INCOME TAX ASPECTS OF LIQUIDATION IN MULTIPERIOD LINEAR GROWTH MODELS

Donald W. Reid, Wesley N. Musser, and Neil R. Martin, Jr.

Since the mid-1960s, agricultural economists have given much attention to research on farm-firm growth. One procedure used in this type of research has been multiperiod linear programming models,[1] e.g. [1, 6, 9, 11, 14, 17]. Several researchers using the multiperiod linear programming framework have compared optimizing criteria [6, 9, 14]. Generally, comparisons were made between maximizing some type of present value criterion and maximizing net worth at the end of the planning horizon. The different assumptions associated with the two maximizing criteria have resulted in different optimal growth patterns.

Theoretically, both criteria are acceptable objectives. Hicks [10, p. 197] states that the firm should choose the plan which maximizes the present value of the stream of expected net receipts. However, according to Lutz and Lutz [13, p. 42], maximizing net worth is consistent with maximizing profits. In an effort to clarify which criterion to use, Boussard [7] supports the net worth criterion by showing that (1) it is equivalent to maximizing the sum of the present values of consumption of the different periods, (2) it guarantees the existence of a planning horizon under constraint of a linear consumption function, and (3) assumptions in connection with its use are no more arbitrary than those required by the present value criterion.

One problem with the net worth criterion, which has not been addressed in the literature on multiperiod linear growth models, arises from income tax considerations of liquidation. Therefore, the specific objectives of this article are to (1) demonstrate conceptually the problem associated with maximizing net worth at the end of the planning horizon when liquidation occurs, (2) consider methods of handling the problem in a multiperiod linear programming model, and (3) present an empirical example showing the effect of the problem on an optimum farm organization over time.

CONCEPTUAL FRAMEWORK

Boussard suggests that the relevant length of a planning horizon is the time needed to make a decision for the first period [7, p. 468]. This definition implies that anything affecting the first period decision should be included in the planning horizon. Therefore, in setting the planning horizon one should consider the appropriate condition of continuity for (1) a firm with indefinite liquidation plans or (2) a firm which ceases operations and liquidates its assets. These two situations are common in the real world. The former firms are the usual going concerns with no plans to terminate operations, and the latter are firms such as farms with retiring owner-operators and limited-life tax shelter farms.[2] The distinction between these two cases is important for income tax considerations in a planning model with the net worth optimizing criterion. Moreover, the tax effect becomes increasingly important as the end of the planning horizon approaches because of selection of the most favored asset values by the objective function.

To illustrate the point conceptually, consider two firms which have exactly the same asset and liability structure just before the end of the planning horizon. One firm will terminate operations and liquidate assets, whereas the other will continue to operate beyond the planning horizon. If one considers the two cases without income taxes, valuing the assets at market value at the end of the planning horizon yields the same net worth for each situation because the cash value equals the market value. When income taxes are considered, however, the net worth of the ongoing firm stays the same as before, but the firm which liquidates is subjected to regular income taxes on profits from sale of produced goods as well as the capital gains tax from the sale of appreciated capital assets. Hence, in terms of market values, the ongoing firm has a greater

Donald W. Reid is Graduate Research Assistant, University of Kentucky, and former Graduate Research Assistant, University of Georgia, Wesley N. Musser is Assistant Professor, University of Georgia, and Neil R. Martin, Jr., is Associate Professor, Auburn University.

[1] Irwin [12], Walker and Martin [18], and Barry [3] discuss the different types of growth models.

[2] Carman [8] discusses limited-life tax shelter farms whose structure requires liquidation of assets at the end of the planning horizon.

value than the liquidated firm.

The foregoing discussion can be formalized in a theoretical model. To simplify the model, the net worth is calculated on January 1 of the year following the last production year, which abstracts from many features considered in retirement models [5, 16]. All previous production expenses have been paid and the only liabilities are loans.[3] In addition, the model involves the typical assumption that taxable income is calculated on a cash basis [2, p. 29]. The net effect of these assumptions is that only sales of capital assets and inventories of production from previous years are relevant for calculating taxable income at liquidation. The model then includes the following equations.

$$(1)\quad V_{oj} = V_{mj} - V_{bj} \quad \text{if } V_{mj} - V_{bj} \leq V_{pj} - V_{bj}$$
$$= V_{pj} - V_{bj} \quad \text{if } V_{mj} - V_{bj} > V_{pj} - V_{bj}$$

$$(2)\quad V_{cj} = V_{mj} - V_{pj} \quad \text{if } V_{mj} - V_{bj} > V_{pj} - V_{bj}$$
$$= 0 \quad \text{otherwise}$$

$$(3)\quad E_c = \sum_i R_i + \sum_j V_{mj} - L$$

$$(4)\quad E_l = \sum_i R_i(1 - t) + \sum_j V_{mj} - \sum_j V_{oj}(t) - \sum_j V_{cj}(\tfrac{1}{2}\, t) - L$$

$$(5)\quad E_c > E_l$$

where

V_{oj} = ordinary income arising from sale of the j^{th} capital asset
V_{mj} = the market value of the j^{th} capital asset
V_{pj} = the original purchase cost of the j^{th} capital asset
V_{bj} = the accounting book value of the j^{th} capital asset, which equals V_{pj} minus accumulated depreciation
V_{cj} = the capital gains for income tax purposes arising from sale of the j^{th} capital asset
E_c = the equity of the ongoing firm at the end of the planning horizon
R_i = the market value of production inventory from the i^{th} farm enterprise
L = the amount of liabilities at the end of the planning horizon
E_l = the equity of the liquidated firm at the end of the planning horizon
t = the average income tax rate.

Equations 1 and 2 define the major income tax effects of sale of capital assets. V_{oj} is the amount of the gain that represents recapture of depreciation on purchased, depreciable assets, and V_{cj} is the amount of the gain that is taxed as capital gains. On raised capital assets such as breeding stock, $V_{oj} = 0$ and $V_{cj} = V_{mj}$ because V_{pj} and V_{bj} both equal zero on cash basis accounting. The terminology and equations also are based on the assumption that the sale of purchased capital assets results in taxable gains rather than taxable losses ($V_{mj} > V_{bj}$). The combination of asset value appreciation arising from inflation and of tax advantage of rapid depreciation makes this assumption realistic.

As summarized in equation 5, the taxable income arising from liquidation clearly reduces terminal net worth of the firm. For a particular set of assets represented by the R_i's and V_{mj}'s and level of debt (L), E_l will be less than E_c because of the income tax due on gains from sale of these assets. However, this liquidation effect does not reflect the full impact of taxes on terminal net worth. With the objective function represented in equation 4, the firm would be expected to have made production-investment decisions throughout the planning period that reflect the income tax effects of liquidation. Thus, the components of net worth (R_i, V_{mj}, and L) at liquidation would be expected to be different from those with an objective function represented by equation 3.

It must be stressed that the objective function for an ongoing firm (equation 3) does have a conceptual weakness. Even though income taxes on the asset value appreciation are not payable at the end of the planning horizon, these taxes will be due when liquidation does occur. However, incorporation of these deferred taxes in equation 3 presents some difficult conceptual and methodological problems. One theoretically correct method would be to include terms in equation 3 that reflect the discounted value of the deferred capital gains. However, this approach requires a judgment on the appropriate discount rate and time until liquidation. For a farm firm in the early or middle phases of its life cycle, the liquidation time can be estimated only with great uncertainty. Furthermore, the appropriate discount rate for a farm firm under the typical circumstances of capital rationing is the opportunity cost of alternative investments; this rate can only be determined simultaneously with the specification of optimal investments [4]. In addition, the impact of the size of the discount rate for distant terminations — for example, the interest factor for discounting deferred taxes for 20 years at 10 percent is .149 and for 30 years is .057 — would reduce the impact of the deferred taxes on the objective function.

An alternative method of correcting the

[3]This assumption represents the situation in the model used in the empirical analysis. Though it is arbitrary, it was included in the theoretical model so the model would correspond to the empirical analysis.

deficiencies of equation 3 is to define the only relevant terminations of the planning horizon as occurring with liquidation and to always use equation 4 as the relevant objective function. This approach not only has the conceptual problem of the uncertain liquidation time but also presents some serious methodological problems. A long planning horizon can result in a large empirical model with associated computational difficulties. To make the size of the model manageable may require abstracting from some of the detail that is important for decisions early in the planning horizon for which the model has the most relevance.

In summary, equation 3 appears to be an objective function that approximates the theoretically correct objective function for a firm which has a very uncertain distant termination date, whereas equation 4 is appropriate for a liquidating firm. The impact of income taxes in equation 4 would be expected to influence terminal equity for the liquidating firm in two ways: (1) the income taxes reduce the value of the objective function and (2) the taxes can result in production-investment decisions that are different from those in the case of the continuing firm. Most important, a decision on the appropriate liquidation time is necessary for multiperiod models which include income taxes. Without income taxes in the model, equation 3 is appropriate both for liquidating and continuing firms.

TABLE 1. SELECTED 1975 PRICE DATA AND ANNUAL TREND RATES USED IN THE MODEL FARM

Item	1975 Value	Inflationary Trend Annual Rate
Land	475/acre	.0717
Soybeans	5.06/bushel	.06
Corn	2.73/bushel	.04
Variable costs - soybeans	74.08/acre	.05
Variable costs - corn	94.18/acre	.05
Feeder pig facilities (purchase price)[a]	542.60/sow	.05
Market hog facilities (purchase price)[a]	444.47/sow unit	.05
Variable costs - feeder pigs (excludes boar and corn costs)	$738.50/sow/year	.05
Variable costs - market hogs (excludes feeder pigs and corn costs)	234.03/sow unit/year	.05

[a]The market value of depreciable assets was estimated by taking both inflation and depreciation rates into account.

METHODOLOGY

A multiperiod linear programming model of a farm in the Georgia Piedmont was used for an empirical consideration of the two different objective criteria. The model farm had 462 acres of total land with 164 acres of cropland; resident labor of 2500 annual hours was available for labor or management of hired labor, subject to a managerial constraint of 25,000 hours of hired labor. At the beginning of the planning horizon, a complement of machinery was available for production of corn and soybeans. Purchase or rental of additional land was not permitted, and expansion was limited to hog enterprises which included farrow-to-finish, feeder pigs, and market hogs. The firm started with zero debt, but was allowed to incur debt up to 30 percent of the total value of the assets. A fixed amount of $18,000 was assumed for consumption with a 5 percent annual increase to accommodate inflation. The methods of Vandeputte and Baker were used to incorporate the basic provisions of federal income taxes into the model [17]. Machinery and hog equipment were decreased in value to reflect straight line depreciation.

The inflationary pattern of the past 15 years was assumed to continue during the planning horizon of the model with a general increase in production and investment costs, product prices, and market values of land and other capital assets. Selected price and trend data in Table 1 illustrate the assumptions incorporated in the model. Hog prices were projected on the basis of historical price cycles with the relationships between market hog and feeder pig prices reflecting historical relationships in Georgia. These two sets of hog prices are shown in Table 2.[4]

TABLE 2. MARKET HOG AND FEEDER PIG PRICES USED FOR THE MODEL FARM

Year	Market Hog	Feeder Pigs
	$/cwt.	$/cwt.
1975	43.90	70.38
1976	41.00	59.85
1977	40.00	57.77
1978	35.00	48.86
1979	41.00	58.64
1980	40.00	56.49
1981	37.00	50.94
1982	35.00	47.05
1983	43.00	60.13
1984	42.00	57.97
1985	39.00	52.30

[4]Reid [15] provides additional detail on the farm model and discusses the methods used in deriving the price projections.

The aforementioned features of the model were held constant for the empirical analysis. The only difference between the liquidating and ongoing firms was the objective function; the former had equation 4 for an objective function and the latter equation 3. It must be stressed that these objective functions incorporate the assumptions of the theoretical model presented in the preceding section: terminal equity was calculated on January 1 after the final production year and cash accounting was used for income tax purposes. The income tax from liquidation was incorporated into the model by entering the taxable income from liquidation into a set of tax activities specifically for the liquidation process, and then transferring the tax liability from those activities to the objective function.

RESULTS

A summary of the results from the model with and without liquidation is presented in Table 3. The most apparent effect of liquidation is the significantly reduced ending net worth in comparison with the situation without liquidation. Net worth was $912,347 with liquidation and $1,165,809 without liquidation. Part of this difference was the income taxes of $364,790 arising from liquidation. However, the net worth of the liquidating firm at the end of 1985 – $1,134,760 – was also lower than that for the continuing firm because of the expected difference in production-investment decisions throughout the planning horizon. These decisions did reduce the income taxes arising from liquidation. Imputed income taxes from liquidation for the continuing firm were $309,304 which would have resulted in a net worth after liquidation of $856,505. Thus, the production-investment decisions for planned liquidation resulted in an increase in net worth of $55,842 over the amount obtained in the imputed unplanned liquidation.[5]

Before consideration of the different production-investment decisions made in the two situations, it is important to stress the similarities in the firm growth. In both situations initial firm growth occurred by expansion of the market hog enterprise. The level of investment in the market hog operation was lower for

TABLE 3. OPTIMAL ORGANIZATION FOR THE MODEL FARM FIRM, 1975-1985

Item	Units	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	Liquidation Period
With liquidation:													
Net worth	dol.	277,756	387,609	500,287	583,407	747,318	908,219	979,091	952,631	1,095.640	1,188,571	1,134,760	912,347
Adjusted gross income	dol.	36,397	125,600	185,600	89,358	223,638	186,681	27,408	7,900	136,316	38,493	0	364,790
Gross federal taxes	dol.	8,191	57,580	97,180	35,520	123,607	97,925	4,959	450	64,438	9,035	0	222,413
Investment tax credit	dol.	3,833	5,579	6,569	3,591	8,605	6,754	0	0	5,006	0	0	
Operator labor	hour	1,201	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	
Hired labor	hour	0	1,247	3,725	5,080	7,327	9,446	9,446	9,446	10,803	10,803	0	
Debt from long-term security	dol.	102,452	149,292	203,486	233,954	294,922	348,423	305,445	348,464	283,275	313,546	379,165	
Feeder pigs													
Raised	sow	0	0	0	0	0	0	0	0	0	0	0	
Purchased	cwt.	684	3,263	5,388	6,495	8,496	10,384	10,384	10,384	11,593	11,593	1,969	
Sold	cwt.	0	0	0	0	0	0	0	0	0	0	0	
Market hogs sold	cwt.	3,551	16,945	27,980	33,727	44,119	53,924	53,924	53,924	60,200	60,200	10,225	
Corn													
Grow	acre	.37	87	110	0	0	0	.64	0	0	0	0	
Buy	bu.	14,301	68,227	107,456	129,225	177,702	217,192	217,182	217,153	242,472	242,472	41,184	
Sell	bu.	0	0	0	0	0	0	0	0	0	0	0	
Store	bu.	22	5,242	6,618	0	0	0	39	0	0	0	0	
Soybeans													
Grow	acre	110	0	0	110	110	110	110	110	110	110	110	
Sell	bu.	3,188	0	0	0	0	0	0	12,936	0	0	12,636	
Store	bu.	0	0	0	3,199	6,397	9,596	12,777	3,039	6,238	9,437	0	
Without liquidation:													
Net worth	dol.	279,321	382,283	504,878	588,965	755,652	918,665	989,895	960,723	1,130,743	1,232,083	1,165,809	856,505[a]
Adjusted gross income	dol.	33,600	137,328	197,077	90,309	237,667	199,041	32,613	8,405	164,839	21,797	0	488,919[a]
Gross federal taxes	dol.	7,100	65,086	105,099	36,071	133,427	106,454	6,745	553	83,078	3,315	0	309,304[a]
Investment tax credit	dol.	3,959	5,956	6,767	3,416	8,906	7,066	0	0	11,518	323	2,216	
Operator labor	hour	1,046	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	2,500	
Hired labor	hour	0	1,449	3,907	5,180	7,657	9,874	9,874	9,874	13,028	13,249	0	
Debt from long-term security	dol.	102,124	152,003	207,830	237,022	302,376	358,247	312,504	358,128	431,764	363,062	459,657	
Feeder pigs													
Raised	sow	0	0	0	0	0	0	0	0	2	13	87	
Purchased	cwt.	706	3,436	5,626	6,678	8,885	10,860	10,860	10,860	13,622	13,587	0	
Sold	cwt.	0	0	0	0	0	0	0	0	0	0	0	
Market hogs sold	cwt.	3,667	17,843	29,212	34,678	46,139	56,396	56,396	56,396	70,767	70,767	1,385	
Corn													
Grow	acre	90	90	90	0	0	0	0	0	0	40	0	
Buy	bu.	10,738	70,519	112,280	134,296	185,835	227,148	227,148	227,148	285,287	286,708	14,157	
Sell	bu.	0	0	0	0	0	0	0	0	0	0	0	
Store	bu.	1,348	5,380	5,380	0	0	0	0	0	0	2,380	0	
Soybeans													
Grow	acre	0	0	0	90	90	90	90	90	90	50	90	
Sell	bu.	0	0	0	0	0	0	0	13,002	0	0	6,651	
Store	bu.	0	0	0	2,600	5,201	7,801	10,402	0	2,600	4,051	0	

[a]Imputed values rather than model results.

[5]The imputed net worth after liquidation for the continuing firm was calculated external to the model. The income tax resulting from liquidation of the terminal assets determined by the model for this case was calculated under the same assumptions as incorporated in the model for the liquidating situation and was subtracted from the terminal net worth for 1985.

the liquidating firm, but the general production-investment decision pattern remained the same in both cases until 1983. As a reflection of the larger market hog production, the adjusted gross income which is the taxable net farm income, was higher in most years for the firm liquidating. In contrast to the market hog production levels, the liquidating firm produced a higher level of row crops. Generally, the liquidating firm produced about 20 acres more row crops each year and retained the necessary investment in cropping equipment. The pattern of row crop production was similar for the two situations; however, in 1975 soybeans were grown by the liquidating firm and corn was grown by the nonliquidating firm.

The major difference in production-investment decisions occurred in 1983-1985 when the firm not liquidating made a transition from a market hog operation with purchased feeder pigs to a farrow-to-finish system whereas the liquidating firm continued with a market hog operation. This transition began with 2 sows in 1983, 13 sows in 1984, and 87 sows in 1985; this expansion after the initial year occurred from raised gilts. The transition was completed in 1985 when no feeder pigs were purchased. The completed transition in 1985 was partially a result of low hog prices of $39.00 per hundredweight (Table 2) combined with continued inflation in corn and other variable costs (Table 1). The low level of profitability of hogs is reflected in the production levels in both situations being just sufficient to utilize operator labor; unlike previous years, no labor was hired. The low profit situation also resulted in the zero adjusted gross income and taxes in 1985 despite the large sales of stored soybeans.

Because both situations had the same price, cost, and technical possibilities, the tax effect in the objective function was the cause of the transition to farrow-to-finish. The particular tax effect was a result of the sow inventory accumulated from pigs raised on the farm being valued in the objective function of the continuing farm at market value without creating an income tax liability. In the liquidating case income taxes would have to be paid on the capital gains arising from the sale. The sow inventory was a major reason for the previously discussed higher terminal net worth before liquidation for the continuing firm but a lower terminal net worth after liquidation than in the planned liquidation situation.

In summary, the results clearly indicate that significant production-investment differences can arise from the tax effects of liquidation. Though the production-investment differences varied throughout the planning horizon, the major differences occurred during the last two to three years. Recommendations for the early periods therefore would be similar with either objective function; only as liquidation approached would the tax effects make major differences. These implications could be altered considerably if the production-investment alternatives were broadened to include other animal enterprises and purchase of additional land.

SUMMARY AND CONCLUSIONS

The findings demonstrate that the tax effects of liquidation should be incorporated into multiperiod linear farm planning models of liquidating firms when net worth is used as the optimizing criterion. This impact of the income tax is shown both conceptually and by empirical example. The logical income tax effect of liquidation is reduction of the net worth at the end of the planning horizon. However, significant reorganization of the production-investment strategy was shown to occur when income tax effects of liquidation were considered even with the limited growth alternatives in the model.

Several logical extensions of the research reported are apparent. First, the differences in organization could logically be even greater if the growth alternatives were wider. In addition, allowing a broader range of liquidation alternatives, such as gradual reduction of operator labor or installment sales of real estate, would also be likely to alter the production-investment choices. Finally, the results indicate that varying enterprise organization before liquidation should be considered along with various liquidation strategies in retirement planning models.

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