples, 33% had a soil P less than 17 ppm. Around 64% of all samples required some amount of P to be applied.

The estimated composite sample is an average of all the grid cells within a field. Given the estimated composite samples for each field, 87 of the fields required P under a uniform rate. The minimum composite sample estimated was 4.68 ppm and the highest was 69 ppm. Compared to a statewide soil test study, the samples in this study had higher soil P on average. The statewide study found an average soil P of 15.3 ppm, with 53.5% of soils having a soil P of less than 17 ppm and 73.9% of soils needing some amount of P (Zhang and McCray, 2018).

However, the data in this study, as well as those in the statewide survey, are convenience samples. A convenience sample is a nonrandom sample taken from participants who are easily available. Since the samples were obtained voluntarily from producers, they are nonrandom. Due to this and being unable to determine the crops or production practices used on these samples, it is unknown whether the samples are representative of Oklahoma wheat production.

The Farmers Coop Association of Snyder in Snyder, OK, charged \$0.27/lb for diammonium phosphate (DAP). The amount of DAP (lb/acre) needed was found by taking the university recommendations for P_2O_5 (lb/acre) and dividing by 0.46. The phosphorus application rate was not restricted to have a maximum or minimum amount applied.

There was no added cost for variable-rate application. In our talks with local fertilizer companies, we were told that the variable-rate technology is already installed so there is no extra application cost. Most variable-rate technology is computerized, and many farmers hire someone to develop and apply their application rates. The management cost to the farmer is assumed to be 0, but in reality it varies from farmer to farmer and from fertilizer company to company. Since a variable-rate application cost was not included, the variable and uniform rate differ only by the application rate and the cost of grid sampling. Note that McClure and Jansen (2018) found a variable-rate application cost of around \$1.50/acre in Nebraska. Including this cost would shift the variable rate in the figures down but would not change the ultimate conclusions.

Above-average and low yields of 60 bu/acre and 40 bu/acre, respectively, were examined. The 40 bu/acre system is around the average wheat grain yield of Oklahoma in 2017 (U.S. Department of Agriculture, 2017a). Sensitivity analysis was done across wheat prices of \$3.00/bu to \$7.00/bu. The cost of the whole-field composite sampling was set to 0 since the per acre cost is quite small. The discount rate used was 0.05¹ and was based on the interest rate for a farm operating loan reported by the USDA Farm Service Agency at the time of the study (U.S. Department of Agriculture, 2017b).

Net Present Value and Phosphorus Application Results

Common Practice

Wheat production was evaluated for yields of 40 bu/acre and 60 bu/acre. Grain prices from \$3.00/bu to \$7.00/bu were considered. For both the 40 bu/acre and 60 bu/acre scenario and across all prices examined, variable rate had a higher net present value on average than the uniform rate (Figure 2(b)). As price increases, the difference between variable and uniform rate increases.

The best predictor of the profitability of variable-rate fertilizer application was the mean available soil P for a field (i.e., the composite sample). In fields with a mean soil P of less than 13 ppm or greater than 35 ppm, a uniform rate had higher returns than a variable rate. Returns to a variable rate were lower in fields with a mean soil P below 13 ppm due to these fields having less soil P variance. Fields with a mean soil P above 35 had a lower return to variable rate because there were not enough yield gains from phosphorus application to cover the cost of grid sampling. In these fields with high soil P, the uniform rate had the highest returns of any scenario examined due to the low associated costs and the small yield gains to P application in these fields.

The difference in returns to a variable rate compared to a uniform rate was highest when mean soil P was around 16 ppm. As mean soil P got closer to the critical value of 27.6 ppm, the difference in NPV of a variable rate compared to a uniform rate decreased. Figure 4 shows the difference in NPV between the common method variable and uniform rate for a 40 bu/acre yield potential scenario and a price of \$5/bu given the mean available soil P for a given field. These results suggest that a composite sample could be a cheap way to predict whether a variable rate or uniform rate should be used. Note that the pattern in Figure 4 does not hold for the other strategies.

Both the uniform and variable rate only use the first-year soil P measurement to determine phosphorus rates for the whole period. This can lead to under-application for variable-rate fertilization if soil P is around 27.6 ppm. For example, if soil P is 27.6 ppm, then no phosphorus is applied in any year from years 1–4. Crop production then reduces soil P to below 27.6 ppm, which causes yield losses. For uniform rate fertilization, if the composite sample is at or above 27.6 ppm,

¹ Sensitivity analysis of various discount rates did not show a significant change in the results. Rankings of the various application strategies did not change and optimal application rate above university recommendations did not change either.

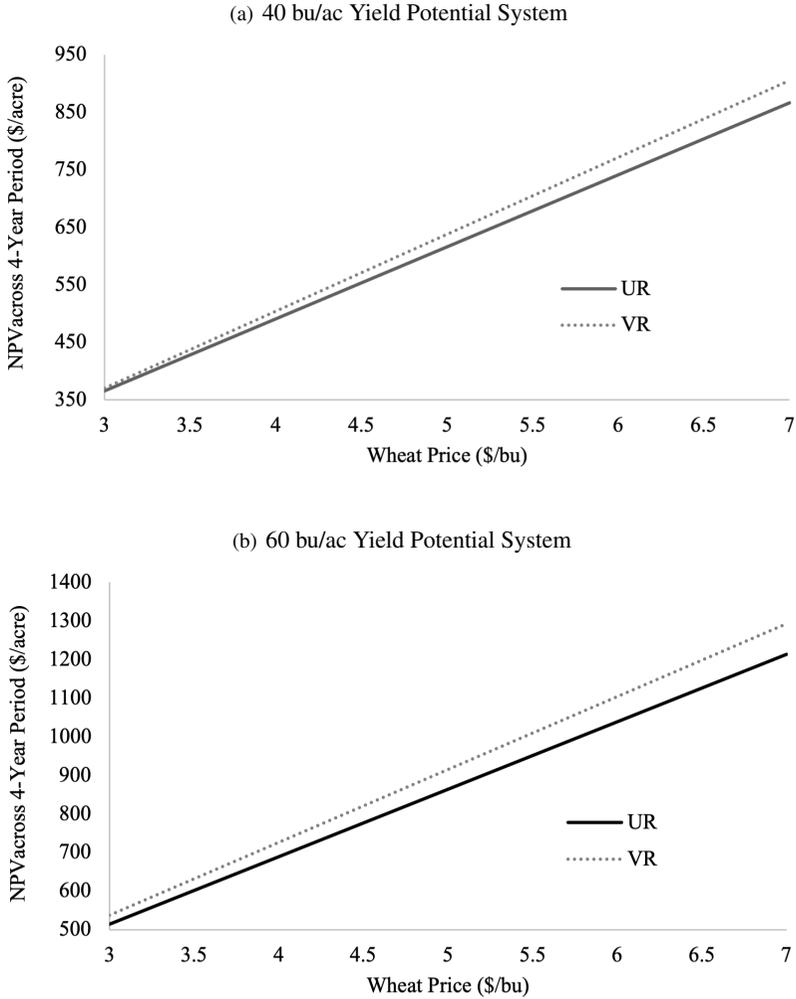


Figure 3. Average Net Present Value for Variable Rate (VR) and Uniform Rate (UR) Phosphorus Application across Prices of \$3/bu to \$7/bu

then no phosphorus is applied in all years. Since the composite sample is an average, then there can be large amounts of the field that are below 27.6 ppm. These areas of the field will experience under-application and yield losses in all 4 years. Yield losses will increase in each subsequent year as crop production further reduces soil P. Therefore, by updating phosphorus rates with yearly sampling, or by applying extra phosphorus, this under-application and resulting yield losses would be reduced.

Optimal Uniform and Variable Rate

To see the impact of applying a higher rate of phosphorus, NPV for uniform rate was calculated for various amounts of phosphorus application above university recommendations. As shown in Figure 5, the amount of overapplication that maximizes NPV on average across all fields is dependent on the gross crop revenue (GCR).² The higher the value of the crop, the more it pays to apply phosphorus above university recommendations because a uniform application rate results

² Gross crop revenue is found by taking potential yield and multiplying by price (i.e., if yield potential is 40 bu/acre and price is \$3/bu, gross crop revenue would be \$120/acre).

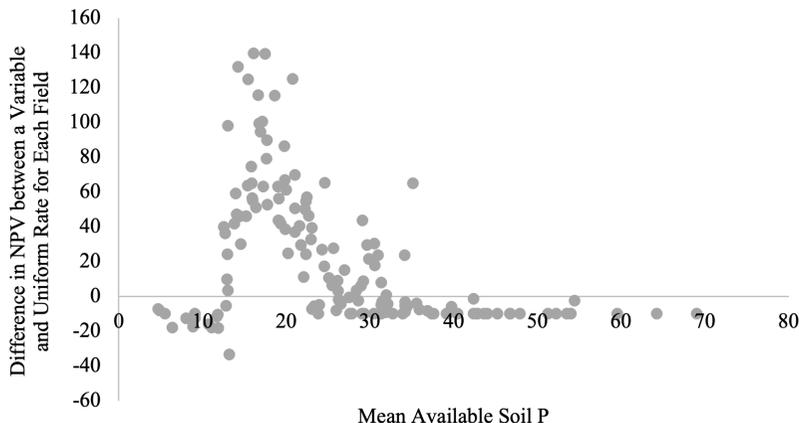


Figure 4. Difference in Net Present Value (NPV) between the Common Methods of Variable and Uniform Rate Given the Mean Soil P for Each Field for a 40 bu/ac System and a Wheat Price of \$5/bu

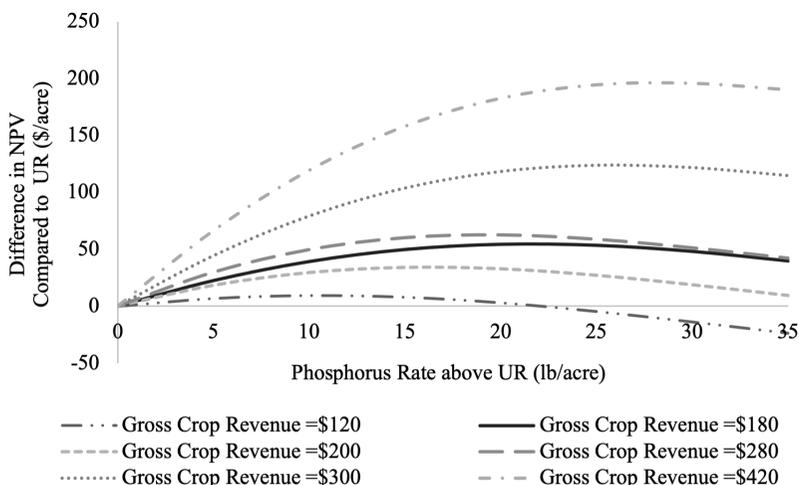


Figure 5. Average Difference in Net Present Value (NPV) When Increasing Phosphorus Rate above University Recommendations Given the Value of the Crop

in underapplication. Many areas of the field are not at the plateau. So, as the value of the crop increases it becomes more valuable to reduce those yield losses from underapplication.

At a GCR of \$200/acre, 40 bu/acre and \$5/bu, profit was maximized, on average across all fields, when 15 lb/acre of P₂O₅ per acre above the university recommendation is applied. When GCR was \$300/acre, profit was maximized when an extra 25 lb/acre of P₂O₅ was applied. In Figure 6, this uniform rate with overapplication is compared to the common practice uniform rate. This rate takes into account that as price increases the amount of phosphorus applied above university recommendations increases.

This method of uniform-rate application had a higher NPV on average at all prices examined than the common practice uniform rate. In a 40 bu/acre yield potential system and with a price of \$5/bu, NPV was increased by \$34.07/acre over the 4-year period by applying 15 lb/acre/year of P₂O₅ above university recommendations. In a 60 bu/acre system and with a price of \$5/bu, NPV was increased by \$123.97/acre over the 4-year period by applying 25 lb/acre/year of above university recommendations.

This method also increased NPV above that of a variable rate. At a yield and price of 40 bu/acre and \$5/bu, NPV for a uniform rate with an additional 15 lb/acre/year of phosphorus was higher than that of the variable-rate NPV by \$12.26/acre over the 4-year period on average. When yield and price were 60 bu/acre and \$5/lb, NPV was higher than that of a variable rate by \$73.07/acre over the 4-year period on average. Therefore, on average, NPV can be increased with a uniform rate by applying an extra amount of phosphorus above university recommendations.

The optimal rate above university recommendation is found given the average NPV across all fields and, as such, results vary from field to field as conditions change. This method does not pay compared to the common method uniform and variable rates in fields with a high mean soil P (>35ppm) or fields with low mean soil P (<13 ppm). Just as with a variable rate, there is a degree of risk associated with this method. However, as has been discussed before, a composite sample can be used to determine which application strategy is optimal.

Further, this analysis does not account for the environmental impacts of higher phosphorus application. With a higher phosphorus rate, the chance of phosphorus runoff increases. This chance is higher for those areas of the field that already have high soil P. But since these environmental impacts do not directly impact NPV, they were not included.

Another uniform rate was examined in which a composite sample is taken every year and a new phosphorus rate determined from this yearly composite sample. This uniform rate with updating approach provides a more accurate phosphorus rate than the common practice. The NPV of this method was higher than the common practice uniform rate at all yields and prices examined (Figure 6).

NPV for this method was higher than the variable-rate NPV in the 40 bu/acre scenario when price was below \$4.00/bu but lower at prices above \$4.00/bu. As the value of wheat increased, the gains from a more accurate application from a variable rate also increased. However, in the 60 bu/acre scenario, a uniform rate with yearly composite sampling had a higher NPV than the variable-rate NPV at all prices. The difference between the 40 bu/acre and 60 bu/acre scenarios was due to the higher amount of nutrient removal seen in a 60 bu/acre scenario. As yield increases, the changes in soil P from year to year increase. This makes a yearly composite sample more valuable as it is able to capture some of these changes. It also suggests that the current method of application is less suited to higher-yielding wheat.

Due to the relatively low cost of a composite sample, yearly composite sampling could be a cost-effective way to increase revenue in higher-yielding wheat. It also decreases over- or underapplication of phosphorus compared to the common practice of keeping the same uniform rate across years. However, phosphorus is still underapplied in parts of a given field, which leads to yield losses. Thus, the optimal uniform rate would involve a yearly composite sample and a phosphorus rate above university recommendation.

The optimal uniform rate was calculated using a yearly composite sample and applying phosphorus above university recommendation. As shown in 7, the optimal amount of phosphorus applied above university recommendations is lower for the optimal uniform rate than for those found in Figure 5. Compared to the common method of uniform rate, the optimal uniform rate increased NPV by \$37.15/acre over the 4-year period when yield and prices were around the Oklahoma average of 40 bu/acre and \$5/bu (Fig. 6a). In a 60 bu/acre system and at a price of \$5/bu, NPV increased on average by \$127.80/acre compared to the common practice uniform rate.

The optimal uniform rate had a higher NPV on average than the common method variable rate at all yields and prices examined. The benefits to the optimal uniform rate increased as yield and price increase. At a price of \$5/bu, the optimal uniform rate's NPV was \$15.33/acre more than a variable rate in a 40 bu/acre system and \$76.90/acre more in a 60 bu/acre system (Figure 6). By using a yearly composite sample and applying an extra amount of phosphorus above university recommendations, a uniform rate can have a higher NPV than that of a variable rate. However, because of the additional P applied, this method did have lower returns than the common method variable and uniform rate

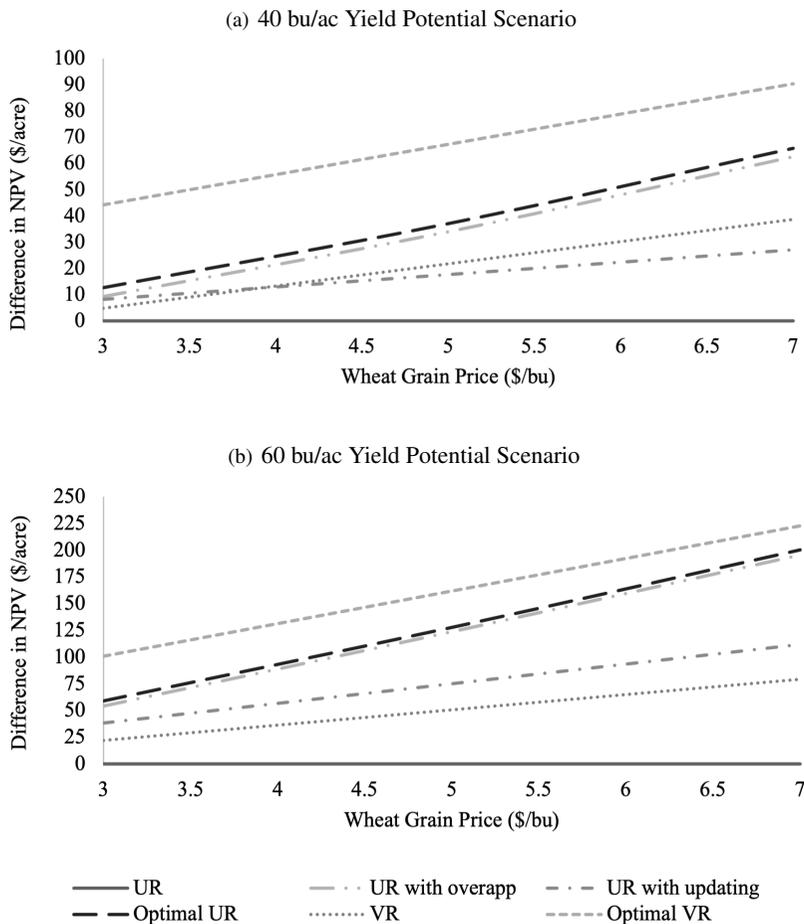


Figure 6. Average Difference in Net Present Value over a 4-Year Period Compared to Uniform Rates and Variable Rates

Notes: Rates are uniform rate (UR), uniform rate with application above university recommendations (UR with overapp), uniform rate with yearly composite sampling (UR with updating), uniform rate with application above recommendations and yearly composite sampling (Optimal UR), variable rate (VR), and a variable rate that uses soil P, phosphorus applied, and yield to update rates yearly (Optimal VR).

when mean soil P was above 35 ppm. Under these conditions, it would be optimal to not apply the additional P.

Last, the optimal variable rate was found by assuming perfect information, where soil P could be determined by using the phosphorus applied, initial soil P, and yield. The variable rate would then change from year to year. For optimal variable rate, NPV was the highest application strategy at any given price or yield. Therefore, there is the potential to greatly increase the profitability of variable rate if soil P levels can be accurately predicted in the subsequent years after grid sampling. However, as with the variable rate, the optimal variable rate did not pay in fields with a high mean soil P (>35 ppm). These fields do not require much phosphorus application and, as such, the yield gains from P application were not enough to offset the cost of grid sampling. However, unlike the common method variable rate, the optimal variable rate did pay compared to the common uniform rate in fields with low mean soil P (<13 ppm). By using a composite sample, it can be determined before grid sampling whether the variable rate should be used, which will reduce the risk of grid sampling when no phosphorus is needed.

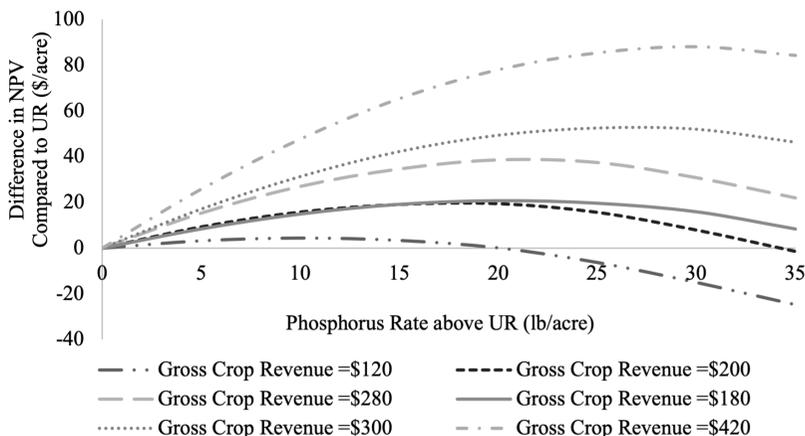


Figure 7. Average Difference in Net Present Value (NPV) When Increasing Phosphorus Rate above University Recommendations (UR) Given the Value of the Crop and When Using a Yearly Composite Sample

Table 3. Average Phosphorus Applied for Variable and Uniform Application Strategies for 4-Year Period (lb/acre)

Yield Potential Scenario (bu/acre)	Wheat Price (\$/bu)	UR ^a	UR with Overapp ^b	UR with Updating ^c	Optimal UR ^d	VR ^e	Optimal VR ^f
40	3	18.76	38.76	30.55	30.55	38.7	40.79
40	5	18.76	78.76	30.55	41.34	38.7	40.79
40	7	18.76	78.76	30.55	53.91	38.7	40.79
60	3	18.76	98.76	53.06	61.49	38.7	64.08
60	5	18.76	118.76	53.06	81.69	38.7	64.08
60	7	18.76	138.76	53.06	81.69	38.7	64.08

Notes: ^a Uniform rate given common method of application.
^b Uniform rate with additional phosphorus application above university recommendations (extra is applied on all fields).
^c Uniform rate with yearly composite sampling.
^d Uniform rate with additional phosphorus application and yearly composite sampling.
^e Variable rate given common method of application.
^f Variable rate that uses soil P, phosphorus applied, and yield to update rates yearly.

Phosphorus Application

Due to the high average soil P of the samples and the common practice of application, a uniform rate applied less phosphorus than a variable rate (Table 3). On average, a uniform rate applied 18.76 lb/acre of P₂O₅ over the 4-year period; a variable rate applied 38.70 lb/acre of P₂O₅ over this period. At low yield and prices, the optimal variable rate applied the highest amount of phosphorus. As the value increased, the optimal uniform rate and a uniform rate that applied phosphorus at rates above university recommendations applied the most phosphorus.

Discussion on Limitations and Further Areas of Research

The yield function has a large impact on the result. Since the interaction terms between different nutrients are not well established, yield functions do not typically include them. These interaction terms can impact what optimal application rates should be. In the case of lime and phosphorus, phosphorus can be a substitute for lime in acidic soils. As shown in Roberts et al. (2006), when considering multiple inputs with variable-rate application, the value of variable-rate technologies can increase. Research on interactions between nutrients, especially in wheat, is rare. For example,

there have only been 11 studies on phosphorus and nitrogen interaction in wheat, with only two of these studies in the last 20 years (Duncan et al., 2018).

There is a large amount of uncertainty in the correct functional form to use, and empirical research has not shown which functional form is best. This makes it difficult to determine the true yield outcomes when simulating yield across time. There may be more variability in the real world than what is assumed here.

Despite the fact that current university recommended rates do not account for soil types, Oklahoma State recommendations were still found to be yield maximizing (Rutter, Arnall, and Watkins, 2017; Watkins, 2017). The inclusion of soil type could increase the profitability of variable rate (Popp et al., 2002). Determining better yield models—and subsequently application rates—would allow for better profit-maximization models.

The composite sample estimated used all grid cell points within the field. This may be more accurate than a typical composite sample taken by a producer. A producer might only take a few samples across the field and thus would have a less accurate test.

These results are impacted by the fields examined. The rate of overapplication above university recommendations used in the optimal uniform rate may be lower if the average soil P is lower since spatial variability decreases when soil P is low.

Further, the fields used in this study were all grid-sampled fields voluntarily sent to us by producers. Therefore, they are a convenience sample and not a random sample. However, most of the research in precision agriculture uses a convenience sample as the field chosen is usually from what is easily available. However, this means that the results may not be representative of the average field. Regardless, this study uses a significantly larger number of fields than past studies and so provides a better picture of the profitability of precision agriculture than those studies that take place over one or two fields.

Conclusion

This study sought to determine when a variable-rate phosphorus application was more profitable than a uniform rate. Data from 147 fields that were grid sampled at 2.5 acres were used to define the distribution of P. The average NPV of alternative application strategies were simulated for a 4-year period. While grid sampling can be profitable, the way grid sampling is usually done was not as profitable as simply applying a little extra phosphorus on the whole field. Dhoubhadel (2021) found little effect of the adoption of precision agriculture on overall farm profitability, which is consistent with our finding that precision agriculture is a marginal investment.

For example, when yield and price are 40 bu/acre and \$5/bu respectively, average NPV over the 4-year period for (i) the same variable rate 4 years was \$638.02/acre, (ii) the same uniform rate for 4 years was \$616.21/acre, (iii) the same uniform rate for 4 years, but with optimal additional phosphorus applied (on all fields) above university recommendations, was \$650.28/acre, (iv) a uniform rate based on a composite sample taken each year was \$633.95/acre, (v) a uniform rate based on a composite sample taken each year and optimal additional phosphorus applied above university recommendations was \$653.35/acre, and (vi) a variable rate using carryover formulas to predict the rate needed on each grid for each year was \$688.02/acre. These results show an improvement in NPV in the last four scenarios over their respective common method scenarios.

The current practice of applying the same amount for a period of 3–4 years means that a variable rate still results in over- or underapplication. This can result in yield losses by year 4 where there is underapplication in certain parts of the field and added costs where there is overapplication. NPV can be increased over this variable rate by using a uniform rate with additional phosphorus applied or with a combination of a yearly composite sampling and additional phosphorus applied.

In order for a variable rate to pay compared to this higher uniform rate, a more accurate variable rate is needed. This can be done using yield monitor data, previous phosphorus application, and soil P data to update variable application rates yearly. Since this information is typically available, a

more profitable variable rate is feasible. A variable rate in which yield, previous phosphorus rates, and initial soil P are used to make yearly phosphorus recommendations had the highest NPV. Such an approach is not commonly used due to its complexity, but the results clearly show that a more precise variable rate is needed in order for variable rate to become more profitable than simply applying a higher uniform rate.

A variable rate applied more phosphorus than a uniform rate. Along with the high price of grid soil sampling, the cost of variable-rate application was higher than that of a uniform rate. Therefore, while variable rates are often marketed as a cost saver, this is not always true. In fields with a mean soil P below 13 ppm and above 35 ppm, a variable rate did not pay on average compared to a uniform rate. Thus, a composite sample could be a cheap way to determine whether to use a variable or a uniform rate. Returns to a uniform rate can be increased by applying a higher rate, yearly composite sampling, or a combination of both.

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