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# THE EFFECTS OF FIRM SIZE AND PRODUCTION COST LEVELS ON DYNAMICALLY OPTIMAL AFTER-TAX COTTON STORAGE AND HEDGING DECISIONS

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## Abstract

Farm size and production costs are varied in a six state variable stochastic dynamic programming model that quantifies monthly hedging, storage, and cash cotton sale decisions for an Alabama cotton producer. State variables considered are: (1) cash cotton price; (2) basis level; (3) before-tax income level; (4) cotton holdings; (5) futures position; and (6) value of futures position. Results indicate that when farm size and production cost level differ, marketing decisions diverge the most for cash cotton sales at the end of the tax year and lower range of cash price (less than \$.65/lb.), basis (less than -.05/lb.), and before-tax income (less than \$0.00) states.

**Key words:** farm size, production costs, stochastic dynamic programming

Due to the discrete and progressive nature of state and federal income tax schedules, farm size and production cost levels have the potential to influence the optimal timing of marketing decisions. This is especially true for storable commodities, since under a cash accounting system the farm can often reduce income tax liabilities and increase after-tax profits by storing a commodity into the following tax year. Even though the 1986 Tax Reform Act (TRA) has decreased the marginal tax rate for most individuals, the sensitivity of optimal marketing decisions to income tax considerations can be magnified at taxable income levels near bracket changes since the 1986 TRA has brought forth larger jumps in marginal tax rate schedules (Tronstad and Taylor, 1989). Tronstad and Taylor (1991) utilized a Stochastic Dynamic Programming (SDP) model to solve for dynamically optimal after-tax grain marketing decisions that generate a lower variance in after-tax income and accumulate more wealth than other mar-

keting strategies which don't consider income tax liabilities. Thus, the purpose of this study was to utilize their modeling framework to solve for dynamically optimal after-tax cotton marketing decisions, then analyze the sensitivity of these results to the level of production costs and farm size. Analyzing the sensitivity of optimal cotton marketing decisions to the level of production costs and farm size will determine when these after-tax marketing recommendations can or cannot be generalized across farm size and production cost levels.

This analysis did not assume that certainty equivalence requirements are satisfied with respect to the nonlinearities of the income tax function like many previous hedging analyses have done (e.g., Berck; Brown; Kahl; and Karp). Subsequently, results of this study provide after-tax decision aid information to producers on marketing decisions that is a step beyond a more traditional hedging analysis, price forecasting model, or outlook and situation report that is often given as an aid for marketing decisions (e.g., Brandt and Bessler; Harris and Leuthold; Helmers and Held; McIntosh and Bessler; and Spriggs).

Monthly cash cotton sales, storage, and hedging with futures are the marketing tools considered in this analysis. Forward contracting was not considered since the results presented for futures transactions can be used to approximate forward contracting decisions. That is, ignoring any differential in transaction costs, information contained in a "forward contract basis" (i.e., forward contract price minus cash price) is very similar to the basis (i.e., futures price minus cash price), provided that both time horizons are equivalent. Because the concept of a minimum selling price is partially captured through the government loan rate and target price, options were not considered.<sup>1</sup> Also, model complex-

<sup>1</sup>This analysis assumes that the cotton producer is eligible for deficiency payments, since most producers find government program participation attractive.

ity was kept manageable without considering options or forward contracting.

Optimal cotton marketing decisions were obtained for an Alabama cotton producer where the presumed objective is to maximize expected wealth. A risk-neutral objective function was utilized, in part because the theoretical underpinnings for incorporating risk in a dynamic setting have not been clearly sorted out (Kreps and Porteus, and Mossin), and many dynamic model decisions yield results that appear risk averse, given a risk neutral objective function (e.g., Antle; Just; and Pope). Furthermore, risk neutrality under a progressive tax structure can appear as risk aversion without incorporating income tax effects since the concavity of a progressive income tax function can yield similar results as the concavity of a utility function for a risk-averse producer (Taylor).

The SDP formulation of the cotton marketing model is presented in the next section. Calculation of transition probabilities for the stochastic variables of cash price and basis are discussed in the third section, then marginal tax schedules and other critical input features are delineated. The fifth section presents optimal decision rules from the SDP model for different farm sizes and production cost levels. Concluding comments are given in the final section.

### STOCHASTIC DYNAMIC PROGRAMMING MODEL

Appropriate specification of stages or time intervals, state variables, decision alternatives, and Markovian relationships between state variables is imperative for solving a stochastic multi-period optimization problem. To analyze marketing decisions within a tax year and keep computation requirements manageable, monthly time intervals were utilized and only December futures contracts were considered. December futures was chosen since: (1) this contract gives the most opportunities (*i.e.*, the greatest time span) to hedge expected production while maintaining a bona fide IRS hedging account, and (2) December futures are currently utilized the most for hedging by Alabama cotton producers.<sup>2</sup> Even though only December futures contracts are considered, market conditions which affect new (old) crop future's contracts will generally affect all other new (old) crop future's contracts in a similar manner, due to the favorable storage characteristics of cotton.<sup>3</sup>

As in Tronstad and Taylor (1991), cash price and basis levels are treated as stochastic in the SDP

model formulation. Other state variables necessary to complete the linkages and dynamics associated with the cotton storage and hedging problem are the state variables of cotton storage, before-tax income, December futures position, and the value associated with any futures position outstanding. Following Tronstad and Taylor (1991), the above SDP model description can be written in the following recursive equation:

$$(1) \quad V_t(P_t, B_t, I_t, S_t, QD_t, VD_t) = \\ \text{MAX}_{XC_t, XD_t} E\{T[R(P_t, B_t, I_t, S_t, QD_t, VD_t)] \\ + \beta \cdot V_{t+1}(P_{t+1}, B_{t+1}, I_{t+1}, S_{t+1}, QD_{t+1}, VD_{t+1})\}$$

Subject to:

- (2)  $0 \leq XC_t \leq S_t + Q_t$  [Cash Cotton Sales Constraint]
- (3)  $QD_{t+1} = QD_t + XD_t$  [Futures constraint]
- (4)  $-XD_t = QD_t$ ; if  $t = 11, 23, 35, \dots$  or the month of November [Futures constraint]
- (5)  $-QD_t \leq XD_t \leq S_t + AQ_t - QD_t$  [IRS Legitimate Hedging Constraint]
- (6)  $0 \leq QD_t \leq S_t + AQ_t$ ; if  $S_t + AQ_t \leq SL$ ; else  $0 \leq QD_t \leq SL$  [IRS Legitimate Hedging Constraint]
- (7)  $0 \leq S_t \leq SL$  [Storage limit constraint]
- (8)  $S_{t+1} = S_t + Q_t - XC_t$  [Storage constraint]
- (9)  $VD_{t+1} = VD_t + XD_t \cdot (P_t + B_t) - VI_t$  [Value of futures constraint]
- (10)  $VI_t = [(P_t + B_t) - VD_t / QD_t] \cdot XD_t$  if  $XD_t < 0$ ; else  $VI_t = 0$  [Loss (-) or profit (+) on short futures position closed]
- (11)  $I_t = 0$ ; if  $t = 1, 13, 25, \dots$  or the month of January [Before-tax income constraint]

<sup>2</sup> Neil R. Martin, Jr., personal communication.

<sup>3</sup> If an inverted market exists, immediate cash cotton sales would most likely be preferred to selling in the futures market, so that any benefit associated with adding another futures horizons is probably quite small.

$$(12) I_{t+1} = I_t + P_t \bullet XC_t + VI_t + DEF_t - C_t$$

[Before-tax income constraint]

$$(13) P_{t+1} = f_1(P_t, e_{1t})$$

[Stochastic Markovian Price Relationship]

$$(14) B_{t+1} = f_2(B_t, e_{2t})$$

[Stochastic Markovian Basis Relationship]

where  $V_t(\bullet)$  = maximum expected present value of after-tax income from the current month  $t$  through the terminal month  $T$  given the initial state described in  $(\bullet)$ ;  $P_t$  = monthly average Alabama cash cotton price, \$/lb. (*Alabama Agricultural Statistics*);  $B_t$  = average closing price of New York Cotton Exchange (NYCE) December futures (*WSJ*) for trading days closest to the 5th, 15th, and 25th of each month minus  $P_t$ ;  $I_t$  = beginning before-tax income state of the producer;  $S_t$  = beginning storage state of the cotton producer;  $QD_t$  = beginning quantity of December futures;  $VD_t$  = beginning value of December futures position (*i.e.* average transaction price of a short futures position multiplied by the quantity short);  $XC_t$  = cash cotton sales for month  $t$ ;  $XD_t$  = December futures transactions [positive (negative) values denote selling (buying) of December futures contracts] incurred in month  $t$ ;  $E$  = the expectation operator;  $T\{R(\bullet)\}$  = after-tax income as a function of before-tax revenue;  $\beta$  = discount factor ( $1/1.005$  or approximate 6 percent annual discount rate);  $Q_t$  = production for month  $t$  (non-zero only for the harvest month of November);  $SL$  = upper storage limit of cotton;  $AQ_t$  = anticipated production;  $VI_t$  = profit or loss generated from closing out a short futures position;  $DEF_t$  = deficiency payments for month  $t$ ;  $C_t$  = costs of production, commission and margin expenses incurred for month  $t$ ;  $f_i(\bullet)$ 's ( $i=1,2$ ) are stochastic Markovian relationships; and  $e_{it}$ 's ( $i=1,2$ ) are random variables.

Bellman's "principle of optimality" is embodied in the recursive structure of equation (1). That is, every decision possible in  $t$  is evaluated using an optimal value for the resulting state in  $t+1$  [*i.e.*,  $V_{t+1}(\bullet)$ ] which has been determined by a backward recursive path of optimal decisions from the terminal period  $T$  to period  $t+1$ . Thus, returns from terminal period  $T$  included in  $V_t(\bullet)$  will have been multiplied by the discount factor  $\beta$ ,  $T-(t+1)$  times. For a more complete description of this backward recursive optimization process, the reader is referred to Howard's book of *Dynamic Programming and Markov Processes*.

Equations (2) through (14) are essentially the same model constraints described and utilized by Tronstad and Taylor (1991), except for dissimilarities in cost figures, farm size, and commodity. Two major cost differentials are: (1) direct storage costs are included above (*i.e.*, \$1.92 per 480 lb. bale of cotton or \$0.004/lb. each month), whereas no direct storage costs were incurred for wheat storage by Tronstad and Taylor (1991), and (2) the above model considers different per acre variable and fixed production costs. Farm size takes on two different values in the above model, so that the level of production ( $Q_t$ ), anticipated production ( $AQ_t$ ), storage limit ( $SL$ ), deterministic state variables ( $I_t$ ,  $S_t$ ,  $QD_t$ , and  $VD_t$ ), and decision variables ( $XC_t$  and  $XD_t$ ) take on two different sets of values.

If a short December futures position exists in November, the position must be closed out as indicated in equation (4) since this is the maturity month of the contract, given the nature of the data. Equations (5) and (6) constrain the producer to having a futures position that is no greater than current storage plus anticipated production ( $QD_t \leq S_t + AQ_t$ ) or the upper storage limit  $0 \leq QD_t \leq SL$ . The producer needs to keep the short futures position of the farm less than current cotton holdings plus anticipated production available for delivery before the December futures contract matures, in order to maintain a legitimate IRS hedging account. Also, the farm needs to keep hedging transactions within "reasonable limits" in relation to normal production [*i.e.*, equation (6)], in order to maintain a bona fide hedging account. Cotton storage is increased by cotton harvested during the month of November ( $Q_t$ ) and decreased by cash cotton sales ( $XC_t$ ), as shown in equation (8). The reader is referred to Tronstad and Taylor (1991) for a further description of the modeling constraints given in equations (2) through (14).

## CASH PRICE AND BASIS TRANSITION PROBABILITIES

In determining Markovian cotton price ( $P_t$ ) transition probabilities, consideration was given to lagged prices, seasonal factors, a time trend, and an autoregressive error structure, comparable to that in Tronstad and Taylor (1991). Own lagged prices are hypothesized to capture current market conditions. Seasonal variables considered were dummy variables associates with each month as well as just a dummy variable associated with months near the U.S. cotton harvest. A time-trend variable is considered to proxy effects that may have occurred from

changes associated with technological advances. As in Tronstad and Taylor (1991), Schwarz's Criteria (SC) was utilized to determine the appropriate variables and lag structure for the model specifications. Also, the Jarque-Bera and goodness-of-fit diagnostic statistics for normality of residuals were utilized to determine whether a linear or log-linear functional form is most appropriate for  $P_t$ .

Utilizing similar methods to determine basis ( $B_t$ ) Markovian transition probabilities, consideration was given to the variables of lagged basis, lagged cash price, and the time of basis to maturity (*i.e.* 0 to 11 months). Own lagged basis is hypothesized to capture information about current market conditions between  $P_t$  and the NYCE futures market. Lagged cash prices and time of basis to maturity are hypothesized to have a positive influence on the basis, due to increased storage costs.

Utilizing the above criteria, the models determined to be appropriate for  $P_t$  and  $B_t$  are as follows:<sup>4</sup>

$$(15) \text{LN}(P_t) = .76248 \bullet \text{LN}(P_{t-1}) - .12934 \bullet \text{LN}(P_{t-2}) \\ (8.8196) \quad (1.5133) \\ - .010843 \bullet \text{LN}(MT_t) + e_{1t} \\ (-2.8415)$$

$$\bar{R}^2 = .8966 \quad DH = .8282$$

$$J-B = 5.8236 [\chi^2, 2 \text{ d.f.}] \quad GF = 8.2174 (\chi^2, 7 \text{ d.f.})$$

$$(16) B_t = .9068 \bullet B_{t-1} + e_{2t} \\ (22.868)$$

$$\bar{R}^2 = .6710 \quad DH = -.6756$$

$$J-B = 1.4852 [\chi^2, 2 \text{ d.f.}]$$

$$GF = 8.5712 [\chi^2, 9 \text{ d.f.}]$$

where LN is the natural log function,  $MT_t$  is a monthly time trend (January 1977=1),  $\bar{R}^2$  is the adjusted coefficient of determination, DH denotes Durbin's-H statistic, J-B denotes the Jarque-Bera diagnostic statistic for normality, GF denotes the goodness-of-fit test for normality of residuals, d.f. designates the degree of freedom associated with each Chi-Squared ( $\chi^2$ ) statistic, and other variables are as described earlier. The monthly data series runs from January 1977 to December 1988 and all prices were deflated by the index of prices paid by farmers, utilizing a base period of December 1988 (*Agricultural Prices*). Four observations (*i.e.*, 8/83, 8/84, 8/86, and 9/86) are missing from the data series, due

to unreported cash prices. Thus, relationships were estimated without observations whenever the lag length for a model resulted in a missing value. The second order Markov process of  $P_t$  was reduced to a first order process, utilizing the reproductive property of a normal distribution in conjunction with the linearity of (15), as described in Burt and Taylor.

A log-linear functional form was chosen to be more appropriate for  $P_t$ , than a linear functional form since the J-B and GF statistics were 20.3620 ( $\chi^2$ , 2 d.f.) and 12.5451 ( $\chi^2$ , 9 d.f.) respectively, for the alternative linear equation chosen. Seasonal factors were not detected to be significant in explaining cash prices. Also, lagged cash prices and the time of maturity variables were found to be insignificant in explaining the basis level.<sup>5</sup>

## MARGINAL TAX SCHEDULES AND OTHER CRITICAL INPUT FEATURES

As noted in Tronstad and Taylor (1991), the 1988 federal income tax code has basically three marginal tax rates for married individuals filing jointly. The marginal tax rates are 15 percent for taxable incomes less than \$29,750, 28 percent for taxable income between \$29,750 and \$71,901, 33 percent for taxable income between \$71,901 and \$171,090, and 28 percent for taxable income exceeding \$171,090. As in Tronstad and Taylor (1991), two personal exemptions (*i.e.* \$3,900) and standard deductions for a married couple (*i.e.* \$5,000) are claimed in determining federal income tax liabilities and social security taxes are included at a rate of 13.02 percent of income, with a maximum social security tax of \$5,702.80.

State taxes for Alabama consist of six marginal tax brackets that are relatively flat. After a \$3,000 deduction, marginal tax rates are 2 percent for incomes less than \$1,000, 2.7 percent for income between \$1,000 and \$5,000, 3.85 percent for income between \$5,000 and \$10,000, 4.4 percent for income between \$10,000 and \$20,000, 4.8 percent for income between \$20,000 and \$50,000, and 5 percent for income greater than \$50,000.

"Large" farm and "small" farm cotton acreage are given to be 750 and 250 acres, respectively. Because a 25 percent acreage reduction program rate is currently in place, planted acreage was limited to 562.5 and 187.5 acres for the large and small farm, respectively. For the large farm, variable production costs

<sup>4</sup> t-values are given in parenthesis.  $\chi^2$  critical values for a .10 (.05) significance level with 2, 7, and 9 degrees of freedom are 4.61 (5.99), 12.02 (14.07), and 14.68 (16.92), respectively.

<sup>5</sup> Although the time-to-maturity variable can be diluted in periods of tight stocks, (*e.g.*, favorable new crop prospects will decrease December futures much more than the spot price), this problem is present for any futures contract available, due to continuous global production and relatively free cotton trade.

were incurred at a rate of \$12.10, \$68.73, \$18.12, and \$131.10 per acre for all planted acres during the months of March, April, May through August, and November, respectively. Fixed production costs occurred at a rate of \$128.00/12 per acre for every month and acre owned by the large farm. The above-cost levels are based primarily on 1989 cost estimates reported by the Alabama Cooperative Extension Service enterprise budgets. Two cost assumptions were considered for the small farm, one at the same level of production costs given for the large farm, and the second was 1.3 and 1.5 times the per-acre variable and fixed production costs, respectively, of the large farm. The adjustments made to variable and fixed costs are essentially the adjustment rates supported and utilized by Mims for the difference between production costs of small and large farms in Alabama.

Grid dimensions utilized for the cotton model described above are as follows: 8 cash price ( $P_t$ ) states (\$.50, \$.55, \$.60, \$.65, \$.70, \$.775, \$.875, \$1.00 per lb.); 7 basis ( $B_t$ ) states (\$.25, \$.10, \$.05, \$0.00, \$.05, \$.10, \$.25 per lb.); before-tax income states ( $I_t$ ) of 9 for the large farm (-\$60,000 to \$100,000 in \$20,000 increments) and 7 for the small farm (-\$60,000 to \$60,000 in \$20,000 increments); storage states ( $ST_t$ ) of 15 for the large farm (0 to 700,000 lbs. in 50,000 lb. increments) and 10 for the small farm (0 to 225,000 lbs. in 25,000 lb. increments); quantity of December futures contract states ( $QD_t$ ) total 8 for the large farm (0 to 700,000 lbs. in 100,000 lb. increments) and 5 for the small farm (0 to 200,000 lbs. in 50,000 lb. increments); and value of December futures states ( $VD_t$ ) of prices from \$.25 to \$1.25 in \$.20 increments, multiplied by  $QD_t$ . Since the CCC loan rate is currently set at \$.50/lb., the lower range of the state space for  $P_t$  was set at \$.50/lb. As in Tronstad and Taylor (1991), the upper end of the distribution of  $P_t$  and both tails of the distribution of  $B_t$  were given a coarser grid space.<sup>6</sup>

The upper limits of before-tax income ( $I_t$ ), cotton storage ( $S_t$ ), and quantity of December futures position ( $QD_t$ ) are set at higher levels for the large farm than the small farm, due to the impacts associated with a larger acreage.  $S_t$  and  $XC_t$  are partitioned in increments of 25,000 and 50,000 lbs. for the small and large farms, respectively.  $QD_t$  and  $XD_t$  were partitioned in 50,000 lb. increments for the small farm since NYCE futures contracts are traded in 50,000 lb. increments. A coarser grid space of 100,000 lb. increments was utilized for  $QD_t$  and

$XD_t$  in the large farm model to keep computation requirements manageable.

Annual Alabama cotton yields from 1944 to 1988 were utilized to estimate cotton yield as a function of a time trend variable. The following equation was estimated for Alabama cotton yields:<sup>7</sup>

$$(17) Y_t = 257.95 + 7.1121 \cdot AT_t$$

$$(9.1851) \quad (6.5428)$$

$$\bar{R}^2 = .493 \quad D-W = 1.648$$

$$J-B = 4.251 [\chi^2, 2 \text{ d.f.}]$$

$$GF = 3.018 [\chi^2, 2 \text{ d.f.}]$$

where  $Y_t$  is yield (lbs./acre),  $AT_t$  is an annual time trend variable (1944=1),  $D-W$  is the Durbin-Watson statistic and all other statistics are as defined earlier. Annual cotton production ( $Q_t$ ) and anticipated production ( $AQ_t$ ) available for hedging is simply  $Y_t$  multiplied by planted acres for both the small and large farm models. Also, it is assumed that \$50.40 (1,008 lbs./acre • \$.05/lb.) in revenue is obtained from cotton seed at each harvest.

As in Tronstad and Taylor (1991), deficiency payments are distributed equally throughout the year as follows:

$$(18) DEF_t = (TP_t - P_t) \cdot FPP / 12; \text{ if } P_t < TP_t \text{ else}$$

$$DEF_t = 0$$

where  $DEF_t$  is the monthly deficiency payment received by the producer,  $TP_t$  is the target price (\$.734, and \$.729 for 1989, and crop marketing years beyond 1989, respectively), and  $FPP$  is farm program production or planted acreage multiplied by 575 lbs./acre. Because  $P_t$  is used to approximate the average national cotton price, equation (18) is a proxy to actual deficiency payments. Also, if  $P_t$  fluctuates above and below  $TP_t$  during the cotton marketing year, the actual annual deficiency payment would be somewhat less than the cumulative monthly payment given by equation (18).

Upper storage limit levels are set at 225,000 and 700,000 lbs. for the small and large farm cotton models, respectively. These levels are more than double anticipated production levels so that the storage constraint imposed should not be a major limitation of the model. Storage costs of \$.004/lb. of cotton were charged for each month of cotton storage. This translates to a figure of \$1.92/bale for a 480 lb. bale of cotton, which is within reason of most warehouse charges for monthly cotton storage.

Each one-way trade of a 50,000 lb. futures contract incurs \$125.00 in commission costs. Also, an initial

<sup>6</sup> Due to the CCC loan rate, the distribution of  $P_t$  is somewhat skewed toward higher prices.

<sup>7</sup>  $t$ -values are given in parenthesis.  $\chi^2$  critical values with 2 degrees of freedom at a .10 and .05 significant level are 4.61 and 5.99, respectively.

margin of \$2,000.00 is required for each contract and margin requirements increase or decrease with futures price movements in order to maintain a \$.04/lb. margin cushion. The margin expense or revenue realized each month is the interest paid (at a rate of .06/12) on the net balance in the producer's margin account.

## OPTIMAL COTTON MARKETING DECISIONS

This section graphically depicts optimal cotton marketing decisions derived from the SDP model for: (1) a large farm size of 750 acres with annual fixed and variable production costs of \$128.00 and \$284.41 per acre, respectively, (2) a small farm size of 250 acres with per acre production costs equal to the large sized farm, and (3) a small farm size with annual fixed and variable production costs of  $1.5 \bullet \$128.00$  and  $1.3 \bullet \$284.41$  per acre, respectively. In comparing the large and small farms, storage levels are set so that the same percent of anticipated production is in storage for both farms. Optimal cotton marketing decisions are given for the months of January, April, August, and December to illustrate converged<sup>8</sup> optimal decision rules for every month and state. Computation requirements were 7.16 and 1.17 hours of CPU time on a CRAY X/MP-48 for the large and small farm models, respectively.

Figure 1 presents results for the month of January where the basis level and cash price vary. Overall, results depict that cash sales are favorable to hedging for basis levels less than  $-.05/\text{lb.}$  and cash price levels above  $$.65/\text{lb.}$  Conversely, hedging is preferred to cash sales for basis levels above  $$.00/\text{lb.}$  and cash price levels above  $$.60/\text{lb.}$  In comparing results across farm size (*i.e.*, Sections A vs. B), cash sales are about 20 percent less for the small farm than for the large farm when cash prices are above  $$.775/\text{lb.}$  and the basis is less than  $$.05/\text{lb.}$  Greater absolute storage and anticipated production of the large farm in conjunction with relatively high prices limit the large farm's ability to reduce and/or defer income tax liabilities by storing into subsequent tax years, relative to the small farm.

A comparison of production cost levels (*i.e.*, Sections B vs. C) for the small farm indicates that cash cotton sales are anywhere from 20 to 60 percent greater for the "low" cost farm than the "high" cost farm when cash prices are above  $$.70/\text{lb.}$  and the basis level is less than  $$.05/\text{lb.}$  This is somewhat contrary to what one might expect on the surface, since higher production costs will result in relatively

lower marginal tax rates on additional cotton sales in the current tax year. However, higher production costs for subsequent tax years also creates the potential for cotton to be sold at lower marginal tax rates in the future. Thus, cash cotton sales decrease while hedging activity increases as production costs rise, at relatively high cash price and lower basis levels.

Figures 2a and 2b reveal optimal cotton marketing decisions for the month of April where the state variables of before-tax income, cash price, and basis level vary. Results in Figure 2a are for a basis level of  $-.10/\text{lb.}$  or a level that is relatively unfavorable to hedging. In comparing the effects of farm size (*i.e.* Sections A vs B), results indicate that the large farm should incur a higher percentage of cash cotton sales than the small farm at the upper price states. This is because additional cotton sales will increase the marginal tax rate relatively more for the small than for the large farm. Futures transactions occur at about the same percentage level for the small and large farms in Figure 2a, except for a cash price of  $$.775/\text{lb.}$  Futures transactions are relatively 25 percent more for the large farm than for the small farm at this price level. This occurs because: (1) a cash price of  $$.775/\text{lb.}$  is the first price level for hedging to become a desirable alternative, given that the basis level is  $-.10/\text{lb.}$ , and (2) hedging is more attractive for the large than for the small farm, given their respective anticipated income tax liabilities.

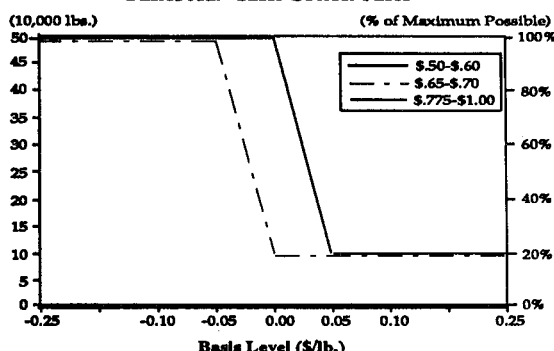
When analyzing the effects of production cost levels (*i.e.*, Sections B vs. C) in Figure 2a, results are similar to those found above for January. That is, cash cotton sales are higher for the low-cost producer than the high-cost producer. As above, storage is more enticing for the high-cost farm because of higher future production costs or lower anticipated marginal tax rates on cotton sales for subsequent tax years, at a given before-tax income level. Futures transactions are identical for the high and low cost farms in Figure 2a.

Figure 2b presents a state space that is the same as Figure 2a, except that the basis level is  $$.10/\text{lb.}$  instead of  $-.10/\text{lb.}$  Because a basis level of  $$.10/\text{lb.}$  is much more favorable to hedging than a basis level of  $-.10/\text{lb.}$ , cash cotton sales and futures transactions are very similar across farm size and production cost levels, except for the lowest before-tax income states. At before-tax income states less than \$20,000, cash cotton sales are relatively higher for the large farm than for the small farm due to the importance of the large farm's capitalizing on low before-tax income states. Similarly, cash cotton sales

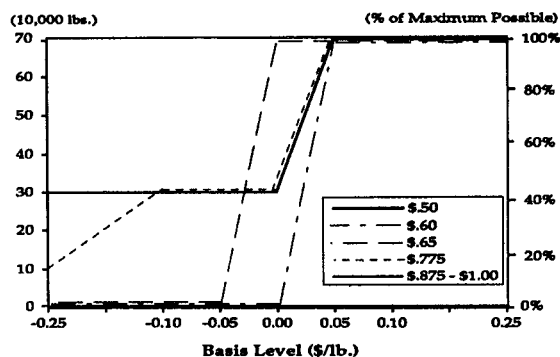
<sup>8</sup> Model convergence was obtained in 48 stages or 4 years.

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 500,000 lbs.**

**Panel A1. Cash Cotton Sales**

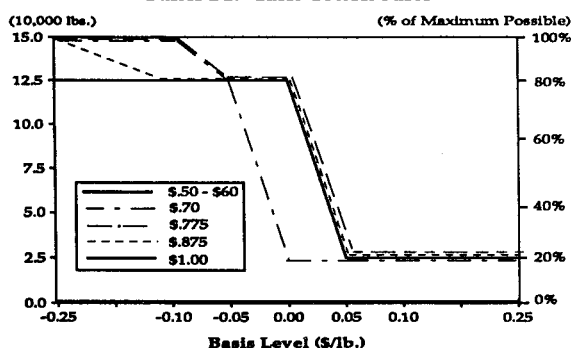


**Panel A2. Futures Transactions**

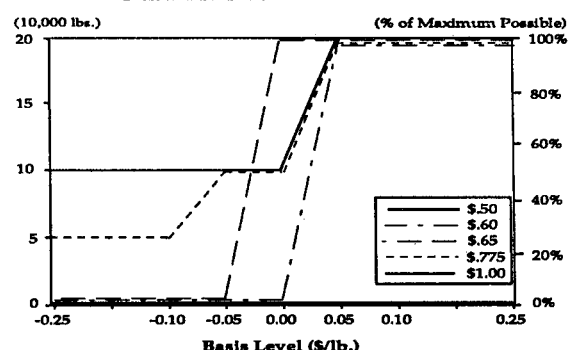


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel B1. Cash Cotton Sales**

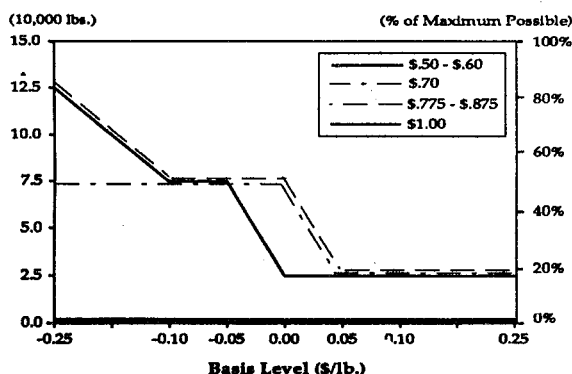


**Panel B2. Futures Transactions**

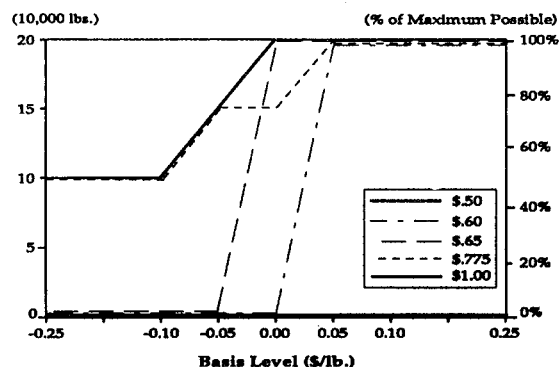


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



**Figure 1. Optimal Cash Cotton Sales and Futures Transactions for the Month of January for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Before-Tax Income Level ( $I_t$ ) = \$0.00, Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Basis ( $B_t$ ), and Cash Price of Cotton ( $P_t$ ). Note, legend lines in each panel are  $P_t$  (\$/lb.).**

occur at a lower price level for the low cost farm than for the high cost farm at a before-tax income level of -\$60,000.

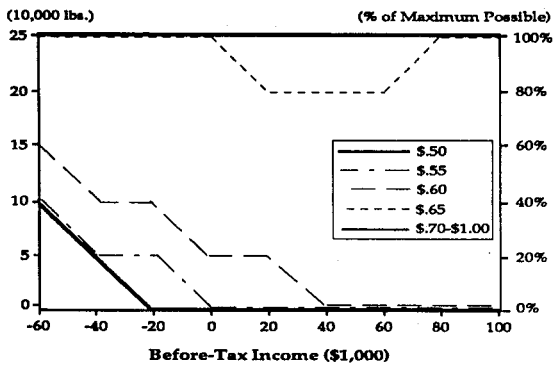
Optimal cotton marketing decisions for the month of August are presented in Figure 3 where the state

variables of basis and cash prices vary as in Figure 1 for January. Because the storage level for Figure 3 is one-half of that in Figure 1, cash cotton sales in relation to the amount of storage are about 20 to 40 percent greater in Figure 1 than in Figure 3 for cash

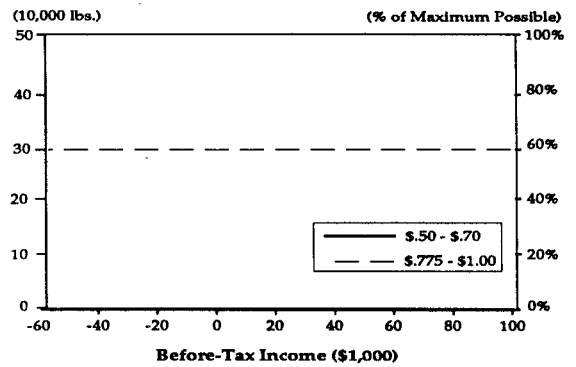


**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 250,000 lbs.**

**Panel A1. Cash Cotton Sales**

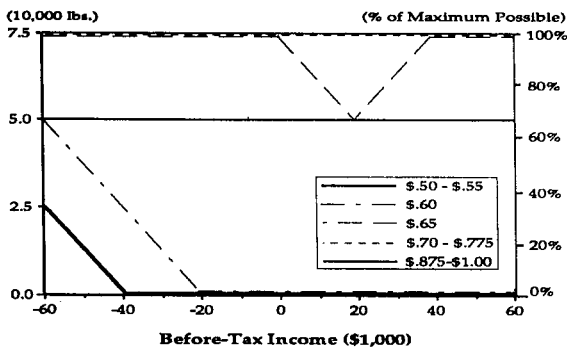


**Panel A2. Futures Transactions**

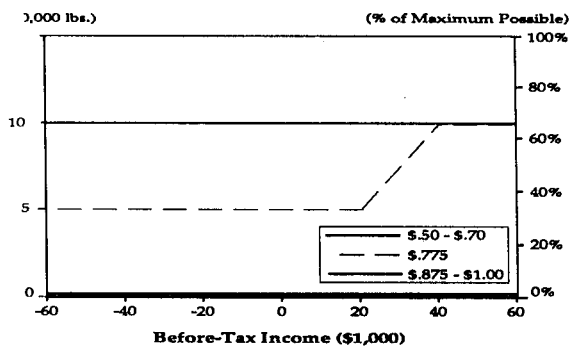


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel B1. Cash Cotton Sales**

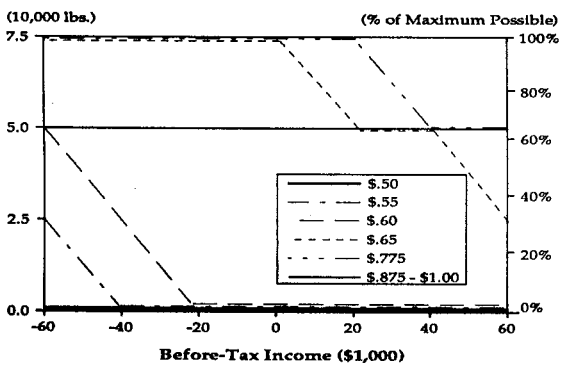


**Panel B2. Futures Transactions**

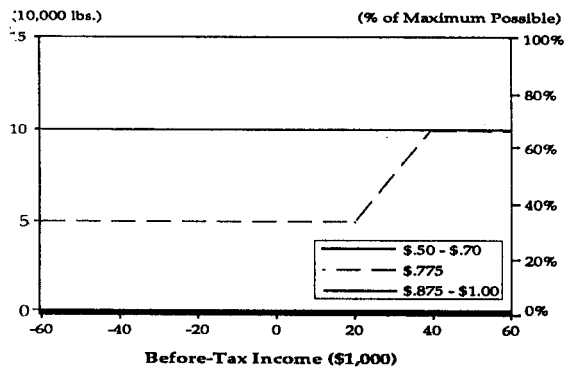


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \cdot \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \cdot \$284.41$ /acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



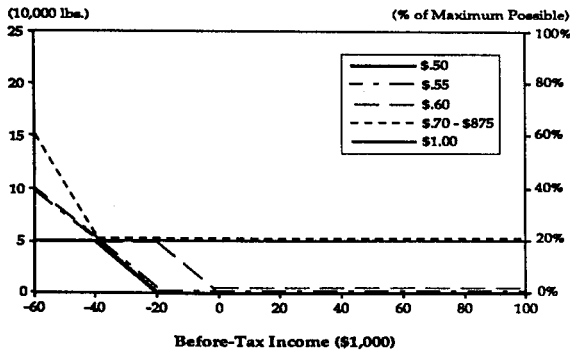
**Figure 2a. Optimal Cash Cotton Sales and Futures Transactions for the Month of April for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Basis Level ( $B_t$ ) = \$.10/lb., Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Before-Tax Income ( $I_t$ ), and Cash Price of Cotton ( $P_t$ ). Note, legend lines in each panel are  $P_t$  (\$/lb.).**

prices between \$.65 and \$.775 per pound. However, futures transactions are about 20 percent more for August than for January in relation to the maximum level possible for cash prices above \$.65/lb. and

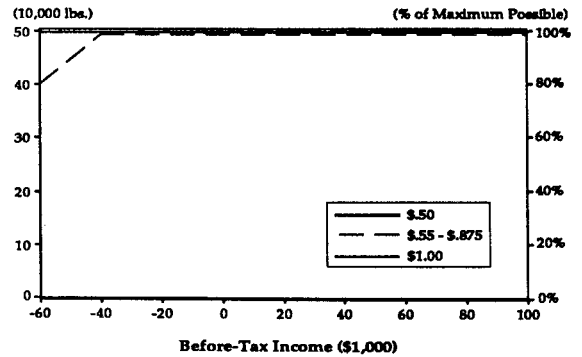
basis levels less than \$.00/lb. This result occurs because the expected after-tax return on the first contracts sold is greater than on later contracts sold, due to the progressive tax structure. Changes in

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 250,000 lbs.**

**Panel A1. Cash Cotton Sales**

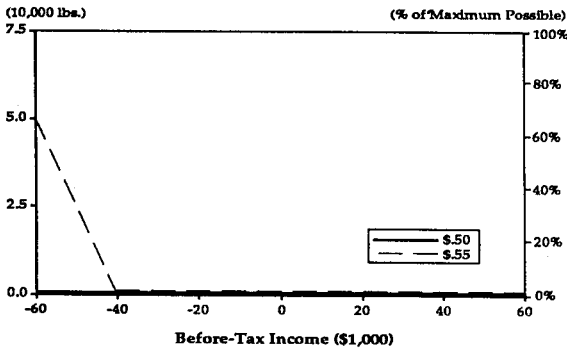


**Panel A2. Futures Transactions**

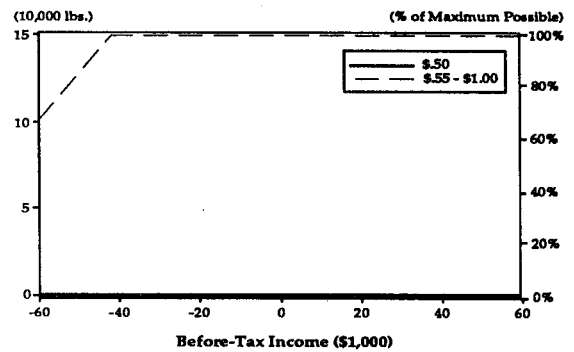


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel B1. Cash Cotton Sales**

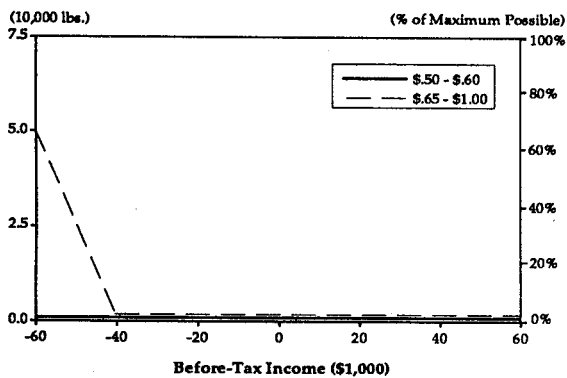


**Panel B2. Futures Transactions**

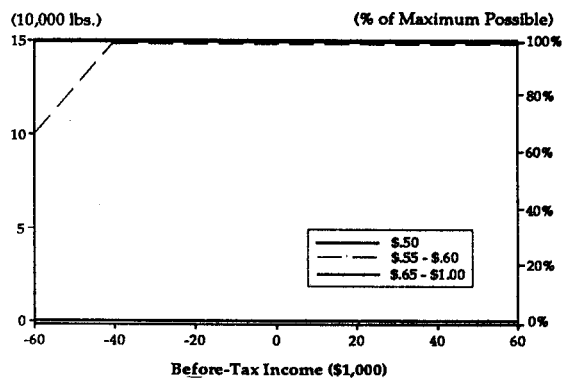


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



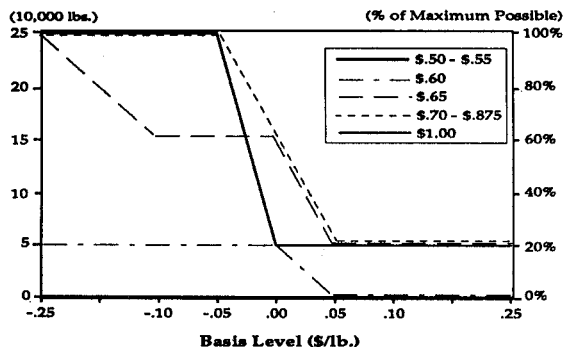
**Figure 2b. Optimal Cash Cotton Sales and Futures Transactions for the Month of April for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Basis Level ( $B_t$ ) = \$.10/lb., Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Before-Tax Income ( $I_t$ ), and Cash Price of Cotton ( $P_t$ ) (\$/lb.).**

marketing patterns across farm size and production-cost levels for August are very similar to those for January.

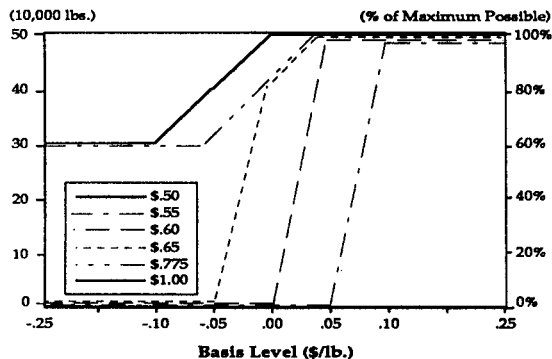
As in Figures 2a and 2b, Figures 4a and 4b present optimal cotton marketing decisions for the month of December where the state variables of before-tax income, cash price, and basis level vary. Changes in

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 250,000 lbs.**

**Panel A1. Cash Cotton Sales**

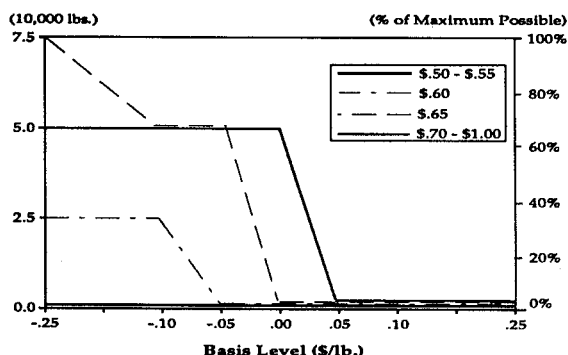


**Panel A2. Futures Transactions**

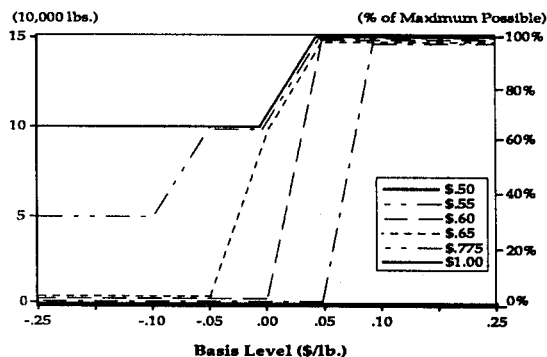


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel B1. Cash Cotton Sales**

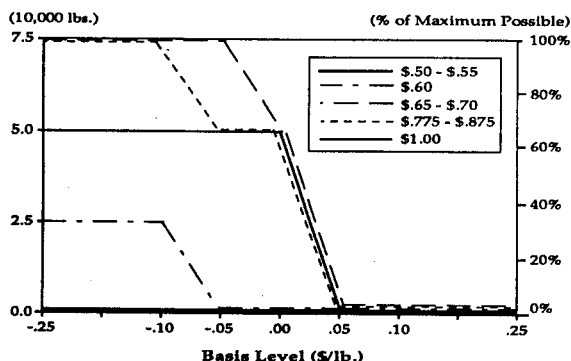


**Panel B2. Futures Transactions**

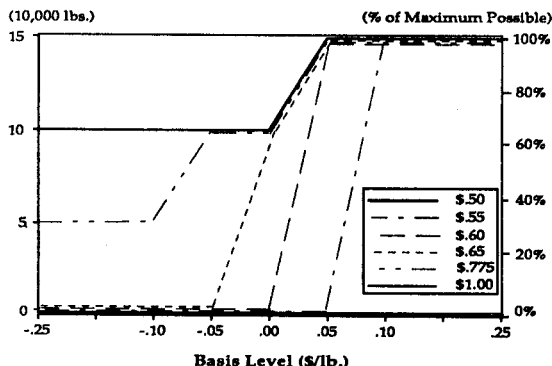


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 75,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



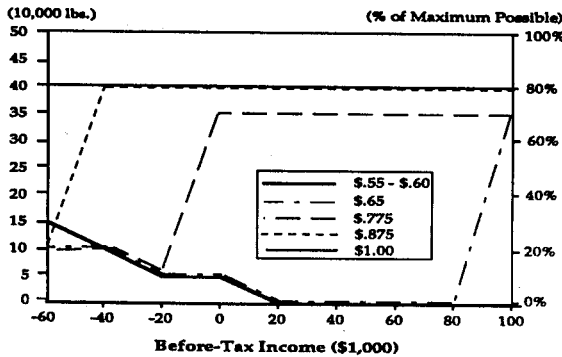
**Figure 3. Optimal Cash Cotton Sales and Futures Transactions for the Month of August for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Before-Tax Income Level ( $I_t$ ) = \$0.00, Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Basis ( $B_t$ ), and Cash Price of Cotton ( $P_t$ ). Note, legend lines in each panel are  $P_t$  (\$/lb.).**

marketing patterns across farm size and production-cost levels for Figures 4a and 4b have some notable differences compared to differences seen in Figures 2a and 2b. First, because December is the last month to capitalize on a low marginal tax rate, the relative

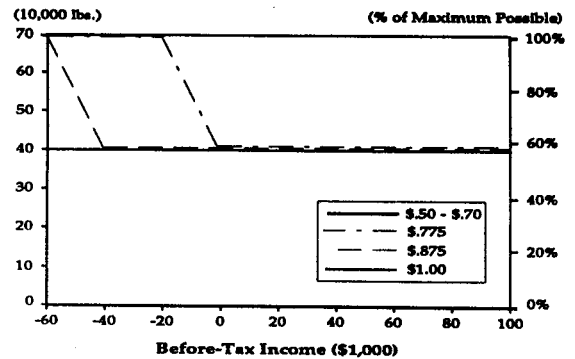
percentage of cash cotton sales is much greater for the small farm than for the large farm for cash prices less than \$0.65/lb. and before-tax income levels less than \$0.00. Also, cash cotton sales are larger for the low-than for the high-cost farm at these lower cash

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 500,000 lbs.**

**Panel A1. Cash Cotton Sales**

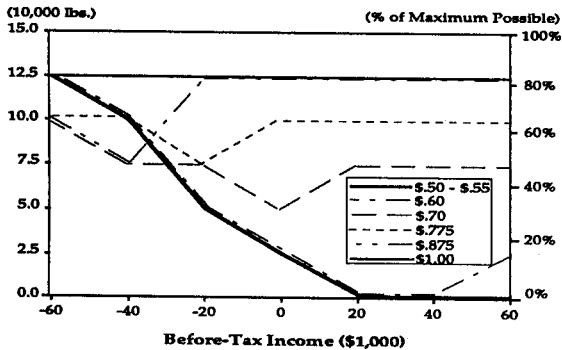


**Panel A2. Futures Transactions**

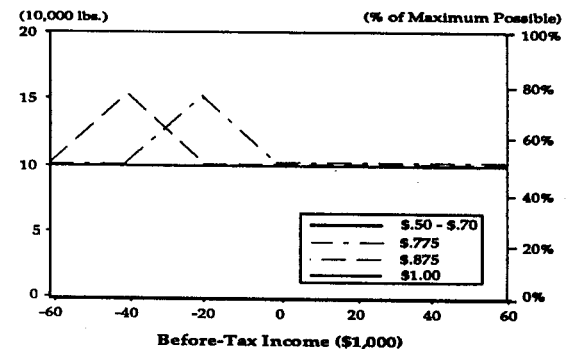


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel B1. Cash Cotton Sales**

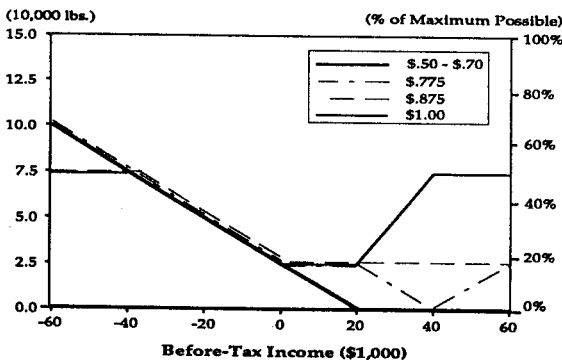


**Panel B2. Futures Transactions**

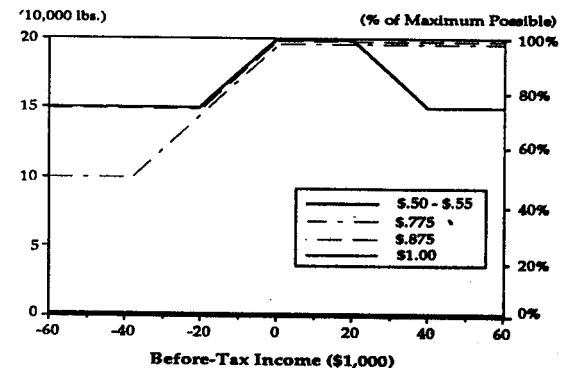


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



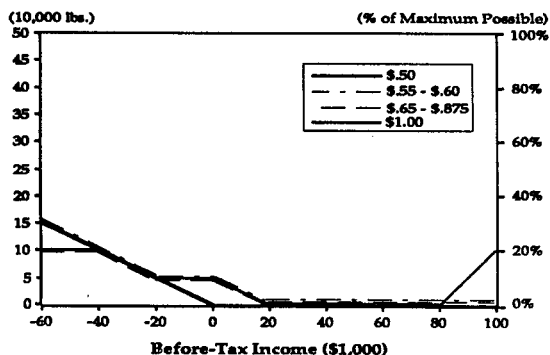
**Figure 4a. Optimal Cash Cotton Sales and Futures Transactions for the Month of December for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Basis Level ( $B_t$ ) = \$.10/lb., Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Before-Tax Income ( $I_t$ ), and Cash Price of Cotton ( $P_t$ ). Note, legend lines in each panel are  $P_t$ (\$/lb.).**

price levels and before-tax income levels, since the higher anticipated earnings of the low-cost farm make it more crucial for the low-than for the high-cost farm to capitalize on low before-tax income states in December.

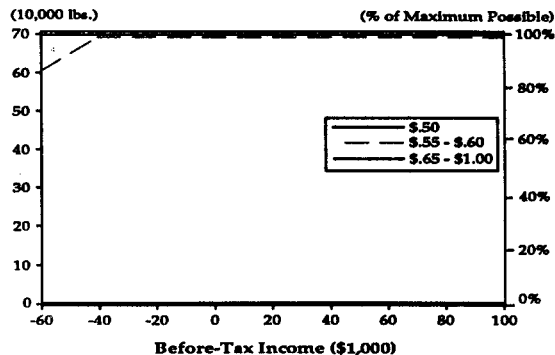
In Panel A1 of Figure 4a, cash cotton sales eventually increase as the before-tax income level of the farm increases for cash prices above \$.775/lb. Optimal cash cotton sales increase with an increase in the before-tax income level since the marginal tax rate

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 500,000 lbs.**

**Panel A1. Cash Cotton Sales**

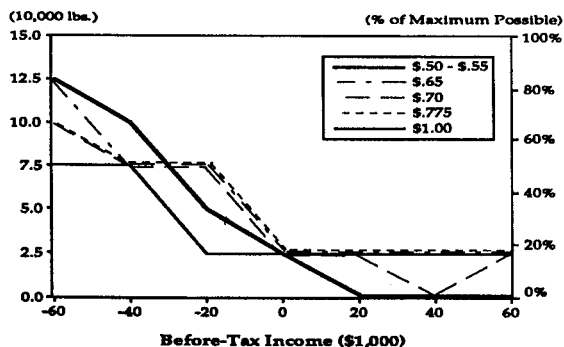


**Panel A2. Futures Transactions**

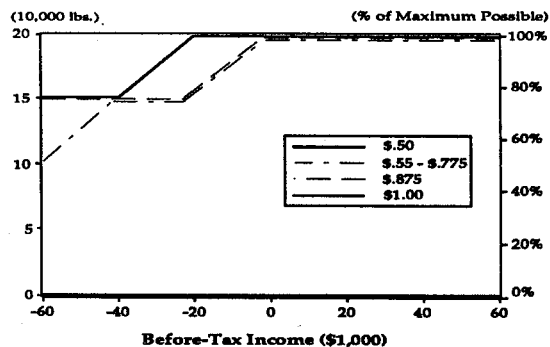


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel B1. Cash Cotton Sales**

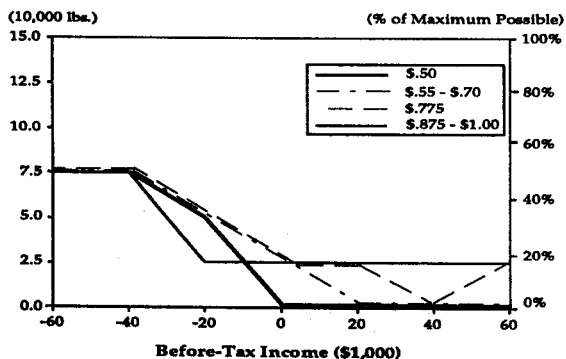


**Panel B2. Futures Transactions**

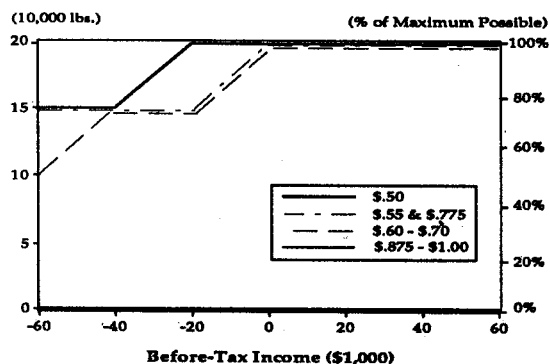


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



**Figure 4b. Optimal Cash Cotton Sales and Futures Transactions for the Month of December for Different Farm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Basis Level ( $B_t$ ) = \$0.10/lb., Quantity of December Futures Position ( $QD_t$ ) = 0 lbs., and Varying Levels of Before-Tax Income ( $I_t$ ), and Cash Price of Cotton ( $P_t$ ). Note, legend lines in each panel are  $P_t$ (\$/lb.)**

becomes very flat for the upper income range (*i.e.*, taxable incomes above \$29,750), the basis level (-\$0.10/lb.) is relatively unfavorable to hedging, and no future production costs will occur for the current

tax year. Thus, even for the month of December, the likelihood of prices declining can outweigh the prospect of reduced income tax liabilities through stor-

age into the subsequent tax year, at upper before-tax income and price levels.

Futures marketing patterns are fairly similar for the month of April and December in that no hedging occurs at a cash price level less than \$.70/lb. when the basis is less than -.10/lb. Also, the maximum amount possible to hedge is always hedge whenever the basis is greater than \$.10/lb., the cash price level is above \$.55/lb., and the before-tax income state of the farm is greater than -\$20,000. However, comparison between hedging transactions in Figures 2a and 4a shows that the high-cost farm tends to hedge relatively more in December than in April for cash prices above \$.775/lb., because the expected gains from deferring income tax liabilities outweigh the probability associated with prices declining more for December than for other months since in December the next tax year is only one month away. This is also why the large farm tends to hedge a relatively larger proportion than the small farm for December than for April.

Figure 5 presents optimal cash cotton sales and futures transactions for the month of December where the state variables of basis and the value of December futures contract(s) outstanding, denoted by Average Transaction Value ( $ATV_t = VD_t / QD_t$ ) for simplicity, vary. Because the  $ATV_t$  of a futures contract can only influence the level of cash cotton sales indirectly by the level of futures transactions and before-tax revenue generated from decreasing a short futures position, cash cotton sales are quite similar across farm size and production cost levels in Figure 5. However, as one would expect, different  $ATV_t$ s result in a more noticeable difference for futures transactions with changes of farm size and production cost levels. In general, the high-cost farm buys back more if its short futures position for basis levels less than -.10/lb. and  $ATV_t$ s greater than \$.65/lb. than the low-cost farm does. However, for an  $ATV_t$  of \$.45/lb., the high-cost farm buys less futures contracts than the low-cost farm. This occurs because each futures contract is bought back for a price less than the  $ATV_t$ , and brings in a positive profit on the futures transaction. Thus, the low-cost farm tends to buy back a smaller percentage of its futures contracts outstanding at lower basis levels (*i.e.*, less than -.05/lb.) than does the high-cost farm.

## CONCLUDING COMMENTS

Results indicate that cash cotton sales are preferred to storage for all farm sizes and production-cost

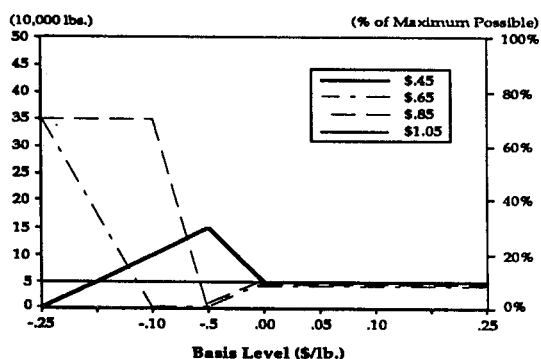
levels when the basis level is less than \$.00/lb., the cash price level is above \$.775/lb., and the before-tax income level of the farm is less than \$0.00. However, as the end of the tax year approaches, storage becomes relatively more attractive since the prospects for deferring income tax liabilities through storage tend to outweigh the probability associated with prices changing unfavorably. Also, given an initial before-tax income level, storage becomes more attractive for the high-cost farm than the low-cost farm as the end of the tax year approaches, since the high-cost farm anticipates lower marginal tax rates on revenue received in the following tax year than does the low-cost farm. In comparing the effects of farm size on cash cotton sales, results suggest that the large farm should incur a relatively larger percentage of storage as cash cotton sales than the small farm at the beginning of the tax year, whereas this situation tends to reverse as the end of the tax year approaches. This result reveals the effect that the progressive tax structure, discounting, and stochastic variables have on marketing decisions for different sized firms.

Hedging storage or anticipated production is a preferred activity for all farm sizes and production-cost levels whenever the basis level is greater than \$.00/lb., and the cash-price level is above \$.65/lb. However, when the basis level is less than -.10/lb., the high-cost farm tends to hedge about 20 percent more than the low-cost farm. This result reflects lower anticipated marginal tax rates for the high-cost farm compared to the low-cost farm, on additional cotton sales in the following tax year. Also, the large farm generally hedges a larger portion of storage than does the small farm when the basis level is less than -.10/lb., since a relatively higher marginal tax rate on additional cash cotton sales is confronting the large farm.

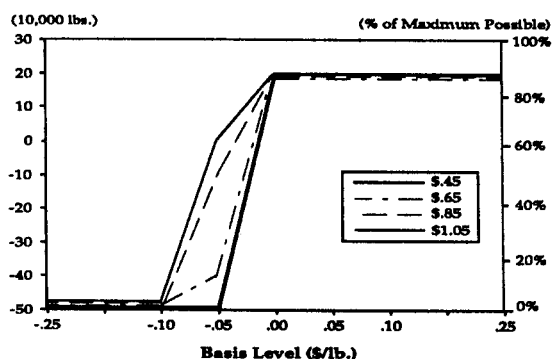
Overall, cotton marketing decisions are much more robust across different farm sizes and production-cost levels than across different levels of cash price, basis, and before-tax income. Nonetheless, marketing decisions can differ quite substantially for different farm sizes and production cost levels. This is most evident towards the end of the tax year for cash cotton sales and the lower range of cash price (less than \$.65/lb.), basis (less than -.05/lb.), and before-tax income (less than \$0.00) states.

**Section A. Farm Size = 750 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 500,000 lbs.**

**Panel A1. Cash Cotton Sales**

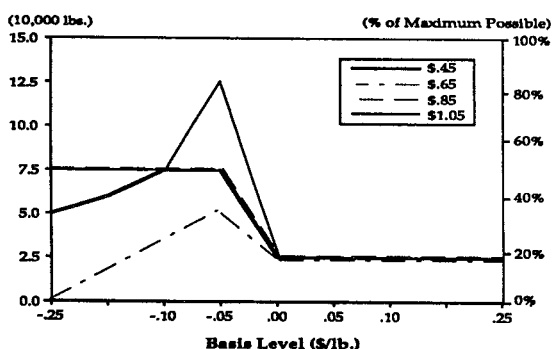


**Panel A2. Futures Transactions**

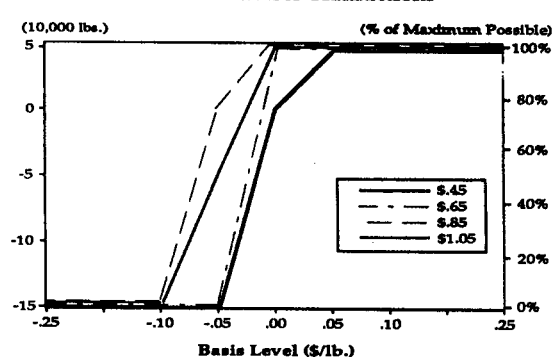


**Section B. Farm Size = 250 acres, Annual Fixed Cost = \$128.00/acre,  
Annual Variable Cost = \$284.41/acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel B1. Cash Cotton Sales**

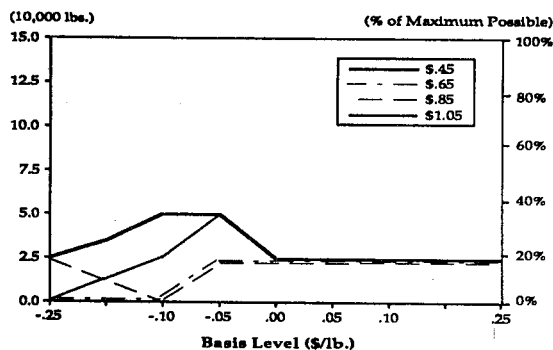


**Panel B2. Futures Transactions**

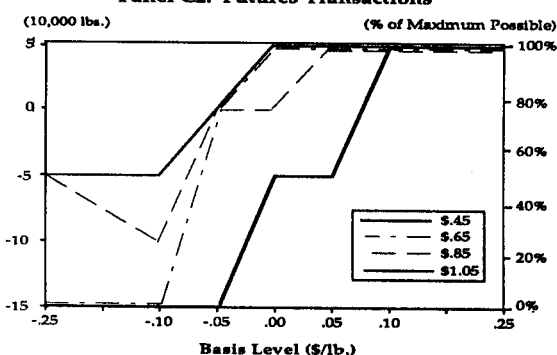


**Section C. Farm Size = 250 acres, Annual Fixed Cost =  $1.5 \times \$128.00$ /acre,  
Annual Variable Cost =  $1.3 \times \$284.41$ /acre, and Storage Level ( $S_t$ ) = 150,000 lbs.**

**Panel C1. Cash Cotton Sales**



**Panel C2. Futures Transactions**



**Figure 5. Optimal Cash Cotton Sales and Futures Transactions for the Month of December for Different Firm Sizes (Sections A vs. B), and Production Cost Levels (Sections B vs. C), Given a Cash Price ( $P_t$ ) = \$0.65/lb., Quantity of December Futures Position ( $QD_t$ ) equal to the Given Storage Level ( $S_t$ ), Before-Tax Income ( $I_t$ ) = \$0.00, and Varying Levels of Basis ( $B_t$ ), and Average Transaction Value ( $ATV_t$ ) of  $QD_t$ . Note, legend lines in each panel are  $ATV_t$  (\$/lb.)**

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