TEMPERATURE PROBABILITIES AND THE BAYESIAN 'NO DATA' PROBLEM

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Weather constitutes an exogenous factor in agriculture which may have considerable influence on production and marketing. For a particular commodity, weather may influence quantity produced, quality of the commodity marketed, and consequently influence prices received (or paid) by various firms associated with that commodity system. Although some has been written about the influence of weather on agriculture [6, 9, 10, 11, 13, 17], little economic analysis is available which attempts to integrate estimated probabilities of some weather phenomenon (a notable exception is McQuigg and Doll [11]). This latter situation may be attributed, at least partially, to the complexities of such an integrative analysis.

This paper examines one possible procedure for integrating temperature probability estimates into an analysis of decisions under uncertainty, which reduces the problem to one of risk. Probabilities of low temperature are utilized in a Bayesian context to illustrate decision-making concerning freeze protection in citrus.

TEMPERATURE PROBABILITIES

Risk and Uncertainty

The two types of outcomes or eventualities which influence plans for the future of every business firm are risk and uncertainty. If each outcome is unknown but occurs with a known probability distribution, the situation is regarded as risk. If each outcome is unknown and the probability of occurrence of each outcome is unknown the situation is regarded as an uncertainty [7].

Freeze Damage

The distinction between risk and uncertainty is a useful one when temperature probabilities and freezes are considered. Freeze damage or loss to a particular commodity may be regarded as an uncertainty while the occurrence of a particular low temperature (or range of temperatures) may be regarded as a risk. This is because actual freeze damage or loss to a crop is a function of other variables, in addition to temperature, and all possible actions which could be taken for prevention of freeze damage. In reality, this simply means that if the outcome is regarded as freeze damage it may be classified as an uncertainty. However, if the outcome is regarded as a temperature occurrence (or range of temperatures) then that outcome may be regarded as a risk.

Some of the other variables which are functionally related to freeze damage in citrus are best stated by Orton [12, p. 19]:

> There are no easily defined criteria by which the severity of freezes in a given region may be judged. The inter-relationship between a very large number of micrometeorlogical and physiological factors are too complex. In the case of citrus, the occurrence of freeze injury in the simplest terms is influenced by a relationship among the critical tissue temperature, the severity and duration of freeze temperatures, the amount of stored heat and the presence or absence of wind during the freeze. Critical temperatures vary with tissue age, type, condition, variety and nutrition. Meteorlogical factors and cultural practices affect dormancy and cold hardiness, which in turn affects critical temperatures.

It is obvious from the above that freeze damage as an outcome with known probability would require estimable relationships among a complex of

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inter-related variables. Each of these other variables possess probability distributions but many would be difficult to quantify.

As a naive model, however, temperature probability could be considered an *indicator* of the outcome freeze damage. This leads directly to a consideration of the quantification of low temperature occurrence.

Computational Aspects

Calculating probabilities of low temperature occurrence in some relevant geographic area may be accomplished by evaluating extreme minimum temperature data utilizing the statistics of extremes. The methodological framework for this approach was originally constructed by Lieblein and others [3, 4, 8]. The appropriate statistical distribution for extreme temperatures is the Fisher-Tippett Type I distribution [16]. The probability density function (pdf) of the Fisher-Tippett Type I distribution for minimums is given by [5, p, 113]:

$$(1) \qquad f(x, \alpha, \beta) = -\frac{1}{\beta} \exp\left[-\frac{1}{\beta}(x-\alpha) - e^{-(x-\alpha)/\beta}\right]$$

where $-\infty < x < \infty$, $-\infty < \alpha < \infty$, and $\beta < 0$. The corresponding cumulative distribution function (cdf) is:

$$(2) \qquad F(x) = 1 - \exp\left[-e^{-(x-\alpha)/\beta}\right]$$

where $\beta<0$. The parameters α and β of the cdf are estimated by Lieblein's fitting procedure [8]. Parameters estimated by Lieblein's procedure are "unbiased and as efficient as possible" [4, pp. 223-226].

The author has written a computer program to calculate the value of the reduced variate $(x-\alpha)/\beta$ of the cdf [14]. Using either extreme minimum temperature input data for a month, a group of months (season), or yearly, the program computes the value of the reduced variate first for the maximum extreme minimum temperature in the data set. The value of the reduced variate is then computed in unit (integer) decrements of x to the minimum extreme minimum. That is, $F(x_o)$ is computed for each integer value of x_o in the data range from[1]:

$$(3) \qquad F(x_o) = \int_{x_o}^{\infty} f(x, \alpha, \beta)\, dx,$$

where x_o represents a particular temperature. This calculation will yield the probability (P) that a temperature equal to or less than x_o will occur as $P(x_o)=1-F(x_o)$. A return period, T(x) can also be computed by:

$$(4) \qquad T(x_o) = 1/P(x_o).$$

$T(x_o)$ is the number of time periods which will elapse, on the average, before a temperature equal to or less than x_o will occur. Examples of $P(x_o)$ and $T(x_o)$ for extreme minimum temperature data for a citrus season from the Weslaco, Texas, weather reporting substation are shown in Table 1.

For example, to interpret the probabilities in Table 1, if $x_o = 21°$ then $P(21°) = .045$ and $T(21°) = 22.1$ This means that the probability of a temperature equal to or less than 21° occurring from November through March in Weslaco is .045. Also, on the average, 22.1 November through March seasons (years) would elapse before a temperature equal to or less than 21° would occur.[2]

THE BAYESIAN "NO DATA" PROBLEM

Temperature probabilities are amenable to integration with the Bayesian decision model. Suppose the most simplistic case of decision-maker faced with choosing an optimal course of action with respect to investing in freeze protection for his citrus grove. Let A_i represent action concerning freeze protection. Let θ_j represent the occurrence of n alternative states of nature with respect to temperature. Then,

$$(5) \qquad \lambda_{ij} = f(A_i, \theta_j)$$

[1]Utilization of this functional form is the common practice in climatology. See [14, pp. 5-8] for a more extensive discussion.

[2]There is a subtle distinction which should not be ignored when interpreting these statistics. Since the input data are extreme minimum temperatures (i.e. lowest temperature recorded per unit time), $P(x_o)$ is technically the probability of a temperature equal to or less than x_o occurring and which is also the extreme minimum per unit time. Another way to compute the probability of occurrence of a low temperature would be to use occurrence of temperatures (rather than minimums per unit time as input data). Such probabilities would always equal or exceed the probabilities computed from minimums. In practice, however, the distinction is not of major import since low temperatures are the focal point of the analysis. This is true because the lower the temperature the more likely it is to be the minimum per unit time.

Table 1. PROBABILITY AND RETURN PERIOD FOR SELECTED MINIMUM TEMPERATURES, WESLACO, TEXAS, NOVEMBER THROUGH MARCH.

Temperature x_o	Probability of Temperature x_o or Below Occurring	Return Period $T(x_o)$
35	.993	1.0
34	.971	1.0
33	.921	1.1
32	.837	1.2
31	.728	1.4
30	.606	1.6
29	.487	2.1
28	.380	2.6
27	.290	3.4
26	.218	4.6
25	.161	6.2
24	.118	8.5
23	.086	11.6
22	.063	16.0
21	.045	22.1
20	.033	30.7
19	.023	42.6
18	.017	59.3
17	.012	82.6
16	.009	115.1

Source: Computed from monthly extreme minimum temperatures for the 50-year period 1920-21 to 1969-70. Data obtained from the Texas Agricultural Experiment Station, Weslaco, Texas.

where λ_{ij} is an outcome. Attention must focus on θ_j.

If temperature probability as computed above is regarded as an indicator of freeze damage then $P(\theta_j)$ may be regarded, from (3), as:

$$(6) \qquad P(\theta_j) = \int_{x_o'}^{\infty} f(x, \alpha, \beta)\, dx - \int_{x_o}^{\infty} f(x, \alpha, \beta)\, dx$$

where θ_j is the occurrence of a temperature between x_o' and x_o, given $x_o' < x_o$. Equation (6) represents Bayesian objective a priori information concerning the probability distribution of the states of nature. Derivation of a Bayesian decision would thus be to select the action A_i for which expected utility, $\hat{u}_i$ is a maximum; where

$$(7) \qquad \hat{u}_i = \sum_j u_{ij} P(\theta_j)$$

and where $u_{ij} = g(\lambda_{ij})$ [3]. This derivation is referred to as the "no data" problem [1, p. 113].

If some a posteriori probability distribution, $P(\theta | \psi)$, can be calculated by performing an experiment ψ (with results ψ_k, k = 1, 2, ..., n) that serves as a predictor of θ then the "data" strategy would be to select the action A_i for which expected utility $\hat{u}_i^k$ is a maximum; where

$$(8) \qquad \hat{u}_i^k = \sum_j u_{ij} P(\theta_j | \psi_k).$$

However, with the case of citrus freeze protection, it is difficult to construct a predictive model in which ψ is a precise indicator of θ.

[3]Where u_{ij} is some linear transformation of λ_{ij}.

An Application

The derivation of a Bayesian decision utilizing a priori information in the form of temperature probabilities may be illustrated by a hypothetical yet realistic example involving a Weslaco, Texas, grapefruit grove. For simplicity, the example is limited to a 2 x 2 matrix where:

A_1 = the action "no freeze protection"
A_2 = the action "freeze protection"
θ_1 = the state of nature "no minimum temperature occurrence below 22°"–no freeze damage
θ_2 = the state of nature "minimum temperature occurrence below 22°"–freeze damage.

In this example, u_{11} and u_{21} are relatively easy to quantify. Considering u_{11} the Texas Agricultural Extension Service [15] has recently computed net return per acre for Texas grapefruit under typical management at \$192. Of course, u_{11} is greater than u_{21} with the difference attributable to the total cost of a freeze protection system.

The magnitude of u_{21} can be readily estimated using costs of a freeze protection system reported by Connolly [2, p. 146]. Assume this system will protect a grove from damage below 22° temperatures. The costs involved are:

1. \$487 per acre original investment in a cold protection system.
2. \$20 per acre annual maintenance costs (considered as depreciation) which retains the value of the original investment at \$487 per acre.
3. \$182 per acre cost of firing cost restoration of the system. Restoration cost is assumed to bring the system back to the original \$487 per acre investment.

With an opportunity cost factor of 10 percent per year assumed, and a 2 percent factor for risk, insurance, and taxes, the per acre annual fixed cost of the freeze protection system would be \$34.09[4]. Including \$20 per acre depreciation, the per acre annual fixed cost of the system would be \$54.09. Thus, u_{21} is estimated at \$137.91, or \$138 by rounding.

The more difficult estimates are u_{12} and u_{22}. For the example, the magnitudes were derived under these assumptions:

1. zero own-price flexibility for Texas grapefruit.
2. net return per acre on a non-protected grove for a freeze year is -\$1500. This includes production costs incurred during the year of the freeze, costs to re-establish grove, and the present value of lost net revenue until grove is back to production level preceding the freeze.

Under the first assumption, the difference between u_{21} and u_{22} will be the cost of firing and restoration of the freeze protection system. Thus, u_{22} is estimated as \$138 - \$182 = -\$44.

The second assumption results in u_{12} = -\$1500 which is at least correct in sign but may not be precise in magnitude. Reliable and typical data are sparse on costs incurred after a severe freeze on Texas groves.

The Bayesian decision under the above conditions is derived utilizing $P(\theta_1) = .955$ and $P(\theta_2) = .045$, from Table 1. With measurement in dollars, the expected utility is a maximum for $\hat{u}_2$, thus action A_2 is selected (Table 2).

Table 2. DERIVATION OF A BAYESIAN "NO DATA" DECISION UTILIZING TEMPERATURE PROBABILITIES FOR A WESLACO, TEXAS, GRAPEFRUIT GROVE.

Action	State of Nature		Expected Utility Measured in Dollars $\hat{u}_i$
	No Freeze $\theta_1 > 21°$	Freeze $\theta_2 \leq 21°$	
A_1 No Freeze Protection	\$192	\$ - 1500	\$115.86
A_2 Freeze Protection	\$138	\$ -44	\$129.81

Limitations

The suggested model is theoretical even though objective information concerning $P(\theta)$ is relatively easily obtainable from extreme minimum temperature analysis. The weakest part is quantification of u_{i2}.

Another difficulty, already noted, is that in the model θ_j is regarded as the occurrence of a particular range of temperatures rather than the occurrence and extent of freeze damage to the commodity. This, however, could be remedied by additional research on the quantification of relationships among all those variables which affect freeze damage.

[4] (1/2 of 10% = 5% + 2% = 7% times \$487 = \$34.09.)

CONCLUSIONS

The usefulness of temperature probabilities in a Bayesian "no data" problem has been illustrated using freeze protection in citrus as an example. Since such probabilities are relatively easy to compute, objective a priori information for a Bayesian model is readily attainable. The advantage of the computational procedure outlined for calculating temperature probabilities is that the geographic area is specific to the location of the commodity and the relevant time period for considering low temperatures is essentially unrestricted. That is, probabilities for a month, a group of months (season), or for a year may be computed. Probabilities such as these can also be a basic input into a simulation model of the costs and returns associated with freeze protection in the citrus industry.

The procedures outlined above provide a logical prerequisite analysis for more complicated models of decisions under uncertainty which involve weather phenomena.

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