WHY DO FARMERS FORWARD CONTRACT IN FACTOR MARKETS?

John J. Haydu, Robert J. Myers, and Stanley R. Thompson

Abstract

This study investigated farmers' incentive to forward purchase inputs. A model of farmer decision making was used to derive an optimal forward contracting rule. Explicit in the model was the tradeoff between the quantity of input to be purchased in advance, and the remaining portion to be purchased later on the spot market. Results indicated that the primary reasons farmers contract inputs are to reduce risk and to speculate on favorable price moves. A numerical example of fertilizer used in corn production indicated that the size of the price discount was the dominant factor in forward contracting decisions.

Key words: forward contract, decision making, price discount, risk aversion

Most research on responses to agricultural production risk has focused on the output side of the production process, particularly when considering forward and futures contracts (e.g., McKinnon 1967; Chavas and Pope 1982; Anderson and Danthine 1983). Some attention has been given to the impact of risk on factors of production. For instance, Batra and Ullah (1974) show how introducing output price risk into a certainty model alters output levels but leaves relative input quantities unchanged. Robison and Barry (1987) evaluate input demand under four conditions: output price risk; input price risk; input quality risk; and production function risk. They also introduce flexibility by allowing the firm to select one input after the uncertainty is resolved (see also Hartman 1975; Holthausen 1976). This approach allows the decision maker to respond to new or changing conditions. In each of these cases, however, the research has assumed spot factor markets only, with no forward contracting of inputs. But many farmers forward purchase a portion of their inputs, and little is currently known about the economic incentives for this behavior. Is it to manage price risk, to ensure timely delivery of supplies, or to guarantee consistent quality? Forward contracting inputs could facilitate the planning process and allow farmers to diversify purchases over time. A fundamental step necessary to the better understanding of forward purchase transactions is to examine the potential gains from a buyer's point of view. Specifically, what is the underlying incentive which entices some farmers to engage in the practice of forward contracting in factor markets?

This paper investigates the incentive issue by incorporating the forward contracting of inputs into a model of farmer decision making. Explicit in the model is the tradeoff between the quantity of input to be purchased in advance (prior to when inputs are actually allocated) at the forward price, and the remaining portion to be purchased subsequently on the spot market. A numerical example of forward contracting fertilizer used in corn production is used to illustrate the model. The optimal forward contracting decision is characterized in terms of the probability distribution of corn and fertilizer prices, as well as other relevant parameters.

The remainder of the paper proceeds as follows. In the next section, a description of the forward contracting problem facing farmers is presented. Of major concern are the decisions facing contract participants, the economic incentives that underlie the agreement, and the possible tradeoffs involved when operating in a risky environment.[1] The following section presents the model and derives a decision rule for optimal forward contracting of inputs. Finally, the numerical example is presented by estimating an optimal forward contract ratio for fertilizer used in corn production. The optimal ratio is computed over a range of different parameter values in

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[1] An environment is considered risky when it consists of various uncertain events whose outcomes may alter the decision maker's well-being.

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order to illustrate key points about the forward contracting decision.

EXCHANGE IN FORWARD CONTRACTS FOR INPUTS

Forward contracting for inputs is a practice which is usually initiated by the manufacturer. The manufacturer's primary incentive to forward sell is to improve the firm's planning capacity. There are substantial risks surrounding some input markets, particularly regarding future prices and demand. By establishing a portion of future demand in a forward market, the manufacturer is able to plan for a minimum production level and cover variable costs.

Farmers participate in forward contracting inputs because it allows them to lock in a certain price earlier than would be possible otherwise, and because it reduces risks surrounding the quality and timeliness of input deliveries. However, these incentives may not be enough to induce entry into forward contracts. Farmers generally also require a price discount to encourage widespread participation.

The equilibrium forward contract price is largely a function of manufacturing costs, current input prices, expected future input prices, and the preferences of farmers and input manufacturers. Although contracts often vary across firms, typically they are of short duration (less than one year), have a fixed price, and may require substantial advance payment. This financial commitment by the farmer is usually compensated for by a price discount below the current spot price. A 5 to 10 percent discount is common. Once the contract is finalized, an increase in the market price implies an *ex post* gain to farmers whereas a price decline implies an *ex post* loss.

THE MODEL

Consider a three period decision environment consisting of an initial period τ in which inputs can be forward contracted; an intermediate period t in which inputs are allocated to the production process; and a terminal period T in which output is realized. The farmer can forward contract all input requirements in period τ if he or she chooses, or choose not to forward contract so that all input requirements are purchased on the spot market at period t. Alternatively, the farmer may choose to forward contract some proportion of his or her requirements at τ and purchase the remainder on the spot market at t. The rest of this section characterizes a model of how this decision can be made optimally.

The farmer is assumed to be producing a known fixed level of output, y_T, using a single input. Thus, total input requirements, x_t, are known in advance

when the forward contracting decision is made. This is an oversimplification for most actual agricultural production processes which have stochastic yields and which allow adjustment of input levels at various points throughout the process. However, the model can be thought of as the second stage of a two-stage decision process, in which the optimal level of total input allocations is chosen first and the optimal proportion of total requirements to forward contract is chosen second. Furthermore, the fixed output assumption leads to a simple forward contracting decision rule which is straightforward to derive and analyze. Thus, the model provides a useful first step in analyzing forward contracting in factor markets. Multiple inputs can be included in the model without changing any of the main results provided that forward contracts exist for only one of the inputs, and the technology is characterized by a Leontief fixed proportions production function.

The farmer is subject to a pair of budget constraints which define terminal wealth after output has been realized. These constraints can be expressed as:

$$(1) \quad W_t = (1 + r_\tau) [W_\tau - f_\tau b_\tau]$$

$$(2) \quad W_T = p_T y_T + (1 + r_t) [W_t - w_t (x_t - b_\tau)]$$

where W_r is initial wealth; W_t is wealth at the input allocation period t; W_T is terminal wealth; r_τ is the interest rate between τ and t; r_t is the interest rate between t and T; b_τ is the amount of the input forward contracted at τ; x_t is the total amount of inputs purchased by time t; f_τ is the forward contract price; p_T is the output price realized in the terminal period (stochastic at period τ); and w_t is the spot price of the input in the input allocation period (stochastic at period τ). Notice that payments for forward contracted inputs are made in full at the time forward contracting takes place, while payments for inputs purchased on the spot market are not made until period t. Of course, no payment is received for output until production is realized in period T.

The farmer's objective is to choose the amount forward contracted to maximize the expected utility of terminal wealth, conditional on information available at time τ :

$$(3) \quad \max_{b_\tau} E_\tau [U(W_T)]$$

where U is an increasing and strictly concave von Neumann-Morgenstern utility function; and E_τ indicates expectation conditional on information available at τ. The maximization is subject to the wealth

constraints (1) and (2) . The first-order condition for this problem is

$$(4) \quad E_\tau [\, U'(\, W_T\,)K\,] = 0$$

where $K = (\, 1 + r_t\,)\,[\, w_t - (\, 1 + r_\tau\,)f_\tau\,]$. Second-order conditions for a maximum are satisfied by the concavity of U.

A theorem noted by Rubinstein (1976) is useful in analyzing the first-order condition further. Rubinstein's theorem shows that if (W_T, K) is multivariate normal, then

$$(5) \quad \begin{aligned} & Cov_\tau [\, U'(\, W_T\,), K\,] = \\ & \quad E_\tau [\, U''(\, W_T\,)\,]Cov_\tau (\, W_T, K\,). \end{aligned}$$

Using this result, the first-order condition can be written

$$(6) \quad \begin{aligned} & E_\tau [\, U''(\, W_T\,)\,]Cov_\tau (\, W_T, K\,) \\ & \quad + E_\tau [\, U'(\, W_T\,)\,]E_\tau (K) = 0 \end{aligned}$$

or, evaluating $Cov_\tau (\, W_T, K\,)$ and $E_\tau (\, K\,)$,

$$(7) \quad \begin{aligned} & A\,[\sigma^2_w (\, 1 + r_t\,)(x_t - b_\tau) - y_T \sigma_{wp}\,] + \overline{w}_t \\ & \quad - (\, 1 + r_\tau\,)\, f_\tau = 0 \end{aligned}$$

where $A = -E_\tau [\, U''(W_T)\,]/E_\tau [\, U'(W_T)]$ is a measure of absolute risk aversion; σ^2_w is the conditional variance of w_t; σ_{wp} is the conditional covariance between w_t and p_T; and $\overline{w}_t$ is the conditional expectation of w_t.

Multiplying the first term in (7) by $\overline{p}_T y_T / \overline{p}_T y_T$ where $\overline{p}_T = E_\tau (p_T)$ does not change anything but allows (7) to be expressed as:

$$(8) \quad \begin{aligned} & R\,[\sigma^2_w (\, 1 + r_t\,)(x_t - b_\tau) - y_T \sigma_{wp}\,] \\ & \quad + \overline{p}_T y_T [\overline{w}_t - (\, 1 + r_\tau\,)f_\tau\,] = 0 \end{aligned}$$

where $R = A\overline{p}_T y_T$. We interpret R as a measure of relative risk aversion evaluated at a wealth level that equals expected gross income from the production process. Further, below we analyze the sensitivity of the optimal forward contracting rule to alternative values of this relative risk aversion measure.

Solving (8) for b_τ gives the optimal forward contracting rule

$$(9) \quad b_\tau = x_t - \frac{y_T \sigma_{wp}}{(1 + r_t)\sigma^2_w} + \frac{\overline{p}_T y_T [\, \overline{w}_t - (\, 1 + r_\tau)f_\tau\,]}{(1 + r_t)R\sigma^2_w}$$

By dividing through by x_t, the optimal forward contracting rule can be expressed as a ratio of total input requirements:

$$(10) \quad \frac{b_\tau}{x_t} = 1 - \frac{\alpha\,\sigma_{wp}}{(1 + r_t)\sigma^2_w} + \frac{\alpha\overline{p}_T [\, \overline{w}_t - (\, 1 + r_\tau)f_\tau\,]}{(1 + r_t)R\sigma^2_w}$$

where $\alpha = y_T / x_t$ is the average product of the input. We might want to constrain the forward contracting decision by requiring $0 \le b_\tau \le x_t$ (i.e. farmers cannot sell the input either forward or on the spot market). In this case we would use (10) whenever the optimal ratio is between zero and one but set the optimal forward contracting ratio to zero (one) if (10) is negative (greater than one).

The first term in the optimal forward contracting ratio (10) can be interpreted as the "hedging" or risk management part of the decision, while the second term can be interpreted as the "speculative" part. To see this, let the forward contract price equal the discounted expected spot price

$$(11) \quad \overline{w}_t - (\, 1 + r_\tau\,)f_\tau = 0.$$

In this case, there is no expected gain or loss from forward contracting and the forward contract decision is aimed solely at reducing risk. This leads to the pure hedging decision rule

$$(12) \quad \frac{b_\tau}{x_t} = 1 - \frac{\alpha\sigma_{wp}}{(1 + r_t)\sigma^2_w}$$

On the other hand, if the forward contract price is less than the discounted expected spot price,

$$(13) \quad \overline{w}_t - (\, 1 + r_\tau\,)f_\tau > 0$$

then the farmer will speculate by forward contracting more than (12) in order to increase the expected gains from forward contracting. Similarly, if the forward contract price is greater than the discounted expected spot price,

$$(14) \quad \overline{w}_t - (\, 1 + r_\tau\,)f_\tau < 0$$

then the farmer will speculate by forward contracting less than (12) in order to reduce the expected losses from forward contracting. In both of the latter two cases, the farmer is trading off higher expected returns against increased risk. Thus, the two primary purposes of forward contracting by farmers are to reduce risks and to speculate on favorable price moves.

The effect of an increase in relative risk aversion is to diminish the size of the speculative adjustment to the pure hedging forward contract ratio (12). That is, as the farmer becomes more risk averse, he or she will revert towards the risk minimizing rule (less forward contracting if $\overline{w}_t - (1 + r_\tau)f_\tau > 0$ and more forwarding contracting if $\overline{w}_t - (1 + r_\tau)f_\tau < 0$). This is the expected result, that speculative activity decreases with an increase in risk aversion. If the expected gain from forward contracting is zero, then the optimal rule is independent of the farmer's degree of risk aversion.

The optimal forward contract ratio depends on the farmer's degree of risk aversion, the value of the forward contract price relative to the discounted expected spot price, interest rates, the average product of the input, the expected output price, and the covariance matrix of the output and spot input prices. In particular, if the covariance between output and spot input prices is positive, then the larger the covariance, the *less* forward contracting takes place. The reason is that if output and input prices move together, then locking in an input price via forward contracting exposes the farmer to the risk of output price declines without a commensurate decline in input prices. In this case, the farmer avoids risk by forward contracting less rather than more.

NUMERICAL EXAMPLE

As a numerical example of how to operationalize the optimal forward contract ratio, the case of fertilizer used in corn production was considered. This example was designed to illustrate various aspects of the model, particularly with respect to relative risk aversion, the average product of the input, and the price discount received from a forward purchase.

Fertilizer data were for anhydrous ammonia and were obtained from two different sources. Six years of monthly spot prices ($/ton) from 1982 through 1988 were purchased from a private fertilizer information service ("Green Markets"). These data represented agricultural fertilizer prices (FOB) for the midwest corn belt. Forward contract price data ($/ton) were obtained directly from a midwest fertilizer manufacturer. Finally, monthly corn prices ($/bushel) for the same seven years were collected from midwest grain elevators.

To operationalize the optimal forward contract ratio, estimates of all of the terms on the right-hand-side of (10) were needed. Table 1 gives a range of parameter values over which the model was simulated. The average product of fertilizer used in corn production was calculated based on results from Vitosh, Lucas, and Black (1979). That study found

Table 1. Parameter Values

Parameter	Range
α	1000-1800
r	0.10
$\overline{w}_t$	156.26
$\overline{p}_T$	2.36
Price discount	0%-15%
R	1-3
σ^2_w	98
σ_{wp}	0.235

that the application of 115 pounds of nitrogen per acre could be expected to yield approximately 100 bushels of corn. However, anhydrous ammonia contains only 82 percent nitrogen, so it takes 140 pounds of anhydrous ammonia to produce 100 bushels of corn. Converting pounds to tons (to be consistent with the pricing units of $/ton) gives an average product of 1430. A range of 1000 to 1800 around this average was used in this study in order to examine the model's sensitivity to changes in this parameter.

The annualized interest rate was chosen to be 10 percent. To compute r_τ and r_t from this rate, the time intervals between forward contracting and fertilizer application, and between fertilizer application and corn harvest must be known. This study assumed that forward contracting occurs in February, that fertilizer is applied in May, and that harvest is in August. Thus, assuming continuous compounding,

(15a) $1 + r_\tau = e^{0.25r} = 1.025$

(15b) $1 + r_t = e^{0.25r} = 1.025$

The expected corn and spot fertilizer prices in Table 1 were computed by taking a simple sample mean of the corn and spot fertilizer price data described earlier. The forward contract price was then calculated by applying a price discount to the expected spot fertilizer price. That is, if the price discount for forward contracting was d, then

(16) $f_\tau = \dfrac{\overline{w}_t}{(1 + d)}.$

The price discount for forward contracting used in the numerical example ranged between 2.5 and 2.7 percent.

Because fertilizer forward contracted is usually sold at a price discount, the optimal forward contract ratio is likely to be quite sensitive to the degree of farmer risk aversion. To examine this sensitivity, relative risk aversion ranged from $R = 1$ to $R = 3$,

values consistent with those estimated by Friend and Blume.

The final piece of information required to simulate the forward contracting rule was the conditional covariance matrix of corn and spot fertilizer prices. These parameters were estimated from a bivariate time-series model of corn and fertilizer prices using the data described earlier. A time-series approach was appropriate because the covariance matrix should be conditional on information available in the forward contracting period (Myers and Thompson 1989). Plots of the estimated correlograms of the data suggested a strong possibility of nonstationarity in both the corn and fertilizer price series, but that they were stationary after first differencing. Unit root tests developed by Dickey and Fuller were also applied, and the null hypothesis of a unit root could not be rejected for either series (Table 2). This indicated that the series were stationary after first differencing. The time-series model was therefore estimated in first difference form. The final specification arrived at was a bivariate vector autoregression (VAR):

$$(17a) \quad \Delta w_t = \gamma_{10} + \gamma_{11}\Delta w_{t-1} + \gamma_{12}\Delta p_{t-1} + \gamma_{13}\Delta w_{t-12} + \gamma_{14}\Delta p_{t-12} + \varepsilon_{1t}, \text{ and}$$

$$(17b) \quad \Delta p_t = \gamma_{20} + \gamma_{21}\Delta w_{t-1} + \gamma_{22}\Delta p_{t-1} + \gamma_{23}\Delta w_{t-12} + \gamma_{24}\Delta p_{t-12} + \varepsilon_{2t},$$

where $\Delta w_t = w_t - w_{t-1}$ and $\Delta p_t = p_t - p_{t-1}$. The 12 month lags were included to account for apparent seasonality in the data. Estimation results are provided in Table 3. Using the Box-Pierce Q-statistic, the null hypothesis of white noise residuals in both (17a) and (17b) could not be rejected (Table 3).

The covariance matrix of the one-step-ahead forecast errors from the VAR is standard regression output in most econometric software. Because the data were monthly, however, these one-step-ahead forecast error covariances were not appropriate for the problem at hand. As discussed earlier, it was assumed that forward contracting of fertilizer occurs in February, that fertilizer application occurs in May, and that harvest is in August. Thus, an estimate of the covariance matrix of the errors from a three-month-ahead forecast of fertilizer prices (February-May) and a six-month-ahead forecast of corn prices were needed (February-August). This is because the analysis required the variance of the spot fertilizer price at application and the covariance between the spot fertilizer price at application and the corn price at harvest, both conditional on information available when forward contracting takes place.

Table 2. Dickey-Fuller Unit Root Test Results

$$\Delta z_t = \delta_0 + \delta_1 z_{t-1} + \delta_2 \Delta z_{t-1} + \delta_3 \Delta z_{t-2} + \delta_4 \Delta z_{t-3} + v_t$$

	$\hat{\delta}_1$	t-Value	p-Value
Fertilizer Price	-0.0511	1.565	>.10
Corn Price	-0.0370	1.240	>.10

Note: > .10 indicates p-value is greater than .10

Table 3. Estimated VAR for Corn and Spot Fertilizer Prices

Independent Variables	Dependent Variables	
	Δw_t	Δp_t
Constant	-0.004 (1.124)	-0.007 (0.020)
Δw_{t-1}	0.433 (0.124)	0.003 (0.002)
Δp_{t-1}	-7.199 (7.12)	0.112 (0.129)
Δw_{t-12}	0.194 (0.131)	-0.002 (0.002)
Δp_{t-12}	-2.226 (6.819)	0.274 (0.122)
Q Statistics of Residuals	5.50 (6 lags) 11.75 (12 lags)	5.81 (6 lags) 10.36 (12 lags)

Note: Numbers in parentheses are standard errors.

The relevant forecast errors were computed by generating three and six month forecasts from the VAR model and subtracting these from actual price realizations. The sample covariance matrix of the errors constructed in this way was then used as an estimate of the required conditional variance and covariance, leading to the estimates in Table 1.

Results from simulating the model over the relevant range of parameters are shown in Table 4. A striking feature of the results is that forward contract ratios were very sensitive to the price discount offered on forward contracts. If the price discount was less than or equal to 2.5 percent, then the optimal forward contract ratio was zero (no forward contracts) over the entire range chosen for other parameters. The reason is that fertilizer prices in period 2 and corn prices in period 3 were positively correlated. Thus, the farmer obtained a natural hedge from his or her open positions by *not* forward contracting. If corn prices fell (rose), then fertilizer prices were also likely to fall (rise), thereby mitigating some of the detrimental (beneficial) effects of the corn price change. This natural hedge was lost if the farmer locked in a fertilizer price via forward contracting, because the farmer was then fully exposed to the risk of corn price decreases (and increases). Thus, in this

Table 4. Optimal Forward Contracting Ratios Under a Range of Alternative Parameter Values

Parameter Values	Price Discount				
	2.5%	2.55%	2.6%	2.65%	2.7%
$R = 1.0$					
$\alpha = 1000$	0	0.45	1	1	1
$\alpha = 1400$	0	0.23	1	1	1
$\alpha = 1800$	0	0.01	1	1	1
$R = 2.0$					
$\alpha = 1000$	0	0	0.45	1	1
$\alpha = 1400$	0	0	0.23	1	1
$\alpha = 1800$	0	0	0.01	1	1
$R = 3.0$					
$\alpha = 1000$	0	0	0	0.45	1
$\alpha = 1400$	0	0	0	0.23	1
$\alpha = 1800$	0	0	0	0.01	1

Note: Optimal forward contracting ratios restricted to lie between zero and one.

example, the risk minimizing forward contracting rule was to forward contract nothing.

Changes in the average product also affected the forward contract ratio. For instance, an increase in the average product would imply that less fertilizer was required for a given level of output. Therefore, total fertilizer needs would decline, including the need to forward contract.

At price discounts above 2.5 percent, small fluctuations in the discount led to wild swings in forward contracting decisions. Forward contracting declined as risk aversion increased for a given price discount. This decline occurred because farmers were using forward contracts to speculate on favorable price movements, with more forward contracting taking place as the price discount increased. Increased risk aversion curbed this speculative activity and caused the forward contracting decision to move towards the risk minimizing choice of zero forward contracts. If the price discount was greater than or equal to 2.7 percent, then all input requirements were forward contracted unless farmers were very risk averse (relative risk aversion greater than three). Thus, the size of the price discount is the key determinant of forward contract decisions for fertilizer in this example.

CONCLUSIONS

Forward contracting of inputs is a growing activity between the suppliers of inputs and the farmers who use them. From a manufacturers' viewpoint, a major incentive to forward sell is an enhanced planning capacity. Not only does this improve production efficiency, but it also reduces potential bottlenecks in distribution during peak periods of demand. Yet little is currently known about the economic incentives of farmers for participating in this form of exchange. As with the manufacturer, is it primarily to reap the benefits of improved planning, or is it (also) to reduce price risk through diversified purchases, or to ensure reliable supplies and quality?

This paper focuses on the decisions facing farmers who forward purchase inputs. A simple model was used to derive an optimal rule for forward contracting. The optimal forward contracting rule indicated that the two primary reasons farmers might participate in forward contracts are to reduce risk (the hedging component) and to speculate on favorable price moves (the speculative component). Speculative activity is curtailed as farmer risk aversion increases. At the limit, as farmer risk aversion increases to infinity, the optimal forward contracting rule reduces to a variance minimizing rule, which depends on the average product of the input and the joint distribution of input and output prices.

A numerical example of forward contracting fertilizer used in corn production indicated that the size of the price discount was the dominant factor in forward contracting decisions. With no price discount, no fertilizer was forward contracted. Furthermore, small changes in the price discount had large effects on the amount of fertilizer forward contracted, tending to swamp the effects of changes in other parameters in the model. This supports the view that price discounts, not risk aversion or hedging potential, are the crucial element in the forward contracting market for fertilizer.

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