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Incorporating risk in a positive mathematical programming framework: a dual approach

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In this study we develop a new methodological proposal to incorporate risk into a farm-level positive mathematical programming (PMP) model. We estimate simultaneously the farm nonlinear cost function and a farmer-specific coefficient of absolute risk aversion as well as the resource shadow prices. The model is applied to a sample of representative arable crop farms from the Emilia-Romagna region in Italy. The estimation results confirm the calibration ability of the model and reveal the values of the individual risk aversion coefficients. We use the model to simulate different scenarios of crop price volatility, in order to explore the potential risk management role of an agri-environmental scheme.

Key words: agri-environmental schemes, farm behaviour, positive mathematical programming, risk aversion.

1. Introduction

Risk is an important component of agricultural activities, since it affects farmer production choices. Hardaker *et al*. (1997) classify risk in agriculture as production, market, institutional, personal and financial risk. Under a risky environment, the decision-maker makes the choices based on his expectations of uncertain outcomes and these expectations are often based on past experiences. Many empirical studies show that the farmer is a risk-averse agent as he is willing to sacrifice some income to ensure against the risky consequences (Feder 1980; Sckokai and Moro 2006). Given the risk-averse attitude of farmers, the recent increase in price volatility on world and European Union (EU) markets is perceived as negative as it makes farmer income uncertain. These unpredictable price variations may lead to nonoptimal production decisions in the short run and may discourage farm investments, leading to a decrease in farm profitability and competitiveness in the medium to the long run. Since risk is a structural component of agricultural production and the farmer is not a risk neutral agent, ignoring risk in modelling farmer behaviour is likely to lead to biased results.

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In a mathematical programming model, risk faced by farmers can be introduced either by randomising the behaviour of input and output prices or by introducing uncertainty in the supply of limiting inputs, as well as in the technical coefficient specification. There are different techniques to accommodate risk in a mathematical programming framework, such as the mean-variance approach (Freund 1956; Coyle 1999), the minimisation of the total absolute deviations (MOTAD) (Hazell 1971), the target MOTAD (Tauer 1983), the chance constrained programming (Charnes and Cooper 1959) and the discrete stochastic sequential programming (Kaiser and Messer 2011).

Besides the inclusion of risk, another important issue in farmer behaviour analyses is the ability of the model to calibrate to the observed base year situation. Although normative mathematical programming models, which lack any calibration, dominated the efforts in agricultural economics modelling for decades, nowadays a wide divergence between the modelled outcome and the observed outcome is unacceptable in policy analysis. Although the addition of a risk term in a normative mathematical programming model may improve the model performance and may overcome the overspecialisation problem, typical of linear programming, it is often not enough to reproduce the observed farmer production decisions. Positive mathematical programming (PMP) is a powerful calibration method formalised in the 1990s with the aim of overcoming the drawbacks of normative models. The PMP method is able to recover a nonlinear cost function, or alternatively a nonlinear yield function, that allows to exactly reproduce the observed activity levels and to provide information about the effect of parameter changes on farm input allocation (Howitt 1995).

In this study, we develop a new methodological approach which partially draws from the few previous attempts of incorporating farm risk in a farm-level PMP model and combines them in an innovative and consistent way. Given the importance of accounting for risk in farm-level analysis and the powerful calibration ability of PMP, the incorporation of risk in a PMP framework is one of the new research frontiers in farmer behaviour analyses. So far, there have been limited attempts in the literature to introduce risk modelling in a PMP framework (Paris and Arfini 2000; Cortignani and Severini 2012; Jansson *et al.* 2014; Petsakos and Rozakis 2015). This may be explained by the difficulties in estimating two different nonlinear terms in the objective function, the cost function and the risk component. The idea of combining risk modelling with PMP relies on the information contained in the farm nonlinear cost function estimated in the PMP procedure. As this cost function incorporates any type of model misspecification, data errors, aggregation biases, price expectations and risk behaviour (Henry de Frahan *et al.* 2007), it should be possible to isolate the risk component from the farm nonlinear cost function in order to study specifically the impact of risk on farmer choices. Our proposal addresses this issue by merging the first linear step with the second nonlinear step of the standard PMP procedure in a farmer expected utility maximisation problem. This allows us to estimate simultaneously the farmer risk aversion coefficient, the farm nonlinear cost function as well as the shadow prices of limiting resources (e.g. land) by exploiting the dual optimality conditions. Our proposal still develops from the original PMP set-up (Howitt 1995; Paris and Howitt 1998) while avoiding its drawbacks and potential inconsistencies (Heckelei and Wolff 2003).

The proposed model is applied to a sample of representative arable crop farms from the Emilia-Romagna region in Northern Italy, with the aim of checking the ability of the model to calibrate to the base year-observed activity levels and to estimate the farmer specific absolute risk aversion coefficient, the parameters of the quadratic cost function and the resource and activity shadow prices. Then, the calibrated model is used to perform some simulations of different crop price volatility scenarios, aiming at checking the farmers' reaction to changes in market conditions and to explore the potential risk management role of a specific agri-environmental scheme (AES), the option to convert a share of cropland to grassland. AESs are the measures which take the highest share of the EU budget allocated to Rural Development Programmes, and several studies have analysed the impact of their adoption on farm performances (Pufahl and Weiss 2009; Arata and Sckokai 2016). However, no study has investigated their potential role as a tool to cope with risk at farm level. The idea is that, since the adoption of AESs guarantees a fixed payment to farmers independent of market conditions and crop yields, these measures may act as an insurance against price and yield risk.

The paper is organised as follows: Section 2 presents the existing literature on PMP and the attempts to incorporate risk in PMP models; section 3 describes our methodological proposal to integrate risk in a PMP model; section 4 details the empirical model and data, while in section 5 we analyse the results of the calibration and of the simulations; section 6 discusses the results and draws the main conclusions.

2. Risk in PMP models

The standard PMP approach is a three-step procedure which uses the dual information provided by the calibration constraints of the first step to recover a farm nonlinear cost function, or alternatively, a nonlinear yield function, which calibrates the model to the observed activity levels. Although PMP was already applied in the 1980s in agricultural economic analyses, the first scientific publication which formalised this methodology dates back to 1995. Following the seminal papers by Howitt (1995) and Paris and Howitt (1998), there have been many methodological developments in the domain of PMP, aiming to improve the standard approach. Paris and Arfini (2000) deal with the problem of zero activity levels in some farms, proposing the self-selection approach. Paris (2001) proposes the symmetric positive equilibrium problem (SPEP) as a way to avoid a linear representation of the technology and to make demand and supply of fixed inputs responsive to output levels and input price changes.

One of the most important developments to the original PMP approach is proposed by Heckelei and Wolff (2003), who argue that the first step of PMP leads to inconsistent parameter estimates when more observations are included. The authors suggest to skip the first step and to employ directly the first-order conditions of the desired programming model to estimate simultaneously the nonlinear cost function and the dual values of the constraints. The work by Heckelei and Wolff (2003) represents a remarkable attempt to join mathematical programming model with econometric techniques within the new framework of 'Econometric Mathematical Programming'. Empirical applications of their method can be found in Henry de Frahan *et al*. (2007), Buysse *et al*. (2007) and Jansson and Heckelei (2011). Röhm and Dabbert (2003) propose, in the standard three-step PMP set-up, a way to account for stricter substitution relationships of the same crop grown under two different technologies (e.g. conventional wheat and wheat under agri-environmental schemes). Kanellopoulos *et al*. (2010) extend the standard PMP approach in order to overcome two important drawbacks. Their proposal avoids a zero shadow value for the least profitable activity, guaranteeing a marginal return of the binding resource equal to the average gross margin of the production plan, and allows the integration of supply elasticities in the calibration process.

Another extension of the PMP approach is represented by the model proposed by Arfini and Donati (2011) and discussed in the book by Paris (2011: 397–404). These authors merge the first linear step with the second nonlinear step of PMP in order to estimate simultaneously the parameters of the nonlinear cost function, the shadow price of resources and the differential marginal costs. The same authors provide also an empirical application to the analysis of the 2008 Common Agricultural Policy (CAP) reform. Doole and Marsh (2014) criticise the standard three step PMP approach by showing that when used to calibrate the New Zealand Forest and Agriculture Regional Model (NZ-FARM), the scenario analysis leads to inconsistent results. The authors provide a set of recommendations to improve the reliability of simulation results.

The use of exogenous supply elasticities in the calibration process is another step towards the calibration of ill-posed mathematical programming models. So far the attempts to use exogenous elasticities in the calibration process are limited and most of them perform a 'myopic' calibration by holding the shadow values of the constraints constant to price changes. Mérel and Bucaram (2010) derive the analytical expression of an exact, non 'myopic', implicit supply elasticity from a programming model. The non'myopic' elasticity accounts for the change in the constraints shadow values due to a crop price change. In addition, the authors derive the necessary and sufficient conditions under which a programming model with Leontief technology and quadratic adjustment costs can be calibrated against a set of exogenous own-price elasticities. A follow-up work is Mérel *et al*. (2011), which derives the closed-form expression of the exact implied supply elasticity of a generalised constant elasticity of substitution (CES) programming model, as well as the necessary and sufficient conditions to make the exact calibration of the model against exogenous elasticities feasible.

As stated above, a relatively new research frontier in the area of mathematical programming concerns the integration of risk modelling in a PMP framework. The idea comes from the information contained in the dual values of the calibration constraints of the standard PMP. As this information captures also the risk behaviour, it should be possible to make the risk component explicit and separate it from the other nonlinear cost components. This would allow to identify the farmer attitude towards risk and the role of risk in farmer choices, as well as to perform simulations under different risk scenarios. So far, there have been a few studies in this direction.

The first attempt is by Paris and Arfini (2000), who introduce risk in a PMP model relying upon the mean-variance approach proposed by Freund (1956). Although their study focused the attention of agricultural economists on this new challenge, the authors simply apply an exogenous absolute risk aversion coefficient in a constant absolute risk aversion (CARA) expected utility framework, and their model still relies upon the standard three-step PMP.

A more recent attempt is proposed by Cortignani and Severini (2012). Their paper develops from the work by Heckelei and Wolff (2003) and directly estimates the parameters of an expected utility maximisation model by applying the generalised maximum entropy (GME) estimation technique on the first-order conditions of the model. Both prices and yields are accounted as a source of risk, but it is not possible to isolate the price risk from the yield risk. The model estimates simultaneously the nonlinear cost function, a farm-specific absolute risk aversion coefficient and the shadow price of land. Their paper presents an illustrative empirical application to a small sample of farms located in the centre of Italy with the aim of evaluating the effect of a revenue insurance scheme on farm production choices and on farm gross margins.

The work by Petsakos and Rozakis (2015) represents the first attempt in the literature to calibrate a nonlinear mean-variance (E-V) model by applying the three-step PMP procedure. The calibration procedure recovers the values of the expected farm income and of its variance–covariance matrix by applying GME. The authors, however, consider risk as the only source of nonlinearity in the cost function as the cost term is kept linear. In addition, they apply a logarithmic utility function, which implies a relative risk aversion coefficient equal to one by construction and leads to the assumption that the degree of absolute risk aversion does not depend on individual farmer risk preferences but only on their individual wealth. Another recent development in estimating the parameters of an E-V mathematical programming model is represented by Jansson *et al.* (2014). The authors apply a Bayesian methodology to estimate the parameters of a farm-level E-V model which exhibits decreasing absolute risk aversion (DARA) preferences in a large-scale application across the EU.

Given the few attempts found in the literature to integrate risk into a PMP framework, and lacking an established consensus on the most suitable one, we propose a different approach which combines in an innovative and consistent way the standard PMP with the estimation of the optimality conditions of the desired programming model.

3. Theoretical model

Our proposal merges the first linear step of PMP with the second nonlinear step by using the dual relationships of a farmer expected utility maximisation problem. The model incorporates the risk term according to the E-V approach and estimates simultaneously the differential marginal cost and the shadow price of resources which usually belong to the first PMP step, as well as the farm nonlinear cost function and the farm-specific coefficient of absolute risk aversion. In addition, no calibration constraints are made explicit in the model.

The primal formulation of a farm-level model incorporating risk in the first PMP step is the following:

$$\max_{\mathbf{x}} EU_f(\tilde{\pi}_f) = E(\tilde{\mathbf{p}}_f)'\mathbf{x}_f - \mathbf{c}_f'\mathbf{x}_f - \frac{1}{2}\alpha_f\mathbf{x}_f'\mathbf{V}\mathbf{x}_f$$

$$\text{subject to} \mathbf{A}_f\mathbf{x}_f \leq \mathbf{b}_f \quad (\mathbf{y}_f)$$

$$\mathbf{x}_f \leq \bar{\mathbf{x}}_f + \boldsymbol{\varepsilon} \quad (\boldsymbol{\lambda}_f)$$

$$\mathbf{x}_f \geq 0$$

where f is the farm index, $\mathbf{x}_f$ is the vector of endogenous activity levels, $\bar{\mathbf{x}}_f$ is the vector of observed activity levels, $\mathbf{c}_f$ is the vector of accounting costs per unit of activity and $E(\tilde{\mathbf{p}}_f)$ is the vector of expected prices. $\mathbf{b}_f$ and $\mathbf{A}_f$ represent the vectors of available resources and the matrix of technical coefficients, respectively, $\mathbf{V}$ represents the variance–covariance matrix of activity prices, α_f is the farmer absolute risk aversion coefficient, $\mathbf{y}_f$ and $\boldsymbol{\lambda}_f$ are the vectors of resource shadow values and of the shadow values of the calibration constraints, respectively, and $\boldsymbol{\varepsilon}$ is the disturbance term vector which prevents linear dependency among the constraints (Paris and Howitt 1998). The model assumes that the random prices are normally distributed.

The dual formulation of the above primal model is as follows:

$$\min_{\mathbf{y},\boldsymbol{\lambda}} \mathrm{TC}_f = \mathbf{b}_f'\mathbf{y}_f + \boldsymbol{\lambda}_f'(\bar{\mathbf{x}}_f + \boldsymbol{\varepsilon}) + \frac{1}{2}\alpha_f\mathbf{x}_f'\mathbf{V}\mathbf{x}_f.$$

$$\text{subject to } \mathbf{c}_f + \boldsymbol{\alpha}_f\mathbf{V}\mathbf{x}_f + \mathbf{A}_f'\mathbf{y}_f + \boldsymbol{\lambda}_f \geq E(\tilde{\mathbf{p}}_f) \quad (\mathbf{x}_f)$$

$$\mathbf{y}_f \geq 0, \boldsymbol{\lambda}_f \geq 0, \mathbf{x}_f \geq 0$$

where TC_f is the value of the dual objective function.

Our estimation model merges the primal first-step PMP model with its corresponding dual model and adds the second step PMP equation as a constraint. The final specification of the estimation model, which can be applied to farms sharing the same technology, is the following:

$$\min_{\mathbf{u}_f,\mathbf{y}_f,\boldsymbol{\lambda}_f,\alpha_f,\mathbf{D},\mathbf{L}} \sum_{f=1}^{F}\sum_{i=1}^{I} y_{fi}b_{fi} + \sum_{f=1}^{F}\sum_{j=1}^{J}\left(\frac{1}{2}u_{fj}^2 + c_{fj}\bar{x}_{fj} + \lambda_{fj}(\bar{x}_{fj}+\varepsilon_f) + \alpha_f \sum_{j'=1}^{J'} \bar{x}_{fj} V_{j,j'} \bar{x}_{fj'} - E(\tilde{p}_{fj})'\bar{x}_{fj}\right) \quad (1)$$

$$\text{subject to} \quad \mathbf{c}_f + \alpha_f \mathbf{V}\bar{\mathbf{x}}_f + \mathbf{A}_f'\mathbf{y}_f + \boldsymbol{\lambda}_f \geq E(\tilde{\mathbf{p}}_f) \quad (\mathbf{w}_f) \quad (2)$$

$$\mathbf{c}_f + \boldsymbol{\lambda}_f = \mathbf{Q}\bar{\mathbf{x}}_f + \mathbf{u}_f \quad (\mathbf{v}_f) \quad (3)$$

$$\mathbf{Q} = \mathbf{L}'\mathbf{D}\mathbf{L} \quad (4)$$

$$\mathbf{y}_f \geq 0, \boldsymbol{\lambda}_f \geq 0, \alpha_f \geq 0 \quad (5)$$

where j is the activity index, i is the resource index and f indicates farms that share the same technology. $\mathbf{Q}$ is the symmetric positive semidefinite matrix of a quadratic cost function, common to all the farms that share the same technology, $\mathbf{L}$ and $\mathbf{D}$ are respectively the unit lower triangular matrix and the diagonal matrix of the Cholesky factorisation, whose elements are restricted to be non-negative, and $\mathbf{u}_f$ is the vector of specific farm deviations from the common marginal cost function, while $\mathbf{w}_f$ and $\mathbf{v}_f$ represent the Lagrange multipliers associated with each constraint. The solution of the model (1)-(5) consists in the parameter estimates $\mathbf{Q}$, $\mathbf{y}_f$, $\boldsymbol{\lambda}_f$ and α_f, which are the values of the parameters that allow the calibration of the model to the observed activity levels.

The objective function of the estimation model is the sum over all farms of the square of the individual farm deviations from the common marginal cost function of each production activity, $\sum_{f=1}^{F}\sum_{j=1}^{J} u_{fj}^2$; as farms share the same technology, the estimation of the parameters is performed by minimising the square of the individual farm deviations, $\mathbf{u}_f$. In other words, $\mathbf{u}_f$ acts as the error term of a standard regression analysis. In the objective function, we also include the sum over all farms of the difference between the objective function of the primal and the dual model of the farmer expected utility maximisation problem, which should be zero by construction. Constraint (2) represents the dual constraint of the economic equilibrium condition stating that the marginal cost must be larger or equal than the expected marginal revenue, while constraint (3) establishes the relationship between the marginal cost of the first step of the standard PMP and the marginal cost of the farm nonlinear cost function to be estimated. This constraint allows us to estimate the implicit cost $\boldsymbol{\lambda}_f$ for each activity. Constraint (4) is the Cholesky decomposition, which guarantees the matrix $\mathbf{Q}$ to be symmetric and positive semidefinite in order to ensure convexity of the cost function.

The use of the dual relationship (constraint (2)) allows us to merge the first two PMP steps, which has some advantages. First, the calibration procedure does not use the first step of the standard PMP to recover the shadow values of the constraints, which raised several critiques in the literature. Second, the model allows the simultaneous estimation of the vector of shadow values of resources, $\mathbf{y}_f$, the vector of the shadow values of activities, $\boldsymbol{\lambda}_f$, the symmetric matrix of the quadratic cost function, $\mathbf{Q}$, the vector of individual farm deviations from the marginal cost function of each activity, $\mathbf{u}_f$, and the farmer absolute risk aversion coefficient α_f. The coefficient of absolute risk aversion is farm specific, and it exhibits CARA preferences[1] . We impose either neutral or risk-averse behaviour by farmers, forcing the absolute risk aversion coefficients to be non-negative.

From models (1)–(5), we can derive the following Lagrangian function:

$$L_f = \frac{1}{2}\mathbf{u}_f'\mathbf{u}_f + \alpha_f\bar{\mathbf{x}}_f'\mathbf{V}\bar{\mathbf{x}}_f + \mathbf{y}_f'\mathbf{b}_f + \boldsymbol{\lambda}_f'\left(\bar{\mathbf{x}}_f + \boldsymbol{\varepsilon}_f\right) + \mathbf{c}_f'\bar{\mathbf{x}}_f$$
$$-E(\tilde{\mathbf{p}}_f)'\bar{\mathbf{x}}_f + \mathbf{w}_f'\Big(E(\tilde{\mathbf{p}}_f) - \mathbf{c}_f - \alpha_f\mathbf{V}\bar{\mathbf{x}}_f - \mathbf{A}_f'\mathbf{y}_f - \boldsymbol{\lambda}_f\Big) + \mathbf{v}_f'\left(\mathbf{c}_f + \boldsymbol{\lambda}_f - \mathbf{Q}\bar{\mathbf{x}}_f - \mathbf{u}_f\right)$$

From the Lagrangian function, we can derive the corresponding set of Karush–Kuhn–Tucker (KKT) conditions, which represents the solution to the model, and their associated complementary slackness conditions:

$$\frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{u}_f} = \mathbf{u}_f - \mathbf{v}_f = 0 \tag{6a}$$

$$\mathbf{u}_f'\frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{u}_f} = \mathbf{u}_f'\left(\mathbf{u}_f - \mathbf{v}_f\right) = 0 \tag{6b}$$

$$\frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{y}_f} = \mathbf{b}_f - \mathbf{A}_f\mathbf{w}_f \geq 0 \tag{7a}$$

[1] Despite the well-known limitations of the mean–variance expected utility function with CARA preferences, which assumes a normal distribution of the payoffs, this approach is widely used to represent agricultural production choices under risk (see among others: Coyle 1992; Oude Lansink 1999; Garrido and Zilbermann 2008).

$$\mathbf{y}'_f \frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{y}_f} = \mathbf{y}'_f\left(\mathbf{b}_f - \mathbf{A}_f\mathbf{w}_f\right) = 0 \tag{7b}$$

$$\frac{\mathrm{d}L_f}{\mathrm{d}\lambda_f} = \bar{\mathbf{x}}_f + \boldsymbol{\varepsilon}_f - \mathbf{w}_f + \mathbf{v}_f \geq 0 \tag{8a}$$

$$\boldsymbol{\lambda}'_f \frac{\mathrm{d}L_f}{\mathrm{d}\boldsymbol{\lambda}_f} = \boldsymbol{\lambda}'_f\left(\bar{\mathbf{x}}_f + \boldsymbol{\varepsilon}_f - \mathbf{w}_f + \mathbf{v}_f\right) = 0 \tag{8b}$$

$$\frac{\mathrm{d}L_f}{\mathrm{d}\alpha_f} = \bar{\mathbf{x}}'_f\mathbf{V}\bar{\mathbf{x}}_f - \mathbf{w}'_f\mathbf{V}\bar{\mathbf{x}}_f \geq 0 \tag{9a}$$

$$\alpha_f \frac{\mathrm{d}L_f}{\mathrm{d}\alpha_f} = \alpha_f\left(\bar{\mathbf{x}}'_f\mathbf{V}\bar{\mathbf{x}}_f - \mathbf{w}'_f\mathbf{V}\bar{\mathbf{x}}_f\right) = 0 \tag{9b}$$

$$\frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{w}_f} = E(\tilde{\mathbf{p}}_f) - \mathbf{c}_f - \alpha_f\mathbf{V}\bar{\mathbf{x}}_f - \mathbf{A}'_f\mathbf{y}_f - \boldsymbol{\lambda}_f \leq 0 \tag{10a}$$

$$\mathbf{w}'_f \frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{w}_f} = \mathbf{w}'_f\left(E(\tilde{\mathbf{p}}_f) - \mathbf{c}_f - \alpha_f\mathbf{V}\mathbf{x}_f - \mathbf{A}'_f\mathbf{y}_f - \boldsymbol{\lambda}_f\right) = 0 \tag{10b}$$

$$\frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{v}_f} = \mathbf{c}_f + \boldsymbol{\lambda}_f - \mathbf{Q}\bar{\mathbf{x}}_f - \mathbf{u}_f = 0 \tag{11a}$$

$$\mathbf{v}'_f \frac{\mathrm{d}L_f}{\mathrm{d}\mathbf{v}_f} = \mathbf{v}'_f\left(\mathbf{c}_f + \boldsymbol{\lambda}_f - \mathbf{Q}\bar{\mathbf{x}}_f - \mathbf{u}_f\right) = 0 \tag{11b}$$

KKT condition (6a) indicates that the dual value vector $\mathbf{v}_f$ associated with the marginal cost function is equal to the farm deviation vector from the marginal cost function $\mathbf{u}_f$; since the model tries to keep the elements of $\mathbf{u}_f$ as small as possible, the elements of $\mathbf{v}_f$ result in a small positive or negative number too. $\mathbf{w}_f$ is the dual value vector of the economic equilibrium constraint (2) and, given $\mathbf{v}_f = \mathbf{u}_f$ from (6a) and $\boldsymbol{\lambda}_f > 0$, Equation (8b) shows that $\mathbf{w}_f \cong \bar{\mathbf{x}}_f$. Substituting $\mathbf{v}_f = \mathbf{u}_f$ and $\mathbf{w}_f = \bar{\mathbf{x}}_f$ in (7a) and (8a), we obtain the resource constraints and the calibration constraints, respectively. Hence, models (1)–(5) implicitly represent the constraints of a first-step model of the standard PMP, and as a consequence, the estimated model can properly calibrate to the base year activity level without making the first step explicit. This avoids the critiques raised in the literature against the presence of the calibration constraint and the recovery of the resource shadow value in the first step of PMP. The other KKT conditions represent a tautology (condition 9a) and the constraints of the estimation models (1)–(5) (conditions 10a) and 11a).

The parameters estimated by the models (1)–(5) allow the calibration of the nonlinear models (12)-(14) to the base year farmer decision variables $\bar{\mathbf{x}}_f$:

$$\max_{\mathbf{x}_f} EU(\tilde{\pi}_f) = E(\tilde{\mathbf{p}}_f)'\mathbf{x}_f - \frac{1}{2}\mathbf{x}_f'\hat{\mathbf{Q}}\mathbf{x}_f - \hat{\mathbf{u}}_f'\mathbf{x}_f - \frac{1}{2}\hat{\alpha}_f\mathbf{x}_f'\mathbf{V}\mathbf{x}_f. \tag{12}$$

$$\text{subject to } \mathbf{A}_f\mathbf{x}_f \leq \mathbf{b}_f. \tag{13}$$

$$\mathbf{x}_f \geq 0 \tag{14}$$

where $\mathbf{x}_f$ is the vector of endogenous farm activity levels, $\hat{\mathbf{Q}}$, $\hat{\mathbf{u}}_f$ and $\hat{\alpha}_f$ have been estimated previously and $E(\tilde{\mathbf{p}}_f)'$, $\mathbf{V}$, $\mathbf{A}_f$ and $\mathbf{b}_f$ are exogenous parameters. Equation (12) is the farmer expected utility to be maximised, which is equal to the expected revenue minus the estimated nonlinear cost function and minus the risk premium, while Equation (13) represents the resource constraints.

Our methodological proposal for the incorporation of risk in a PMP framework represents an innovative approach compared to previous studies. Our model differs from the work by Paris and Arfini (2000) as we estimate endogenously the farmer coefficient of absolute risk aversion and we do not rely upon the standard three-step PMP. Although our model presents some similarities with the model by Cortignani and Severini (2012), some differences should be mentioned. First, the first-order conditions of our estimation model (equations (1)–(5)) reproduce exactly the first step of the standard PMP procedure while avoiding its well-known weaknesses. Second, by minimising over $\sum_{f=1}^{F}\sum_{j=1}^{J} u_{fj}^2$ we apply a least square estimator instead of a GME estimator. Third, we introduce the farm-specific error term on the common marginal cost function and not on the observed output quantity as in Cortignani and Severini (2012), and we minimise this farm deviations given the assumption of a common technology across farms. Finally, we do not impose any upper bound to the coefficient of absolute risk aversion, thus making the model more flexible in defining the individual (farm-specific) risk preferences. Our approach differs also from Petsakos and Rozakis (2015) as we estimate two nonlinear terms, the cost function and the risk term, and we consider more than one farm simultaneously.

4. Empirical model and data

In this section, an empirical application of the theoretical model presented in Section 3 is provided considering crop price risk only. First, models (1)–(5)

are estimated, and the ability of the model to calibrate to the base year activity levels and to estimate individual specific absolute risk aversion coefficients is checked. Then, the calibrated models (12)–(14) are applied in simulating the land allocation impact of different crop price volatility scenarios. In each scenario, a special attention is paid to the land committed to a specific AES, the option to convert a share of cropland to grassland, and to its potential role in coping with price risk.

The model is applied to a sample of 42 representative arable crop farms representing a combination between six size classes and seven provinces (Piacenza, Parma, Reggio-Emilia, Modena, Bologna, Ferrara and Ravenna) in the plain area of the Emilia-Romagna region in Italy[2]. The use of representative farms instead of the real ones is mainly linked to a technical motivation, since they guarantee the absence of zero activity levels, thus reducing the computational difficulty in testing the novelty of our approach. However, the use of average farms, or alternatively of 'regional' farms, to analyse the effects of policy measures is a common practice in many empirical mathematical programming studies (Schmid *et al*. 2007; Louhichi *et al*. 2010; Chiron *et al*. 2013; Kirchner *et al*. 2016).

The size classes are based on the available farmland and are defined according to the standard classification adopted by the Italian Official Statistics (Istat): 1–10 hectares, 10–30 hectares, 30–50 hectares, 50–100 hectares, 100–300 hectares, >300 hectares. The representative farm is the average arable crop farm in terms of size and land allocation in each territory and size class. Crop production levels are the farmer decision variable, while the exogenous per unit crop-specific variable costs and prices of the representative farms are obtained averaging the corresponding farm-level data for each class–province combination.

The crop-specific variable costs include fertilisers, seeds, crop protection products, electricity, water, heating fuels, insurance, motor fuels, services by agricultural contractors and other specific crop costs. Five crops are included in the empirical model: sugar beet, common wheat, corn, barley and grassland under environmental commitments. The first four crops are the most widely grown crops in the area under study, while the committed grassland is the AES considered in our model. For the farms where the AES is not in place in the baseline, we set very small initial allocation to take into account this scheme as an option, following common practice in the literature (Cortignani and Severini 2009; Arfini and Donati 2013). The payment for grassland under AES is set at 240 euro/ha, according to the Rural Development Program (RDP) of the Emilia-Romagna region (Regione Emilia Romagna 2005). The source of the data is the RICA-AGREA

[2] The Emilia-Romagna region is one of the most important regions in Italy in terms of agricultural production, which is also very similar to that of other regions in Northern Italy (e.g. Lombardia, Veneto and Piemonte), as well as to other intensive arable crop areas around the EU. Thus, our empirical results can be considered representative of all these areas.

database (a merged database that combines data from the Italian Farm Accounting Data Network with those provided by the Integrated Administration and Control System of agricultural payments) for the Emilia-Romagna region.

Wheat represents the most grown crop in each farm size class, and it covers around 50 per cent of the area, followed by corn and sugar beet (Table 1). The largest share allocated to grassland under agri-environmental commitment is around 1.5 per cent[3]. The average variable unit costs of barley and corn range between 52 euro/tonne and 88 euro/tonne and between 57 euro/tonne and 98 euro/tonne, respectively, according to the farm size class, while the ones of wheat fluctuate in a smaller range, between 81 euro/tonne and 94 euro/tonne. The sugar beet variable costs fall in the 17–23 euro/tonne range.

We introduce the land constraint as the only resource constraint, and we adopt a quadratic cost function, the most frequently used functional form in PMP works. Expected output prices, accounting variable cost per unit of activity, observed activity levels, the amount of farmland and the matrix of technical coefficients are all farm-specific exogenous variables, while the variance–covariance matrix of output prices is common to all farms and computed from annual series of crop prices over the period 2002–2008

Table 1 Descriptive statistics of the representative farms by size class (means)

	class 1*	class 2	class 3	class 4	class 5	class 6
Total farmland (ha)	7.0	15.5	41.6	73.9	192.6	361.9
Land allocation (ha)						
Barley	0.6	0.9	2.0	3.3	8.6	18.9
Corn	1.9	4.3	12.1	23.1	70.4	125.1
Sugar beet	0.4	1.8	5.6	11.4	24.7	48.3
Wheat	3.9	8.5	21.7	36.0	88.7	169.0
Grassland under AES	0.1	0.1	0.1	0.1	0.2	0.6
Production (tonnes)						
Barley	2.6	4.1	8.8	15.5	45.8	98.6
Corn	15.9	37.7	110.2	204.2	594.8	1044.4
Sugar beet	23.7	95.9	317.4	615.4	1201.3	2741.8
Wheat	20.4	46.3	119.6	205.2	485.1	929.8
Crop variable costs (euro/tonne)						
Barley	84.5	88.4	65.2	70.5	51.7	59.4
Corn	79.9	97.7	77.8	86.0	56.9	70.9
Sugar beet	21.6	22.6	18.1	19.7	17.2	20.3
Wheat	84.3	93.8	82.8	76.4	82.4	81.1
Price (euro/tonne)						
Barley	194.6	199.4	218.1	198.8	200.1	226.6
Corn	216.0	214.7	216.1	213.3	197.7	215.5
Sugar beet	40.2	42.5	40.2	40.1	38.4	42.2
Wheat	216.5	222.5	220.6	221.5	213.8	232.8

*class 1: 1–10 hectares, class 2: 10–30 hectares, class 3: 30–50 hectares, class 4: 50–100 hectares, class 5: 100–300 hectares, class 6: >300 hectares.

[3] Since the share of committed AES grassland is very small, we have assumed a common technology between farms adopting AES and farms not adopting AES.

(Table A1). Wheat is the crop with the highest price volatility, while sugar beet shows the lowest price variation. The different price–risk scenarios are constructed by inflating the variance–covariance matrix of the base year by a scalar which takes different values in each scenario. The volatility scenarios are set according to the average change in crop price volatility in Italy between the two time periods 2002–2008 and 2009–2014 for wheat, corn and barley. As the variance in the second period increased by 17 per cent for wheat and by 95 per cent for corn and barley, we have simulated three scenarios with a price variance ranging between the baseline level and a 100 per cent increase. More specifically, in the three scenarios, volatility is set to 20, 50 and 100 per cent higher than the baseline, respectively. For simplicity, and due to data limitation, we assume that the output of grassland under agri-environmental commitments is not sold on the market. Thus, this activity bears no risk, since it affects farm income only through the fixed agri-environmental payment.

5. Results

This section presents the results of the model calibration to the base year activity levels and the parameter estimates. Then, the results of the simulations due to an increase in price volatility are discussed, emphasising the changes in the crop mix and the impact on the adoption of the AES.

5.1 Calibration and estimation

The model is able to reproduce the base year-observed crop allocation for each farm in the sample. The percentage deviations between the observed activity levels and the level reproduced by the model are lower than 0.1 per cent for almost all farms, and only one farm shows a deviation higher than 1% (1.4%). The model estimation of the symmetric matrix of the nonlinear cost function shows the substitution relationships among crops; the only exception is sugar beet, which turns out to be a complement of barley and of grassland under environmental commitment (Tables A2). The model is able to estimate a farm-specific absolute risk aversion coefficient, and seven farms exhibit a coefficient equal to zero (Table 2). Six out of seven risk neutral farms are the largest farms in the sample. The correlation coefficient between farm revenue and the risk aversion coefficient is -0.5, indicating a negative relationship between these two variables.

The shadow values of land range between zero and 240 euro/ha. The zero value is due to small calibration errors (see Section 3). In a model setup where the farm cost function is simultaneously estimated with the shadow price of land, some farms may use less land than their total land endowment due to the small calibration errors. However, the share of nonused land remains extremely small (no more than 0.03 per cent of the land).

Table 2 Farm size, absolute risk aversion coefficient, land shadow value and land allocation to AES grassland under different scenarios

Farm number	Farm size (ha)	Absolute risk aversion coefficient (per euro)	Shadow value of land (euro)	Land allocation to AES grassland (% share)			
				Baseline	Scenario 1*	Scenario 2	Scenario 3
1	4.6	0.00592	0	2.2	5.8	10.7	18.6
2	5.1	0.00317	240.0	2.0	5.7	10.6	17.6
3	6.1	0.00269	0.0	1.6	4.5	8.2	13.6
4	7.6	0.00225	240.0	1.3	4.4	8.5	14.4
5	8.1	0.00143	0	1.2	2.5	4.2	6.7
6	8.6	0.00335	33.9	1.2	3.9	7.5	13.2
7	9.1	0.00286	32.3	1.1	3.5	6.8	11.5
8	12.0	0.00185	0.0	0.8	2.8	5.5	9.7
9	13.0	0.00178	0	0.8	2.7	5.3	9.0
10	14.0	0.00190	0	0.7	2.8	5.6	9.7
11	15.0	0.00000	164.5	0.7	0.7	0.7	0.7
12	17.0	0.00136	43.7	0.6	2.6	5.2	9.1
13	18.0	0.00088	204.8	0.6	3.0	6.2	10.8
14	19.0	0.00123	0	0.5	1.8	3.5	6.0
15	32.6	0.00057	240.0	0.3	1.2	2.4	4.2
16	38.0	0.00090	240.0	0.3	2.1	4.6	8.2
17	40.1	0.00052	240.0	0.2	1.0	1.9	3.2
18	41.1	0.00043	240.0	0.2	1.4	2.8	5.2
19	44.0	0.00029	240.0	0.3	1.0	1.9	3.1
20	46.5	0.00029	240.0	0.4	0.9	1.6	2.5
21	48.6	0.00054	240.0	0.2	1.0	2.0	3.5
22	58.6	0.00045	240.0	0.2	1.0	2.1	3.6
23	61.5	0.00030	0	0.2	0.7	1.3	2.2
24	65.6	0.00032	202.9	0.2	0.8	1.6	2.8
25	72.5	0.00028	0	0.2	0.6	1.0	1.6
26	80.1	0.00025	147.5	0.1	0.7	1.4	2.3
27	88.0	0.00050	28.4	0.3	1.4	2.8	4.9
28	91.1	0.00030	96.0	0.1	0.6	1.2	2.1
29	120.1	0.00019	133.7	0.1	0.4	0.8	1.4
30	155.6	0.00013	169.3	0.1	0.3	0.6	1.0
31	187.1	0.00005	240.0	0.1	0.2	0.5	0.9
32	190.1	0.00005	238.6	0.1	0.2	0.5	0.9
33	210.6	0.00004	99.9	0.0	0.2	0.3	0.6
34	235.0	0.00004	75.5	0.4	0.5	0.7	1.0
35	250.0	0.00002	57.0	0.1	0.1	0.2	0.4
36	310.1	0.00004	221.6	0.0	0.2	0.4	0.7
37	322.1	0.00000	36.9	0.0	0.0	0.0	0.0
38	350.6	0.00000	0	0.0	0.0	0.0	0.0
39	360.5	0.00000	0	0.1	0.1	0.1	0.1
40	380.0	0.00000	0	0.2	0.2	0.2	0.2
41	400.1	0.00000	0	0.0	0.0	0.0	0.0
42	410.0	0.00000	0	0.6	0.6	0.6	0.6

*Scenarios 1, 2 and 3 correspond to a price volatility respectively 20%, 50% and 100% higher than the baseline volatility.
Source: simulation results.

5.2 Simulation results

The simulation results focus on the changes in land allocation among crops under different price volatility scenarios and a special attention is paid to the farmland share committed to grassland. While risk neutral farmers do not change their crop mix under different scenarios, risk-averse farmers increase the share of AES grassland as a response to a rise in crop price volatility. As most of the largest farms in the sample are risk neutral, the effect is detected only for small/medium size farms. If volatility is set to 20 per cent larger than the baseline (scenario 1), the smallest risk-averse farms would commit to AES more than 2 per cent of their land (Table 2). In scenario 2, where volatility is 50 per cent larger than the baseline, most of the smallest farms would contract more than 8 per cent of their land under AES. Finally, by doubling volatility (scenario 3), the percentage of farmland subject to grassland would increase to more than 10 per cent in around 1/5 of the risk-averse farms, and it would be between 4 and 10 per cent in another 1/3 of the farms. Thus, the AES scheme seems to work as income stabiliser tool. Indeed, when crop price fluctuations become larger, risk-averse farmers are willing to convert some share of high-income and high-risk crops to a lower-risk activity.

In parallel with the changes in AES grassland, we also observe a change in the crop mix as a response to an increase in crop price volatility. The crop mix under different scenarios is shown in Figure 1, where the land allocation is aggregated among all the 42 farms. The change in the crop mix across scenarios is the result of a direct and a cross effect. The former consists of the decrease of production of high-risk crop when price volatility rises, while the second is due to the relationships among crops, shown by the cross terms of the symmetric matrix of the nonlinear cost function (i.e. the derivative of the marginal cost of production of a crop with respect to the quantity produced of another crop) and by the covariances of output prices. When crop price

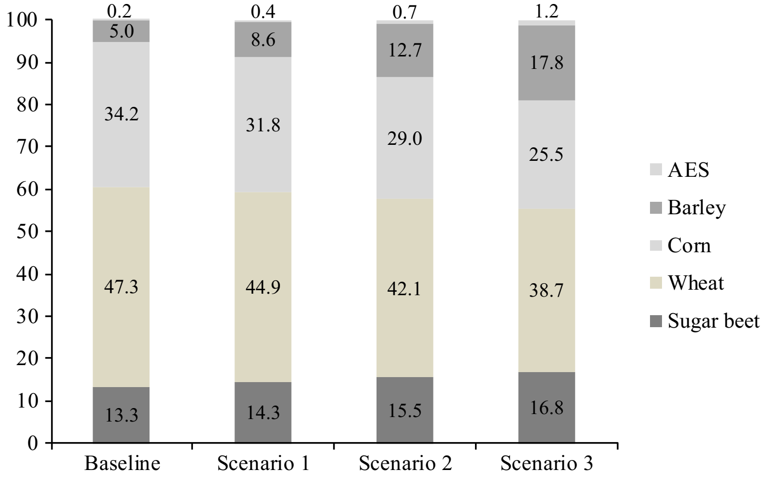

Figure 1. Land allocation to each crop in the sample (% shares).

volatility rises, the share of total farmland allocated to sugar beet in the sample increases from 13.3 per cent in the baseline to 16.8 per cent in the most risky scenario, since sugar beet is the lowest risk crop in the sample (lowest price variance). On the other hand, the share of the highest risk crop, wheat, drops from 47.3 in the baseline to 38.7 per cent in the high volatility scenario, and a similar drop (from 34.2 to 25.5 per cent) is observed for corn. Although barley is the second most risky crop, its share rises across the three scenarios, likely as a consequence of the negative cross term between barley and sugar beet in the cost function matrix, such that the marginal cost of barley production decreases with an increase in land allocated to sugar beet.

The simulation results confirm the importance to account for both the direct and the indirect effects of a change in crop price volatility. In addition, the increasing share of committed grassland supports the potential role of AES as income stabiliser tool especially for the smallest farmers that are also the most risk averse[4].

6. Discussion and conclusions

In this study, we develop a new methodological approach which accommodates risk in a PMP framework, introducing some innovations with respect to the available literature. The individual farmer absolute risk aversion coefficients are estimated endogenously in the model together with the farm nonlinear cost function, the shadow prices of resources and the shadow prices of activities. The model calibrates to the base year-observed activity levels without making any calibration constraint explicit.

We provide an empirical application of our model on a sample of representative farms from the Emilia-Romagna region in Italy. We show that the model calibrates to the base year-observed activity levels for all farms, and it is able to reveal the individual farmer absolute risk aversion coefficients. In this respect, we identify a negative correlation between risk aversion coefficient and farm size, with the largest farms in the sample exhibiting risk neutral behaviour. Our model is consistent with the standard PMP foundations (Paris 2011: 340–411), but it avoids the critiques raised in the literature by directly estimating the above shadow prices. Unlike the model of Cortignani and Severini (2012), where an upper bound on the coefficient of risk aversion is imposed, our method is more flexible in estimating that coefficient, while the error terms can be interpreted as deviations from the common farm cost function. Differently from

[4] We have also carried out an additional simulation in order to find the level of the AES payment in the baseline situation which produces the same environmental benefit of an increase in price volatility, measured in terms of AES grassland share. We consider twenty simulated levels of price volatility. If volatility increases by 5% compared to the baseline situation, the AES payment should increase by around 3.5 euro/ha in order to produce the same adoption of AES grassland induced by the 5 per cent increase in volatility. The per-hectare payment should increase by around 15 per cent to provide the same AES adoption caused by a 50 per cent rise in the baseline volatility and by 30 per cent to replicate the same adoption of a doubling in volatility.

Petsakos and Rozakis (2015), we estimate both a nonlinear cost function and a nonlinear risk term.

We perform some simulations to test the farmers' response to changes in price volatility and to investigate the role of agri-environmental grassland as a farmers' strategy to cope with risk. The idea is that the grassland under agri-environmental contract may represent an income risk management tool for farmers, since it guarantees a fixed payment independent of market conditions. The simulation results confirm the potential role of the grassland program as income stabilisers for the risk-averse farms, namely the small and medium size farms, since they increase the share of farmland under AES commitment as a response to increased crop price volatility. When the crop price volatility doubles, the smallest farms would contract around 10 per cent of their land under AES grassland, and in some cases, such share would increase to over 15 per cent. The impact of price volatility on the crop mix depends not only on farmer risk preferences, but also on the complementarity/substitutability relationships among crops.

We believe that our calibrated farm-level model has potential for being used in policy simulations involving the impact of risk, such as, for example, the analysis of the newly introduced income stabilisation tool of the CAP. This tool may be modelled by introducing an additional cost (i.e. the annual insurance payment) together with a threshold on farm revenue that activates the revenue insurance scheme. The simulation of different volatility scenarios and the consideration of a multiple-year time horizon would show the volatility degree which makes the option beneficial for each farmer.

Despite this potential, our model still has some important limitations. For example, we assume that farmers exhibit CARA risk preferences and that income volatility is due only to price changes, while yields are kept constant over time. It would be interesting to further develop the model by assuming DARA preferences and by introducing variable crop yields over time. Indeed, DARA preferences may capture the influence of CAP payment changes on farmer risk aversion (Koundouri *et al.* 2009).

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Appendix

Table A1 Variance–covariance matrix of prices in the baseline computed on 2002–2008 data (€^{2}/tonne^2).

	Sugar beet	Wheat	Corn	Barley	AES
Sugar beet	0.19	−0.98	−0.64	−0.88	0.00
Wheat	−0.98	11.02	7.82	9.77	0.00
Corn	−0.64	7.82	6.04	6.95	0.00
Barley	−0.88	9.77	6.95	8.88	0.00
AES	0.00	0.00	0.00	0.00	0.00

Source: own elaboration on data by the Chambers of Commerce of Bologna, Milan and ISMEA.

Table A2 Coefficients of the quadratic term of the farm nonlinear cost function

	Sugar beet	Wheat	Corn	Barley	AES
Sugar beet	6.43×10^{-05}	3.14×10^{-05}	9.76×10^{-05}	-4.64×10^{-06}	-2.29×10^{-05}
Wheat	3.14×10^{-05}	1.22×10^{-03}	9.94×10^{-04}	1.27×10^{-03}	1.20×10^{-04}
Corn	9.76×10^{-05}	9.94×10^{-04}	9.15×10^{-04}	9.22×10^{-04}	1.23×10^{-04}
Barley	-4.64×10^{-06}	1.27×10^{-03}	9.22×10^{-04}	1.57×10^{-03}	-3.89×10^{-06}
AES	-2.29×10^{-05}	1.20×10^{-04}	1.23×10^{-04}	-3.89×10^{-06}	7.03×10^{-03}

Source: estimation results.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Data S1 Dataset and code.