INSTITUTE OF AGRICULTURAL
AND FOOD ECONOMICS
NATIONAL RESEARCH INSTITUTE

MULTI-ANNUAL PROGRAMME
2015 - 2019

Exogenous determinants of agricultural production – prices of the production factors and the selected macroeconomic indicators

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69.1
MONOGRAPHS
OF MULTI-ANNUAL
PROGRAMME
WARSAW 2017

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MULTI-ANNUAL PROGRAMME 2015 - 2019

**THE POLISH AND THE EU AGRICULTURES 2020+
CHALLENGES, CHANCES, THREATS, PROPOSALS**

Warsaw 2017

The authors are the researchers from the Institute of Agricultural and Food Economics – National Research Institute

The paper was prepared under the topic: **Sources of growth and the expected evolution of structures and the role of the agri-food sector until the year 2020 and beyond** in the task: *Investments, efficiency and new technologies as the sources of economic growth in agriculture until and after 2020.*

The paper aimed at assessment of the sources of economic growth in agriculture, with particular focus on verification of the character and efficiency of the sources. Emphasis was put on assessment of exogenous sources – relation of prices of the factors of production (capital, labour and land). This was captured in the convention of the analysis of price and degree of its rarity. The assessment covered also the relationship between the relations of prices of factors of production and their relations as for their engagement in production. In fact, the analysis covered the relationship between the price relations of production factors and the used production techniques. The attention was also focused on exogenous impacts in the field of monetary policy.

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Leszek Ślipski

ISBN 978-83-7658-718-9

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Introduction

In the monograph, we assume that the choices of agricultural producers, regarding production techniques they use, i.e. the relationships of used production factors, are affected by the prices of these factors and elements of the economic policy, including the monetary policy. This forms the relationships of these variables, i.e., for example, between the factor prices and production techniques for the agricultural sector. This is, in a sense, a hypothesis or rather a research assumption.

In the monograph, we refer to the relationships of production factor prices determined exogenously in the market, according to the level of their scarcity. We treat them as external conditions of the production process, including (being the subject of our interest) the formation of the production factor relationships in the agricultural sector. These relationships have a further impact on the productivity of these factors, and finally on income obtained and production profitability[1]. We point to this so as to stress that the analysed relationships are a condition and cause of these latter processes occurring in the phenomena.

As the methodological basis for the used research technique, we adopt the relationships arising from the production function. The reasoning is carried out based on an analytical aspect according to the Neo-Classical theory of economics. We also refer here to the tradition of formulating the price relationships, production factors or production techniques and effectiveness coming from the Main School of Planning and Statistics in Warsaw (currently, the Warsaw School of Economics), where the theoretical foundations in this area have been established and developed by the economists such as J. Rajtar, F. Tomczak and A. Woś, S. Gburczyk.

This formulation is illustrated by the hypothetical relationships, and then verified empirically using statistical material. This is to potentially confirm the principles resulting from analysis based on the analytical aspect and the quoted literature. We use the principle of deduction and generality in the analytical process as the basis. This is the basis for reasoning, and not empirical analysis itself, which is primarily used for the purposes of illustration and some verification.

In pursuing the objective, i.e. the identification and assessment of exogenous macroeconomic conditions of the production in agriculture related

[1] These issues are not analysed in this monograph.

mainly to the monetary policy, we used the instruments of both micro- and macroeconomic analytics. Empirical analyses have also been supported by quantitative data analysis methods – in particular, non-linear models were used, so was econometric analysis of time series, also in multivariate terms. We also used the vector autoregression models, and their economic interpretation was determined by the obtained functions of the response to the impulse from the monetary policy area.

The first chapter showed the mechanism of functioning of the production factor market. It also defined the factor price relationships as an endogenous condition. The second chapter showed the relationships of production factor prices (capital, labour and land). This is formulated as price analysis and the level of its scarcity. In the third chapter, we analysed the relation between production factor price relationships and their relationships as regards their employment in the production. In fact, the relation between the production factor price relationships and used production techniques has been analysed. This an essential part of studies in the analysis when it comes to the factor prices. The basis was our own analytical aspect. We also indicated the relation between the price relationships and the production factor productivity relationships. The fourth chapter showed the impact of the monetary policy on the selected price indices in agriculture and their changes as a result of positive interest rate shock.

Chapter I
Production factor market as an exogenous condition

We assume that the availability of production factors, their market scarcity level and resulting prices affect the relationships of these factors in agricultural producers or in agriculture as a sector of the economy. It is important to assume that the relationship of the scarcity level of production factors in the market is illustrated by their price relationships. In addition, these prices are more associated and more simply defined with the scarcity relationships for the labour and capital factors, than for the land factor, simply due to specific regulations in trade with the latter. Apart from this issue, the analysis is limited to the labour and capital factor prices and their impact on the factor relationships in agricultural producers or in agriculture. These factor relationships are production techniques. We assume that their changes are determined by changes in the production factor price relationships. They, i.e. production factor price relationships, are, in turn, shaped by the production factor market mechanism. This is, as we assume, the classical exogenous condition, as each individual agricultural producer (also as it seems – all producers as the sector) does not affect these prices i.e. production factor prices. This, in turn, affects the level of the productivity of these factors and consequently determines their remuneration in agricultural producers. The latter is an endogenous condition, which was the subject of the previous monograph of the Multi-Annual Programme[2]. The foundations of this division are given in the next chapter. In order to refer to the problem formulated in this way, we start with determining the production factor market mechanism shaping the prices of these factors. Here, we rely on the approach strictly embedded in the theory of microeconomics.

1.1. Production factor market mechanism

We define production factors, obviously, as the variables which are necessary for the process of producing final or intermediate goods in companies[3]

[2] Bezat-Jarzębowska A., Rembisz W. (2016), *Techniki wytwarzania jako endogenne uwarunkowanie produkcji i jej zmian w rolnictwie krajów UE*. Monografie Programu Wieloletniego 2015-2019, nr 32 IERiGŻ-PiB, Warszawa.

[3] Even less generalised aspects, whose message is more useful, will consider the roles of the production factor relationships as a basis for all management process. For example, we find that "From analysis of historical and economic phenomena it results that the objective

in the economy. Of course, this also applies to agricultural producers. In classical terms, particularly as regards agricultural producers, three production factors are defined: land, labour and capital[4]. We also point to such conditions of the more or less effective use of these production factors as enterpreneurship or organisation. The latter, just as innovation, is sometimes considered as additional, unconventional production factors. In the paper, we refer only to these real and measurable production factors and their combinations, or links, as the effect of exogenous i.e. market conditions, as one of the more important factors affecting the combination of factors and resulting production efficiency.

As the labour factor, in a sense of the general theory of production factors derived from the production function concept, we should understand a worker together with their skills, experience, etc. and specified work time. In turn, the capital factor, as a general category, in these considerations will be material capital which consists of fixed capital and working capital (cf. Rembisz, Sielska,

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grounds for the occurrence and development of enterprises are mainly included in the mechanism of shaping an optimal combination of production factors..." and "production activity is related to the continuous collation of production factors in various configurations and proportions". Then, in individual combinations, there are specific relationships between them which determine the efficiency of these links". J. Klimek (2009), *Hermeneutyka przedsiębiorczości*. Wyd. A. Marszałek, Toruń, p. 8.

[4] Reasoning in the paper is conducted in terms of the theory of production factors, derived from J. B. Say, where the concept of the production factors ist the most general classification of material production foundations made at the highest level of abstraction, while three production factors are general ahistorical categories which for each economic entity (in agriculture) at any place and time are necessary for the production process. The worth of each result of these factors is the same as its contribution into the product and its profit from its "services" in the production created. The most abstract and general concept of material production elements, i.e. the concept of the production factors is, to some extent, an anatomical basis for assessing the resulting general relationships, such as e.g. production efficiency, remuneration. The production efficiency always and in all conditions is connected with and results from the way of combining and using the production factors. This way of combining, these proportions among the production factors, production techniques, determine the economic results, including the remuneration of the factors. On the other hand, the price of these factors results from the proportion of their scarcity, i.e. from the market price of the given factor specified by the supply and demand relationships of the factor concerned. Technical progress, human capital, knowledge and all qualitative characteristics are also embodied in the factor supply relationship. These are the exogenous conditions of the production (now apart from the specific agricultural policy, economic policy and other macroeconomic conditions). These dependencies are governed by specific laws and regulations, i.e. the relationship between average and marginal productivities, typically determined based of production function analysis (cf. Kozłowski Z., Przedmowa do pracy Harleman, Satmer [in:] Herlemann H., Stamer H. (1963), *Rolnictwo w dobie technizacji,* PWRiL, Warszawa.

p. 83)[5]. With similar assumptions as to the generality, the land factor in this paper should be understood as a resource of agricultural land-utilised agricultural area (UAA), along with its qualitative dimension, which is used for the agri-food production in the agricultural producer and in the entire sector as a set of these producers. Since, as mentioned at the beginning, this paper follows analyses characteristic of microeconomics in the concept of the Neoclassical school, the land factor is a type of capital and will not (basically) be considered here as a separate production factor (cf. e.g. Stigler, pp. 278-279).

The mechanism shaping the condition of equilibrium in the production factor market defines the factor price mutually acceptable by both sides (demand and supply). We should bear in mind the fact that the supply side of the production factor market is represented by consumers (in our case, farmers, farm owners), who start a quantitative flow of factors by offering the producer (in our case-offering themselves, due to the unity of the term consumer and producer, which is specific of the farm and distinguishes agricultural economics within economics) the production factor service ready for being employed in the production process. The market side reporting the demand for these factors are, in this case, producers (in the case of the farm-the owner, the farmer is also identical to the consumer). Therefore, in relation to farms, as we stated before, consumers as owners of production factors (including labour) are also producers. The farmer is simultaneously a consumer who expects remuneration for provided services using owned production factors (labour, land, capital) and is also a producer who must pay for these services, i.e. remunerate for them (wages, ground rent, interest). The source of this remuneration can only be the effective use of these factors i.e. their productivity. The productivity is the relationship between the production and the factor resource employed on the farm. The basis for this reasoning and assessment as to the productivity are the production function assumptions. Of course, the productivity is the basis for assessing the degree of using production factor resources being at the producer's disposal[6].

At the microeconomic level of the farm, it is, as we stressed and still do, difficult to separate these processes. Hence, there are many misunderstandings in the literature. Nevertheless, these processes described in the circular flow and

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[5] Rembisz W., Sielska A. (2015), *Mikroekonomia współczesna.* VIZJA PRESS&IT, Warszawa.

[6] Such a basis for the assessment are not the factor inputs. This itself to some extent expresses the profitability of production, the inputs from the given factor resources can be higher depending on the profitability, including also depending on the demand, as there is implicitly an assumption about the production sold, i.e. useful.

indicated above, take place and each farmer, consciously or not, compares his remuneration for labour on own farm with non-agricultural income, ground rent obtained and profit derived from employed material capital with interest on, e.g. deposits or government bonds. On the agricultural sector scale, however, these processes are more visible and this is the subject of attention throughout the paper.

Fundamental principles in the production factor market, when it comes to shaping the price in relation to the demand for and supply of these factors, are quite simple, just as the market mechanism itself[7].

We should adopt the following designations:

L - labour factor,

K - capital factor,

Z - land factor,

c_L - labour factor price,

c_K - capital factor price,

c_Z - land factor price,

∂ - increase in the specific variable,

D - demand for the given factor,

S - supply of the given factor.

The fundamental principles of the production factors market, in analytic terms, referring to the evolution of the factor prices are as follows: price (of the service) of the labour, capital or even land factor changes in relation to the increasing demand, most often in line with (cf. Rembisz, Sielska; 2015)[8]:

$$\frac{\partial c_L}{\partial L^D} > 0, \text{and} \frac{\partial c_K}{\partial K^D} > 0 \ (1.1)$$

With some restrictions (e.g. regulatory-legal and other), for the land factor we have:

$$\frac{\partial c_Z}{\partial Z^D} > 0, (1.2)$$

for increasing: ($\partial L > 0, \partial K > 0, \partial Z > 0$).

[7] The market mechanism is so simple and effective, that we do not think about it, just as we do not think about the air, as noticed once by M. Fridman. The market pricing mechanism leads, in accordance with this classical statement, to their most effective use in the production (to the cheapest production methods), thus according to our line of reasoning. Friedman M., (1994), *Wolny wybór.* Aspekt, Sosnowiec.

[8] Rembisz W., Sielska A. (2015), *Mikroekonomia…*, op. cit.

Of course, for the decreasing demand for the given production factor, its price by assumption should decrease, e.g. for the labour factor $\frac{\partial c_L}{\partial L^D} < 0$.

In relation to the increasing supply of these factors, their prices and prices of their services change most often in line with[9]:

$$\frac{\partial c_L}{\partial L^S} < 0 \text{ and } \frac{\partial c_K}{\partial K^S} < 0. \text{ (1.3)}$$

Almost identically, for the land factor:

$$\frac{\partial c_Z}{\partial Z^S} < 0, \text{ (1.4)}$$

with: $(\partial L > 0, \partial K > 0, \partial Z > 0)$.

And in relation to the decreasing supply, which is currently a real case in the reality of agriculture, especially with respect to labour and land factors, the price of services of these factors is changing in accordance with:

$$\frac{\partial c_L}{\partial L^S} < 0 \text{ and } \frac{\partial c_Z}{\partial Z^S} < 0, \text{ (1.5)}$$

with: $(\partial L < 0,, \partial Z < 0)$.

Therefore, we are dealing with the mechanism of increasing the price with the increasing demand and decreasing the price with the increasing supply and the other way round. Consequently, we determine the price of equilibrium c_L^*, which in a simplified manner can be illustrated as follows:

$$c_L^S \underset{t}{\Rightarrow} c_L^* \underset{t}{\Leftarrow} c_L^D, \text{ (1.6)}$$

for the given period *t,* supply *S* and demand *D*. In the subsequent iterations, to some extent according to the Walrasian Auctioneer Mechanism, equilibrium is disturbed and restored, for other price levels of production factor services. In terms of our analysis, this is unimportant. Important is the impact of such factor service price on the factor productivity and the resulting level of remuneration of the given factor which will be discussed later on.

Therefore, the level of equilibrium determined in the production factor market, i.e. the level of prices of given factors (and consequently the prices of their services) affects their level of employment, utilisation rate (production efficiency) and remunerations in agricultural producers. These are remunerations of factors underpinned by wages, profit and ground rent. This

[9] More generally, we have: $c_K = \frac{1}{K_t}$, where each increase in the factor supply (here the material capital factor) leads to the decrease in its price which is equivalent to the notations above.

determines income of farmers as consumers. This also affects their expenses for production factor services as producers and specifies the demand for production factors (cf. Rembisz, Sielska, pp. 84)[10]. This complicated system results, as we know, from the fact that the farmer is both the worker and the employer. Nevertheless, this is subject to the same production factor market's laws as in relation to all producers.

The market mechanism also shapes the factor price relationships, including the mutual relationship, e.g. between the labour and capital factors and between the capital and land factors:

$$\frac{c_L}{c_K}; \frac{c_K}{c_Z}. \quad (1.7)$$

This proportion (1.17) affects the relationships of the productivity of factors employed by farmers as producers and finally affects the relationships as to the level of remuneration of factors by enforcing the specific level of their productivity in both absolute and relative terms (i.e. in mutual relationship to other production factors).

These dependencies are illustrated on the specific isoquant curve for the adopted production function, which we will not discuss or illustrate empirically here. These dependencies are either strengthened or undermined, most often by the agricultural policy as a classical exogenous condition and macroeconomic conditions, including e.g. interest rate conditions. The increase in e.g. the interest rate increases, in fact, the requirements as to the productivity of the capital and land factors, whose remunerations, i.e. profit and ground rent are, naturally, referred (comparatively) to the interest rate.

1.2. Production factor price and its relation to the productivity

As it has been mentioned, the final remuneration for production factor services employed in the production process is determined by the productivity index of the given production factor. This is an endogenous factor. The production factor service price is, in turn, an exogenous factor inducing the endogenous factor. The latter i.e. the remuneration is determined by the productivity of the given factor. We assume that these two values interact with each other. Remunerations may not, in principle, deviate from the price level of

[10] Rembisz W., Sielska A. (2015), *Mikroekonomia…*, op. cit.

services of these factors shaped in the market. This, depending on the relationship between the price and remuneration, affects or not the obligation to improve their productivity. This directly results from the regulatory mechanism of the market and producer's choice, from its equilibrium conditions (cf. Rembisz, Sielska, 2015)[11].

We are talking here about the productivity of the last factor unit, i.e. the marginal productivity, which is or should be at the level close to the market price of the given factor[12]. Using the differential calculus, these dependencies for the labour and capital factors, resulting from the producer's choice and determining its equilibrium, can be presented as follows:

$$\frac{\partial y}{\partial L} = c_L, \ (1.8)$$

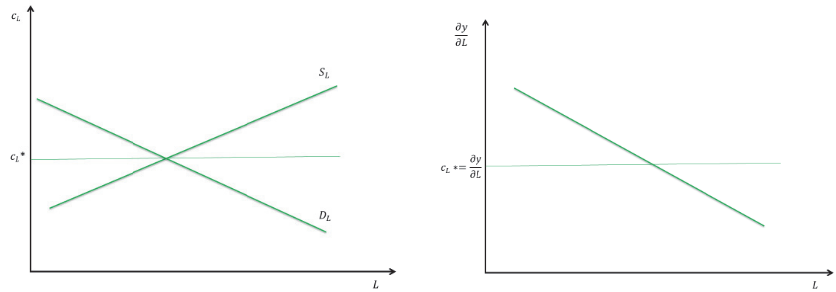
$$\frac{\partial y}{\partial K} = c_K, \ (1.9)$$

where:
y-agricultural production,
$\frac{\partial y}{\partial L}$-marginal productivity of the labour factor,
$\frac{\partial y}{\partial K}$-marginal productivity of the capital factor.
other designations as above.

The remuneration of the production factor, determined in this way, is affected, as shown above (and in the reference), by the exogenous condition, i.e. the price of this factor established in its market. By adopting the *ceteris paribus* principle, we can illustrate this using Figures 1 and 2.

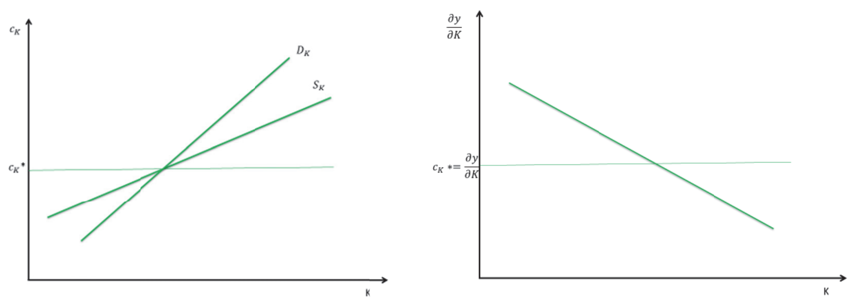
[11] Rembisz W., Sielska A. (2015), *Mikroekonomia…*, op. cit.
[12] This is the issue of allocation in the Walras equilibrium model, where producers have no influence on their products, they adapt the use of employed production factors according to their prices, using them properly, so that the productivity at least covered their price (remuneration). This leads to the situation of so-called zero profits, thus the state of general equilibrium.

Figure 1. Relationship between the production factor price and its marginal productivity



Source: own study.

Figure 2. Relationship between the capital factor price and its marginal productivity



Source: own study.

As it can be seen, the price of equilibrium of production factor services, as established in the market, should hypothetically be related to the marginal productivity of the given factor (it should even determine or enforce this productivity). In fact, the producer determines the production factor remuneration in the sphere of rational management[13], i.e. from the point in which the marginal and average factor productivity become equal with the

[13] This results from the above-mentioned condition *zero profit conditions* and, in the technical sense, is defined in production function analysis for the designation of rational management zones as well as ranges of the variability of flexibility of the product in relation to the increase in inputs, cf. Rembisz W., Sielska A. (2012), *Mikroekonomiczna funkcja produkcji – właściwości analityczne wybranych jej postaci*, Vizja Press&It, Warszawa.

ceteris paribus principle, which, *inter alia*, applies to the exogenous factor i.e. product prices. So we have:

$$c_L^* \Rightarrow D_L \approx \frac{\partial y}{\partial L} = \frac{y}{L} \text{ (1.10)}$$

where:

D_L- endogenous remuneration of the production factor.

This should be considered as a reference point in assessing real processes from the point of view of the efficiency of the relationship between production factors in agriculture.

1.3. Concept of exogenous factors

In order to determine the exogenous factors we are interested in, we will start with the production efficiency (PE) expressed in current prices, which is known as the economic efficiency or even profitability (however, it does not include costs which are not factor inputs, such as taxes and other charges).

This can be expressed as a quotient between income and the cost of using production factors, i.e., capital, labour and land factors for the given level of the agricultural production on the scale of the producer or agriculture at current prices (Bezat-Jarzębowska, Rembisz, 2013)[14]:

$$EP = \frac{p_i y_i}{K_i c_K + L_i c_L + Z_i c_Z}, \text{ (1.11)}$$

where:

i – index determining the agricultural producer,

p – prices of agricultural products,

c_K – capital factor price,

c_L – labour factor price,

c_Z – land factor price.

We ignore the time subscript *t*.

The factor price in fact means the price for services of the given factor in the production process, which is the term known in the circular flow, and this price is designated in the market regardless of the producer.

[14] Bezat-Jarzębowska A., Rembisz W. (2013), *Ekonomiczny mechanizm kształtowania dochodów producentów rolnych*, IERiGŻ-PIB, Warszawa.

In conditions of competitive equilibrium in the product market and assuming the homogeneity of the function over the given period of time, instead of the above formula we have[15]:

$$p_i y_i = K_i c_K + L_i c_L + Z_i c_Z. \ (1.12)$$

Finding the bilateral logarithm of the above formula makes it possible to write the production efficiency in value terms as a total:

$$lnp_i + lny_i = lnK_i + lnc_K + lnL_i + lnc_L + lnZ_i + lnc_Z. \ (1.13)$$

Determination of fractional derivatives and exclusion of time indices allows to make the following notation:

$$\frac{\partial y}{y} - \left\{\frac{\partial K}{K} + \frac{\partial L}{L} + \frac{\partial Z}{Z}\right\} = \left\{\frac{\partial c_K}{c_K} + \frac{\partial c_L}{c_L} + \frac{\partial c_Z}{c_Z}\right\} - \frac{\partial p}{p}, \ (1.14)$$

where:
∂ – increase (fractional derivative) of the given variable.

By analysing the above equation, it can be divided (Bezat-Jarzębowska, Rembisz, 2016)[16].

The left side of this equation is responsible for endogenous factors, which are conventional and dependent on agricultural producers in a sense of their choices for maximising their objective function. These factors are related to the production efficiency and its changes in a sense of TFP:

$$\frac{\partial y}{y} - \left\{\frac{\partial K}{K} + \frac{\partial L}{L} + \frac{\partial Z}{Z}\right\} \cong \Delta TFP. \ (1.15)$$

These relationships, as endogenous, are excluded from further analysis (they were analysed in the previous monograph; Bezat-Jarzębowska, Rembisz; 2016)[17].

The factors listed on the right side of the equation (1.15) are exogenous factors of our interest. These are relationships between product prices (obtained or received) to production factor prices (in fact, their services resulting from given employment of the factor) defined in the production factor market as we showed above (this corresponds to the essence of price scissors, xp):

$$\left\{\frac{\partial c_K}{c_K} + \frac{\partial c_L}{c_L} + \frac{\partial c_Z}{c_Z}\right\} - \frac{\partial p}{p} = xp. \ (1.16)$$

[15] This corresponds to the *zero profit conditions* just as we have indicated.
[16] Bezat-Jarzębowska A., Rembisz W. (2016), *Production techniques…, op cit.*
[17] Ibid.

In the view of the above equations, the exogenous factor is the relationship between the product prices and production factor services. The relationships between production factor prices are also of interest to us. The agricultural producer has no influence on them. In Neoclassical terms, it is said that the producer is a price-taker also when it comes to production factor service prices when the conditions of competitive equilibrium are met. There is no need to make great effort to assume that this condition is met in the case of agricultural producers (individual farms). The operation, i.e. the purchase of the capital factor, in a greater or smaller quantity, or employing the labour factor by the single agricultural producer does not have – or even better – absolutely does not have any influence on the state of equilibrium in the market of these factors i.e. on the prices of their services.

The analysis of the formula (1.14) makes it possible to note that between endogenous and exogenous factors there are substitution relationships. This applies not only to the relationships between changes in the production efficiency to changes in production factor service prices in relation to product prices (price scissors). This also applies to the relationships, in pairs, of changes in employing the given factor and changes in its prices for given income (product of the production growth rate and product prices)[18]:

$$\left\{-\frac{\partial K}{K}-\frac{\partial L}{L}-\frac{\partial Z}{Z}\right\} = \left\{\frac{\partial c_K}{c_K}+\frac{\partial c_L}{c_L}+\frac{\partial c_Z}{c_Z}\right\}. \ (1.17)$$

Individual pairs of the substitution relationships are:

$$-\frac{\partial L}{L} = +\frac{\partial c_L}{c_L}; -\frac{\partial Z}{Z} = +\frac{\partial c_Z}{c_Z}; \text{and} + \frac{\partial K}{K} = -\frac{\partial c_K}{c_K}. \ (1.18)$$

These relationships will be the subject of further empirical studies.

Assuming the conditions of competitive equilibrium in the market of products manufactured and sold by agricultural producers, the price is determined for each of them. The producer is by the same token (or above all) the price-taker here. If the product price is determined, therefore it is the constant variable (horizontal line for the producer) in a given period. Thus, we have below:

$$\frac{\partial p_t}{p_t} = 0, \ (1.19) \text{ and}$$

$$\frac{\partial c_L}{c_L}+\frac{\partial c_K}{c_K}+\frac{\partial c_Z}{c_Z} = xp. \ (1.20)$$

[18] In fact, we have $\left\{\frac{\partial K}{K}+\frac{\partial L}{L}+\frac{\partial Z}{Z}\right\} - \left\{\frac{\partial c_K}{c_K}+\frac{\partial c_L}{c_L}+\frac{\partial c_Z}{c_Z}\right\} = \frac{\partial p}{p}+\frac{\partial y}{y}$ for $\frac{\partial p}{p}+\frac{\partial y}{y} = n$ – constant.

Difference quotients, rates of changes in production factor service prices, on the left side of the above equation are exogenous factors. In this aspect, their impact on the relationships of employing corresponding production factors is analysed in two further chapters. We do not analyse the impact on the factor productivity.

The above notation and interpretation is, in some sense, a methodological and theoretical basis for the concept of exogenous factors, as used in this paper. It can be clarified that, in conditions of competitive equilibrium in the market of goods, i.e. when the producer has no influence on the price of his product, important are only the relationships between production factor service prices, whose evolution we discussed above and defined them as the exogenous factor. They necessitate changes in the relationships of factors used, i.e. production techniques and their remuneration through their productivities. Moreover, these processes are interconnected.

The concept of the exogenous factor can also be extended, building on the slightly different production function as proposed by Fuglie et al. (2012)[19]. They divided production factors into conventional and unconventional. Conventional factors (or conditions) are defined as internal to the given producer (endogenous and material such as land, labour and capital factors) and unconventional factors are exogenous (external) factors, such as, for example, climate change, technological progress (probably induced in the sense of Hayami-Ruutana), social infrastructure, knowledge and its dissemination. This is somehow an extension of the concept of exogenous factors affecting the choices made by the agricultural producer. We do not take this issue at the present stage of analysis. We refer to this so as to outline the wider context of the concept of exogeneity. Production functions in the mentioned aspect for the analysed period t can be written as follows (cf. Bezat-Jarzębowska, Rembisz, 2015)[20]:

$$y = f(X, NX) \quad (1.21)$$

where:

X – conventional (endogenous) factors

NX – unconventional (exogenous) factors.

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[19] Fuglie K.O., Wang S.L., Ball V.E., (2012). *Productivity Growth in Agriculture: An International Perspective*, CABI.

[20] Bezat-Jarzębowska A., Rembisz W. (2016), *Wprowadzenie do analizy inwestycji, produktywności, efektywności i zmian technicznych w rolnictwie*. IERiGŻ-PIB, Warszawa.

Analytically, the production function (1.22) for conventional and unconventional factors can be written as:

$$f(X, NX) = h(X) + g(NX) \quad (1.22)$$

where:

$h(X)$- conventional (endogenous) factors

$g(NX)$- unconventional (exogenous) factors.

The proposed approach to the production function is also inseparably related to the concept of the production efficiency in the TFP formula (*Total Factor Productivity*). In classical terms, TFP is defined as (cf. Rembisz, 2008)[21] the functional relationship between the production and conventional (endogenous) and aggregated production factors (in general), but taking into account the effect of the impact of exogenous factors.

The indicated dependencies may be presented as follows, using the adopted function (Fuglie et al., 2012)[22]:

$$TFP = \frac{y}{h(X)} + g(NX) \quad (1.23)$$

Due to the fact that the production efficiency, consequently as the endogenous (producer-dependent) factor, in a quantitative sense, is composed, to some extent, of the productivity of individual production factors, the *TFP* equation can be presented in a form of a total of productivity products of individual production factors and their weights:

$$TFP = a\frac{y}{K} + b\frac{y}{L} + c\frac{y}{Z} \quad (1.24)$$

where:
a, b, c – weights of the share in total inputs,
y – agricultural producer's production,
K – capital factor,
L – labour factor,
Z – land factor,

[21] Rembisz W. (2008), *Mikro- i makroekonomiczne podstawy wzrostu w sektorze rolno--spożywczym*, Vizja Press&IT, Warszawa.
[22] Fuglie K.O., Wang S.L., Ball V.E. (2012). *Productivity Growth…*, op. cit.

$\frac{y}{K}$ – capital factor productivity,

$\frac{y}{L}$ – labour factor productivity,

$\frac{y}{Z}$ – land factor productivity.

Other notations as before.

1.4. Production factor prices as the exogenous factor

The above comments and findings can be deepened by clarifying the importance of the production factor service prices as the exogenous factor, i.e. these conditions, in this case the specific parameters, which the prices are, on which the producer has no influence. In this sense and in classical terms, this means, as we remind, that the producer is the price-taker, not only when it comes to prices of his own products, but also, or above all, from the point of view of our analysis, he is the price-taker in relation to the production factor service prices.

The identification of conditions (1.15) and (1.16) allows to analyse the behaviour of agricultural producers, including, as to the choice of production techniques, either in a holistic manner (after introducing indicators of the structure $\frac{K}{K+L} + \frac{L}{K+L} = 1$) or separately.

As we stressed, the basis of considerations in this paper will be only the right side of the equation (1.14), i.e. the formula (1.16), i.e. relationships between the production factor service prices. We assume implicitly, which results from the assumption on competitive equilibrium and „price-taking" of the producer, that product prices, similarly as at the producer level, also at the sectoral level are arranged horizontally[23]. If we go deeper in our analysis, these prices can be shaped by the market or are determined pursuant to institutional decisions. They can, therefore, be objective (shaped by the law of supply and demand) or subjective. However, their common feature is that they are not shaped by the agricultural producer and are valid for the sector in the given analysed period[24]. It is the essence of the exogenous factor.

[23] At the microeconomic level of the agricultural producer, due to the fact that he is the price--taker, and thus the product price is constant for him, the derivative at the product price is equal to zero, therefore the production factor price relationships are sufficient to determine the exogenous factor.

[24] This assumption would require deeper justifications. Here, we assume e.g. that for the given demand as, in fact, the growth restrictions in agriculture, there is a relatively constant level of prices for the sector irrespective of the occurring friction and is horizontal and these prices do

From the „exogeneity" of the formula (1.16) it follows[25] that the rise in product prices (which, in principle, is not assumed in the major part of reasoning) may also compensate for the cost effect caused by the rise in the production factor prices. This would require an assumption of the zero-rate increase in the production efficiency. This would mean abolishing the possibility of effective compensation for adverse changes in relationships between the production factor service prices and product prices. In turn, in the event of a negative rate of the increase in the efficiency (its decrease) and with the rise in the production factor prices, a possible rise in the production goods prices will be subject to double compensation – there will be a need to balance the decreased efficiency and the cost effect of the rise in the production factor prices. Such mechanisms correspond to the concept of inflation pushed by costs. This could in particular be the case if there was a formula or possibility of adapting the product prices to production costs, which is, for obvious reasons, demanded by farmers' organisations and associations and adopted by some agricultural economists. This possibility in our reasoning is by assumption rejected, as we indicated in one of the last references. In practice, however, this may take place in a form of direct payments and selected intervention programmes. This replaces the mechanism dangerous to the economy and particularly to consumers, namely, the direct mechanism of adapting the product prices to costs, resulting in essentially the same effect: the lack of triggering pressure on improving the production efficiency as the endogenous factor.

In turn, where there is a positive and increasing rate of the increase in the production process efficiency (enforced by the impossibility of compensating for the price rise) and this rate is higher than the cost effect of the rise in the production factor prices resulting in the increasing production cost, there is no need to raise the agricultural product prices[26]. Income of agricultural producers can even increase when the following condition is met:

$$\frac{\partial p}{p} = 0 \ (1.25)$$

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not follow proportionately the possible rise in the production factor service prices (which could be a special case and what we refer to in the text).

[25] Bezat-Jarzębowska A., Rembisz W. (2013), *Ekonomiczny…*, op. cit., p. 140-141.

[26] The obligation to improve the efficiency is most essential for the growth of general prosperity, also in agriculture, based on natural market regulation mechanisms leading to the improved production efficiency.

This option is a desired situation both for consumers (no rise in the prices of consumer goods of agricultural origin) and for producers (increase in income which results from efficiency improvement processes) and does not charge the taxpayer with the costs of subsidies. This is due to the implicit presence of the following dependencies:

$$\frac{\partial y}{y} - \frac{\partial K}{K} \Rightarrow \frac{\partial c_K}{c_K} \quad (1.26)$$

and

$$\frac{\partial y}{y} - \frac{\partial L}{L} \Rightarrow \frac{\partial c_L}{c_L} \quad (1.27)$$

The morphology of the rate at which the production efficiency improves is determined by the ratio of the rate at which the capital factor productivity is changing (left side of the equation 1.27) and the rate at which the labour factor productivity increases (left side of the equation 1.28). Assuming further that there is no improvement, or there is stagnation of the production efficiency, i.e. the left side of the equation (1.14) is equal to zero, then we have:

$$\frac{\partial p}{p} \Leftarrow \frac{\partial c_L}{c_L} + \frac{\partial c_K}{c_K} \quad (1.28)$$

The dependency (1.28) illustrates the process of „pushing" the product prices by costs (supply inflation where the prices are pushed by costs). The rise in the product prices is „pushed" by the rise in the production factor prices, i.e. the relationship of our interest which throughout the monograph is referred to as the exogenous factor.

This mechanism, pointing to the process of compensating the cost effect of the rise in the production factor prices by means of the rise in product prices, assuming *ceteris paribus* the rate at which the production process efficiency improves, is possible to be obtained in conditions of the full monopoly or uncompetitive market. This is true from the perspective of a single entity i.e. the agricultural producer[27]. In the case of the sustainable market, this mechanism is not simply translated into achieving a new balance between the demand for and

[27] In the case of the fairly homogeneous market, which is the specific agricultural product, agriculture as a sector is in fact a monopolist, the importance of the import is different here than in the market in e.g. apples or poultry, etc.

supply of productive goods, hence, sources of income growth should be sought in the repeatedly indicated need to improve the management efficiency, including the price risk management efficiency.

The above-listed dependency (1.28) has far-reaching political implications. It shows that the prices obtained in the market adapt to the change in the production factor costs. This means a steady rise in the prices of consumer products of agricultural origin. It is a widely accepted „tool" of price claims that are still reflected in the „justified" market intervention announcing a programme for the growth and protection of agricultural producers' income.

Chapter II
Production factor price relationships

In this chapter, we will discuss the above-mentioned and obvious exogenous factor for agricultural producers and the entire sector i.e. production factor prices and their relationships. We exclude from analysis their relationships with the product prices, by adopting them as determined. We will analyse in pairs the levels of these prices and their changes in relation to employment of production factors in the sector. We will show the impact of these relationships on the choice made by the agricultural producer and as a result in the entire sector, in analytical terms. This choice refers to production techniques, i.e. relationships among employed production factors. Implicitly, and this is an assumption or research hypothesis, this choice is implied by the production factor price relationships. We refer this empirically to the scale of the sector.

2.1. Theoretical aspect

To recapitulate, we will get back to the equation (1.16) expressed as $\left\{\frac{\partial c_K}{c_K}+\frac{\partial c_L}{c_L}+\frac{\partial c_Z}{c_Z}\right\}-\frac{\partial p}{p}=xp$. The obtained dependencies are indices of the most important economic parameters for agricultural producers. They are, as we indicated above, the exogenous factor for agricultural producers. This corresponds to the fact that agricultural producers as the sector are, in fact, price-takers. The system of changes in price scissors can result from both a self--regulating market mechanism and from certain policies, even interventions. Thus, their evolution is determined, first of all, by the laws of supply and demand (they are, in this sense, objective and predetermined for the producer). They may also be established by institutions and, in this sense, are the effect of their impact within certain agricultural policy instruments, directly and indirectly. According to the Jovens' interpretation, producers adapt to prices, not *vice versa*. Producers adapt their individual costs determined by the relationship between the productivity of employed factors and their remunerations (prices) and, as a result, adapt to the prices of products being permanent to them, as shown above. This relationship is not transitive, i.e. it is difficult to expect that the product prices would adapt to production costs, although this is the case in a form of specific support schemes or direct actions e.g. interventions under the agricultural policy. These issues are left aside. For us, from the point of view of this analysis, relevant is the fact that an important role in this process of choices made by agricultural producers and as a result, in the entire sector, as to the proportion of employing production factors is played by the relationship of these prices.

Agricultural producers adapt to the production factor price relationships with the determined product prices due to the improved production efficiency, including the productivity of individual factors, so this is done through endogenous factors. Production factor price relationships and their changes are not dependent on the agricultural producer. Price scissors, regardless of whether we are dealing with the conditions of competitive equilibrium and the full market regulation provided by the Smith „invisible hand" or with prices specified in the process of intervention or other administrative conditions, will always be exogenous to the agricultural producer.

For the full understanding of the above considerations, as in the previous chapter, we may assume that endogenous factors (cf. 1.16) can be identified with the production efficiency. The improved production efficiency can happen through the effective substitution of the labour factor by increasing employment of the capital factor[28] and by intangible forms of progress, such as innovation and knowledge, management efficiency and overall biological and natural progress, etc. In situations where the production efficiency is not improved, a prerequisite for improving the profitability of production become the market (exogenous) conditions. This restrictive assumption essentially implies a neutral impact of the efficiency factor on the profitability of production, which is always a bad message for assessing the rationality of management, particularly, when cost effects of the rise in the production factor prices must be compensated for by the rise in the product prices.

We are, therefore, facing the possibility of balancing costs increased due to the rise in the production factor prices by raising the product prices[29]. Thus, by converting the equation (1.16), it is possible to illustrate „pushing" the

[28] Rembisz W. (2005), *Wynagrodzenie czynników wytwórczych w gospodarstwach rolnych*. Zagadnienia Ekonomiki Rolnej, nr 4, Warszawa: IERiGŻ-PiB, p. 24-43.

[29] Bezat-Jarzębowska A., Rembisz W. (2015), *Endo- i egzogenne źródła wzrostu gospodarczego w rolnictwie – zarys probematyki*. Roczniki Naukowe SERiA, vol. 17, iss. 6, p. 19-24.

product prices by costs[30] resulting from the rise in the prices paid for production factor services[31].

$$\frac{\partial p}{p} \Leftarrow \left\{\frac{\partial c_K}{c_K} + \frac{\partial c_L}{c_L} + \frac{\partial c_Z}{c_Z}\right\} \quad (2.1)$$

Assuming the zero rate of improvement in the production efficiency, the equation (2.1) indicates a need to compensate for the cost-related increase in the remuneration of the labour and capital factors (implicitly induced by the exogenous rise in the prices of services of these factors)[32], by raising the agricultural product prices. The possibility of such transmission of the cost effect to final prices in markets characterised by competitive equilibrium is limited (both in the Polish market and in most EU countries). This transmission is only possible in the case of the market where the agricultural producer would be the price-giver rather than the price-taker (e.g. monopoly, oligopolistic markets, price cartels). The above formula is logical mainly in relation to the sector as a set of agricultural producers. Then, pushing the product prices by the production factor prices is possible, which does not change the fact that each agricultural producer is, in fact, the price-taker, as even the higher price resulting from this process is determined for him, not by him. This applies, however, to the levels, not rates of this rise. Nevertheless, we can assume that in the entire sector the rise in product prices is impossible, hence the left side of the above dependency is zeroed $\frac{\partial p}{p} = 0$ and attention is paid to changes in the production factor prices and their relationships:

$$\frac{\partial c_K}{c_K} : \frac{\partial c_L}{c_L} : \frac{\partial c_Z}{c_Z} \quad (2.2)$$

The prices of these factors are further analysed in relation to the applications of production factors and, as a matter of fact, to the level of services of these factors and changes in their relationships.

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[30] In the case of farms, the labour factor cost is related to its remuneration.

[31] Bezat-Jarzębowska A., Rembisz W. (2015), *Wprowadzenie do analizy inwestycji, produktywności, efektywności i zmian technicznych w rolnictwie*, op. cit. s. 24-25.

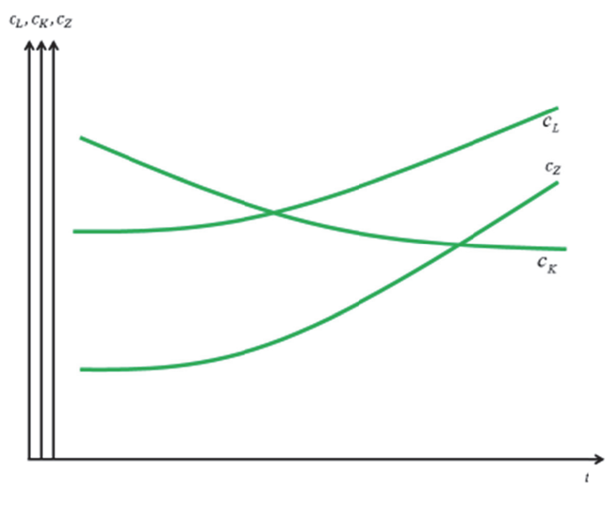
[32] To make the considerations simpler, the land factor price has been excluded.

2.2. Hypothetical relationship of the capital, labour and land factor prices

The consequence of the assumptions made in chapter II as to the evolution of the labour and capital factor prices is hypothetical (Figure 3). It presents a ratio of the production factor input costs – the price of the labour factor whose remuneration increases, and the capital factor which becomes cheaper and cheaper as well as the land factor price. A hypothetical change in the land factor price is also included. In fact, we can refer this to prices of services of these factors in the production process.

Such hypothetical assumptions as to the production factor prices are justified by the theories and observations of the economic growth and development and the resulting relationships of scarcity of these production factors. This applies to the land factor in a natural way, in connection with the non-agricultural demand for its use, urbanisation, residential construction, industrialisation, servicisation, recreation, environmental protection, etc. Identically, the non-agricultural demand for the labour factor, as the economic development proceeds, does not require any justifications as it is a fact. As the economic development proceeds, the supply of the capital factor to be used is increasing. Deeper analysis of this issue is omitted.

Figure 3. Hypothetical assumption as to the evolution of the capital, labour and land factor prices



Source: own study.

2.3. Objective function and producer equilibrium condition vs production factor prices

The agricultural producer can be treated as any entrepreneur whose objective is to maximise the profit function (in agricultural economics, however, it has been agreed that this function is to maximise income). This task is to optimise employing production factors. The income function D for $i -$ of this agricultural producer can be written as:

$$D_i = py_i - (K_i c_K + L_i c_L) \text{ (2.3)}$$

while its maximisation for the given level of production y is formulated as:

$$max_y\, D_i = max_y\, [py_i - (K_i c_K + L_i c_L)] \text{ (2.4)}$$

The solution (2.4) is, according to mathematical analysis, equating the first derivative of the statement in square brackets to zero:

$$[py_i - (K_i c_K + L_i c_L)]' = 0 \text{ (2.5)}$$

or:

$$(py_i)' - (K_i c_K + L_i c_L)' = 0 \text{ (2.6)}$$

Finally, (2.6) may be written as:

$$(py_i)' = (K_i c_K + L_i c_L)' \text{ (2.7)}$$

Assuming the continuity and differentiability of the function of income and cost of using (services) of production factors in the entire field, we have:

$$(py_i)' = \partial y_i p - \text{derivative of total income (2.8)}$$

$$(K_i c_K + L_i c_L)' = \partial K_i c_K + \partial L_i c_L - \text{derivative of total cost, (2.9)}$$

which is also the marginal cost.
Therefore, the equation (2.7) is as follows:

$$\partial y_i p = \partial K_i c_K + \partial L_i c_L \text{ (2.10)}$$

Using the equilibrium condition in the market with competitive equilibrium (marginal yield is equal to the product price), we get:

$$(py_i)' = \partial y_i p = p \text{ (2.11)}$$

This is in line with the situation of the producer for whom, in conditions of free competition, the price obtained from the market is a straight line parallel to the the axis of abscissae (cf. Figure 1 and 2), i.e. also the exogenous variable.

When considering the above dependencies in the concept of producer equilibrium (i.e. his optimal choice due to the production factor price and productivity), it can be assumed, for further considerations, that employing the capital or labour factors will be permanent[33]. Then, we have the following dependencies:

$$\partial K_i c_K = 0 => \partial y_i p = \partial L_i c_L \quad (2.12)$$

or

$$\partial L_i c_L = 0 => \partial y_i p = \partial K_i c_K \quad (2.13)$$

In relation to (2.12) and (2.13), it should be noted that the rise in costs and income, triggered by the increased employment of the production factor, is equal to each other. The above conditions, according to the assumptions of the Neoclassical theory of division, result from producer equilibrium. It can be demonstrated that:

$$\frac{\partial y_i p}{\partial L_i} = c_L \quad (2.14)$$

and

$$\frac{\partial y_i p}{\partial K_i} = c_K \quad (2.15)$$

Therefore, the quotient of marginal income to the change in employment of the production factor determines the level of its remuneration. Naturally, this applies to a short period in which no technical and technological changes are possible as a result of the investment.

2.4. Production factor prices and productivity

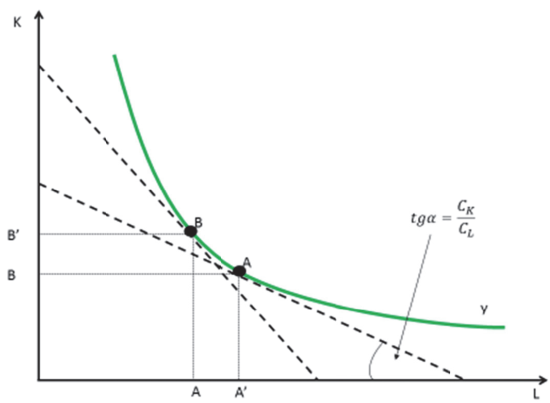
It follows from the foregoing that the relative ratios of the capital and labour factor prices to the relationship of marginal productivities of the production factors can be presented by the following formula:

$$\frac{\partial K}{\partial L} = \frac{\partial y/_{\partial L}}{\partial y/_{\partial K}} = \frac{c_L}{c_K} \quad (2.16)$$

[33] Bezat-Jarzębowska A., Rembisz W. (2013), *Ekonomiczny mechanizm…,* op. cit., p. 71.

The relationship between the prices of these factors and their productivity is also presented graphically in Figure 4. The equation (2.16) determines the tangent of the inclination angle of the straight line tangential to the production isoquant. Each point in this isoquant means the productivity of the capital and labour factors and the shift between them (continuous and incremental) means marginal productivities. The shifts from point A to B, and *vice versa* illustrate changes in the described relationship (2.16) i.e. relationship between the price ratio and the productivity ratio. The rise, e.g. in the labour factor price in relation to the capital factor price will enforce the increase in the labour factor productivity from the point A to B.

Figure 4. Price relationships and productivity of capital and labour factors



Source: own study.

This tangent line is a cost constraint:

$$I^K - Kc_K + Lc_L = 0 \ (2.17)$$

where:

I^K- cost constraint line.

This results from the fact that:

$$\frac{I^K}{c_K} = K \ (2.18)$$

and

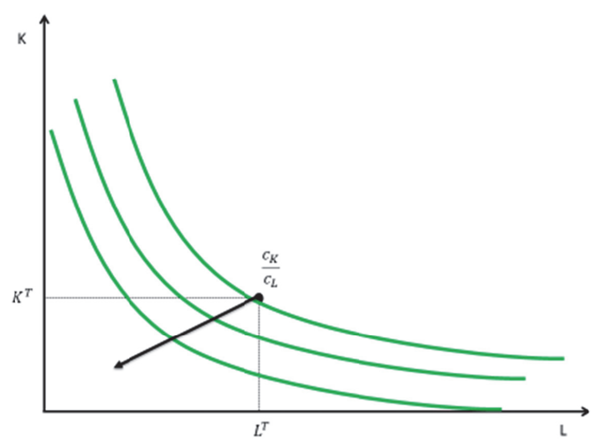
$$\frac{I^K}{c_L} = L \ (2.19)$$

After dividing the sides of (2.18) and (2.19), we have:

$$\frac{K^T}{L^T} = \frac{c_L}{c_K} \; (2.20)$$

Whereby K^T and L^T are the relationships of costs of using the production factors describing the production technique, illustrated in Figure 5. These are the relationships of the factors for the given relationship of their prices with the given straight line of total costs. The statement $\frac{K^T}{L^T}$, being graphically the point of tangency between the production isoquant and the production cost constraint curve illustrates the optimum production technique in terms of minimising production costs.

Figure 5. Relationships of capital and labour factor prices vs factor relationships describing the production technique

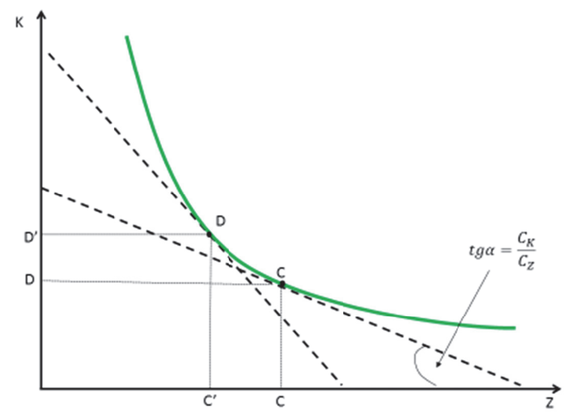


Source: own study.

The production technique shown in the above figure, i.e. relationship of production factors for the given production (as determined) disregards here the cost constraint as in the previous figure. The downward movement of the isoquant curves toward the beginning of the coordinate system illustrates the improvement in the production efficiency in the sense of TFP.

Of similar nature, regardless of differences in the slope scale, is the relationship between the capital factor and the land factor. In academic economics of agriculture, it is characteristic of the capital-intensive intensification[34]. Currently, this is the relationship determined the increased land factor productivity. In fact, the capital factor embodies technical and technological progress determining the disclosure of other types of progress, such as biological and natural progress. At the current stage of the agricultural development, the capital factor replaces the land factor per production unit, i.e. in a relative sense. This is due, *inter alia*, to the relative cheapness of the capital factor in relation to the land factor. This is illustrated in Figure 6 in a form of shifting the equilibrium point from point C to D. This corresponds to increasing capital intensity and reducing land use intensity, which corresponds to the increased land factor productivity at the expense of capital, at the same TFP level.

Figure 6. Price relationships and productivity of the capital and land factors



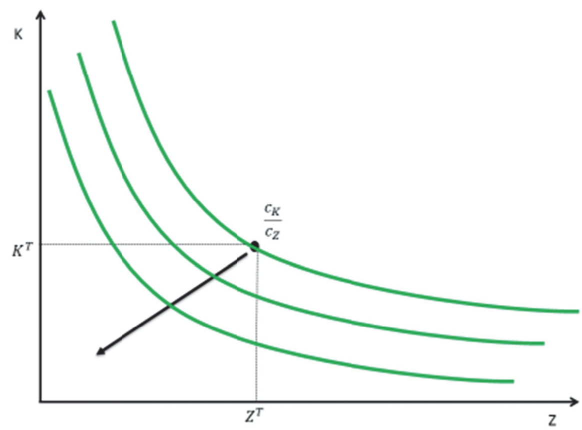
Source: own study.

When the *TFP* improves, we have the situation illustrated in Figure 7 of isoquants[35]. The curves of the same production level (isoquants) for increasingly efficient production functions are moving downwards. The smaller and smaller

[34] Woś A., Tomczak F. (1983), *Ekonomika rolnictwa. Zarys teorii*. PWRiL, Warszawa.
[35] Isoquant in the formal sense is the contour line of the specific production function, here – two-factor, that is the intersection of this hyperplane and its projection on the plane of axis K and Z. In the case shown in Figure 7 these are the intersections (contour lines) of the subsequent steeper (more efficient) production function in relation to these two production factors.

quantity of both factors is needed for the same production level. Without changing the price relationship, the production techniques remain the same but are more and more effective. In the case of changes in the relationships of the prices of these factors, the production techniques are changing and becoming more and more effective.

Figure 7. Relationship of capital and land factor prices vs factor relationships describing the production technique



Source: own study.

This and previous dependency will not be subject to the empirical verification. It has been included in the monograph because it has an indirect impact on the price relationships and employment of production factors.

2.5. Demand and supply in the production factor market

The production factor prices are, naturally, determined in the market. The basis for this is a specific mechanism. On the side of the demand for these factors, it is related to the producer's choice conditions, the effect of which is the specific, conditional demand for the production factors. We put it somehow in brief, highlighting only the most important points of this mechanism. We exclude the mechanism of the evolution of the land price as a slightly different problem, whose essence goes beyond the following formulation.

From the producer equilibrium condition in the theory of economics (microeconomics), for the given (determined in the market) factor prices: (c_L, c_K) and total costs for: $W(c_L,\ c_K, K, L)$, it is possible to determine the conditional demand for the production factors. The equations of this demand can be written as:

$$K = L(c_L, c_K, y) \ (2.21)$$

$$L = K(c_L, c_K, y) \ (2.22)$$

These are conditional equations due to the fact of adopting for the agricultural producer the given product prices and the given level of production as well as employing another factor. The equations (2.21) and (2.22) show the relationship between the optimal choice of a combination of production process factors (as made by the producer) and which corresponds to the concept of the production technique, and production factor prices at the given production volume and product prices.

The presented hypothetical relationship responds to the relevant cognitive problem – the size of each of these two production factors (K^*, L^*) with the use of the other, which would be used by the agricultural producer if he wanted to produce the given production volume in the most cost-effective way, without raising prices of agricultural products[36]. The solution to this issue is the system of conditions for optimal employment of two production factors[37]:

$$K^* = (c_L, c_K, y) \ (2.23)$$

$$L^* = (c_L, c_K, y) \ (2.24)$$

and with the given cost constraint I^K as:

$$I^K(c_L, c_K, y) = c_K \cdot K^*(c_L, c_K, y) + c_L \cdot L^*(c_L, c_K, y) \ (2.25)$$

This relationship also results from the theory of the marginal productivity, which in this case is also the theory of division and at the same time the theory of demand for production factors[38] (cf. Milewski, 2002).

Therefore, the production factor prices will be shaped as follows:

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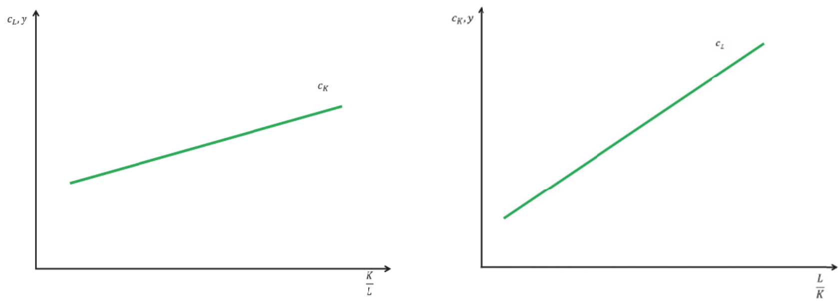
[36] Varian H.R. (1997), *Mikroekonomia*, Wydawnictwo Naukowe PWN, Warszawa, p. 361-363.
[37] Rembisz W. (2005), *Wynagrodzenie czynników wytwórczych…*, op. cit. s. 25.
[38] Milewski R., ed. (2002), *Podstawy ekonomii*. Wydawnictwo Naukowe PWN, Warszawa, p. 274-275.

$$c_K = \frac{K}{L}(c_L, y) \; (2.26)$$

$$c_L = \frac{L}{K}(c_K, y) \; (2.27)$$

where the prices of these factors are also affected by a possibility of financing it under the given production technique.
The above dependencies can also be presented graphically in a simplified way.

Figure 8. Graphic presentation of production factor prices in the context of the theory of division



Source: own study.

As a consequence of dependencies (2.26) and (2.27), we can derive, in an analytical way, conditional equations of the demand for production factors, while maintaining the cost constraint I^K in a form of (2.25): c_K, y

$$L = \frac{I^K}{c_L} - \frac{c_K}{c_L} K \; (2.28)$$

$$K = \frac{I^K}{c_K} - \frac{c_L}{c_K} L \; (2.29)$$

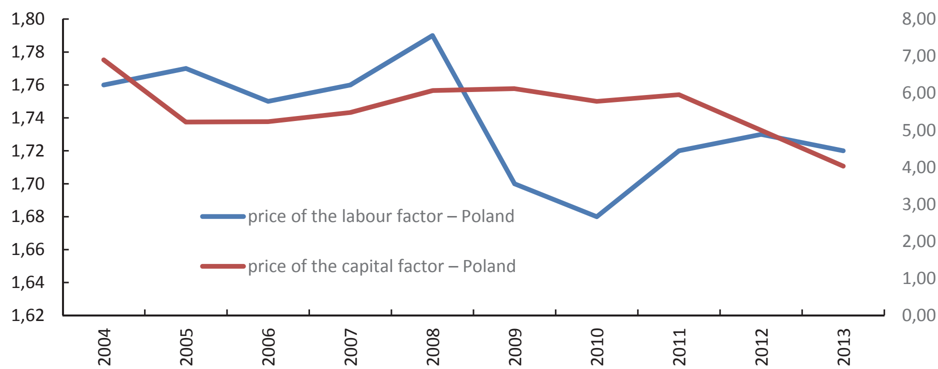
These equations show the size of the demand for the given production factor, depending on its relative price, i.e. its level in relation to the cost constraint and in relation to the price of the substitution production factor at the given level of employing this factor in the production process.

These dependencies are clearly conditional and substitute. This means that with the rise in the price of one factor with the given financial (cost) constraint and employment of the other factor (implicitly – other factors), we will observe a decrease in the demand for this factor and its substitutability.

2.6. Empirical aspect

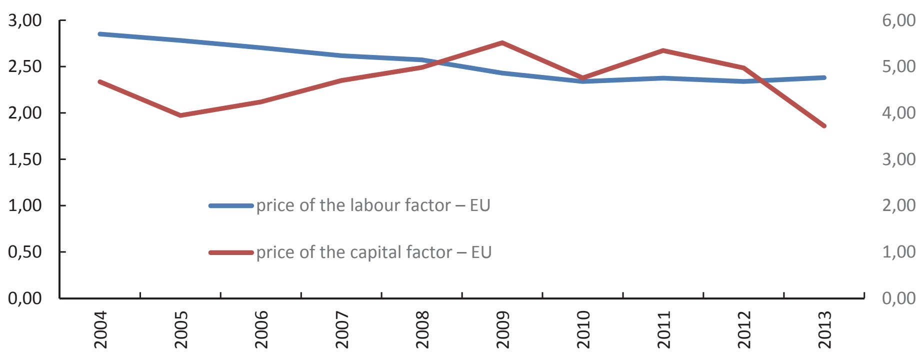
In the first step of empirical analysis, an attempt has been made to verify a hypothetical assumption as to the evolution of the capital and labour factor prices as well as land and capital factor prices. The same time series as described in Chapter 1 (subchapter 1.5) have been used. The verification covered the years 2004-2013, which is dependent on the availability of data. The following figures show the capital and labour factor prices for the selected EU countries.

Figure 9. Labour and capital factor prices in agriculture in Poland



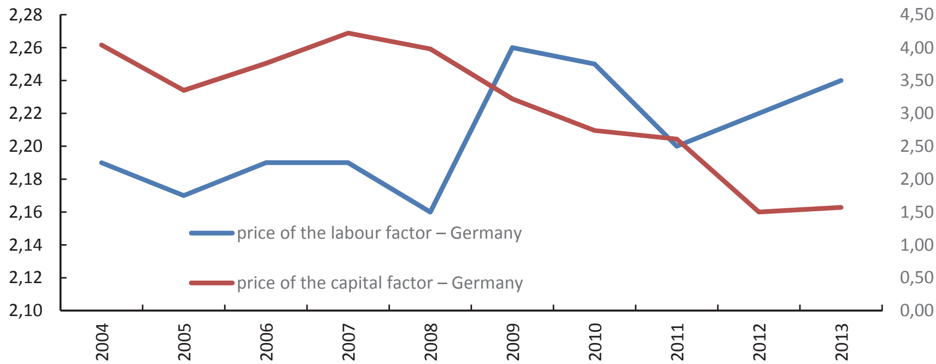
Source: own study based on the Eurostat data.

Figure 10. Labour and capital factor prices in agriculture in EU



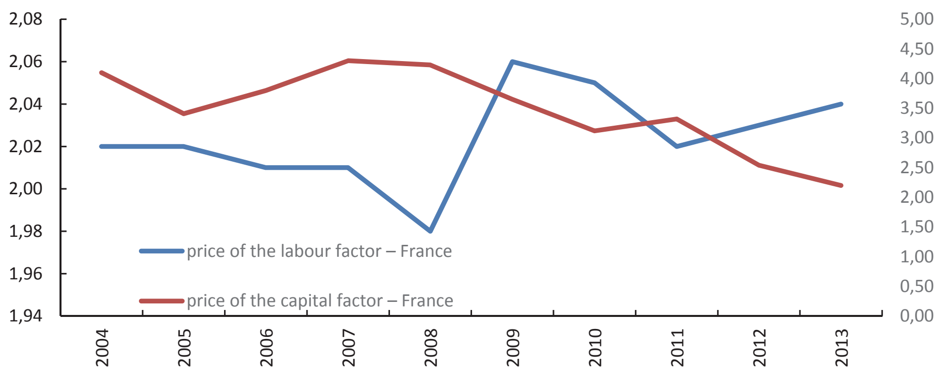
Source: own study based on the Eurostat data.

Figure 11. Labour and capital factor prices in agriculture in Germany



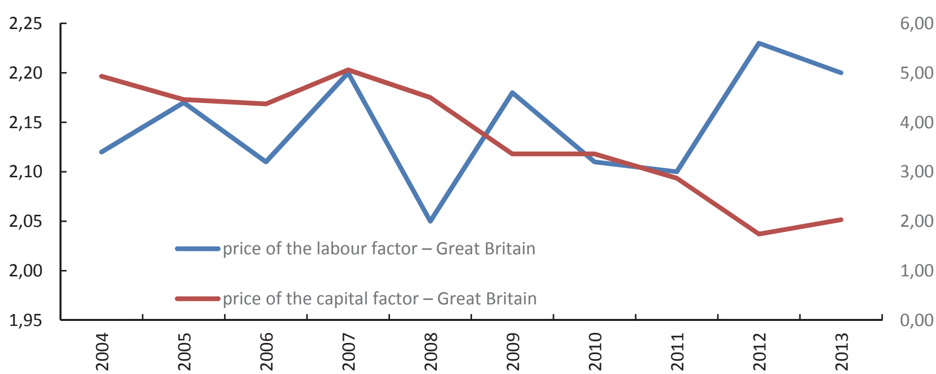
Source: own study based on the Eurostat data.

Figure 12. Labour and capital factor prices in agriculture in France



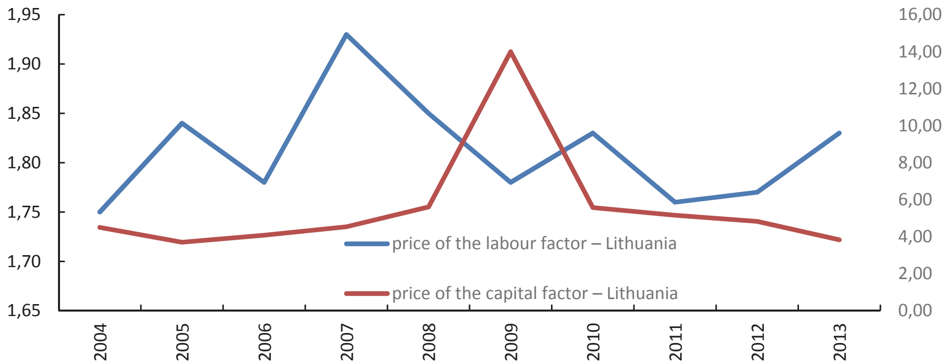
Source: own study based on the Eurostat data.

Figure 13. Labour and capital factor prices in agriculture in the Great Britain



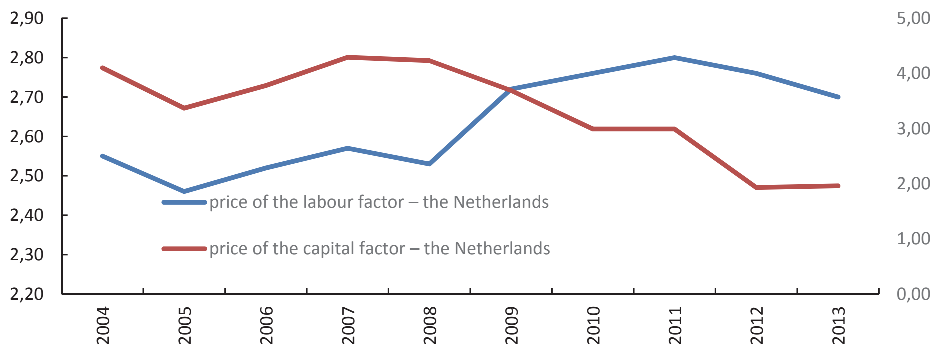
Source: own study based on the Eurostat data.

Figure 14. Labour and capital factor prices in agriculture in Lithuania



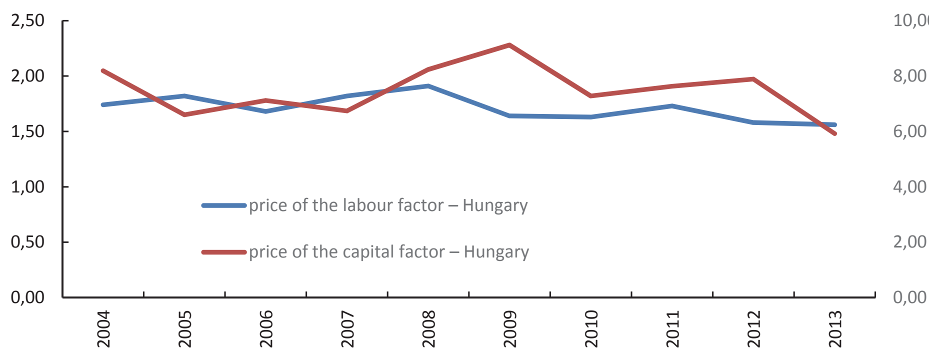
Source: own study based on the Eurostat data.

Figure 15. Labour and capital factor prices in agriculture in the Netherlands



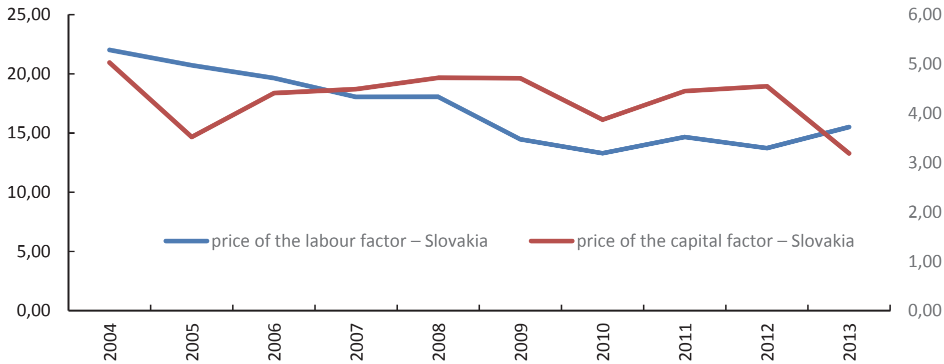
Source: own study based on the Eurostat data.

Figure 16. Labour and capital factor prices in agriculture in Hungary



Source: own study based on the Eurostat data.

Figure 17. Labour and capital factor prices in agriculture in Slovakia



Source: own study based on the Eurostat data.

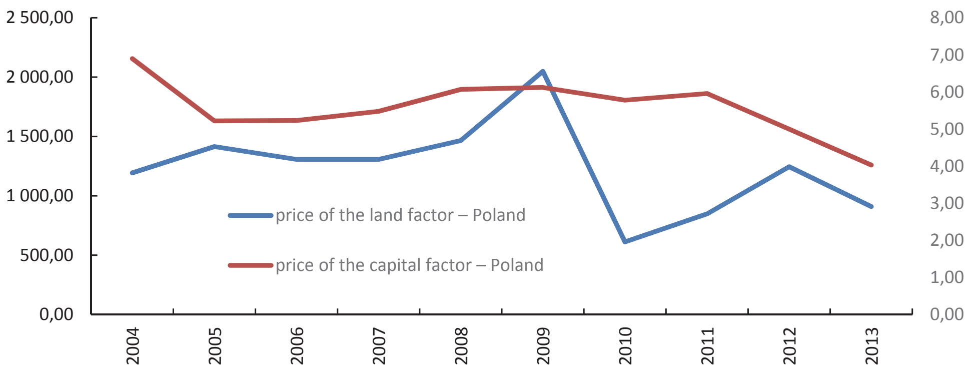
Analysis of the received visualisations confirms the adopted hypothetical assumptions derived from the analytical aspect as to the trends. It is evident that the labour factor price in relation to the capital factor price becomes higher and higher. In the periods since 2008, we can see clear opposing trends in changes in prices of both analysed production factors, indicating the occurrence of substitution processes, when it comes to production techniques. This is consistent not only with the analytical aspect presented and assumed hypothetical price relationships. This is consistent with all known models of growth in agriculture[39], including the theory of intensification[40], known in academic economics of agriculture, often invoked Hayami-Ruttan model and more widely-Kuznetz model. The general explanation for the evolution of the presented factor price relationships arising from the quoted literature sources is in line with our previous comments, and is as follows. The capital factor is growing in quantity which results simply from the industrial and economic development as such, hence, on the level of scarcity basis, the basic law of the market, it is increasingly cheaper in absolute terms. It is also cheaper in relative terms with respect to the labour factor, as it becomes more expensive due to the general development, decrease in its availability for agriculture as a result of competitive employment outside this sector. These changes in production factor price relationships are also determined, as shown in the above analytical aspect, by the improved productivity of both factors. By assumption, the increased productivity should result from the rise in the factor price, if we assume that

[39] Rembisz W., Floriańczyk Z. (2014), *Modele wzrostu gospodarczego w rolnictwie*, IERiGŻ--PIB, Warszawa.
[40] Woś A., Tomczak F. (1983), *Ekonomika rolnictwa…*, op. cit.

endogenous relationships are induced by exogenous relationships. We leave it for the further stage of studies.

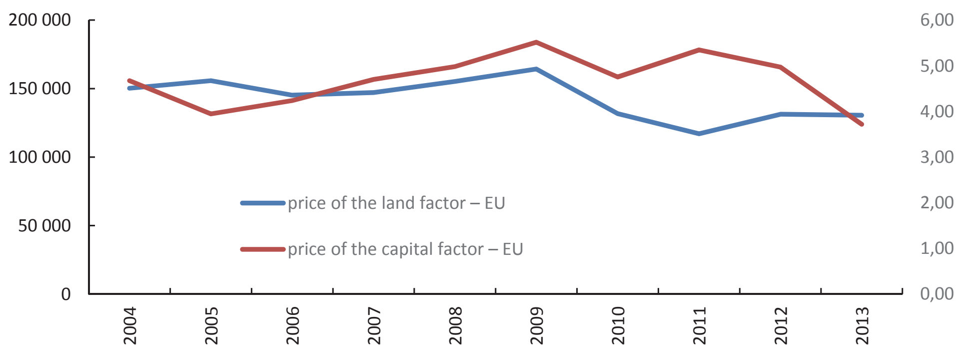
Relationships between the capital and land factor prices are presented in the following figures.

Figure 18. Land and capital factor prices in agriculture in Poland



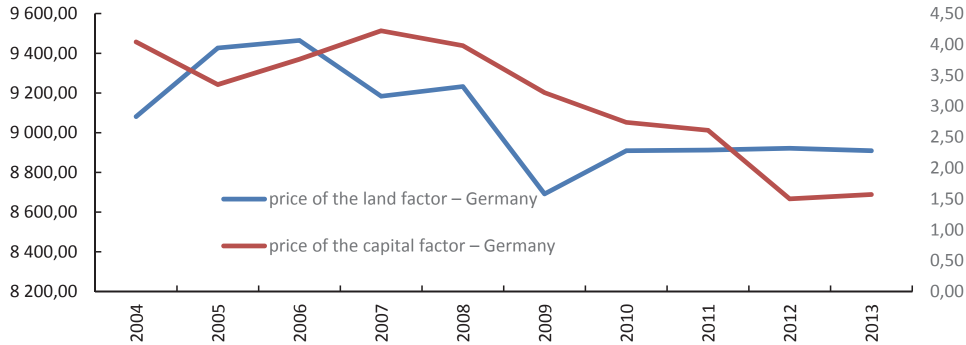
Source: own study based on the Eurostat data.

Figure 19. Land and capital factor prices in agriculture in EU



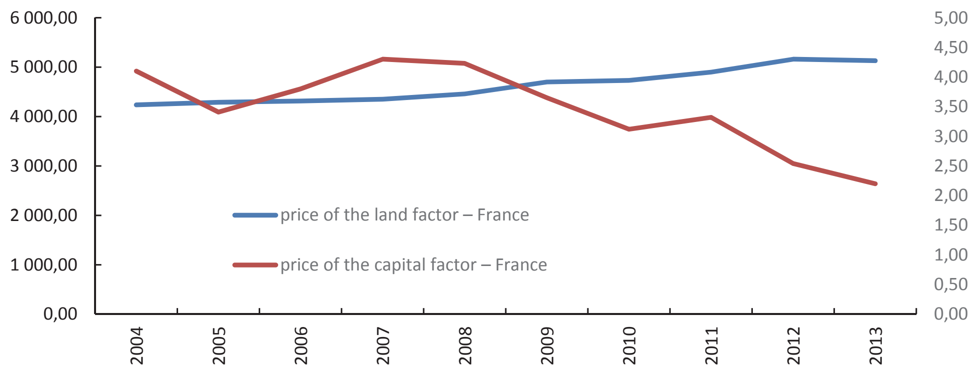
Source: own study based on the Eurostat data.

Figure 20. Land and capital factor prices in agriculture in Germany



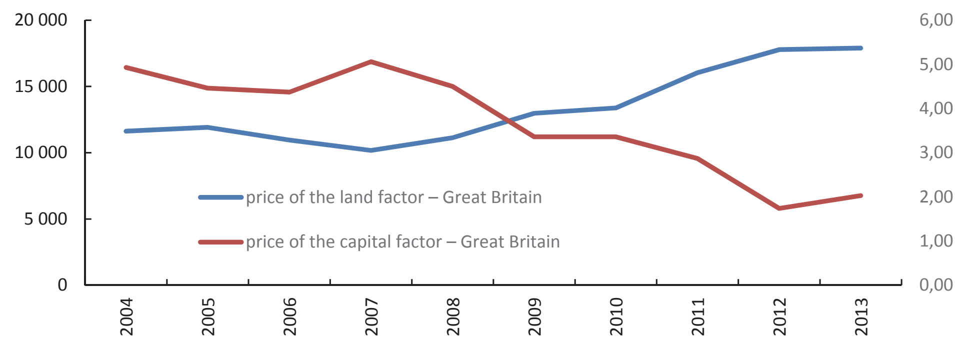
Source: own study based on the Eurostat data.

Figure 21. Land and capital factor prices in agriculture in France



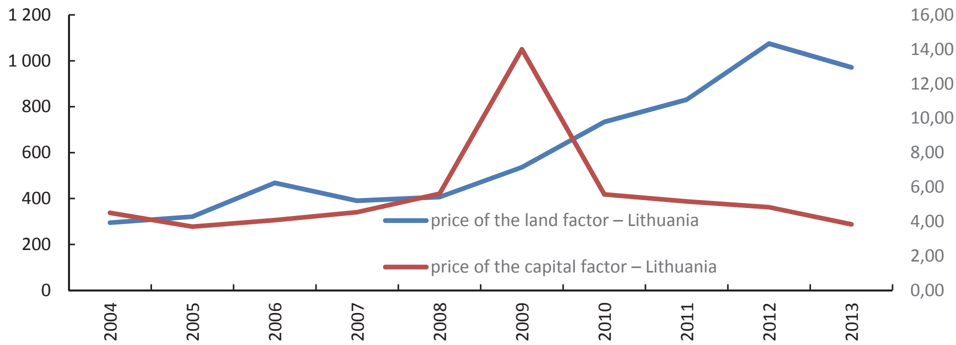
Source: own study based on the Eurostat data.

Figure 22. Land and capital factor prices in agriculture in the Great Britain



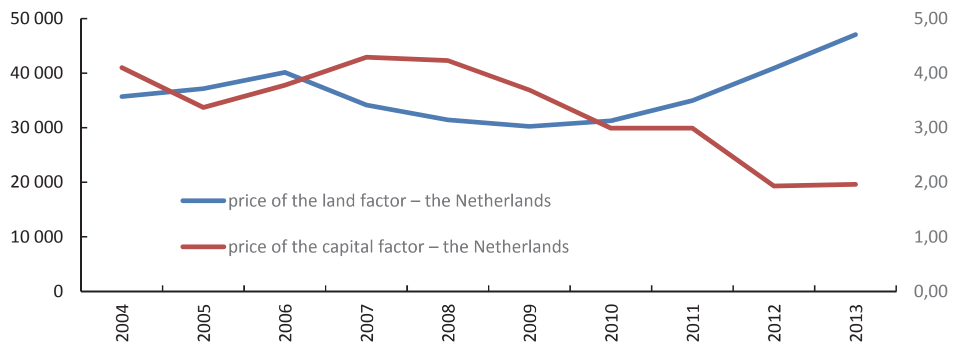
Source: own study based on the Eurostat data.

Figure 23. Land and capital factor prices in agriculture in Lithuania



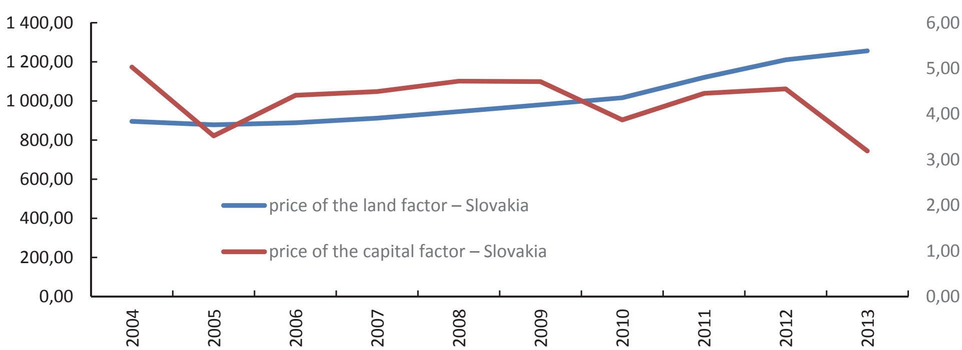
Source: own study based on the Eurostat data.

Figure 24. Land and capital factor prices in agriculture in the Netherlands



Source: own study based on the Eurostat data.

Figure 25. Land and capital factor prices in agriculture in Slovakia



Source: own study based on the Eurostat data.

The obtained empirical illustrations regarding price relationships of these two factors over time are arranged in the analysed countries according to the adopted analytical assumptions and hypothetical charts. Generally speaking, the trends in changes in both factors are opposing and intersecting. The reasons are the same as those provided in analysis of relationships between the capital and labour factor prices. We do not go into details as regards the extent to which certain regulatory restrictions in trade in the land factor could affect the possible flattening of outlined intersecting systems of prices of these factors. Of importance for us are basic relationships, and these are consistent with our aspect and literature.

Chapter III
Relationships of prices and employment of production factors

The effect of certain factor price relationships and their changes, to which we referred in the previous chapter and which we identify with the exogenous factor, are changes in the relationship of these used factors. This is the endogenous factor because it is the result of the producer's choice. The producer has independent influence and control of it. These production factors relationships are defined as production techniques. This chapter is dedicated to analysing changes in the relationship of employing production factors as a result of changes in their price relationships. We refer it to each factor individually. We do not define the causality here, but we only show the relationships between the price and use of the given production factor. We stick to the same analytical convention as in previous chapters.

3.1. Employment of the capital factor in relation to its price

The analytical formulation of the production function allows to consider it in the following univariate and intensive form:

$$y = f(K, p_K) \text{ (3.1)}$$

where:

$p_K = \frac{y}{K}$ – capital factor productivity.

From the point of view of the agricultural production volume, of key importance is employment of the capital factor in relation to the production – i.e. the capital factor productivity coefficient. An important role is also played by the capital factor price, being – as shown above – the exogenous factor. This exogenous factor, or the production factor price, implies or enforces the marginal productivity of the employed factor:

$$\frac{\partial y}{\partial K} = c_K \text{ (3.2)}$$

The capital factor price is, of course, determined exogenously in the market of this factor (Figure 27). The employed capital factor in the production, by definition, should be bring return on this employment or use, equal at least to the interest rate. Thus, in essence, the production factor price in dynamic terms in the current production account and in the cost account results, to some extent, from the interest rate and is financed from profit whose source is the productivity of that factor after deductions, for example, depreciation. However, in our reasoning, we use the microeconomic categories of production factors as some generalisation.

With this in mind, we actually have the function of employing the capital factor defined by two arguments, i.e. productivity and price of this factor, i.e. as[41]:

$$K = f(p_K, c_K) \text{ (3.3)}$$

We can also, according to the tradition of the Kalecki[42] school, of economic thought put it separately, i.e. as:

$$K = f(p_K) \text{ (3.4)}$$

and

$$K = f(c_K) \text{ (3.5)}$$

We will further illustrate it with figures of hypothetical dependencies in this regard.

When in the function (3.1) we substitute the factor price for the productivity, assuming the same direction of their changes, and for the given production we have the following function:

$$y = f(K, c_K) \text{ (3.6)}$$

and its basis :

$$K = \frac{y}{c_K} \text{ (3.7)}$$

for $C_K > 0, y = const.$

[41] While maintaining the higher level of formality as a basis, we can adopt: $y = f(K)$ – classical production function and $K = g(y)$ as the function of employment of the capital factor to obtain this production: *y,* with $g = f^{-1}$.

[42] For example, we find such formulations attributing the effect in a form of national income to a single factor or labour factor, or capital factor with the determination of their quantitative and productive contribution, which is known as the intensive and extensive factors – cf. Nasiłowski W. (1974), *Analiza czynników rozwoju gospodarczego PRL*. PWE, Warszawa.

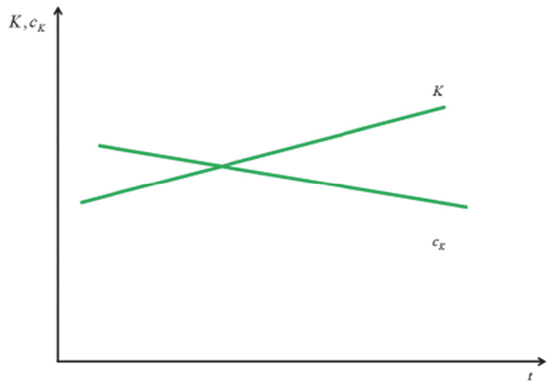
This means that the decrease in the capital factor price must be accompanied by the increase in its use in the production at its given level.

In developed countries, the characteristic feature of the production process and growth is the labour and land effectiveness as well as capital-intensity of production techniques. The latter may mean, as in hypothetical Figure 26, greater employment of the capital factor than the production level obtained. Increasing the use of the capital factor in agriculture in agricultural producers performs not only growth functions. The capital factor is heavily burdened by the substitution functions in relation to the labour factor, which is natural and relative to the land factor, and this is specific to agriculture. In the latter case, the capital factor substitutes for the absence of the increase in the land factor which would be necessary to „handle" the given increased production and also substitutes for the absolute loss of the land factor. In both cases, it leads to the increased land productivity which in fact should be attributed to the capital factor. Hence, as we have mentioned, a better option may be to perceive the land factor as the capital factor.

The consequence of the above-defined relationship (3.2) is the capital factor productivity. It can be believed that the level of this productivity will be the function decreasing over time, which is a typical assumption as regards the agricultural production function. However, since the productivity is the endogenous category, we will not devote much attention to it in this monograph.

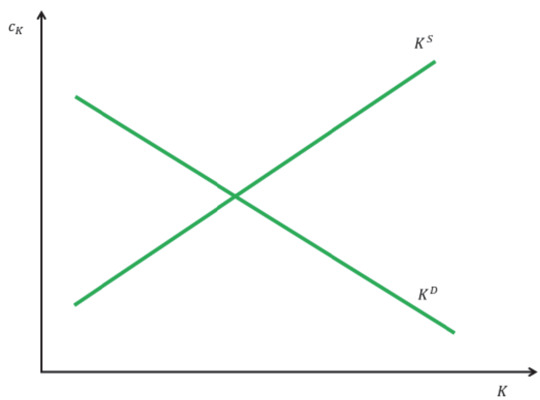
It also seems that such formulation will give rise to an assumption on the existence of the substitution relationship between the price, as the exogenous factor, and employment of the capital factor so as to obtain the given production level. Its hypothetical course is illustrated in Figure 28, which is, to some extent, an extension of the hypothetical assumption illustrated in Figure 26.

Figure 26. Hypothetical assumption on the relationship between the capital factor price and the employment level of this factor



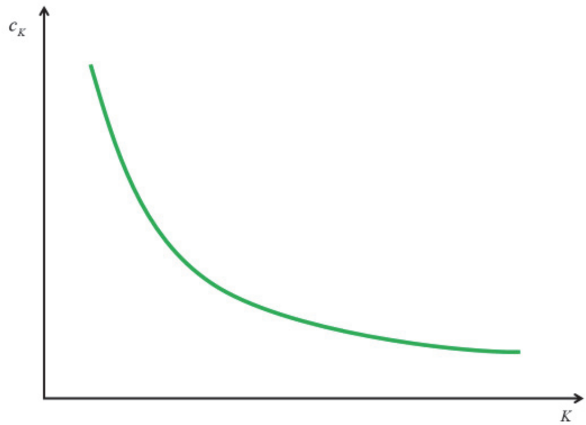
Source: own study.

Figure 27. Capital factor price in its market-hypothetical aspect



Source: own study.

Figure 28. Hypothetical assumption on the substitution relationship between the capital factor price and its employment in agriculture for the given production



Source: own study.

3.2. Employment of the labour factor in relation to its price

Similarly, as above, when considering the production function, we can also consider analytically its univariate form, by focusing on the relationship between employment of the labour factor and the production, as well as its productivity and price. The latter also for the agricultural producer (e.g. income in other sections, which is known as the parity issue) is determined exogenously.

Therefore, we adopt the initial function expressed as:

$$y = f(L, p_L) \ (3.8)$$

where:

$p_L = \frac{y}{L}$ - labour factor productivity.

The level of the labour factor remuneration (endogenous factor) in relation to its price (exogenous factor) is essentially in line with:

$$\frac{\partial y}{\partial L} = w_L \Leftarrow c_L \ (3.9)$$

where:

w_L- labour factor remuneration.

As we can see, the labour factor remuneration is determined internally by its productivity (as a source of its financing), which, however, is also affected by the price level of that factor in the labour market. In other words, the remuneration in the given field of use (in the agricultural sector) is referred to its price resulting from other uses (supply and demand structure in the market of that factor). This is a direct reference to the aforementioned parity issue i.e. reference of the labour factor remuneration in agriculture to its remuneration in other economic activities. These remuneration relationships determine the conditions of equilibrium in the labour market and the resulting labour factor price in that market. This is known as one of the foundations of the Lewis, Schultz and, more widely, Kuznetz growth models. They are essentially based on shifting the labour factor resources from agriculture into sections with the higher productivity of this factor where the driving force pushing from agriculture and pulling the labour factor is its price (in the market). Thus, the problem of the factor price as the exogenous condition, for each producer in each sector, not only in the agricultural sector, has its theoretical origins, inter alia, in the above-mentioned growth models.

Hypothetical Figure 29 depicts the relationship between employment of the labour factor and its price. Its higher price leads to the reduction of its employment in agriculture (*vide* the above-mentioned models) and enforces its remuneration increase financed in the productivity and determined by the marginal productivity. However, the price of this factor itself is governed by the market laws as shown in Figure 30. The lower supply is the higher price of this factor and *vice versa*.

In this reasoning, we assume that the higher price of the labour factor, e.g. in non-agricultural employment, enforces the increase in its remuneration in agriculture. However, this is determined by the endogenous factor i.e. the increase in its productivity. As we know, this process is, to some extent, interrupted by payments and income support for agriculture under the Common Agricultural Policy (CAP). This undermines the obligation to finance the increase in remuneration by the increased labour productivity[43].

A reflection of these processes is what is shown in Figure 31. It shows the hypothetical substitution relationship between the labour factor price in agriculture and its productivity for the given production level. The existence of this substitution relationship will be the subject of empirical verification. This

[43] Incidentally, it also undermines the natural development mechanisms described in the quoted models by Lewis, Schultz, Kuznetz, Todaro and others.

may result implicitly from the assumption on the existence of the function of employing the labour factor as[44]:

$$L = f(p_L, c_L),\ (3.10)$$

or separately:

$$L = f(p_L),\ (3.11)$$

and

$$L = f(c_L).\ (3.12)$$

Thus, employment of the labour factor is the function of both its price and productivity, implicitly in the same direction. The increased productivity of this factor allows to reduce its employment. Similarly, the rise in its price also leads to a reduction in its employment. Employment as the function of the labour factor price will be subject to further empirical analysis.

Assuming the same result and increase in the productivity and price of the labour factor, we can assume the following function as equivalent to (3.6):

$$y = f(L, c_L)\ (3.13)$$

and its basis :

$$c_L = \frac{y}{L}, L = \frac{y}{c_L},\ (3.14)$$

for $L > 0, y = const.$

This implies that reducing employment of the labour factor in the given production leads to the rise in its price and, the other way round, the rise in its price should lead to a reduction in its employment in the sector (moving to alternative employment in other sectors). This will be further illustrated by hypothetical figures.

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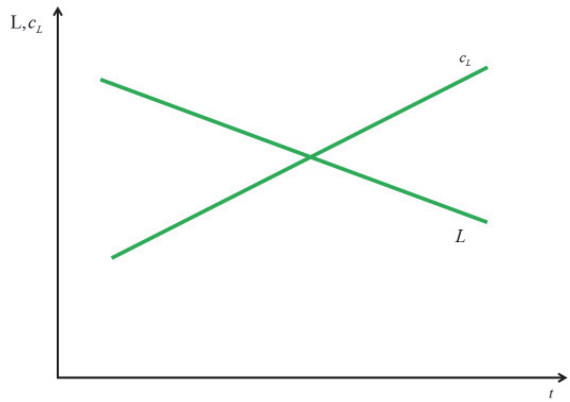
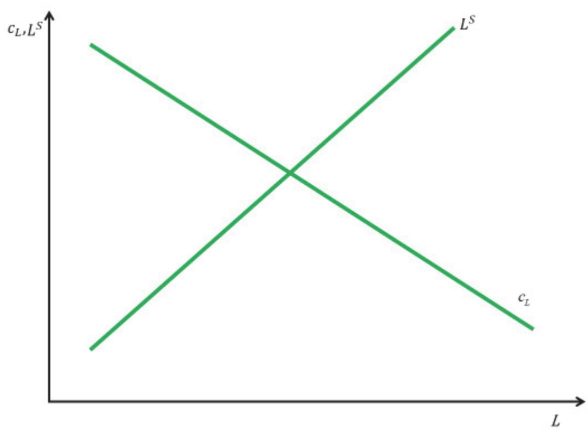
[44] Just like in the case of the capital factor, as formal evidence of it we can adopt: $y = f(L)$ – classical production function and $L = q(y)$ – as the function of employment of the capital factor to obtain this production with $q = f^{-1}$.

Figure 29. Hypothetical assumption on the relationship between the labour factor price and the level of employment of this factor



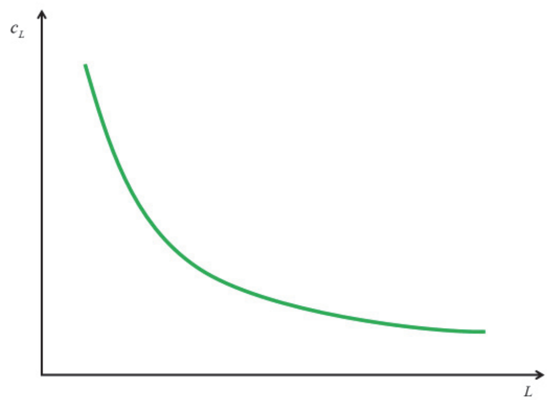
Source: own study.

Figure 30. Price and supply of the labour factor-hypothetical aspect



Source: own study.

Figure 31. Hypothetical assumption on the substitution relationship between the labour factor price and the level of employment of this factor in agriculture



Source: own study.

The dependencies shown are, to some extent, classical trends in changes in factor relationships or production techniques (cf. Rembisz W., Floriańczyk Z., 2014)[45], as known from academic economics of agriculture (cf. Woś A., Tomczak F., 1983)[46], also known from the fundamentals of the production factors market in microeconomics.

3.3. Employment of the land factor in relation to its price

When we continue the reasoning under the same approach as above, the analytical formulation of the production function in relation to the land factor, with other determined factors, allows to consider this function in the univariate and intensive form[47]:

$$y = f(Z, p_Z), (3.15)$$

where: $p_Z = \frac{y}{Z}$-land factor productivity.

[45] Rembisz W., Floriańczyk Z. (2014), *Modele wzrostu…,* op. cit.
[46] Woś A., Tomczak F. (1983), *Ekonomika rolnictwa…,* op. cit.
[47] A justification for such formulation of this function may also be the identity approach used in the literature (Nasiłowski 1974). In adopting it properly, we can formulate the following dependency: $y = Z \cdot \frac{y}{Z} = Z \cdot p_Z$.

As shown previously, in terms of the agricultural production volume, of great importance is employment of the land factor in relation to the production – i.e. the land factor productivity coefficient. This equation is a reference to the theory of intensification of agriculture, strongly highlighted in economics of agriculture in the second half of the last century (Woś, Tomczak 1983, Tomczak 2005). This theory emphasised the role of increasing the land factor productivity in relation to its loss associated with the urbanisation and industrialisation process. We also refer to this below, in the graphic hypothetical assumptions. The obligation to intensify i.e. increase the land factor productivity, in line with this theory, also, or even, first of all, resulted from the unsatisfied demand for food. In this regard, a significant role is played by the land factor price, which is the exogenous factor, which by assumption[48] implies the productivity, particularly marginal, of the employed factor:

$$\frac{\partial y}{\partial Z} = c_Z \quad (3.16)$$

The land factor price is, naturally, determined exogenously in the market of this factor (Figure 33) with all its conditions and restrictions. With this in mind, we actually have the function of employing the land factor, for the given production, defined by two arguments i.e. productivity and price of this factor, i.e. as:

$$Z = f(p_Z, c_Z) \quad (3.17)$$

According to the previously presented approach to the given production, we can also formulate separately employment of the land factor as:

$$Z = f(p_Z) \quad (3.18)$$

and

$$Z = f(c_Z) \quad (3.19)$$

Therefore, it is formulated as the productivity functions and as functions of its price. This last formulation will be subject to empirical analysis.

[48] This assumption arises from the producer's equilibrium model, the higher the price of the given factor enforces it more efficient, and at least more and more intense, use, in economics of agriculture it is known from the Harleman-Stamer triangle.

On this basis we can assume, with great approximation, that the changes, p_Z and c_Z are unidirectional (correlated – which we will examine separately). Therefore, we can modify the formula (3.15) as follows:

$$y = f(Z, c_Z) \quad (3.20)$$

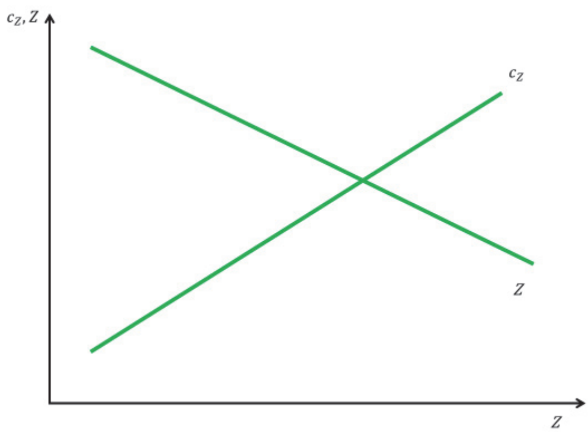
In the analytical form we have:

$$c_Z = \frac{y}{Z} \quad (3.21)$$

for $Z > 0, y = const.$

Therefore, reducing the land factor resources in the given production leads to the rise in its price (and formally, also the other way round). This will be further illustrated by hypothetical figures. They refer to the relationship in this regard, in the same convention as for the capital and labour factors.

Figure 32. Hypothetical assumption as to the relationship between the land factor price and the level of its employment

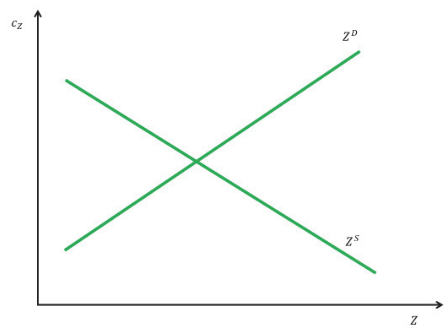


Source: own study.

The meaning of this hypothetical assumption is obvious, it results straight from the hypothesis – the level of scarcity of the given good affects its price with the *ceteris paribus* principle. It appears that all views or specific conditions of the price of that factor must take this rule into account. We do not go deeper into this, because at this level of generality this assumption is sufficient.

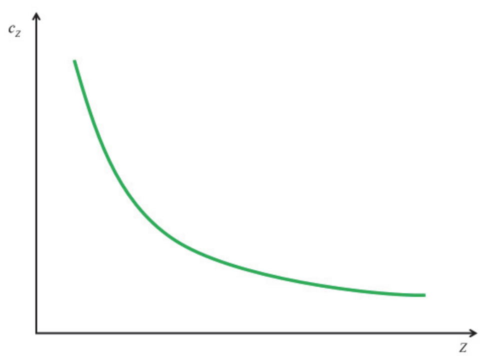
Likewise, without discussion, we adopt another assumption on the evolution of the land factor market price, as shown in Figure 33. On a basis of these two assumptions and formula 3.17, we adopt another hypothetical assumption, further verified, shown in Figure 34 and in formula 3.19, relating to possible relative substitution between the price and employment of the factor, or in other words, the relationship shown in Figure 33.

Figure 33. Land factor price in its market-hypothetical aspect



Source: own study.

Figure 34. Hypothetical assumption as to the substitution relationship between the land factor price and its use in agriculture for the given production



Source: own study.

3.4. Empirical aspect-employment of the capital factor in relation to its price

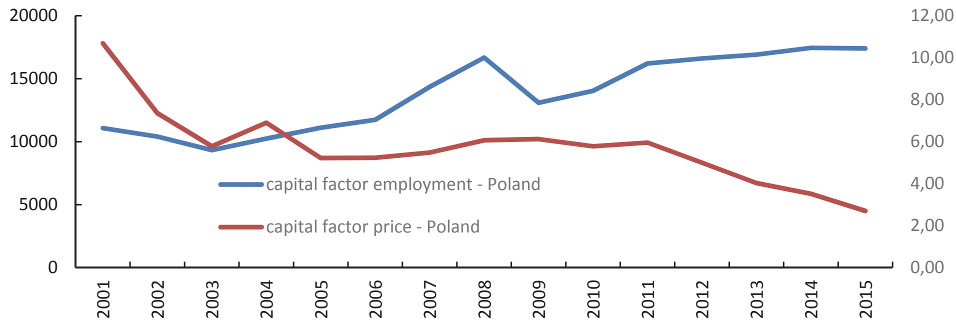
The relationships shown in the above subchapters and formulated analytically in the formulae 3.1-3.7 and Figures 26 to 28 have been verified empirically. For this purpose, the following time series have been used:

- capital factor K – determined based on the Eurostat data as the total intermediate consumption, fixed capital consumptions and gross fixed capital formation. The capital factor is expressed in fixed prices of million EUR;
- capital factor price c_K – determined, on a *proxy* basis, as the base interest rate on a basis of alternative employment – lost benefits (cf. Kleinhanss, 2014)[49].

In the first step, the hypothetical relationship presented in Figure 26 was verified. According to it, even in the absence of the increased productivity, the relative cheapening of the capital factor should lead to the rational increase in its agricultural employment. Charts in all Figures 35-43 and especially in 37 and 41 seem to give a basis for positive verification of this assumption. Charts in these figures, are arranged in a very similar manner and properly reflect this hypothetical assumption. This also refers clearly to agriculture in Poland (Figure 35) and to a large extent in the case of agriculture in other EU countries. This is also some positive verification of the derived analytical formulation (as in the above formulae from 3.1 to 3.7) compliant with the tradition of academic economics of agriculture, whose principles are positively verified by these statistical observations. This is not only of theoretical and cognitive significance, but is essential for real management to understand changes in production techniques and in remunerations of production factors, such as labour and capital. This is only empirical observation, however, as we showed, it has significant theoretical foundations. This is (as it seems) a sustainable process, and the economic development and technical progress and innovation will even make it faster, because, irrespective of aspects, capital is their medium which grows cheaper and cheaper.

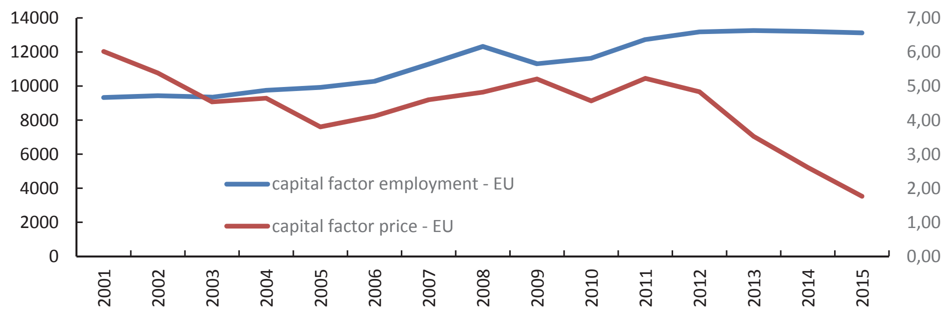
[49] Kleinhanss W. (2014), *Analiza konkurencyjności głównych typów gospodarstw rolnych w Niemczech,* [in:] A. Kowalski, M. Wigier, B. Wieliczko (ed.), *WPR a konkurencyjność polskiego i europejskiego sektora żywnościowego*, IERiGŻ-PiB Program Wieloletni 2011--2014, no. 14.

Figure 35. Capital factor price and its employment in the production in agriculture in Poland



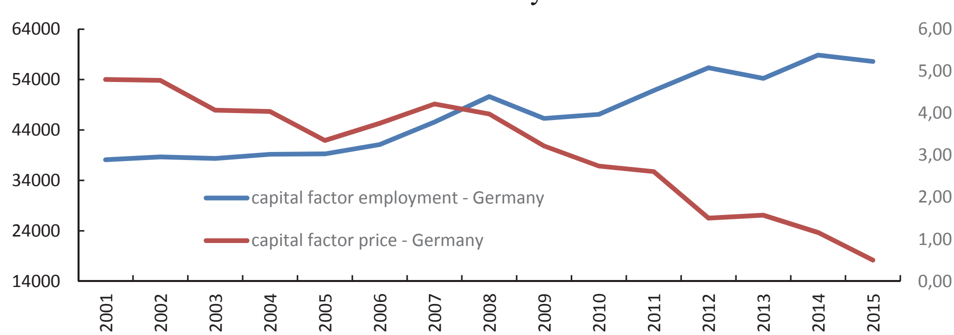
Source: own study based on the Eurostat data.

Figure 36. Capital factor price and its employment in the production in agriculture in EU



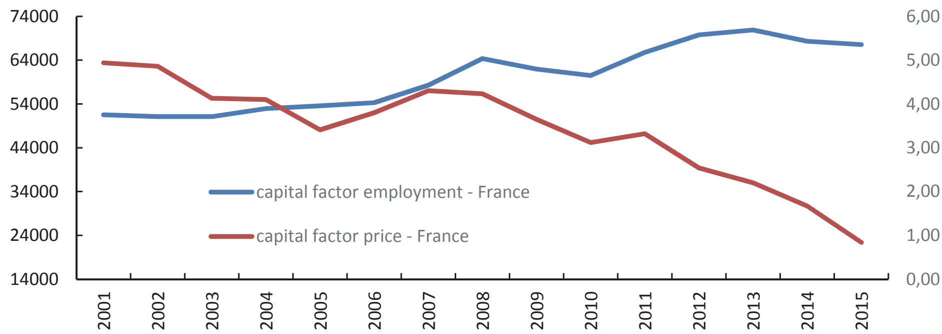
Source: own study based on the Eurostat data.

Figure 37. Capital factor price and its employment in the production in agriculture in Germany



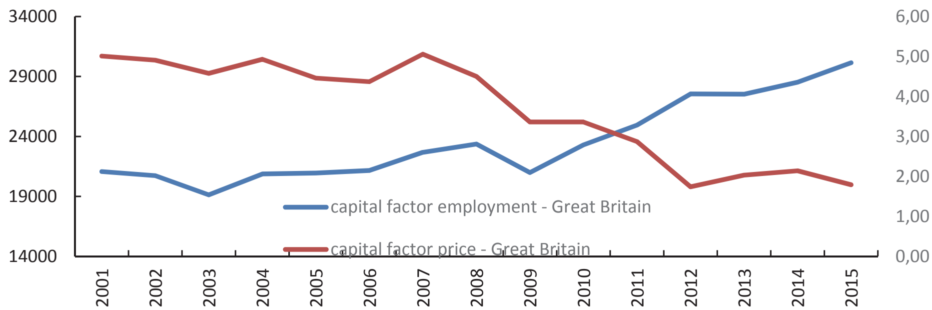
Source: own study based on the Eurostat data.

Figure 38. Capital factor price and its employment in the production in agriculture in France



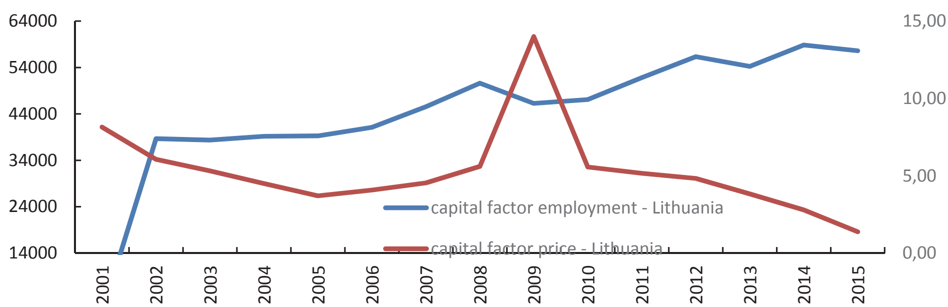
Source: own study based on the Eurostat data.

Figure 39. Capital factor price and its employment in the production in agriculture in the Great Britain



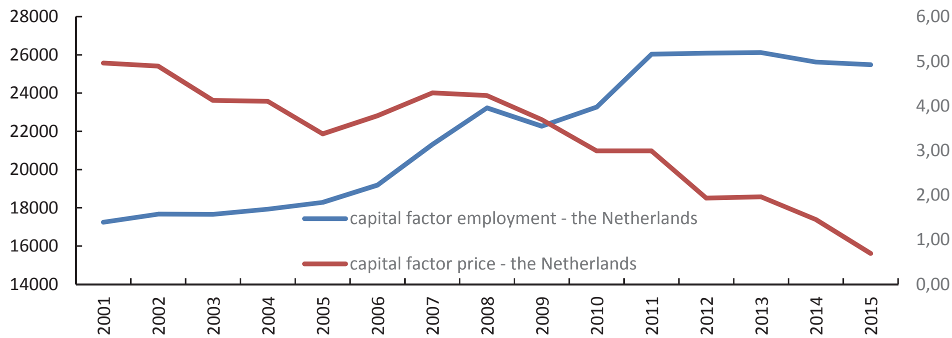
Source: own study based on the Eurostat data.

Figure 40. Capital factor price and its employment in the production in agriculture in Lithuania



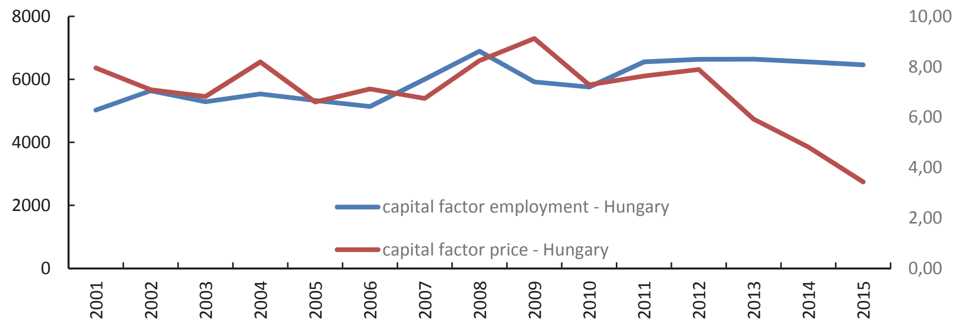
Source: own study based on the Eurostat data.

Figure 41. Capital factor price and its employment in the production in agriculture in the Netherlands



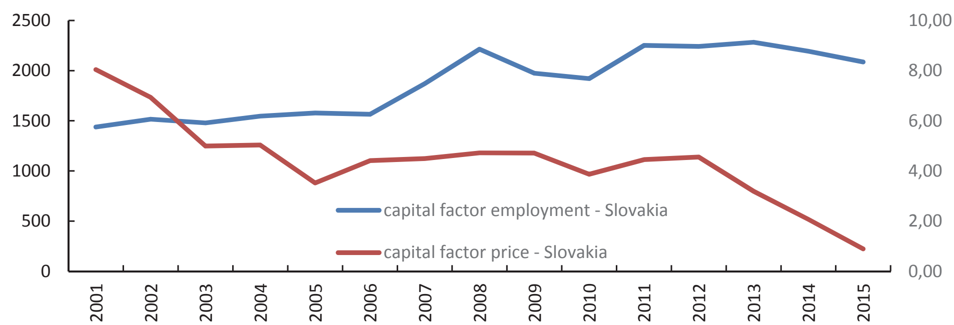
Source: own study based on the Eurostat data.

Figure 42. Capital factor price and its employment in the production in agriculture in Hungary



Source: own study based on the Eurostat data.

Figure 43. Capital factor price and its employment in the production in agriculture in Slovakia



Source: own study based on the Eurostat data.

The overview or detailed description of the above figures does not make any sense and is redundant. Of importance are the relationships and trends we showed. We can notice that in richer countries of the so called „old Union", the „scissors" between the decrease in the price and employment of the capital factor „spread" more than in the new countries of the Union, and therefore those relatively less developed. In the former, the capital factor supply is undoubtedly higher, therefore, the price is relatively lower, and consequently employment of this factor is higher when compared to the new countries. This, as we mentioned, is of cognitive and practical significance, e.g. as to production costs it can be assumed that there should be no pressure on the rise in prices of agricultural products.

In a clear reference to the above empirical aspect, an attempt was made to verify and estimate the hypothetical assumption on the substitution relationship between the price and employment of the capital factor implicitly so as to obtain the given production with respect to 28 EU countries for 2015, which was hypothetically shown in Figure 28. Figure 44 illustrates the empirical relationship for the EU countries in total, and this relationship was further estimated. What was estimated, were the nonlinear linearised models, i.e. exponential, logarithmic, and hyperbolic model. Ultimately, the model best matched to this empirical data[50] proved to be the following model:

$$y = a + \frac{b}{X} \quad (3.22)$$

The results of this estimation confirm the existence of the statistically significant substitution relationship, which can be approximated by the equation of the following hyperbolic curve limited to the first quarter of the coordinate system (parentheses contain standard errors of the estimated parameters)[51]:

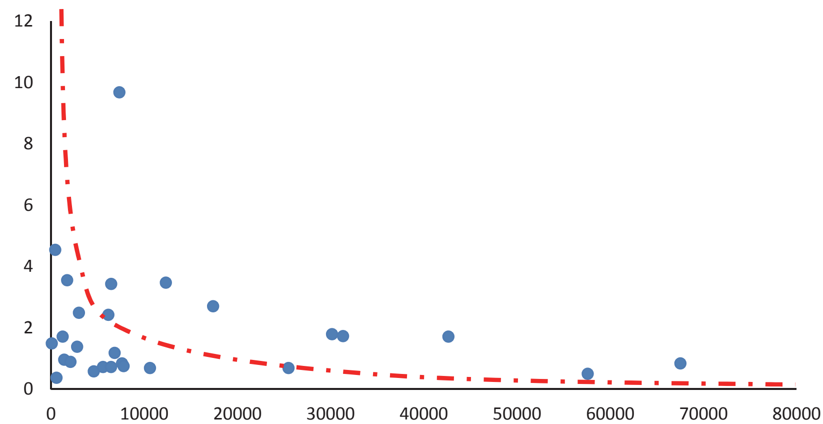
$$\left(\widehat{C_K}\right) = 2{,}18 + \frac{1{,}92 \cdot 10^{-5}}{K} \quad (3.23)$$

$$(0{,}366)\ (2{,}18 \cdot 10^{-8}).$$

[50] The comparison was made on the basis of the corrected value of the determination coefficient and the Akaike information criterion.

[51] The results of the empirical studies presented have been presented more widely during the 14th International Scientific Conference "Global Problems of Agriculture and Food Economy" and will be printed in a form of an article entitled *Capital factor as an endogenous source of growth in agriculture* in the journal *World Agriculture Issues*.

Figure 44. Capital factor price and level of its employment in agriculture in the EU countries, 2015



Source: own study based on the Eurostat data.

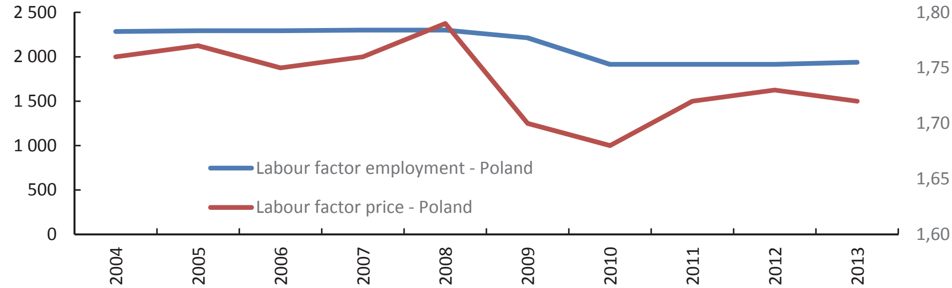
3.5. Empirical aspect – employment of the labour factor in relation to its price

Just like in the case of verifying the hypotheses of the previous section, the data (time series) derived from the Eurostat database have been used. They were:

- labour factor L-determined on a basis of Eurostat data as total labour force input in thousands of AWU,
- labour factor price c_L-defined as the average hourly remuneration expressed in EUR.

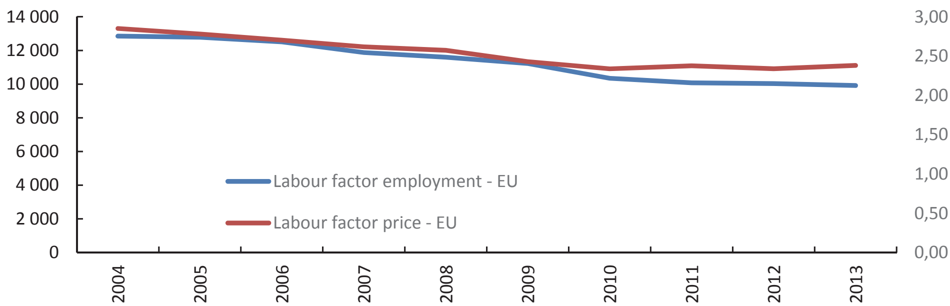
The first step in empirical verification of the theoretical assumptions and analytical aspects described in subchapter 3.2 was the assessment of the relationship between the labour factor price and the level of its employment. This was illustrated in hypothetical Figure 20. Figures 45-53 from empirical data show the price and employment of the labour factor in agriculture in the selected EU countries.

Figure 45. Labour factor price and its employment in the production in agriculture in Poland



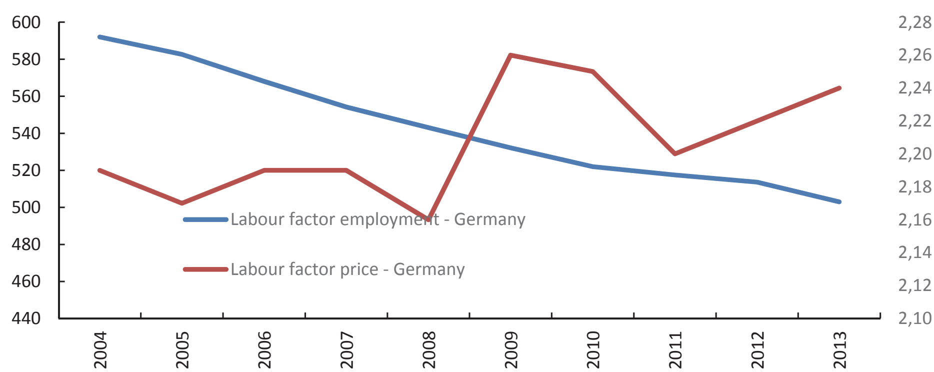
Source: own study based on the Eurostat data.

Figure 46. Labour factor price and its employment in the production in agriculture in EU



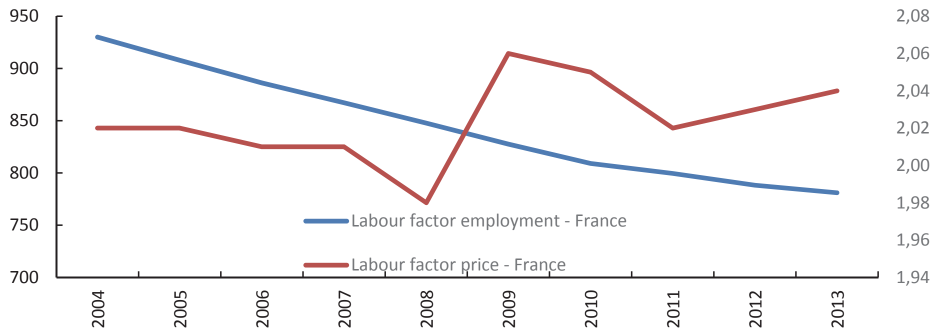
Source: own study based on the Eurostat data.

Figure 47. Labour factor price and its employment in the production in agriculture in Germany



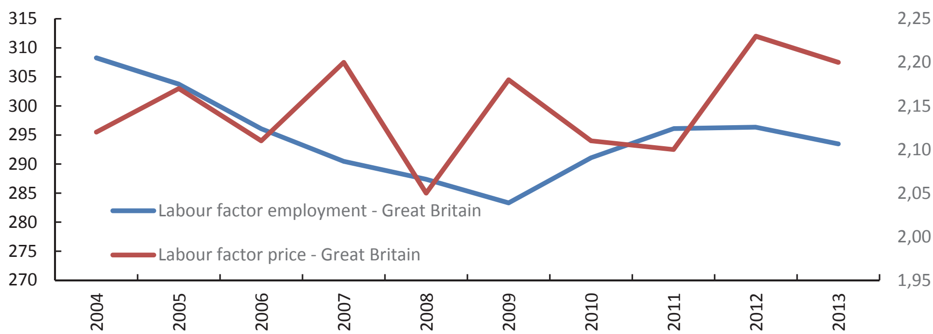
Source: own study based on the Eurostat data.

Figure 48. Labour factor price and its employment in the production in agriculture in France



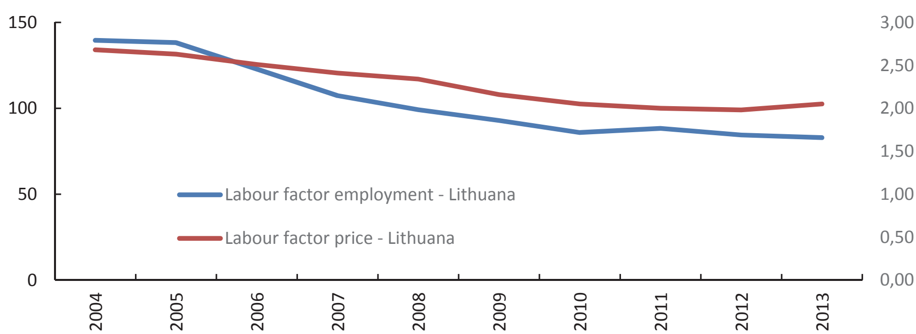
Source: own study based on the Eurostat data.

Figure 49. Labour factor price and its employment in the production in agriculture in Great Britain



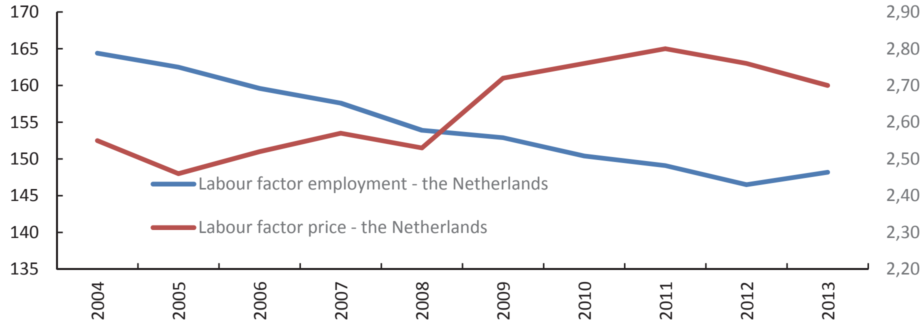
Source: own study based on the Eurostat data.

Figure 50. Labour factor price and its employment in the production in agriculture in Lithuana



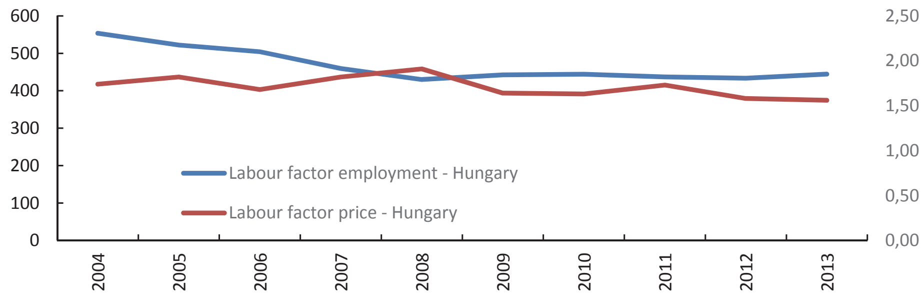
Source: own study based on the Eurostat data.

Figure 51. Labour factor price and its employment in the production in agriculture in the Netherlands



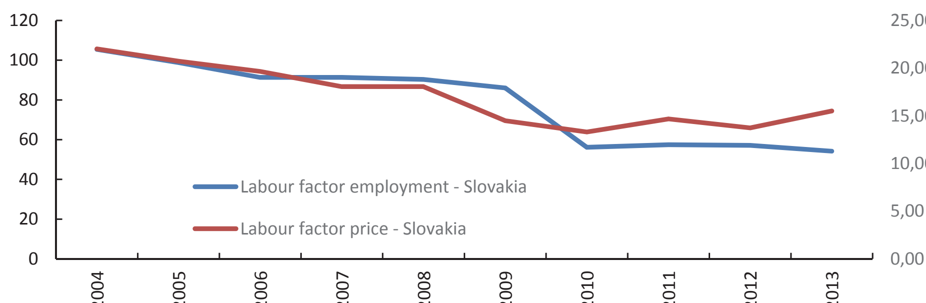
Source: own study based on the Eurostat data.

Figure 52. Labour factor price and its employment in the production in agriculture in Hungary



Source: own study based on the Eurostat data.

Figure 53. Labour factor price and its employment in the production in agriculture in Slovakia



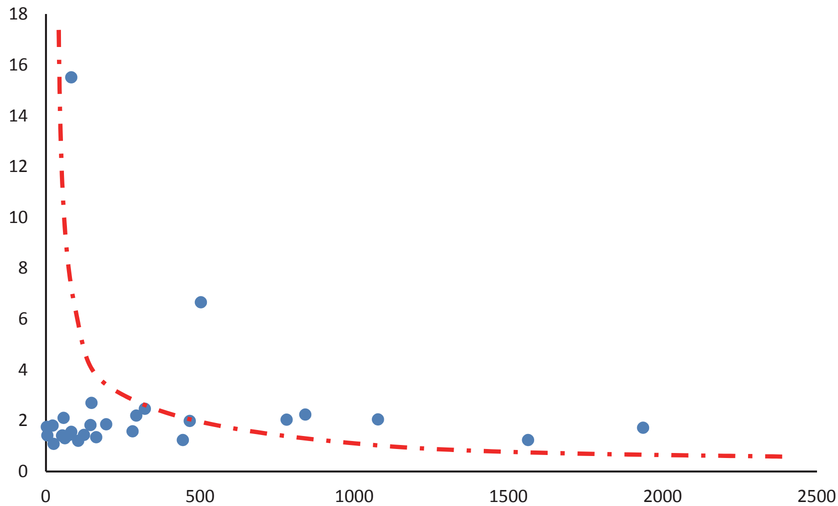
Source: own study based on the Eurostat data.

The visualisations obtained above give rise to ambiguous yet positive verification of the adopted hypothetical assumptions and derived analytical aspect. This depends on whether we refer these assumptions to empirical data and visualisations for Poland and other new or old European Union countries. With regard to the old EU countries, and therefore those more developed, the higher price of the labour factor in its market corresponds to its increasingly lower agricultural employment. This verifies empirically the invoked principles of agricultural economics, reflected in the quoted models of development. The higher price of this factor must mean, in fact, its shifting to more efficient types of employment. It was certainly determined by the unemployment rate, which is clearly lower than in the new EU countries. This is well illustrated by the visualisations for these countries.

In Poland, these proportions are not so clear, but they are visible. This may result relatively from the very stable agrarian structure and related slow loss of employment of the labour agent. This may be determined by the high unemployment rate in non-agricultural activities, as observed throughout the analysed period, and the lack of full possibilities of allocating the labour factor in relation to its price relationship. In addition, the actual labour price in the economy has not risen significantly, *inter alia*, due to the aforementioned conditions. Similarly, although more clearly, empirical visualisations verify the derived analytical aspect and hypothetical assumptions with regard to other new EU countries adopted in this analysis.

Adopting another assumption in accordance with the hypothetical Figure 31, the hypothesis regarding the existence of the substitution relationship between the exogenous factor i.e. the labour factor price and its employment was verified. For this purpose, estimation was made for one year (2013 – last available data) for 28 EU countries in total. The non-linear hyperbolic model was used for this purpose.

Figure 54. Capital factor price and level of its employment in agriculture in the EU countries in 2013



Source: own study based on the Eurostat data.

As a result of the estimation attempts, the following estimations of the model parameters have been obtained:

$$(\widehat{C_L}) = 3{,}01 + \frac{1{,}66 \cdot 10^{-5}}{L} \quad (3.24)$$

$$(0{,}215)\ (1{,}18 \cdot 10^{-7}).$$

The parameters obtained and the quality of matching the model to empirical data are a poor rationale which, with a high degree of caution, allows us to adopt a hypothesis regarding the existence of the substitution relationship between the exogenous labour factor price and its level of use.

3.6. Empirical aspect – employment of the land factor in relation to its price

The relationships shown in the above subchapters and formulated analytically in the formulae 3.15-3.21 and Figures 32 to 34 have been verified empirically. For this purpose, time series from the Eurostat database have been used. The following definitions have been assigned to them:

- land factor Z has been designated on a basis of Eurostat data as the total of utilised agricultural area,
- land factor price c_Z has also been taken from the Eurostat database for the years available: 2005, 2007, 2010 and 2013.

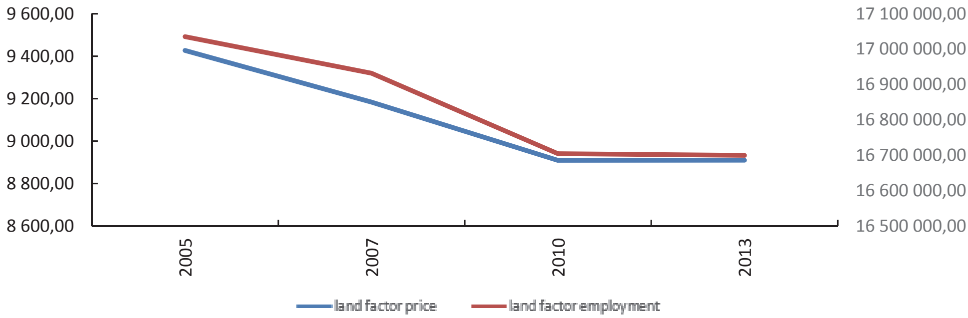
Figure 55. Land factor price and level of its employment in the production in agriculture in Poland

Source: own study based on the Eurostat data.

Figure 56. Land factor price and level of its employment in the production in agriculture in the EU

Source: own study based on the Eurostat data.

Figure 57. Land factor price and level of its employment in the production in agriculture in Germany



Source: own study based on the Eurostat data.

Figure 58. Land factor price and level of its employment in the production in agriculture in France

Source: own study based on the Eurostat data.

Figure 59. Land factor price and level of its employment in the production in agriculture in Great Britain

Source: own study based on the Eurostat data.

Figure 60. Land factor price and level of its employment in the production in agriculture in Lithuania

Source: own study based on the Eurostat data.

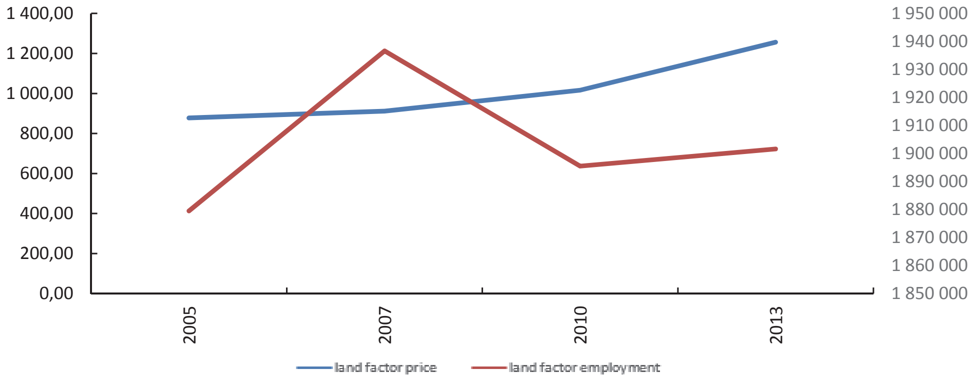
Figure 61. Land factor price and level of its employment in the production in agriculture in the Netherlands

Source: own study based on the Eurostat data.

Figure 62. Land factor price and level of its employment in the production in agriculture in Hungary

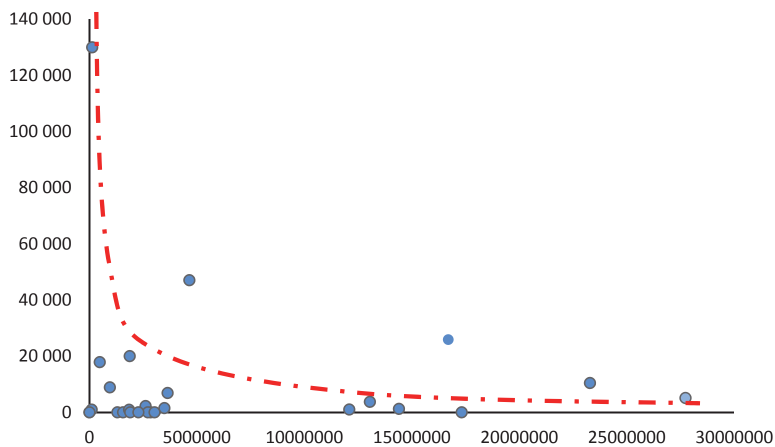
Source: own study based on the Eurostat data.

Figure 63. Land factor price and level of its employment in the production in agriculture in Slovakia



Source: own study based on the Eurostat data.

Figure 64. Land factor price and level of its employment in agriculture in the EU countries in 2013



Source: own study based on the Eurostat data.

The obtained substitution relationship was subject to estimation. The following form of curve was obtained:

$$\left(\widehat{C_Z}\right) = 1254{,}58 + \frac{9{,}24 \cdot 10^{-7}}{Z} \quad (3.25)$$

$$(0{,}1981)\ (1{,}01 \cdot 10^{-8}).$$

The parameters obtained are also a poor rationale allowing us to adopt a hypothesis regarding the existence of the substitution relationship between the exogenous land factor price and its level of use.

Chapter IV
Monetary policy as a part of the exogenous mechanism of price formation in agriculture

As indicated in the first chapter of this monograph, external (exogenous) factors affecting the price relationships analysed and, consequently, the production (according to the relationship 1.21) may also include the impact of the economic policy. The macroeconomic policy affects not only the level of the domestic prices of agricultural products, but also the prices of means of production.

The above reasoning can be written in a mathematical form (by extending equation 1.21 in relation to 1.14), by specifying that the right side of the equation 1.14 is a function (*inter alia*[52]) of the interest rate:

$$\left\{\frac{\partial c_K}{c_K}+\frac{\partial c_L}{c_L}+\frac{\partial c_Z}{c_Z}\right\}-\frac{\partial p}{p}=f(i)\ (4.1)$$

where: i – nominal interest rate.

The interest rate which is the primary tool of the monetary policy affects price indices of production factors, since it is the price of monetary capital which is potentially convertible to these factors and their use. Consequently, it affects the above-mentioned price relationships. Finally, with the given productivity, in the sense of TFP, it determines the profitability of the agricultural production[53]. The mechanisms of processes related to transmission of effects of the economic policy, including monetary policy, to the real and nominal sphere have been presented in the „Pendulum model" and discussed in the paper by Czyżewski and Kułyk[54].

From the analytical point of view and possibility of building agricultural scenarios, four options of the economic policy impact can be identified. The point is the monetary and fiscal policy, and each can be pursued in an expansive

[52] In our considerations, we will limit ourselves only to the variability of price indices resulting from the change in the variance of the interest rate.

[53] Kata R. (2011), *Wpływ polityki fiskalnej i monetarnej na zadłużenie gospodarstw rolnych w Polsce*. Warszawa, Roczniki Nauk Rolniczych, Seria G, vol. 98, iss. 3, p. 73-83.

[54] Czyżewski A., Kułyk P. (2010), *Relacje między otoczeniem makroekonomicznym a rolnictwem w krajach wysokorozwiniętych i w Polsce w latach 1991-2008*. Warszawa, Ekonomista, no. 2, p. 189-214.

or restrictive manner. In this monograph, attention will be focused on assessing the monetary policy impact on agricultural price indices[55].

The impact in question, or the system of links, which is between decisions of the Monetary Policy Council (MPC) and the economy, is defined in the theory of economics as the monetary transmission mechanism (MTM). This system is a path of impulses resulting from the pursued monetary policy. It starts with an instrument which, most commonly, is the interest rate. Finally, we observe the response of macroeconomic variables, such as the production, inflation or exchange rate. From our perspective, essential will be the response of indices informing about the change in the agricultural product prices.

These paths of this impact caused by the decisions from the monetary policy area are defined in the literature of the subject as transmission channels. Polański[56] defines monetary impulses as changes in the prices of financial instruments, e.g. changes in interest rates, in foreign exchange rates or in securities exchange rates, as well as in their supply itself, including also the money supply. On the macroeconomic scale, a key role is played in this regard by central banks and impulses they generate and in the light of credit money, changes in interest rates are, therefore, key monetary impulses.

An important issue is also to assess the economic vulnerability, also in sectoral terms, to shocks resulting from rapid fluctuations in the demand and supply and to assess the ability of the given sector or the whole economy to return to the state of equilibrium after the occurrence of disturbances.

4.1. Monetary policy impact – theoretical aspect

In the monograph, the assessment of the monetary policy impact on the agricultural sector (more specifically, on price indices om the sector) will be embedded around mainstream economics. In relation to new Keynesian economics[57], the approach to explaining fluctuations in the production is based

[55] Chapter IV (subchapters IV.1–IV.5) is an extension of the unpublished dissertation by the co-author of this monograph on the impact of the monetary policy on prices in agriculture (cf. Waszkowski A. (2017), *Mechanizm transmisji impulsów polityki pieniężnej do sfery realnej na przykładzie polskiej gospodarki*).

[56] Pietrzak B., Polański Z., Woźniak B. (2008), *System finansowy w Polsce*. Tom 1. Wydanie drugie zmienione. Wydawnictwo Naukowe PWN, Warszawa.

[57] Some authors question the very term "New Keynesian economics" due to its identification difficulty (cf. Wojtyna, 2000). Others described it as the "new Neoclassical synthesis" (cf. e.g. Goodfriend, King, 1998).

on the pillars of microeconomics which accompanies us and organises the analytical approach applied in this paper.

The most important relationships to determine the directions of dependencies in the monetary policy model in this aspect are related to three assumptions presented below (Kokoszczyński et al., 2002[58]; Kokoszczyński, 2004[59]).

If companies or farms maximise their profits, current prices are treated by them as the weighted average of historical prices and prices which have just been changed (in the infrequent price adjustment Calvo model[60]). They are subject to fluctuations over a given period. This is tantamount to stating that the level of the price index (or its growth rate) depends on entrepreneurs' and producers' expectations as to the future price level and on the difference between the real current marginal cost and the level of this cost in long-term equilibrium in the sector or economy. The difference between these marginal costs is then a reflection of the relationship between the current production level and its optimal value in a situation of elastic prices. This is defined in the literature of the subject as the output gap. In an analytical manner, this dependency is known as the New Keynesian Phillips curve[61] (I.20) and was derived for the Calvo Model[62] (1983):

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi x_t, \quad (4.2)$$

where:
π_t – inflation rate in the period *t*,
E_t – expected value of the inflation rate π in the period t,
x_t – output gap in the period *t*,
β and φ – model parameters.

[58] Kokoszczyński R., Łyziak T., Pawłowska M., Przystupa J., Wróbel E. (2002), *Mechanizm transmisji polityki pieniężnej – współczesne ramy teoretyczne, nowe wyniki empiryczne dla Polski*. Materiały i Studia 151, Narodowy Bank Polski.
[59] Kokoszczyński R. (2004), *Współczesna polityka pieniężna w Polsce*. Wydawnictwo Naukowe PWE, Warszawa.
[60] Prices are determined in such a way that the weighted average of anticipated markups is equal to the optimal markup value in a situation where there is no stickiness in the sector. Calvo (1983) introduced a concept of infrequent price adjustment which gives an effect of stickiness of their level. A probability to change the price is defined using the Poisson distribution with the permanent probability of price adjustment by the farm or company.
[61] Different variants of the Phillips curve are presented, inter alia, in the paper by Urbańska A. (2002), *Polityka monetarna: współczesna teoria i analiza empiryczna dla Polski*. Materiały i Studia 148, Narodowy Bank Polski.
[62] Calvo G. A. (1983), *Staggered prices in a utility – maximizing framework*. Journal of Monetary Economics, Vol. 12, Issue 3, pp. 384-398.

Past expectations of present inflation from the new classical Phillips curve (4.2) are in this case replaced by present expectations of future inflation. Woodford[63], taking into account the fact of the high correlation of inflation expectations, notes that the new classical Phillips curve enables the occurrence of permanent effects of monetary shocks. In the case described by the relationship (4.2), the inflation rate is ahead of the production rate[64].

Similarly, farms and companies perform optimisation. Then, we can observe the relationship between the production volume and the interest rate. Analytically, it can be presented by the dynamic IS curve equation as:

$$IS: y_t = -\sigma^{-1}(i_t - E_t(\pi_{t+1}) - i^{\rho}) + E_t(y_{t+1}) + \varepsilon_t^D, \quad (4.3)$$

where:
y_t – production logarithm in the period *t*,
ε_t^D – demand shock (exogenous component of the total demand in the period *t*),
i_t – nominal interest rate in the period *t*,
i^{ρ} – bank rate corresponding to the real interest rate in the long-term equilibrium exclusive the permanent economic growth,
σ – model parameter.

Then, the IS curve presented in the categories of the output gap (x_t) has the following form (4.4):

$$IS: x_t = -\sigma^{-1}(i_t - E_t(\pi_{t+1}) - i^{\rho}) + E_t(x_{t+1}) + \varepsilon_t^D. \quad (4.4)$$

The equations (4.2)-(4.4) are a structured, aggregated image of the transmission mechanism for the small closed economy. It follows from the equation (4.2) that an external monetary impulse may affect the production if the current short-term real interest rate or its expected future value is changed under its influence. When we assume the lack of the simultaneous adjustment of all prices[65], the change in the nominal short-term interest rate also affects the level of the real interest rate.

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[63] Woodford M. (2002), *Interest and Prices. Foundations of Theory of Monetary Policy*. Princeton University Press, Princeton.

[64] Urbańska A. (2002), *Polityka monetarna…*, op cit.

[65] This statement results from analysis of the production response to monetary disorders, which was carried out in the category of examining random effects of the money supply disorders. The modern monetary policy uses not a monetary aggregate, but a short-term interest rate. As Kokoszczyński notes (2004) „the expansive monetary shock works in the presented models in the same way as tax reduction in the traditional business cycle models" cf. Waszkowski A. (2017), *Mechanizm transmisji…,* op cit.

The discussed relationships (4.2)-(4.4) are limited to and define the mechanism of impact in the case of the closed economy only. The „opening of the economy" results in a necessity of its completion. The current literature trend, belonging to so-called „new macroeconomics of the open economy" also uses dynamic models of overall equilibrium with clearly defined microeconomic foundations for the behaviour of economic operators, nominal stickiness and imperfect competition. Attempts to include the openness of the economy in macroeconomic concepts underlying the monetary policy most often apply to the approach consisting in the maximal use of the analysis trend looking for the optimal monetary policy in the closed economy[66]. The models considered in these papers assume infrequent price adjustments according to Calvo[67] and include the monetary policy defined as the interest rate policy, not limiting solely to the reaction of monetary shocks. The formal description of the open economy uses the derived relationships for the closed economy and also takes into account parameters such as the degree of openness of the economy concerned and the rate of substitution between foreign and national goods.

The IS curve (4.4) for the open economy in the category of the output gap is expressed by the following formula:

$$IS: x_t = -\sigma^{-1}\omega_{\alpha^o}[(i_t - E_t(\pi_{t+1}) - i^\rho + f_{\alpha^o}E_t(\Delta y_{H,t+1})] + E_t(x_{t+1}), \tag{4.5}$$

where:
ω_{α^o}, f_{α^o} model parameters are dependent on the level of the openness of the economy,
$\Delta y_{H,t+1}$ applies to foreign countries.

To describe the change in the nominal exchange rate we use most often the equation of uncovered interest rate parity (UIP). This results in a fact that the changes in the exchange rate depend functionally on the current and expected differences in interest rates between the analysed open and closed economy.

[66] Exemplary papers: Lane P.R. (1999), *The new open economy macroeconomics: a survey*. Journal of International Economics, Vol. 54 (2001), pp. 235–266 and McCallum B.T., Nelson E. (2001), *Monetary policy for an open economy: an alternative framework with optimizing agents and sticking prices*. External MPC Unit Discussion Paper Vol. 5, Bank of England, Londyn.
[67] Calvo G. A. (1983), *Staggered prices...* op cit.

The presented relationships: The IS curve and the Phillips curve, supplemented by the uncovered interest rate parity, are considered in the literature of the subject as key elements of the models for describing the monetary transmission mechanism. The state of long-term equilibrium of the open economy differs from that of the closed economy in this way that the evolution of inflation depends on the level of openness of the economy and on changes in the foreign exchange rate. In addition, the level of the output gap is affected by the above factors along with the degree of substitution between national and foreign goods.

The presented theoretical trend is described as a new macroeconomic synthesis. It is treated in terms of consensus in macroeconomics. It seeks to combine the strengths of the competing modern approaches to analysing the economy. From the new classical economics and the real business cycle school, this trend „borrowed" a concept of intertemporal optimisation of companies and households, endogenous way of modelling rational expectations and permanently balanced markets. These trends provide the macroeconomic description of the economy with dynamic microfoundations based on the behaviour of a representative operator. In turn, from Neo-Keynesianism the new synthesis borrowed the assumption on the monopolistic competition where nominal prices change occasionally[68]. This transitional price stickiness is a reason for which the level of economic activity is determined by the size of aggregate demand that can be influenced by the shocks from the monetary policy area[69]. The new synthesis implies that fluctuations in the production volume and employment are not a problem requiring intervention and are a natural response of the economy to real disturbances (cf. Goodfriend, King, 1998)[70].

In formal terms, the basic model of the new synthesis can be presented by means of the equations (4.2)-(4.4). This model was quickly recognised as an agreement between the competing theoretical positions on the issue of key macroeconomic phenomena. It enables analysis of price and production decisions (one of the areas of interest of the Keynesian economists) as well as decisions on the consumption and supply factors (a sensitive area of the new

[68] Bludnik I. (2010), *Nowa synteza neoklasyczna w makroekonomii*. Bank i Kredyt, Vol. 41(2), p. 43-69.

[69] A major source of fluctuations in the economy are therefore not fluctuations in the money supply.

[70] Goodfriend M., King R. (1998), *The new neoclassical synthesis and the role of monetary policy*. Working Paper 98-05, Federal Reserve Bank of Richmond.

classical economics and the real business cycle school). This model allows to analyse both short-term fluctuations and the long-term trend of the growth of economic activity.

4.2. Monetary policy impact-measurement methods

The same concept of time series modelling divides the history of econometrics into two periods, which are separated by the paper by Sims published in 1980[71]. They started abandoning the multiple-equation structural modelling to the benefit of atheoretical models. This is particularly important in view of the fact that those models were resistant to criticism by the Cowles Commission[72] (Kusideł, 2000):

a) did not require the prior division into the exo- and endogenous variables,
b) did not impose zero restrictions,
c) there is no priority, in relation to modelling, theory of economics which was a foundation to build classical models.

The modelling method proposed by Sims assumes that each variable constructs a separate equation within the model. There is, therefore, no need to divide the variables into exo- and endogenous ones. In addition, since the role of exogenous variables is played only by lags of all variables used in the study, it is not necessary to impose zero restrictions in order to obtain the model identification[73]. The lack of the priority of the theory of economics arises from two previous principles. Since there are no endogenous variables in the model, and none of the variables can be described as exogenous, „everything is the cause of everything", so it is impossible to impose any economic hypotheses.

The new approach to time series analysis initially encountered great criticism determining the Sims' approach to modelling as „atheoretical". However, this criticism was not fully reasonable. Admittedly, within the modelled system there is

[71] Sims C. A. (1980), *Macroeconomica and reality*. Econometrica, Vol. 48, No 1, pp. 1-48.
[72] Cowles Commission for Research in Economics – its original objective was to take studies on determining stock exchange prices, after which the ultimate focus was on the theory of econometrics. The members of the Commission focused, above all, on formulating of structural multiple-equation models. Cf. Kusideł E. (2000), *Modele wektorowo-autoregresyjne VAR. Metodologia i zastosowanie*, [in:] Dane panelowe i modelowanie wielowymiarowe w badaniach ekonomicznych, ed. Suchecki B., vol. III. Wydawnictwo Absolwent, Łódź.
[73] *The identification problem does not exist in vector autoregression models*, cf. Rubaszek M. (2012), *Modelowanie polskiej gospodarki z pakietem R*. Oficyna Wydawnicza SGH w Warszawie.

no space to include economic hypotheses, but the mere selection of variables for the system of equations results from the economic knowledge of the phenomenon being studied. VAR models used nowadays are a bridge between traditional econometrics and atheoretical analysis[74]. Continuators of the work by Sims introduce into the model the exogenous variables, such as: absolute term, linear trend, and dichotomous variables. In addition, treating the VAR model as a reduced form of the structural model allows for an interpretation of economic phenomena[75] without having to give up information on data-generating stochastic processes. It should be noted that since the early 90s, VAR models have become the primary tool to examine co-integration of variables (vector error correction model, VECM) ousting the Engle and Granger procedure[76].

The efficiency and effectiveness of the monetary policy, as well as its impact on the price level and dynamics, is most often based on the mechanism of vector autoregression and vector cointegration.

The usefulness of VAR models is confirmed by the work carried out as part of the studies by the National Bank of Poland. The description of the MTM using the VAR models can be found in the papers by Łyziak et al.[77], Demchuk et al.[78], Postek[79], Kapuściński et al.[80] or in collective comparative analysis by Kokoszczyński[81]. Vector autoregression systems are also presented in the project of the European Central Bank[82] and the paper by Hericourt[83]. The

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[74] Kusideł E. (2000), *Modele wektorowo-autoregresyjne…,* op cit.

[75] *These are deviations from the assumptions proposed by Sims*.

[76] Engle R.F., Granger C.W.J. (1987), *Co – integration and error correction: representation, estimation and testing*. Econometrica, Vol. 55, No. 2 (Mar., 1987), pp. 251-276.

[77] Łyziak T. (2012), *Oczekiwania inflacyjne w Polsce*. Materiały i Studia Narodowego Banku Polskiego, Vol. 271, Warszawa oraz Łyziak T., Przystupa J., Wróbel E. (2008), *Monetary policy in Poland: a study of the importance of interest rates and credit channels*. SUERF Studies, The European Money and Finance Forum, Vol. 2008/1.

[78] Demchuk O., Łyziak T., Przystupa J., Sznajderska A., Wróbel E. (2011), *Mechanizm transmisji pieniężnej w Polsce. Co wiemy w 2011 roku*? Raport Instytutu Ekonomicznego NBP.

[79] Postek Ł. (2011), *Nieliniowy model mechanizmu transmisji monetarnej w Polsce w latach 1999-2009. Podejście empiryczne*. Materiały i Studia 253, Narodowy Bank Polski.

[80] Kapuściński M., Kocięcki A., Kowalczyk H., Łyziak T., Przystupa J., Stanisławska E., Sznajderska A., Wróbel E. (2015), *Mechanizm transmisji polityki pieniężnej w Polsce. Co wiemy w roku 2015?* Raport Instytutu Ekonomicznego NBP.

[81] Kokoszczyński R., Łyziak T., Pawłowska M., Przystupa J., Wróbel E. (2002), *Mechanizm transmisji…,* op cit.

[82] Peersman G., Smets F. (2001), *The monetary transmission mechanism in the Euro area: more evidence from VAR analysis*. Working Paper Series 91, European Central Bank.

[83] Hericourt J. (2006), *Monetary policy transmission in the CEECs: a comprehensive analysis*. Economic and Business Review, Vol. 8(1), pp. 37-82.

specifications of the selected models are presented in Table 1. It is worth pointing to the fact that in the Polish and world literature, to the knowledge of the authors, there is no sectoral approach in this area. In this regard-the monetary policy impact on changes in prices in the agricultural sector, this monograph is trying to bridge the gap.

Table 1. Characteristics of the selected VAR models

Author of the model	**Endogenous variables**	**Exogenous variables**
Peersman, Smets, (2003)	industrial production, CPI, interest rate, foreign exchange rate	prices of raw materials, industrial production in the USA,
Kokoszczyński et al. (2002)	level of industrial production sold, unemployment rate, CPI, WIBOR1M, M1 aggregate, profitability of 12-month treasury bills, nominal effective foreign exchange rate	
Hericourt (2006)	industrial production, CPI, interest rate, foreign exchange rate, monetary aggregate, national credits	industrial production in the euro zone, interest rate in the euro zone, prices of raw materials
Łyziak et al. (2008)	industrial production, CPI, interest rate, foreign exchange rate, monetary aggregate, national credits	
Demchuk et al. (2011)	CPI, investments, consumption, PKB, WIBOR 1M, nominal effective foreign exchange rate	
Kapuściński et al. (2015)	PKB, HICP, credits and loans for households and companies, WIBOR 3M, real effective foreign exchange rate	GDP in the euro zone, EURIBOR 3M, additional dichotomic variables

Source: Waszkowski A. (2017), Monetary transmission… op cit.

4.3. Comments on time series modelling – stationarity and autocorrelation

Prior to estimating any model built based on time series, it is necessary to examine whether the data generation process is stationary or not[84]. In practice of time series econometrics, the situation in which the analysed variables are stationary, is rather unlikely. They are often characterised by a stochastic or deterministic trend, as well as by seasonality. Failure to take account of these phenomena may result in the inadequacy of the statistics of the model quality factor or apparent regression, manifesting itself in estimations of parameters of questionable precision, despite satisfactory matching of the model with empirical data[85].

The stochastic process $\{x_t\}$ is defined as stationary in the strict sense (strong stationarity), in a situation where the total probability distribution for $\{x_{t1}, x_{t2}, \dots, x_{tn}\}$ is the same as for $\{x_{t1+h}, x_{t2+h}, \dots, x_{tn+h}\}$ for any observations $t_1, t_2, \dots, t_n$ and a time lag which is described using the parameter h. It is not possible to verify strong stationarity in an empirical manner. Therefore, the concept of time series stationarity in the wider sense (weak stationarity) was introduced into the modelling practice. It is then required only that the expected time series value, variance and co-variance are constant over time. The above-mentioned definition can be written as (4.6)-(4.8):

$$E(x_t) = \mu \text{ dla t} = 1, 2, \dots \quad (4.6)$$

$$var(x_t) = \sigma^2 < \infty \quad (4.7)$$

$$Cov(x_t, x_{t+h}) = Cov(x_{t+k}, x_{t+k+h}) = \gamma(h) \quad (4.8)$$

where:
$E(x_t)$ – expected value of the variable x_t,
$var(x_t)$ – variance of the variable x_t,
$Cov(x_t, x_{t+h})$ – co-variance of the variable x_t.

For empirical verification aimed at answering the question whether the examined time series is stationary in the wider sense we use the so-called unit

[84] These issues have been shown, inter alia, in the monograph by Syczewska E.M. (1999), *Analiza relacji długookresowych: estymacja i weryfikacja*, op. cit.
[85] Welfe A. (2009), *Ekonometria*, op. cit.

root tests[86]. The reasoning (verification of the hypothesis) is as follows. Analysis covers the original series of the series x_t. Where it is stationary, we define it as integrated of order zero: $x_t \sim I(0)$. Where it is not, then its first increment is calculated: $\Delta x_t = x_t - x_{t-1}$. If the variable Δx_t is stationary, then we define the variable x_t as integrated of order 1: $x_t \sim I(1)$, and $\Delta x_t \sim I(0)$. In the case of non--stationarity Δx_t we calculate another increment designated as $\Delta^2 x_t = \Delta x_t - \Delta x_{t-1}$ and examine its stationarity. This is continued until we obtain a difference which is stationary. If for $k = 1,2,\dots,z-1$ the variable $\Delta^k x_t$ is non-stationary while the variable $\Delta^d x_t$ is stationary then the series Δx_t is non-stationary, integrated of order d: $x_t \sim I(z)$[87].

One of the most commonly used tests to verify time series stationarity is the Dickey–Fuller test (DF)[88]. In the basic version of the test, we estimate the model expressed as[89]:

$$\Delta x_t = (\rho - 1)x_{t-1} + \vartheta_t, \quad (4.9)$$

where: x_t-time series being the implementation of the stochastic process $\{x_t\}$ at the moment t,
ρ – parameter,
ϑ_t – white-noise random component.

The time series x_t is defined as stationary when $|\rho| < 1$. On the other hand, when x_t has the unit root (is, therefore, integrated of order 1), then after its differentiation the parameter ρ is equal to zero. The DF test verifies the null hypothesis assuming time series non-stationarity in relation to the alternative hypothesis (4.10):

$$H_0: (\rho - 1) = 0,$$

$$H_1: (\rho - 1) < 0. \quad (4.10)$$

[86] Time series stationarity analysis can be found, inter alia, in the paper by Witkowska D., Matuszewska-Janicka A., Kompa K. (2012), *Wprowadzenie do ekonometrii dynamicznej i finansowej*, Wydawnictwo SGGW w Warszawie.
[87] Cf. Rubaszek M. (2012), *Modelowanie polskiej gospodarki z pakietem R*, op. cit.
[88] Dickey D. A., Fuller W. A. (1981), *Likelihood ratio statistics for autoregression time series with a unit root.* Econometrica, Vol. 49, No. 4 (Jul., 1981), pp. 1057-1072.
[89] Cholewiński R. (2008), *Wpływ zmian kursu walutowego na dynamikę procesów inflacyjnych*. Materiały i Studia 226, Narodowy Bank Polski.

The test statistics of the DF test is similar to the statistic of the t test and is formulated as follows[90]:

$$DF = \frac{\hat{\rho}}{s_\rho}, (4.11)$$

where: $\hat{\rho}$ – estimation of the parameter ρ of the model expressed as (4.9),
s_ρ – standard error of the parameter estimation.
DF statistical distribution expressed as (4.11) is, however, asymmetric and impossible to obtain analytically.

An extension of the model expressed as (4.9), in the case of autocorrelation[91] of the random component ϑ_t, is the equation (4.12):

$$\Delta x_t = \rho x_{t-1} + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \cdots + \delta_K \Delta x_{t-K} + \vartheta_t, (4.12)$$

where:
K – lag order,
δ – distribution parameters.

Estimating the parameter ρ of the equation (4.12) is used to build the statistics of the augmented Dickey-Fuller test. The distribution of this statistics is asymptotic, while including the distribution by augmenting the test eliminates the issue of autocorrelation of the random component. The received estimators are deprived of structural bias. The lag order K for the model is usually determined in practice using the information criteria.

An important practical aspect of using tests from the DF family is the possibility to take into account various reasons for time series non-stationarity. Hence, for the ADF test, it is also necessary to take into account, in addition to

[90] Charemza W.W., Deadman D.F. (1997), *Nowa ekonometria*. PWE, Warszawa.
[91] In the case of the autocorrelation phenomenon, estimation of the parameter ρ of the equation (5.8) becomes biased. Then between the average value of the estimator and the parameter value there is a significant difference. This results finally in the fact that the population parameter is estimated with the structural non-random error. It ultimately impedes or even prevents the correct process of verifying the null hypothesis and, consequently, making type I or II error.

variable lags, the possibility of absolute term and trend. Then, the initial, the most general model, is as follows[92]:

$$\Delta x_t = \mu_0 + \mu_1 t + \rho x_{t-1} + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + \cdots + \delta_K \Delta x_{K-1} + \vartheta_t. \quad (4.13)$$

where: μ_0 – constant,
μ_1 – estimation of the parameter with the time variable t.

Only after verifying the significance of the model parameters (4.12) with regard to the presence of the absolute term and development trend, we can adopt the final form of the model and, therefore – of the test statistics.

A separate group of tests allowing to examine time series stationarity, are tests which assume stationarity in the null hypothesis. An example of such test is the KPSS test (Kwiatkowski, Phillips, Schmidt and Shin[93]). The KPSS test assumes that the time series has a component of the deterministic trend, random walk process, and random component. We will not discuss here the details regarding function forms and assumptions themselves. The KPSS test itself has two versions[94]. The first one tests the null hypothesis of stationarity around a linear trend, relative to the alternative hypothesis of the occurrence of the unit element. In this case, we test the LM statistics[95] expressed as:

$$LM = \sum_{t=1}^{T} \frac{S_t^2}{\hat{\sigma}_u^2}, \quad (4.14)$$

where: $S_t^2 = (\sum_{i=1}^{t} e_i)^2$ for t=1, 2, …, T-residual partial sums,
e_t for t=1, 2, …, T-regression residuals y_t relative to the constant and trend *t*,
$\hat{\sigma}_t^2$-estimation of variance equal to the residual sum of squares divided by the number of observations T.

[92] Welfe A. (2009), *Ekonometria*, op. cit.
[93] Kwiatkowski D., Phillips P. C. B., Schmidt P., Shin Y. (1992), *Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?* Journal of Econometrica Vol. 54, Issues 1-3, pp. 159-178.
[94] Based on the paper by Syczewska E.M. (1997), *Badanie empirycznej mocy testu Kwiatkowskiego-Phillipsa-Schmidta-Shina*. Roczniki Kolegium Analiz Ekonomicznych Szkoły Głównej Handlowej w Warszawie, Vol. 5, s. 47-65.
[95] In the KPSS test, the test statistics is the univariate LM (Lagrange multiplier) statistics, cf. Kwatkowski et al., (1992), *Testing the null hypothesis…,* op cit, p. 162.

The second version of the KPSS test tests the null hypothesis of time series stationarity around the average relative to the alternative hypothesis of the occurrence of the unit root. Components e_t are regression residuals y_t relative to the constant.

The statistics of the stationarity test in relation to the average $\hat{\eta}_u$ is convergent to:

$$\hat{\eta}_u \overset{w}{\rightarrow} \int_0^1 V(r)^2 dr, \quad (4.15)$$

where:
$V(r) = W(r) - rW(1)$– standard Brownian bridge defined for the standard Wiener process $W(r)$,
$\overset{w}{\rightarrow}$ – poor convergence of probability measures.

In contrast, the statistics of the KPSS test relative to the trend $\hat{\eta}_r$ is poorly convergent to the limit:

$$\hat{\eta}_r \overset{w}{\rightarrow} \int_0^1 V_2(r)^2 dr, \quad (4.16)$$

where: $V_2(r)$-second order Brownian bridge.

The critical values of the KPSS test were obtained by Kwiatkowski et al. using the Monte Carlo simulation procedures.

When performing the „recipe" for testing the existence of unit roots, we should bear in mind the fact that the strength of these tests is relatively low[96]. This means that in the case of variables generated by the stationary stochastic process, it may happen that the null hypothesis, which is in fact false, will not be rejected, i.e. type II error will be made. Rubaszek[97] notes that this is particularly true for long-term memory processes (persistent processes).

The next step in initial analysis of time series is to verify the existence of the autocorrelation phenomenon. Its analysis allows to understand the structure of the ARMA process, which is helpful in selecting the appropriate lag order in cointegration analysis[98]. For this purpose, the ACF autocorrelation function is

[96] DeJong D. N., Nankervis J. C., Savin N. E., Whiteman C. H. (1992), *The power problems of unit root tests in time series with autoregressive errors*. Journal of Econometrics Vol. 53, Issues 1-3, pp. 323-343.
[97] Rubaszek M. (2012), *Modelowanie polskiej gospodarki...*, op. cit.
[98] Cf. Witkowska D., Żebrowska-Suchodolska D. (2009), *Zastosowanie testów autokorelacji do weryfikacji hipotezy o słabej efektywności rynku*, [in]: Metody matematyczne, ekonometryczne i komputerowe w finansach i ubezpieczeniach 2007 (the multi-author work, edited by P. Chrzan and T. Czernik). Wydawnictwo Akademii Ekonomicznej w Katowicach.

commonly used (to examine the structure of the autocorrelation process). It measures the correlation coefficient between the variable and its subsequent lag:

$$ACF(s) = cor(x_t, x_{t-s}) = \frac{cov(x_t, x_{t-s})}{\sqrt{var(x_t)\, var(x_{t-s})}} = \frac{cov(x_t, x_{t-s})}{var(x_t)}, (4.17)$$

where: x_t – analysed variable,
s – lag order (Cholewiński, 2008).

The autocorrelation phenomenon is transferred to subsequent lags. We shall consider the following model:

$$x_t = \rho x_{t-1} + \vartheta_t, (4.18)$$

where: ϑ_t – white-noise process.
If $E(x_t) = 0$, for $t = 0,1, \dots T$, ρ is the first order autocorrelation coefficient then:

$$x_t = \rho^t x_0 + \sum_{i=0}^{t} \mu^{t-i}\, \vartheta_i. (4.19)$$

Therefore, the autocorrelation coefficient ρ for s (ρ^s) order lag decreases geometrically. This is due to the fact that for the stationary stochastic process we have the following dependency: $-1 < \rho < 1$. In this case, the series which is the second component of the equation sum is therefore convergent. The issue to solve is to determine which lag orders are responsible for the emergence of autocorrelation. In this case, useful is the partial auto-correlation function (PACF). It shows to what extent autocorrelation of the given time series does not result from „transferring" previous autocorrelation, but from the occurrence of relevant order autocorrelation[99].

4.4. Vector autoregression models – estimation and verification methods

For the **N** vector of macroeconomic variables expressed as:

$$\mathbf{y}_t = [y_{1t}, y_{2t}, \dots, y_{Nt}]', (4.20)$$

between the components of which there is a relationship determined by the theory of economics, the autoregression vector model VAR(P), where P is the autoregression order, can be presented as[100]:

[99] Cholewiński R. (2008), *Wpływ zmian…* op cit.
[100] Lütkepohl H., Kratzig M. (2004), *Applied time series econometrics*. Cambridge University Press. Cambridge.

$$\mathbf{y}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \cdots + \mathbf{A}_P\mathbf{y}_{t-P} + \boldsymbol{\varepsilon}_t, (4.21)$$

where:
$\boldsymbol{\varepsilon}_\mathrm{t}$ – N-dimensional white noise process with the expected value equal to 0 and covariance matrix Σ,
$\mathbf{A}_0$ – matrix with dimensions Nx1 – is the matrix containing absolute terms for individual equations,
$\mathbf{A}_\mathrm{p}$ for $p = 1, 2, \dots, P$ – matrices with dimensions NxN-present the impact of p-th lag of all dependent variables on their current values.

The number of unknown parameters contained in $\mathbf{A}_\mathrm{p}$ matrices where $p = 0,1,2, \dots, P$ equals to the product $(1 + PN) \cdot N$. In addition, the elements of the covariance matrix are unknown, which gives $N(N + 1)/2$ of unknown parameters. The desired characteristic of the VAR(P) model is its stationarity which can be described as the disappearance of the impact of $\boldsymbol{\varepsilon}_\mathrm{t}$ shock on the values of the dependent variable vector:

$$lim_{k\to\infty} \frac{\partial \mathbf{y}_{t+k}}{\partial \boldsymbol{\varepsilon}_t} = 0. (4.22)$$

The only source of shock in the VAR(P) model are, therefore, random components, and the fulfillment of the stationarity requirement is equivalent to the existence of the long-term value μ for y_t, to which the process returns:

$$\mu = A(L)^{-1}A_0, (4.23)$$

where: (.)– lag operator.
The pace of this return is determined by the roots of the equation*:*

$$|A(z)| = 0, (4.24)$$

whose number is equal to the product *P•N*.

The process described by the equation (4.21) is stationary if all roots of the equation (4.24) are outside of the unit circle:

$$\{|z_k| > 1 \text{ dla } k = 1,2, \dots, PN\}. (4.25)$$

If the VAR(P) model is the model of the stationary process, it is possible to convert it to the form of the infinite-order vector moving average (VMA):

$$\boldsymbol{y}_t = \boldsymbol{A}(L)^{-1}(\boldsymbol{A}_0 + \boldsymbol{\varepsilon}_t) = \boldsymbol{\mu} + \sum_{k=0}^{\infty} \boldsymbol{\theta}_k \boldsymbol{\varepsilon}_{t-k}, (4.26)$$

where:

L – lag operator,
$\boldsymbol{\theta}_k$ – matrices determining responses of endogenous variables contained within the vector $\mathbf{y}_t$ to random components for k periods,
whereby: $\boldsymbol{\theta}_k = \frac{\partial y_{t+k}}{\partial \boldsymbol{\varepsilon}_t}$. (4.27)

Parameters of the autoregression vector model can be estimated[101] using the LSM, separately for each equation, due to the fact that exogenous variables are predetermined variables, independent[102] of the random component. If we write the VAR model as:

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t, \text{ (4.28)}$$

$$cov(\boldsymbol{x}_t, \boldsymbol{\varepsilon}_t) = 0 \text{ (4.29)}$$

where: $\mathbf{x}_t = [1, y_{t-1}, y_{t-2}, \dots, y_{t-P}]'$ – vector of exogenous variables with dimensions *P•N+1*, $\mathbf{A} = [\mathbf{A}_0, \mathbf{A}_1, \dots, \mathbf{A}_\mathrm{P}]$-matrix with dimensions *N(1+N•P)* containing all model parameters, then the LSM estimator for the parameters of the *n*-th VAR equation is expressed as:

$$y_{nt} = a_n x_t + \varepsilon_{nt}, \text{ (4.30)}$$

where: a_n is the *n*-th row of the matrix **A**.

Its estimation is equal to:

$$\hat{a}_n = (\sum_{t=1}^{T} y_{nt} x_t')(\sum_{t=1}^{T} x_t x_t')^{-1}. \text{ (4.31)}$$

On the other hand, the LSM estimator for the entire matrix of parameters is determined by the equation:

$$\hat{A} = (\sum_{t=1}^{T} y_t x_t')(\sum_{t=1}^{T} x_t x_t')^{-1}. \text{ (4.32)}$$

[101] The description of the VAR estimation procedure is described in detail by Lütkepohl H. (2007), *Econometric analysis with vector autoregressive models*. Economic Working Papers ECO 2007/11. European University Institute, pp. 14-25.
[102] Independence in the multivariate space means the orthogonality of vectors. The model expressed as (5.20) can be estimated using the classical least squares method (CLSM), applied to each equation separately. The CLSM estimators are compatible and asymptotically effective. If the random components of the VAR model are correlated between equations, and this possibility is allowed in the VAR models, then the effectiveness of the estimators can be increased by using the total estimation of all equations. However, for the VAR models, this procedure does not increase the effectiveness of the estimators as all equations have identical variables on the right, which is due to the assumption that the matrices $\mathbf{A}_0$ $\mathbf{A}_1$ ... $\mathbf{A}_\mathrm{P}$ do not contain zero elements.

The VAR model can also be estimated using the maximum likelihood estimation[103] (MLE) method. If the random component of the equation (4.21) has the multivariate normal distribution, the likelihood function is:

$$\boldsymbol{\mathcal{L}} = \prod_{t=1}^{T}(2\pi)^{-\frac{N}{2}}|\Sigma|^{-\frac{1}{2}} exp[-\tfrac{1}{2}(\mathbf{y}_t - \mathbf{A}\mathbf{x}_t)'\Sigma^{-1}(\mathbf{y}_t - \mathbf{A}\mathbf{x}_t)]. \quad (4.33)$$

Where the covariance matrix is a diagonal matrix, the estimator obtained by the MLE method corresponds to the LSM estimator.

From the point of view of the VAR(P) specification, it is important to determine the lag order P. Since the theory of economics does not provide information on the dynamic relationships between variables, for their determination we often use information criteria, such as: Akaike AIC, Hanna-Quinn HQ, Bayesian Schwarz BIC or Final Prediction Error FPE. The value of the likelihood function logarithm at maximum is:

$$\mathcal{L}(\hat{A}, \hat{\Sigma}) = -\frac{TN}{2}(1 + ln2\pi) - \frac{T}{2}ln(|\hat{\Sigma}|). \quad (4.34)$$

As the number of estimated parameters of the VAR(P) model equals to $N(1 + N \cdot P)$, the value of information criteria can be determined using the following formulae (Lütkepohl and Krätzig, 2004):

a) Akaike criterion: $\text{AIC} = |\hat{\Sigma}_r| + \frac{2}{T}N(1 + NP)$, (4.35)
b) Hannan-Quinn criterion: $\text{HQ} = |\hat{\Sigma}_r| + \frac{2\,log(log\,T)}{T}N(1 + NP)$, (4.36)
c) Bayesian Schwarz criterion: $\text{BIC} = |\hat{\Sigma}_r| + \frac{logT}{T}N(1 + NP)$, (4.37)
d) FPE criterion: $\text{FPE} = \frac{1+\frac{N(1+NP)}{T}}{1-\frac{N(1+NP)}{T}}|\hat{\Sigma}_r|$, (4.38)

where: $\hat{\Sigma}_r$ – determinant of the estimator of the random component's covariance matrix,
T – time series length.

The desired aspect of econometric modelling is to obtain the system which is best suited to empirical data, which is mathematically equivalent to maximising the likelihood function (4.34) with the lowest number of estimated

[103] Estimation of the VAR models using MLE and the procedure of testing lag orders and statistical properties of the random component was prepared in this paper based on the paper by Rubaszek, 2012. The convergent description of the diagnostic procedures in modelling multivariate time series is included in the paper by Lütkepohl H. (2007), *Econometric analysis...* op cit.

parameters. Therefore, we must select lag P for VAR(P) for which the value of the information criteria (4.35-4.38) is lowest.

A hint in selecting maximum lag may also be the test of relevance of subsequent lags of the VAR model[104]. The null hypothesis $H_0: \mathbf{A}_P = 0$ is verified based on the likelihood ratio test, whose statistics is formulated as follows:

$$LR = T(ln|\hat{\Sigma}_{re}| - ln|\hat{\Sigma}_{ur}|), (4.39)$$

where:
$\hat{\Sigma}_{re}$ and $\hat{\Sigma}_{ur}$ – covariance matrices of random components for the VAR model with and without restrictions.

When the null hypothesis is true, the LR statistics has the asymptotic distribution χ^2 with N^2 degrees of freedom, equal to the number of parameters on which a zero restriction has been imposed.

The improper specification consisting in selecting too small values for the VAR model may result in residual autocorrelation, and further to bias in the covariance matrix estimator. Bias in the LSM estimator leads to incorrect conclusions from the verification of hypotheses based on the *T*-student test or the likelihood ratio.

Autocorrelation can be detected using the multivariate Ljung-Box test[105] or the Box-Pierce test. The covariance matrix for the stationary *n*-dimensional economic process $\boldsymbol{y}_t$ is formulated as:

$$\mathbf{\Gamma_J} = Cov(\mathbf{y_t}; \mathbf{y_{t-J}}), (4.40)$$

While the estimator of this matrix for the sample is equal to:

$$\hat{\Gamma}_J = \frac{1}{T}\sum_{t=J+1}^{T} y_t\, y_{t-J}'. (4.41)$$

The null hypothesis of the lack of autocorrelation of the random component to order *J* inclusive is verified, i.e.:

$$H_o: \forall_{1\le j\le J}\mathbf{\Gamma}_j = 0, (4.42)$$

[104] Rubaszek M. (2012), *Modelowanie polskiej gospodarki...*, op. cit.

[105] In the literature of the subject, the Ljung-Box test is also referred to as adjusted portmanteau test and the Box-Pierce test as portmanteau test. To verify the hypothesis of the absence of autocorrelation of the random component, we can also use the Breusch-Goodfrey test which is the multivariate version of the Lagrange multiplier test.

relative to the alternative hypothesis:

$$H_1: \exists_{1 \leq j \leq J} \boldsymbol{\Gamma}_j \neq 0 \ (4.43)$$

where: $\boldsymbol{\Gamma}_{\mathrm{j}}$ – covariance matrix of order j for the random component of the estimated VAR(P) model[106].

The multivariate statistics of the Ljung-Box test is formulated as:

$$LB = T^2 \sum_{j=1}^{J} \frac{1}{T-j} tr(\hat{\Gamma}_j' \hat{\Gamma}_0^{-1} \hat{\Gamma}_j \hat{\Gamma}_0^{-1}), \ (4.44)$$

and when the null hypothesis is true, it has the distribution χ^2 with $N^2(J-P)$ degrees of freedom. On the other hand, the statistics of the Box-Pierce test is formulated as:

$$BP = T \sum_{j=1}^{J} \frac{1}{T-j} tr(\hat{\Gamma}_j' \hat{\Gamma}_0^{-1} \hat{\Gamma}_j \hat{\Gamma}_0^{-1}) \ (4.45)$$

and when the null hypothesis is true, it also has the distribution χ^2 with $N^2(J-P)$ degrees of freedom.

Often, during testing many statistical hypotheses, it is assumed that the random component of the model has the multivariate normal distribution. Then, the statistics used have defined distributions, such as e.g. *t*-Student's statistics has the *t* distribution, and the statistics of the likelihood ratio test - χ^2 distribution. Verification of the null hypothesis assuming the multivariate normal distribution of the random component:

$$H_0: \boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Sigma}) \ (4.46)$$

relative to the alternative hypothesis:

$$H_1: \boldsymbol{\varepsilon}_t \nsim N(0, \boldsymbol{\Sigma}) \ (4.47)$$

can be made using the Dornik-Hansen omnibus test[107]. In this test, we initially calculate the root of the covariance matrix from the sample $\hat{\boldsymbol{\Sigma}}^{1/2}$ and then we standardise the model residuals:

$$\boldsymbol{\mu}_t = \hat{\boldsymbol{\Sigma}}^{-1/2} \mathbf{e}_t. \ (4.48)$$

[106] Rubaszek M. (2012), *Modelowanie polskiej gospodarki...*, op. cit.
[107] The Dornik-Hansen test is the multivariate generalisation of the Jarque-Ber test.

The individual components of the vector $\boldsymbol{\mu}_t$ are orthogonal in relation to each other, which results from the fact that the matrix $var\ \boldsymbol{\mu}_t$ is the unit matrix. For the normal distribution, skewness is equal to 0 while kurtosis is 3. Using this, we designate these characteristics of the distribution for the individual components of the vector $\boldsymbol{\mu}_t$. The statistics of the Dornik-Hansen is expressed as:

$$DH = \frac{T}{6}\sum_{n=1}^{N} S_n^2 + \frac{T}{24}\sum_{n=1}^{N}(K_n - 3)^2, \quad (4.49)$$

where: S_n, K_n – respectively, skewness and kurtosis for $n = 1,2, \dots, N$.
When the null hypothesis is true for the Dornik-Hansen statistics, the presented relationship (4.49) has the distribution χ^2 with $2N$ degrees of freedom.

In the case of the VAR models, their stability is also tested. The most formalised method is the multivariate Chow test[108]. We can identify three versions of this test: *break-point*, *sample-split* and *Chow forecast.* In the case of the break-point test, estimated are three models with the same specification for the defined moment T^*, i.e.

a) model I: subsample *{1, 2, ..., T^*}* for which we have T_1 observations, residuals $\hat{\boldsymbol{\varepsilon}}_{t(1)}$ and covariance matrix

$$\widehat{\boldsymbol{\Sigma}}_1 = \boldsymbol{T}_1^{-1} \sum_{t=1}^{T_1} \hat{\boldsymbol{\varepsilon}}_{(1)} \hat{\boldsymbol{\varepsilon}}'_{(1)t}, \quad (4.50)$$

b) model II: subsample $\{T^* + 1, \dots, T\}$ for which we have T_2 observations, residuals $\hat{\boldsymbol{\varepsilon}}_{t(2)}$ and covariance matrix

$$\widehat{\boldsymbol{\Sigma}}_2 = \boldsymbol{T}_2^{-1} \sum_{t=1}^{T_2} \hat{\boldsymbol{\varepsilon}}_{(2)} \hat{\boldsymbol{\varepsilon}}'_{(2)t}, \quad (4.51)$$

c) model III: subsample *{1, 2, ..., T}* for which we have *T* observations, residuals $\hat{\boldsymbol{\varepsilon}}_t$ and covariance matrix

$$\widehat{\boldsymbol{\Sigma}}_3 = \boldsymbol{T}^{-1} \sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}'_t. \quad (4.52)$$

Verification covers the null hypothesis that the parameters of the I, II, III models are identical. If this hypothesis is true, the statistics of the likelihood ratio λ_{BP} of the *break-point* test, expressed as:

$$\lambda_{BP} = T ln\left|\widehat{\boldsymbol{\Sigma}}_3\right| - \left(T_1\, ln\left|\widehat{\boldsymbol{\Sigma}}_1\right| + T_2\, ln\left|\widehat{\boldsymbol{\Sigma}}_2\right|\right) \quad (4.53)$$

[108] Por. Lütkepohl H., Kratzig M. (2004), *Applied time series…* op cit.

has the distribution χ^2 with the number of degrees of freedom equal to the total number of the VAR model parameters, including the parameters of the random component covariance matrix

The *sample-split* test requires defining two substrings expressed as: $x_{1(t)} = x_t \; for \; t \leq T^*$, $x_{1(t)} = 0 \; for \; t > T^*$ and $x_{2(t)} = x_t \; for \; t \leq T^*$, $x_{2(t)} = 0 \; for \; t > T^*$ where $x_{1(t)} + x_{2(t)} = x_t$. For the model expressed as:

$$\boldsymbol{z_t} = \begin{bmatrix} \boldsymbol{A}_{(1)} \; \boldsymbol{A}_{(2)} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_{1(t)}} \\ \boldsymbol{x_{2(t)}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{(1)t} \\ \boldsymbol{\varepsilon}_{(2)t} \end{bmatrix}, \quad (4.54)$$

we test the null hypothesis assuming that $A_{(1)} = A_{(2)}$. The testing statistics λ_{SS} in the *sample-split* version, expressed as:

$$\lambda_{SS} = T\{ln|\hat{\boldsymbol{\Sigma}}_{\boldsymbol{3}}| - ln\,|T^{-1}(T_1\, ln|\hat{\boldsymbol{\Sigma}}_{\boldsymbol{1}}| + T_2\, ln|\hat{\boldsymbol{\Sigma}}_{\boldsymbol{2}}|)\} \quad (4.55)$$

when the null hypothesis is true, has the distribution χ^2 with degrees of freedom equal to $p \cdot n^2$. The last Chow test-*forecast test*-has a slightly different structure consisting in estimating the VAR model based on $\mathrm{t} = 1, 2, \ldots, \mathrm{T}^*$ observations. Then, we make an ex-post forecast for other observations. The test verifies the hypothesis whether the variance of errors of ex-post forecasts is not statistically significantly bigger than its theoretical value. Chow tests in this paper were carried out for the break date falling on the middle of the time series.

4.5. Structuring of the VAR models

To obtain the economic interpretation of the VAR model, it is necessary to structure it. The structural vector autoregression model (SVAR) has the following form:

$$\mathbf{A}\mathbf{y}_t = \mathbf{C}_0 + \mathbf{C}_1\mathbf{y}_{t-1} + \mathbf{C}_2\mathbf{y}_{t-2} + \cdots + \mathbf{C}_P\mathbf{y}_{t-P} + \mathbf{B}\boldsymbol{\eta}_t, \quad (4.56)$$

where:
random components of individual equations have normal distributions and are orthogonal to each other ($\boldsymbol{\eta}_\mathrm{t} \sim N(0, \mathbf{I}_N)$),
matrices: A, B – define concurrent dependencies among the variables forming the vector $\mathbf{y}_\mathrm{t}$,
matrices $\mathbf{C}_i$ for $i = 1,2, \ldots, P$-define the dynamic properties of the model.

The direct estimation model of the SVAR expressed as (4.56) may be impossible due to the problem of identification. Therefore, the parameters of the

SVAR model are obtained by estimating the vector autoregression model and then structuring it. As the number of parameters for the SVAR model is higher than for the VAR model by $N^2 + \frac{N(N-1)}{2}$, then, in order to obtain the parameters of the SVAR model, it is necessary to impose the same number of restrictions on it.

As the random components are independent of each other, they are interpreted economically by defining them as structural shocks, i.e. for example: demand, supply or monetary shock. Analysis of responses of the endogenous variables $\{y_i: i = 1,2, \dots, N\}$ to stimuli in a form of structural shocks $\{\eta_j: j = 1,2, \dots, N\}$ is described by the impulse-response function (IRF) and its value after the lapse of the k-periods is described by the following dependency:

$$IRF_{k(i,j)} = \frac{\partial y_{i,t+k}}{\partial \eta_{j,t}}. \quad (4.57)$$

Therefore, analysis covers the change in the value $\{y_{i,t+k}: k = 0,1,2 \dots\}$ in response to the stimulus in a form $\eta_{j,t} = 1$.

The values of the impulse-response function of all endogenous variables included in the SVAR model, with respect to all structural shocks, can be written in a form of the matrix as:

$$\Psi_{\boldsymbol{k}} = \boldsymbol{IRF_k} = \frac{\partial \mathbf{y}_{t+k}}{\partial \boldsymbol{\eta}_t}. \; (4.58)$$

To calculate the values of the impulse response function, in the first step we should write the SVAR model as a structural infinite vector moving average[109] SVMA (∞):

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{k=0}^{\infty} \Psi_k \boldsymbol{\eta}_{t-k}. \; (4.59)$$

The transformation of the SVAR model into the SVMA is made by writing the SVAR as the reduced VAR:

$$\mathbf{y}_t = \mathbf{A}^{-1}\mathbf{C}_0 + \mathbf{A}^{-1}\mathbf{C}_1\mathbf{y}_{t-1} + \dots + \mathbf{A}^{-1}\mathbf{C}_P\mathbf{y}_{t-P} + \mathbf{A}^{-1}\mathbf{B}\boldsymbol{\eta}_t. \; (4.60)$$

The random component of the VAR model (4.60) is equal to:

$$\boldsymbol{\varepsilon}_t = \mathbf{A}^{-1}\mathbf{B}\boldsymbol{\eta}_t. \; (4.61)$$

Finally, the SVMA (∞) is shown by the following equation:

[109] Unlike the multivariate Koyck model, which is the model with the infinite distribution of lags.

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t + \sum_{k=1}^{\infty} \boldsymbol{\theta}_k \boldsymbol{\varepsilon}_{t-k}. \ (4.62)$$

Therefore, the relationship between the random components of the SVAR and VAR models is as follows:

$$\Psi_k = \boldsymbol{\theta}_k \mathbf{A}^{-1}\mathbf{B}, \text{ dla k=0,1,2,...} . \ (4.63)$$

In the case where the dependent variable y_i is defined in terms of the growth or growth rate $y_i = \Delta z_i$, dynamic analysis of the model properties is based on the description of the accumulated impulse response function (AIRF). The accumulated impulse response function determines the dynamic impact of j-th structural shock η_j per variable z_i:

$$AIRF_{k(i,j)} = \frac{\delta z_{i,t+k}}{\delta \eta_{j,t}}. \ (4.64)$$

In addition, as:

$$z_{it} = z_{i,t-k-1} + \sum_{m=0}^{k} y_{i,t-k+m} \ (4.65)$$

$$\Rightarrow$$

$$\frac{\delta z_{it}}{\delta \eta_{j,t-k}} = \sum_{m=0}^{k} \frac{\delta y_{i,t-k+m}}{\delta \eta_{j,t-k}}, \ (4.66)$$

it means that:

$$AIRF_{k(i,j)} = \sum_{m=0}^{k} IRF_{k(i,j)}. \ (4.67)$$

The literature of the subjects identifies recursive and non-recursive structuring, which will be subject to further considerations as well as long-term structuring by Blanchard and Quah[110], which will not be used in this paper.

In recursive structuring, it is assumed that for $i < j$ the variable y_{it} is the exogenous variable for y_{jt}. The variable y_{jt} does not occur in the equation for y_{it}. This type of restrictions was suggested by Sims[111]. In general, we can write them as follows:

[110] Blanchard O., Quah D. (1989), *The dynamic effects of aggregate demand and supply disturbances*, American Economic Review Vol. 79(4), pp. 655-673.
[111] Sims C.A. (1980), *Macroeconomica and reality*. Econometrica, Vol. 48, No. 1 (Jan., 1980), pp. 1-48.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \vdots & 0 \\ * & 1 & \vdots & 0 \\ \dots & \dots & \ddots & \dots \\ * & * & \dots & 1 \end{bmatrix} \mathbf{B} = \begin{bmatrix} * & 0 & \vdots & 0 \\ 0 & * & \vdots & 0 \\ \dots & \dots & \ddots & \dots \\ 0 & 0 & \dots & * \end{bmatrix}. (4.68)$$

If the restrictions expressed as (4.68) are imposed, the parameters marked as (*) in matrix **A** and **B** can be obtained by solving the non-linear system of equations with $\frac{N(N+1)}{2}$ of unknowns and the same number of parameters expressed as:

$$\mathbf{\Sigma} = \mathbf{A^{-1}BB'(A^{-1})'}. (4.69)$$

The equation (4.69) is usually solved by the use of the so-called Cholesky decomposition of the matrix $\mathbf{\Sigma}$. Each positively defined symmetric matrix $\mathbf{\Sigma}$ can be presented as the following product:

$$\mathbf{\Sigma} = \mathbf{PP'}, (4.70)$$

where the matrix **P** is the nonsingular lower triangular matrix. The designation of the matrices **A** and **B** boils down to solving the matrix equation expressed as:

$$\mathbf{A^{-1}B} = \mathbf{P}. (4.71)$$

The most common solution with recursive restrictions is the system of equations:

$$\mathbf{A} = \mathbf{I}\ (4.72)$$

$$\mathbf{B} = \mathbf{P}. (4.73)$$

4.6. Vector autoregression model-empirical aspect

The assumption of recursive structuring of the VAR model expressed as (4.72)-(4.73) always determines the adoption of a specific set of variables for modelling. In this monograph (which also results from the availability of data), it has been decided to estimate six vector autoregression models[112]. For this purpose, the following time series have been used, limited to the Polish economy only:

[112] Those models were presented and discussed during the 18th Scientific Conference on Quantitative Methods in the Economic Studies 2017 at the Center for Nature and Forestry Education in Rogów.

– π_t^P – indices of wheat buying-in prices (exclusive of seed grain),

– $\pi_t^{\dot{Z}}$ – indices of rye buying-in prices (exclusive of seed grain),

– π_t^B – indices of cattle buying-in prices (exclusive of calves),

– π_t^T – indices of swine buying-in prices ,

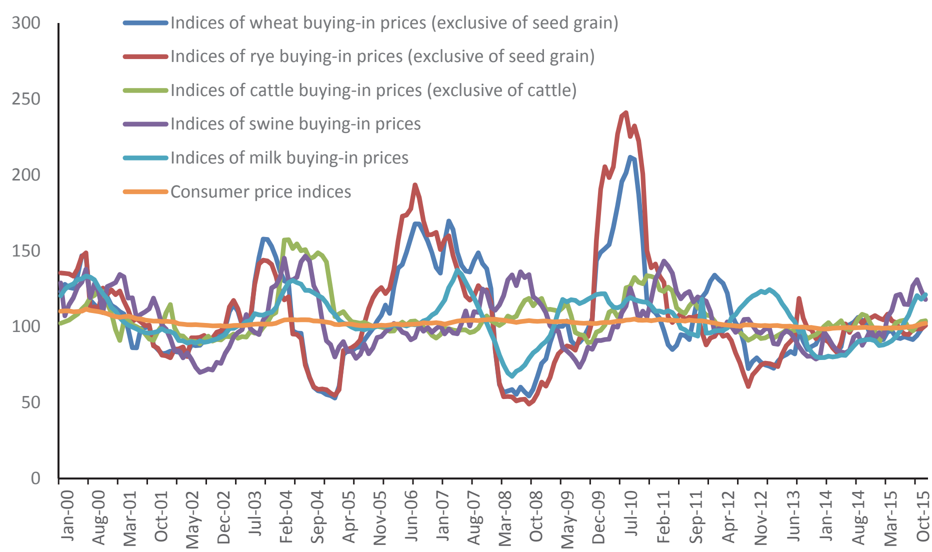
– π_t^M – indices of milk buying-in prices ,

– π_t^{CPI} – consumer price indices,

and these time series are the natural logarithms of the chain index, where the analogous period of the previous year = 100, and they are calculated based on the CSO data. For the purpose of shock simulation, the interest rate, defined as the natural logarithm of the marginal lending rate, marked as i_t has also been used (data source: NBP and own calculations).

All-time series used in analysis are monthly time series and relate to the period 01.2000-02.2017 (206 observations). The price indices have been shown in Figure 65.

Figure 65. Price indices used in analysis



Source: own study.

The use of the price indices in further analysis enables the comparability of variables. In turn, using the logarithms eliminates the risk of apparent regression, which is manifested by the overstated statistics of matching the model to empirical data. The additional advantage of the data prepared in this way is the fact that due to the adoption of the average value as the base value, we eliminate the duplication of the seasonality from the base year. Finally, it is also important that the data on index volatility is often the only information available. Hence, it is so popular in econometric modelling based on time series.

Analysis of time series stationarity and autocorrelation

For empirical verification of the hypothesis, whether the analysed time series is stationary, we used the above-discussed unit root tests. The time series stationarity is based on the ADF Dickey-Fuller test for two versions: test with the absolute term and the version extended by the linear trend. The results of verification of the hypothesis for the defined time series are summarised in Table 2.

Table 2. Results of the ADF test for the initial time series

Time series	p-value for the test with the absolute term	p-value for the test with the absolute term and trend	Conclusion
π_t^P	0,00716	0,03733	stationary ~I(0)
$\pi_t^{Ż}$	0,00052	0,00379	stationary ~I(0)
π_t^B	0,03060	0,10940	non-stationary
π_t^T	0,00116	0,00907	stationary ~I(0)
π_t^M	3,943e-006	4,58e-005	stationary ~I(0)
π_t^{CPI}	0,08533	0,27350	non-stationary
i_t	0,26830	0,08971	non-stationary

Source: own study.

Based on the reasoning procedure, it was found that three time series are non-stationary: π_t^B, π_t^{CPI} and i_t. Therefore, it was decided to verify stationarity of their first increments. To do this, the ADF test procedure has been repeated and its results are shown in Table 3 (Δ represents the first increment).

Table 3. Results of the ADF test for the first increments

Time series	p-value for the test with the absolute term	p-value for the test with the absolute term and trend	Conclusion
$\Delta\pi_t^B$	1,631e-005	0,00015	stationary ~I(1)
$\Delta\pi_t^{CPI}$	0,00926	0,03482	stationary ~I(1)
Δi_t	0,00805	0,03454	stationary ~I(1)

Source: own study.

Finally, modelling will cover the time series integrated of order 0 ~I(0) or of order 1 ~I(1). In order to assess the impact of the monetary policy on agricultural price indices, the models of the following forms will be built:

– model 1: $\mathbf{Y}_t = [\pi_t^P, \Delta\, i_t]'$,

– model 2: $\mathbf{Y}_t = [\pi_t^{\dot{Z}}, \Delta i_t]'$,

– model 3: $\mathbf{Y}_t = [\Delta\pi_t^B, \Delta i_t]'$,

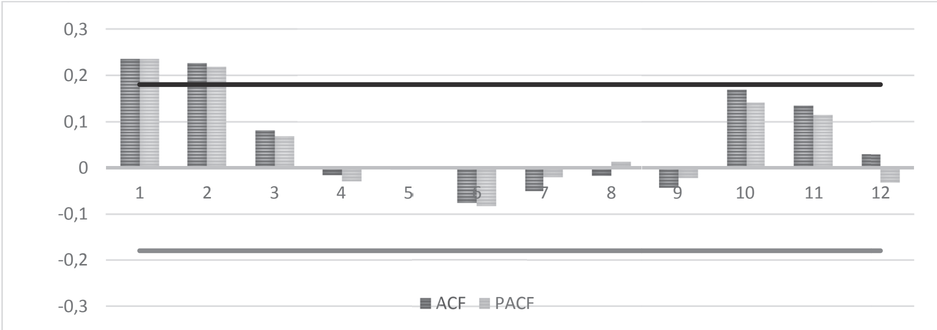
– model 4: $\mathbf{Y}_t = [\pi_t^T, \Delta i_t]'$,

– model 5: $\mathbf{Y}_t = [\pi_t^M, \Delta i_t]'$,

– model 6: $\mathbf{Y}_t = [\Delta\pi_t^{CPI}, \Delta i_t]'$.

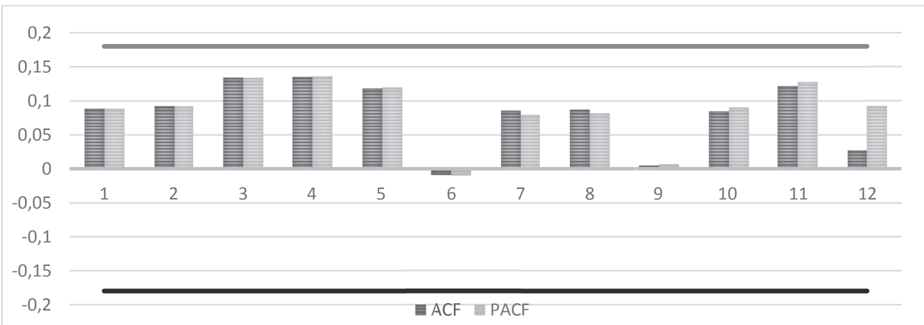
The autocorrelation functions were used to identify the structure of the autoregression process, thus examining the autocorrelation coefficient between the variable and its subsequent lags. In order to determine what orders are responsible for the resulting autocorrelation, the partial autocorrelation function has been used. Analysis used 12 lags, due to the fact that the econometric models are based on monthly data. The results obtained have been visualised as correlograms (Figures 66-72).

Figure 66. ACF and PACF functions for the time series π_t^P



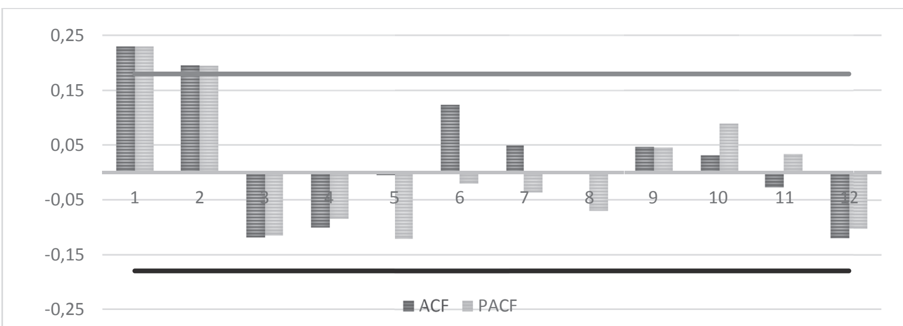
Source: own study.

Figure 67. ACF and PACF functions for the time series $\pi_t^{\dot{z}}$



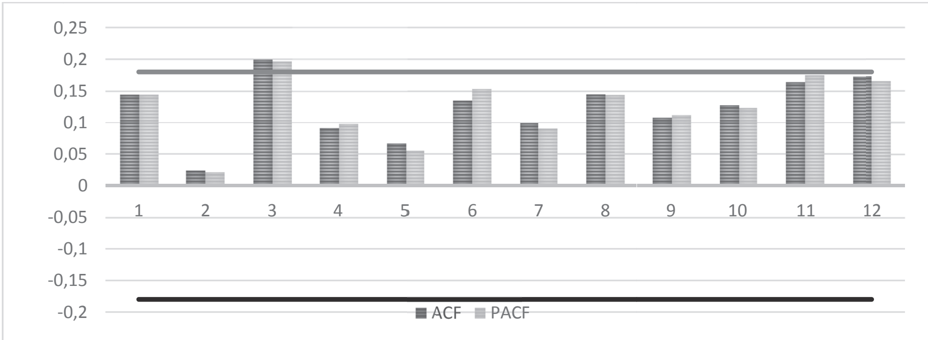
Source: own study.

Figure 68. ACF and PACF functions for the time series $\Delta\pi_t^B$



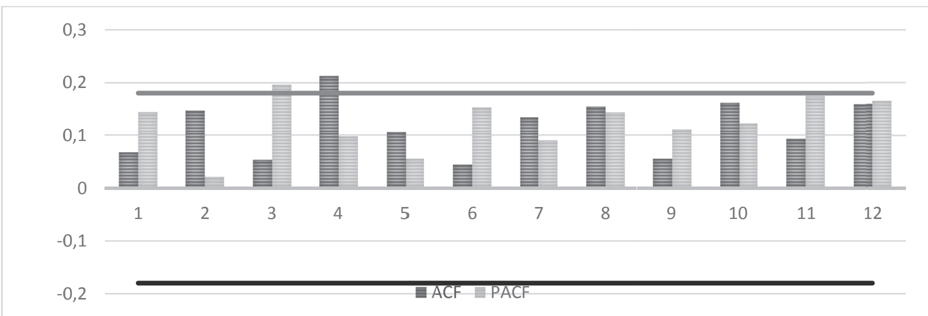
Source: own study.

Figure 69. ACF and PACF functions for the time series π_t^T



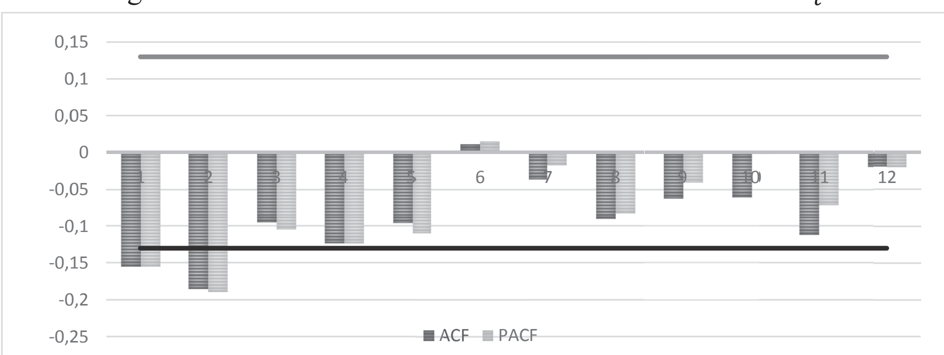
Source: own study.

Figure 70. ACF and PACF functions for the time series π_t^M



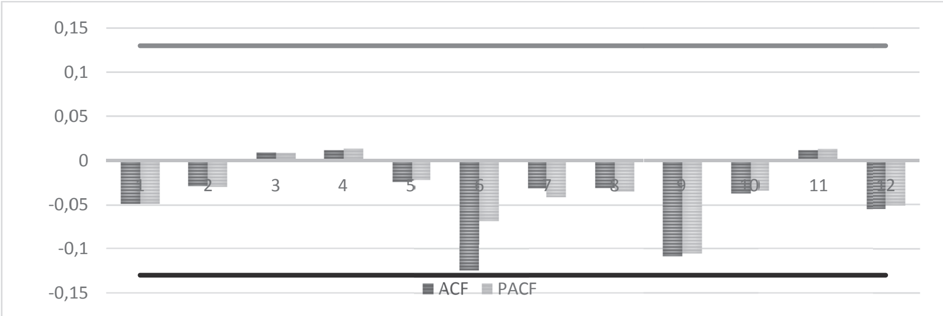
Source: own study.

Figure 71. ACF and PACF functions for the time series $\Delta\pi_t^{CPI}$



Source: own study.

Figure 72. ACF and PACF functions for the time series Δi_t



Source: own study.

Analysis of correlograms shows that the implementations of stochastic processes, after appropriate differentiation, are ARMA type processes with the lag order not exceeding two. This is an important hint in specifying the models, and more precisely, in selecting the lag order ot the vector models.

Estimation and structuring

In the case of estimating each of 6 models, the same procedures were used. In the first step, based on the information criteria (4.35)-(4.38), the lag order of the VAR model has been specified. The significance of this lag was also verified by the formal *LR* test (4.39). The results of the specification are presented in Table 4.

Table 4. Specification of the VAR models

Model	AIC	HQ	BIC	Selected lag	Statistics *LR*	Conclusion on the lag significance
1	4	3	3	3	16,07	significant
2	4	2	2	2	21,33	significant
3	2	2	1	2	16,93	significant
4	2	2	1	2	17,54	significant
5	3	2	2	2	15,90	significant
6	2	23	2	2	20,01	significant

Source: own study.

The following models have been obtained:

–model 1 VAR(3): $\mathbf{Y}_t = [\pi_t^P, \Delta\, i_t]'$

$$\begin{bmatrix} \pi_t^P \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 1{,}42 & 0{,}06 \\ 0{,}08 & 0{,}09 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^P \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} -0{,}5 & 0{,}05 \\ -0{,}15 & 0{,}12 \end{bmatrix} \begin{bmatrix} \pi_{t-2}^P \\ \Delta i_{t-2} \end{bmatrix} +$$

$$\begin{bmatrix} 0{,}02 & -0{,}05 \\ 0{,}11 & 0{,}09 \end{bmatrix} \begin{bmatrix} \pi_{t-3}^P \\ \Delta i_{t-3} \end{bmatrix} + \begin{bmatrix} 0{,}32 \\ -0{,}19 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

–model 2 VAR(2): $\mathbf{Y}_t = [\pi_t^{\dot{Z}}, \Delta i_t]'$

$$\begin{bmatrix} \pi_t^{\dot{Z}} \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 1{,}34 & 0{,}09 \\ 0{,}04 & 0{,}12 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^{\dot{Z}} \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} -0{,}39 & 0{,}11 \\ -0{,}01 & 0{,}14 \end{bmatrix} \begin{bmatrix} \pi_{t-2}^{\dot{Z}} \\ \Delta\, i_{t-2} \end{bmatrix} + \begin{bmatrix} 0{,}267 \\ -0{,}139 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

–model 3 VAR(2): $\mathbf{Y}_t = [\Delta\pi_t^B, \Delta i_t]'$

$$\begin{bmatrix} \Delta\pi_t^B \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 0{,}37 & -0{,}02 \\ 0{,}15 & 0{,}16 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-1}^B \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} -0{,}19 & 0{,}05 \\ -0{,}01 & 0{,}21 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-2}^B \\ \Delta\, i_{t-2} \end{bmatrix} + \begin{bmatrix} 0{,}001 \\ -0{,}007 \end{bmatrix} +$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

–model 4 VAR(2): $\mathbf{Y}_t = [\pi_t^T, \Delta i_t]'$

$$\begin{bmatrix} \pi_t^T \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 1{,}02 & 0{,}18 \\ 0{,}09 & 0{,}17 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^T \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} -0{,}11 & 0{,}2 \\ -0{,}07 & 0{,}17 \end{bmatrix} \begin{bmatrix} \pi_{t-2}^T \\ \Delta\, i_{t-2} \end{bmatrix} + \begin{bmatrix} 0{,}42 \\ -0{,}11 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

–model 5 VAR(2): $\mathbf{Y}_t = [\pi_t^M, \Delta i_t]'$

$$\begin{bmatrix} \pi_t^M \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 1{,}26 & 0{,}03 \\ 0{,}06 & 0{,}09 \end{bmatrix} \begin{bmatrix} \pi_{t-1}^M \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} -0{,}31 & -0{,}02 \\ -0{,}02 & 0{,}11 \end{bmatrix} \begin{bmatrix} \pi_{t-2}^M \\ \Delta\, i_{t-2} \end{bmatrix} + \begin{bmatrix} 0{,}22 \\ -0{,}38 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

–model 6 VAR(2): $\mathbf{Y}_t = [\Delta\pi_t^{CPI}, \Delta i_t]'$

$$\begin{bmatrix} \Delta\pi_t^{CPI} \\ \Delta i_t \end{bmatrix} = \begin{bmatrix} 0{,}34 & -0{,}02 \\ 2{,}1 & 0{,}11 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-1}^{CPI} \\ \Delta\, i_{t-1} \end{bmatrix} + \begin{bmatrix} 0{,}05 & 0{,}01 \\ 1{,}72 & 0{,}18 \end{bmatrix} \begin{bmatrix} \Delta\pi_{t-2}^{CPI} \\ \Delta\, i_{t-2} \end{bmatrix} + \begin{bmatrix} -0{,}001 \\ -0{,}006 \end{bmatrix} +$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

We also verified stationarity of the obtained models and using (4.24) we calculated the roots of the characteristic equation. The roots of these equations (Table 5) for the estimated models 1-6 are located outside the unit circle. Therefore, we can conclude that the obtained VAR models are stationary.

Table 5. Roots of the characteristic equations for the estimated VAR models

Model	$\lvert z_1 \rvert$	$\lvert z_2 \rvert$	$\lvert z_3 \rvert$	$\lvert z_4 \rvert$	$\lvert z_5 \rvert$	$\lvert z_6 \rvert$
1	1,5521	1,5521	1,9847	1,6612	1,8455	1,5454
2	1,6553	1,6553	2,008	2,0100	-	-
3	1,3201	1,2298	1,1035	1,0021	-	-
4	2,1501	2,0303	2,1011	1,9978	-	-
5	1,5654	1,6545	1,6545	1,0235	-	-
6	1,9789	1,8845	1,4655	1,0050	-	-

Source: own study.

The next stage of statistical verification of the models was analysis of autocorrelation of the multivariate random component and its compatibility with the normal distribution. To this end, test statistics have been used, described using (4.44)-(4.45) and (4.49). The empirical values obtained are presented in Table 6.

Table 6. Results of the tests of autocorrelation and the compatibility of the distribution of the multivariate random component with the normal distribution

Model	*p-value* L-B (5.41)	*p-value* B-P (5.42)	*p-value* D-H (5.46)
1	0,94	0,75	>0,0000
2	0,97	0,77	>0,0000
3	0,98	0,61	>0,0000
4	0,94	0,71	>0,0000
5	0,90	0,65	>0,0000
6	0,62	0,40	>0,0000

Source: own study.

The obtained *p-value* values indicate no grounds for rejecting the null hypothesis of the absence of autocorrelation to the order of sixteen, including at any typical significance level for the *Portmanteau* test. The value of the *adjusted Portmanteau* statistics leads to similar conclusions.

In the case of the compatibility of the multivariate random element's distribution with the normal distribution, the Dornik-Hansen test has been carried out, whose test statistics with the *p-values* for each model close to zero allow to conclude that the residuals have no normal distribution. The resulting multivariate random component distributions in each model are skewed and leptokurtic.

The final phase of statistical verification of the VAR models obtained were the Chow stability tests. The bootstrap *p-values* for the statistics versions (4.50)-(4.52) are provided in Table 7. Finally, it turned out that for the models built there are no grounds for rejecting the null hypothesis of the stability of the model at the significance level of 5%.

Table 7. Results of the Chow stability tests

Model	Break point	Sample split	Chow forecast
1	0,09	0,66	0,21
2	0,09	0,67	0,21
3	0,09	0,72	0,17
4	0,09	0,70	0,23
5	0,08	0,55	0,30
6	0,07	0,90	0,24

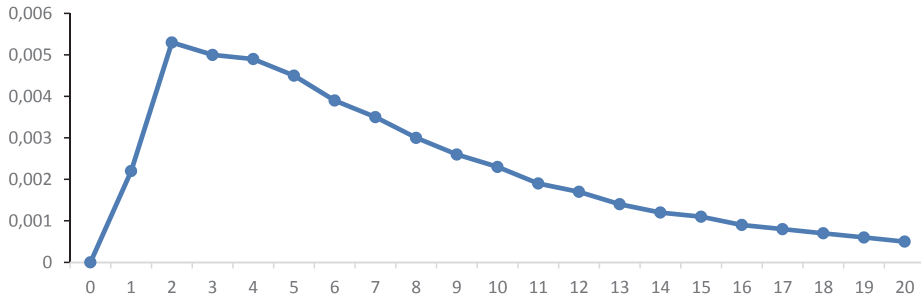
Source: own study.

Finally, for the purpose of economic interpretation, the models were structured where a key role is played by structural monetary shock. The Cholesky decomposition was used for this purpose.

Impulse response functions

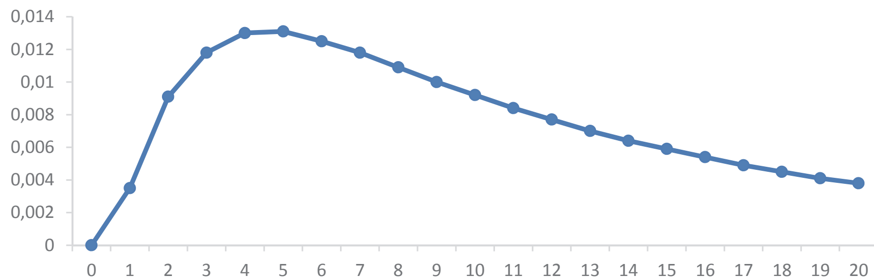
Structuring applied allows to present the function of responses of the price indices in agriculture to the impulse coming from the monetary policy area (Figures 73-78).

Figure 73. Functions of the response of the wheat buying-in price index to the interest rate impulse



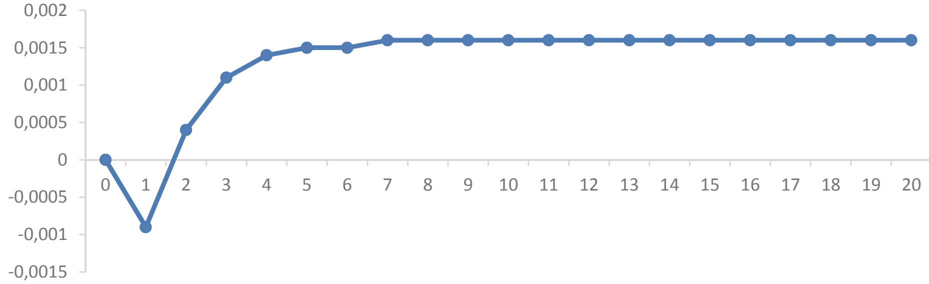
Source: own study.

Figure 74. Functions of the response of the rye buying-in price index to the interest rate impulse



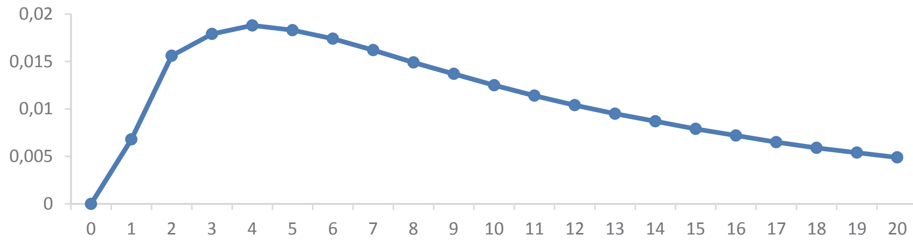
Source: own study.

Figure 75. Functions of the response of the cattle buying-in price index to the interest rate impulse



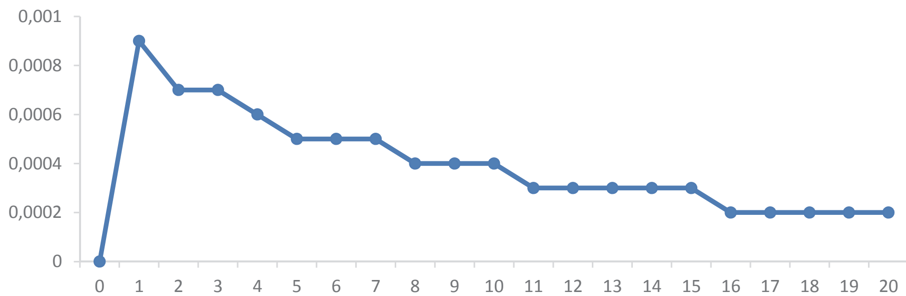
Source: own study.

Figure 76. Functions of the swine buying-in price index to the interest rate impulse



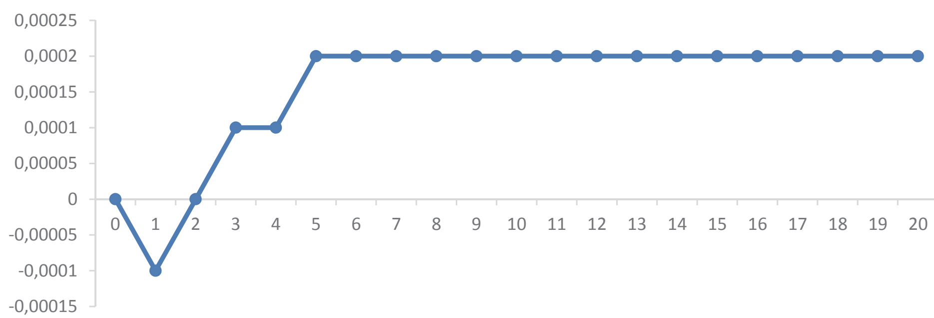
Source: own study.

Figure 77. Functions of the milk buying-in price index to the interest rate impulse



Source: own study.

Figure 78. Functions of the consumer price index to the interest rate impulse



Source: own study.

It should be expected that as a result of positive interest rate shock, we will observe, although with some lag[113], the decrease in price indices. The results obtained are not consistent with the theory cited.

The wheat buying-in price index, as a result of positive monetary shock, responds with an immediate increase which is statistically significant. The impulse response function returns to its base path only 20 months after the occurrence of monetary shock. In terms of quality, the similar IRF shape was obtained for the rye, swine and milk buying-in price indices.

Therefore, we observe the *price puzzle* phenomenon, which is defined as the rise in the prices due to the increased interest rate. The existence of „strong positive correlation between the nominal interest rate and the price level" was called Gibson's paradox by Keynes as „one of the most fully established empirical facts in the entire area of quantitative economics" (cf. Keynes (1930), A Treatise on Money, part 2). Attempts to clarify the price puzzle phenomenon can be found in the papers by, inter alia: Barsky and Summers (1988), Balke and Emery (1994) or Castelnuovo and Surico (2009). The price puzzle phenomenon is often explained by the existence of supply inflation „pushed by the costs".

[113] The existence of lag in the reaction of the price indices can be explained pursuant to positive economics in the context of "stylized facts". In the economic literature, the term "stylized facts" defines the rules as to the behaviour of the economy in the short, medium and long term. In this case, we talk about the fluctuations in the economic activity and related leading and lagging indicators. Leading variables are those which record the change in the phase of a cycle earlier than the reference variable (variables that lead real GDP in a cycle are, for example, stock market indices or real money supply). Lagging variables are defined as those towards which the change in the phase of a cycle takes place later than in the reference variable (for GDP, lagging variables are, inter alia, inflation rate and unemployment rate). Most stylized facts can be found in the paper by Skrzypczyński (2010).

Summary

In the monograph, we carried out analysis based on the adopted theoretical and analytical approach based on the production function. Two groups were identified based on the original approach: endogenous or conventional (related to the production factors, which illustrates production techniques) and exogenous related to market regulation with the given institutional and legal solutions and economic policy, in particular, the monetary policy. The main focus was on the exogenous factors, which were determined based on the originally derived relationships (mathematical formulae) or were adopted from the literature of the subject. These formulae, for better understanding, have been visualised as charts in figures. These show the hypothetical assumptions as to the evolution of the production factor price relationships and its relation with employment of the production factors in production i.e. with production techniques. Then, in the next step, these assumptions, being to some extent research hypotheses, were empirically verified, also using the econometric and statistical apparatus.

In the first chapter, presenting the market of production factors as the exogenous condition, a substantial part was devoted to the concept of exogenous factors, and the factor production prices were formulated analytically as those which define these conditions. We are based here on the assumption that the relationship of the level of scarcity of production factors in the market is illustrated by prices. Analysis was concentrated on the labour and capital factor prices and their impact on the factor relationships i.e. production techniques.

The very production factor price relationships have been formulated both theoretically and by means of hypothetical figures showing the capital, labour and land factor price relationships in Chapter 2. In this chapter, we also refer to the objective function and the producer's equilibrium condition in the context of the production factor prices. Also, we introduced the definition of productivity and documented the relationships of factors describing the production technique in relation to the capital and labour and capital and land factor price relationships. We also included a description of the supply and demand functions in the production factor market. The chapter is ended with empirical verification of the hypothetical assumptions made. We verified the relationships between the labour factor and capital factor prices and between the land factor and capital factor prices for Poland, EU countries, Germany, France, the Great

Britain, Lithuania, the Netherlands and Slovakia. Analysis of the visualisations obtained confirmed the adopted hypothetical assumptions derived from the analytical aspect. The price relationships result from the level of their scarcity.

Chapter 3 presents, in the analytical convention, the capital, labour and land factor prices in relation to the price, which results from the specific price relationships and their changes. In the analytical convention, we point to their exogenous nature, whereas changes in the relationships of the factor inputs are derived from the producer's choice, and therefore belong to the endogenous factors. The analytical reasoning based on mathematical formulae as well as quantitative verification of the discussed dependencies for the selected European Union countries have been consistently conducted. Finally, for each production factor, the empirical substitution relationship between its price and level of employment has been presented and estimated based on the non-linear hyperbolic model. Summing up, it has been confirmed that the production factor price relationships correspond to the relationships of their employment, i.e. production techniques.

As part of the exogenous factors, it was decided to complete the considerations based on the production function with the functional approach, indicating the price dependency in the economy, including the agricultural sector, on the interest rate. This transmission mechanism was presented in Chapter 4. The analytical aspect is based on the Neoclassical model of the monetary policy impact to the nominal sphere through the transmission channels. The main attention was focused on the interest rate channel. This impact has also been measured using time series econometrics. We estimated 6 vector autoregression models, then we structured them and, using the impulse response function, we showed the impact of the monetary policy through the interest rate on the change in the following price indices: buying-in of wheat, rye, cattle, swine, milk as well as consumer price (as a benchmark).

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