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A New and Dynamic Approach for Forecasting the Margin Protection Program for Dairy Producers

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The 2014 Farm Bill offers dairy-producers a new safety net. The Margin Protection Program considers differences between average national prices of milk and feed (corn, soybean meal, and alfalfa). A web-based tool forecasts this margin using derivatives, considering shocks to a commodity's futures price as differences between the futures price and its terminal/expiration price. Shocks are constructed per time to maturity (delivery horizon); considering one-month up to one-and-half years ahead (18 different shocks per commodity). Rank correlations among shocks are maintained when forecasting prices. However, these correlations are static, ignoring a crop growing season's new information. We incorporate this new information using time-varying correlations. Moreover, we model dynamic copulas of joint time-varying correlations among newly constructed one-month delivery horizon shocks. Forecasts indicate relative improvement.

Key words: Dairy Margin Forecast, Dynamic Conditional Correlation (DCC) Copulas, MPP Dairy Margin, Time-Varying Correlations

The latest Agricultural Act (Farm Bill) of 2014 includes a new safety net program for dairy producers. The Margin Protection Program (MPP) for dairies explicitly considers a national dairy "average" margin—accounting for the difference between milk and feed prices—and compares it to a producer's selected preference for a certain margin threshold of his or her yearly production history. If the national market margin is below the threshold selected by the producer in his or her particular contract, the producer receives an indemnity.

The national margin considered by the U.S. Department of Agriculture (USDA) for this program is defined by an equation making use of national monthly average prices of all milk, corn and alfalfa, as well as a Midwestern price for soybean meal. The first three values are obtained with (average monthly) transaction data directly from the USDA's National Agricultural Statistics Service (NASS), and the last one from average monthly prices reported at Decatur, Illinois, through the USDA Market News-Monthly Soybean

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Meal Price report. (USDA, Notice MPP-1, 2014). The following equation pertains to the national Actual Dairy Producer Margin (ADPM), expressed in \$/hundredweight (cwt) of milk:

$$\text{(1)} \quad \text{ADPM \$/cwt of milk} = \text{All Milk \$/cwt} - \{[1.0728 \times \text{Corn \$/bu}] + [0.0137 \times \text{Alfalfa \$/ton}] + [0.00735 \times \text{SBM \$/ton}]\}$$

Figure 1 illustrates this margin from 2000 onwards, which considers prices since the implementation of the Federal Milk Marketing Order Reform.[1] More recently, dairy producers experienced increased volatility in the prices of these feed commodities which, in turn, directly resulted in higher variability in their margin returns (without considering other production factors).

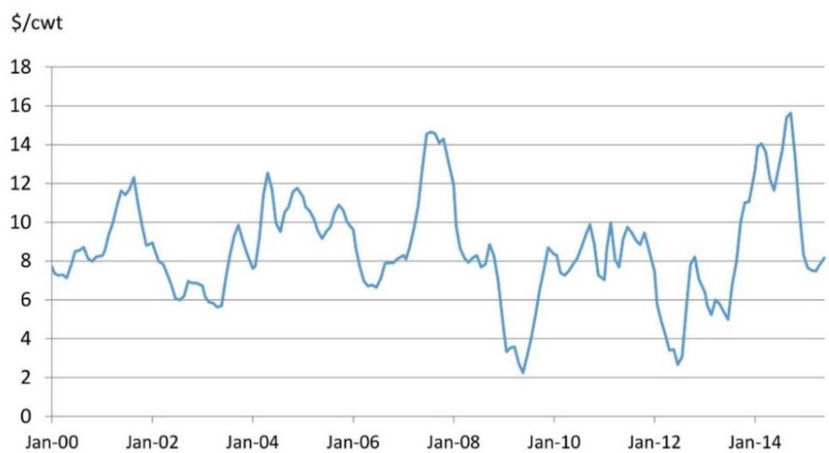

Figure 1: Actual Dairy Producer Margin for MPP Dairy Program.

The evolution of the prices of each of these commodities, considering the product of its specific factor, is illustrated in Figure 2. As may be seen from this figure, correlations among these prices appears to be varying both during a calendar year and through the years. Table 1 provides summary statistics for correlations between corn and soybean meal, between soybean meal and alfalfa, and between corn and alfalfa. These correlations are for periods of four years and show that correlations between pairs of commodities change substantially from period to period.[2] Time-varying correlations among the commodities are a critical matter that are taken into account in this study.

[1] A report by Jesse and Crop (2001) provides details on policy decisions and their implications.
[2] At the time the study was conducted, data was available until June 2015. A period of four years was taken arbitrarily to illustrate the changing correlations after each period; correlations for other time periods arrive at different numerical results, but also indicate how these change after each period.

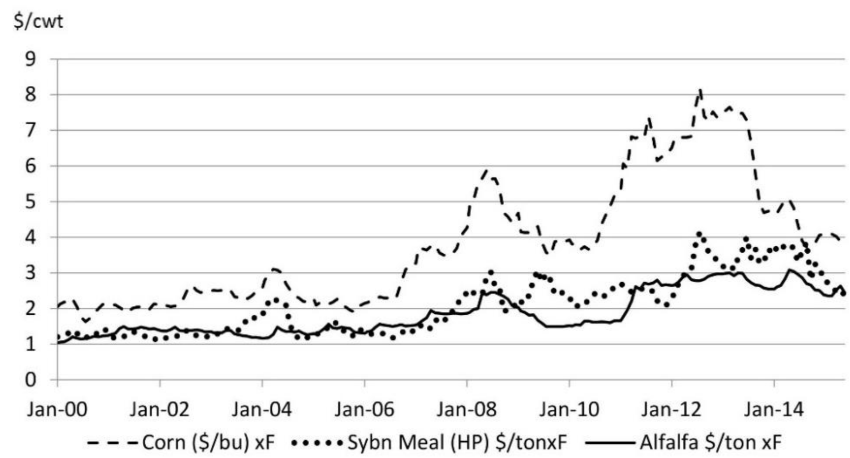

Figure 2: National Average Feed Prices for MPP Dairy Program, multiplied by its specific factor.

Table 1. Correlations between Commodities, every four years.

	Correlations		
	Corn - Soybean Meal	Soybean Meal - Alfalfa	Corn - Alfalfa
Jan 2000 to Dec 2003	0.42552	-0.28104	0.12998
Jan 2004 to Dec 2007	0.57066	0.16649	0.71459
Jan 2008 to Dec 2011	0.15465	-0.05834	0.84338
Jan 2012 to Jun 2015	0.25135	0.42559	0.64246

Given equation (1) for the national dairy margin and its direct effect on determining eligibility of an indemnity, dairy farmers are able to estimate forward potential dairy margin values using futures contracts for most of the markets.[3] The program covers a calendar year of milk production, and its enrollment deadline date is by the end of September of the previous year.[4]

The policy evaluates the average national margin in regard to a producer's insured margin every two months from the beginning of the (calendar) year. That is, the monthly national margins are subsequently averaged every two months for a two-month average in February, April, June, August, October, and December of a contract year. This two-

[3] Alfalfa does not have a futures contract. For a future study, we consider the effect from varying inter-relationships between this and the other two feed commodities.
[4] In the first year of the program, enrollment for 2014 and 2015 was extended until December 19, 2014, because Congressional passage and presidential signing of the farm bill was delayed. In 2015, enrollment for 2016 was extended until November 20.

month average margin is compared to the insurer's selected (annual) margin to determine if there is a payment (indemnity) that should be made for one-sixth of the annual production (i.e., covering the two months of production when the national margin is below the insured margin). Thus, the relevant future months that are considered in terms of (average) margin values correspond to the ones where the policy may trigger a payment (i.e., every two months from February until December.) Price values of the national dairy margin that can be selected for insurance purposes range from the lowest of $4 to the highest of $8, in increments of $0.50; and the premium paid at each level increases accordingly as a nonlinear function.

Presently, there is a web-based tool that forecasts this margin by making extensive use of futures (and options) markets data (Newton, Thraen, and Bozic, 2015; Newton and Nicholson, 2014). This method applies futures and options prices of milk (Class III and Class IV), corn and soybean meal, as well as their historic cash prices and of alfalfa to predict the margin price. The futures prices are used to predict the cash prices, and alfalfa uses its own historic prices to predict its cash prices. Following Bozic et al. (2014), the web-based tool takes into consideration differences between the expected price of a commodity—given by its futures contract price at a certain date—and its settled (terminal) price at expiration. These differences are denoted as shocks to a commodity's market. They are considered unexpected price deviates of each commodity, occurring at different periods during the year (e.g., the difference in prices of December contracts purchased in September with their terminal, expiring price in December; or differences in prices of March contracts purchased in December with their terminal, expiring price in March), and with different spans of time or time to maturity, (3 months, 12 months, etc.) The inter-relationship between the different time to maturity deviates among these commodities' futures prices is calculated via rank correlation. These rank correlations are incorporated in the tool's forecasting method by applying an empirical copula that maintains the ranking correlation order among the price deviation series, in a similar manner to Bozic et al. (2014).

The present forecasting tool does not distinguish correlations between price deviates of commodities with similar time to maturity (e.g,. 6 months), but that may occur at a different time of the year. That is, the tool treats the correlation calculated by the deviation price from a September contract purchased in March the same as the correlation computed by the price deviation from a March contract purchased in September. This, however, seems to leave out the additional information available for corn and, indirectly, to soybean meal (derived from soybeans) during the northern hemisphere spring when sowing is already underway, in comparison to the prior end of fall which is the harvesting stage.

This paper seeks to address this matter by considering the effect of dynamic correlations during different periods of the year among shocks on the commodities' futures prices, and its eventual impact on the margin forecast. Our forecasting method uses a similar approach in regard to the use of commodities' futures market data; however, the computation of our price shocks differs by making more use of information contained in the expected futures prices at a specific date. These newly computed shocks are used in conjunction with dynamic copula methods[5] that identify the time-varying correlations among the markets to provide forecasts for the prices considered in the Actual Dairy Producer Margin.

Our purpose is to provide another method for the forecast estimation of the dairy margin that further assists producers in determining the level of "insurance" coverage they may prefer. In other words, this study aims to shed additional light on the potential future values of the dairy margin by considering a different method in the forecasting approach. This may be of added benefit for producers in helping them decide their safety level (in this program) in the same manner that there are different weather models applied for stakeholders involved in other safety and risk management programs. Moreover, this approach may likewise aid other dairy stakeholders—financial agents, policy makers, etc.—in their decision-making regarding this program. It is relevant to note that this paper does not address potential effects of changes in rate-making of premiums; this and other matters are subjects for further study. As mentioned, the objective in this study is to address the forecasting of the dairy margin by using a different, novel approach and, to that end, we evaluate the differences with the current method by applying pertinent forecast error measurements, as well as by comparing expected net payments under both methods. Our results indicate relative improvement over the current method. We briefly explain the method from the web-based tools currently in use, as well as some of its parameter estimates being applied. We then present our methodology and contrast some parameter estimates with those from the current tool. We provide new dairy margin forecasts and compare these with current forecasts by applying forecast error measurements that cover different aspects of our model's forecasting ability, as well as propose future lines of study.

[5] Woodard et al. (2011) argue that copulas permit the use of broader and more flexible tools for modeling the relationship or dependence configuration among series in probabilistic settings, in comparison to more conventional methods used such as Iman and Conover (1982) or Phoon, Quek and Huang (2004). Patton (2006b) provides evidence of modeling advantages provided by time-varying copulas.

Methods

The aim is to forecast the national average cash prices for all milk, corn, soybean meal, and alfalfa that are part of the ADPM. Employing futures markets for this purpose enables us to apply up-to-date market information of transactions that are to be completed in the nearby future, making use of current readily available information. Efficient markets are assumed.

The current method (Newton, Thraen, and Bozic, 2015; Newton and Nicholson, 2014) considers futures prices for milk (classes III and IV), corn, and soybean meal. In order to establish the (basis spread) relationship between the cash and futures prices, the national cash average prices for milk, corn, and soybean meal are regressed (via Ordinary Least Squares, or OLS) with the respective futures prices. The alfalfa cash prices are modeled as a function of current average milk prices and prior alfalfa prices.[6]

For the three commodities with futures contracts, shocks or price deviates are constructed according to their time to maturity, e.g., a September 8 price of a futures contract on corn with expiration in December is subtracted to that contract's December 8 price. The deviation period (time to maturity) here would be three months. This is done for each commodity's contracts up until the full estimated period ahead, in a similar manner to Bozic et al. (2014)For example, Class III milk would have a deviate for one month ahead, two months ahead, and so forth until the last estimated month ahead. The same approach is taken for Class IV milk, corn, and soybean meal.[7]

Thus the method accounts for different price deviate series of futures contract price data, considering the period from January 2001 to March 2013, and then a rank correlation is calculated among each of these series. For instance, a corn contract with 12-month deviates (corn's sixth nearby contract) would have a rank correlation calculated with each Class III deviate (first nearby (month) contract until the 18th nearby contract), with each Class IV deviate (first nearby through 18th), each soybean meal deviate (first nearby until 13th), and, finally, with each of its other corn contract deviates (first nearby until 8th).

Despite the large number of rank correlations computed, these correlations are static. The method may overlook timely market information that could affect the relationship between deviates of the same nearby time-to-maturity but which are at a distinctly different point in time of the year (e.g., deviates for a crop at sowing periods may be

[6] In this study, and for forecast comparison purposes, we consider these estimations as given and taken from Table 2 of the appendix in Newton, Thraen, and Bozic (2015). We leave addressing the correlations between cash prices of alfalfa and the other commodities, and their effect on the margin forecast, for future study.
[7] Newton, Thraen, and Bozic (2015) consider up to 18 months ahead, starting from July of the prior year until January of the year following the end of the contract period.

substantially different from those near harvesting periods, given the more recent (crop) information available.) Our study considers this by applying a Dynamic Conditional Correlation (DCC) model (Engle, 2002) among price deviations, which take into account a new, evolving, one-month shock for each commodity's futures price.

The current method also estimates the probability distribution function for each commodity by applying a log-normal distribution to the futures prices. The method considers each distribution's variance by calculating the implied volatilities of at-the-money option premiums via inverting the binomial option pricing model (Cox, Ross, and Rubinstein, 1979; Miranda and Fackler, 2002). With the commodities' marginal (probability) distributions, the authors then apply an empirical copula method (Bozic et al., 2014) and simulate futures prices.

The empirical copula incorporates the marginal distributions and maintains the previously computed rank relationships among them. Simulated draws of 5,000 futures prices for each commodity according to their time-to-maturity are obtained. Moreover, for months where a commodity does not have a futures contract, a weighted average of nearby months is used to extrapolate its prices. With these simulated futures prices, the corresponding cash prices are obtained by using the parameter estimates of the OLS equations initially estimated (Table 2, appendix of Newton, Thraen, and Bozic, 2015). This then leads to calculating the dairy margin according to equation (1). The projected dairy margins can be accessed directly from the website http://dairymarkets.org/MPP/Tool/. Available year-ahead projections are from July and, more importantly, from the end of September (date by which dairy producers must make the program's contract decision covering production for the coming year) in 2007 until last year, as well the current date's estimation.

Table 2. Forecast Measurement Errors for Actual (current) Method of Forecasting the Dairy.

Margin and the new (proposed) method, for years 2008 and 2009.

	2008						2009					
	ME		MAE		RMSE		ME		MAE		RMSE	
	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New
December*	0.64	1.09	0.76	1.09	0.99	1.21	0.38	0.82	0.67	0.84	0.68	1.02
February	0.47	0.57	0.76	1.01	0.92	1.1	-0.88	-0.64	1.58	1.63	2.05	1.98
April	-0.03	-0.04	0.96	1.17	1.11	1.26	-1.53	-1.32	2.05	2.03	2.49	2.33
June	-0.33	-0.34	1.07	1.22	1.21	1.3	-2.13	-1.99	2.55	2.54	3.02	2.9
August	-0.55	-0.51	1.17	1.23	1.3	1.29	-2.46	-2.2	2.81	2.66	3.23	2.96
October	-0.7	-0.51	1.23	1.15	1.35	1.24	-2.47	-1.99	2.77	2.37	3.14	2.75
December	-0.89	-0.52	1.36	1.07	1.51	1.19	-2.25	-1.49	2.51	2.28	2.96	2.65
	2010						2011					
	ME		MAE		RMSE		ME		MAE		RMSE	
	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New
December*	0.34	0.08	0.34	0.95	0.44	1.09	-0.33	-0.49	0.47	0.49	0.63	0.71
February	0.3	0.47	0.3	1	0.39	1.09	-0.02	-0.26	0.67	0.71	0.88	0.86
April	0.03	0.29	0.41	0.76	0.51	0.92	0.61	0.37	1.14	1.07	1.51	1.3
June	-0.03	0.2	0.39	0.71	0.48	0.85	0.93	0.6	1.35	1.14	1.71	1.4
August	-0.02	0.24	0.35	0.65	0.44	0.8	1.29	0.92	1.64	1.36	2.01	1.62
October	0.08	0.42	0.4	0.78	0.48	0.93	1.34	1.04	1.64	1.41	1.97	1.63
December	-0.09	0.35	0.5	0.78	0.69	0.92	1.34	1.21	1.6	1.54	1.91	1.74
	2012						2013					
	ME		MAE		RMSE		ME		MAE		RMSE	
	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New	Actual	New
December*	1.18	2.18	1.18	2.18	1.38	2.27	0.45	0.98	0.81	0.98	0.91	1.07
February	0.57	1.55	1.06	1.62	1.28	1.87	-0.07	0.43	0.91	0.74	0.99	0.88
April	-0.21	0.84	1.43	1.42	1.69	1.67	-0.31	0.27	0.94	0.81	1.06	0.92
June	-0.91	0.23	1.89	1.53	2.24	1.74	-0.39	0.25	0.89	0.67	1.01	0.81
August	-1.52	-0.29	2.34	1.73	2.77	1.94	-0.41	0.28	0.88	0.74	1	0.88
October	-1.48	-0.03	2.18	1.67	2.63	1.94	-0.07	0.83	1.04	1.22	1.22	1.8
December	-1.4	0.25	2.01	1.73	2.48	1.97	0.4	1.6	1.37	1.94	1.74	2.93
	2014											
	ME		MAE		RMSE							
	Actual	New	Actual	New	Actual	New						
December*	1.05	1.76	1.05	1.76	1.09	1.9						
February	2.14	3.24	2.14	3.24	2.69	3.78						
April	2.97	4.31	2.97	4.31	3.58	4.93						
June	3.14	4.61	3.14	4.61	3.64	5.1						
August	3.44	5.04	3.44	5.04	3.89	5.49						
October	3.91	5.73	3.91	5.73	4.42	6.29						
December	3.81	5.82	3.81	5.82	4.3	6.32						

* Prior year.

Our method begins by calculating one-month price deviates for each commodity by taking into account current futures prices with up-to-a-year of time to maturity.[8] The intent is to make use at a point in time of the market's expected (price) information for all the future months and then account for how each future month's value specifically changes from month to month. This is in contrast to considering just the information contained in the current futures prices and the difference with respect to its terminal value (at expiration). Our method of considering just a one-month variation for all futures prices with up to 15 months of time to maturity is in lieu of findings by Irwin and Good (2015) and Westhoff (2015). Those studies compared 10-year price projections for corn, wheat, and soybeans from the USDA's World Agricultural Supply and Demand Estimates reports and the futures markets, as well as those from the Food and Agricultural Policy Research Institute at the University of Missouri, respectively. Despite these studies being yearly projections instead of monthly projections, as is our case, they found that these futures markets forecasts tend towards a "steady state". This result is in conformity with the theory of futures markets for storable commodities, where the positive difference between current and deferred futures prices is limited by the cost of carry and, thus, leads to converging prices in a steady-state. When the actual margin did deviate substantially from a relative steady state (fluctuating substantially away from the forecasted margin bands), it may be most likely responding to supply or demand shocks—as mentioned by Irwin and Good (2015).

We directly interpolate futures prices for months in which a commodity does not have a futures contract, and we consider one-month shocks as the deviation between a futures price at the current date and its price for delivery the prior month. These one-month shocks are grouped separately according to the initial futures price time to maturity, e.g., a December futures price in September is differenced with the November futures price in September. This shock is included in a series with a January futures price in October which is differenced with its December futures price in October. Another series of shocks are of two-month time to maturity periods, as is the case of a November futures price in September that is differenced with its October futures price in September. Thus we have a monthly price deviation series for each commodity considering one month ahead, two months ahead, etc. futures prices and estimate time-varying correlations among these commodities' price deviations according to their time to maturity. The data were obtained from Brian Gould's "Understanding Dairy Markets" website (http://future.aae.wisc.edu/).

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[8] In rigor, given that the projections we probe start from September of a particular year and run from October until December of the following year, we consider each commodity's futures prices with up to 15 months of maturity.

We apply the DCC model to these series of deviations of futures prices considering data from September 2000 to June 2015.[9]

The Dynamic Multivariate GARCH model (Engle, 2002) specifies the dynamic conditional covariance matrix $\boldsymbol{H_t}$ as a nonlinear combination of univariate conditional variances. More specifically,

$$(2) \qquad \boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t$$

where D_t is a diagonal matrix of time-varying standard deviations $\sqrt{h_{it}}$ with i =1,2…k (number of variables), and $\boldsymbol{R}_t$ is a matrix of time-varying conditional correlations. Estimation is in two steps, and consistent and asymptotically efficient estimates are obtained. First, each series may be estimated individually with an AR, ARMA or other, such as MA, e.g. $\boldsymbol{y}_t = \emptyset_o + \sum \emptyset_i \boldsymbol{y}_{t-i} + \boldsymbol{\varepsilon}_t$ and the residuals for the *i*th series in y_t can be obtained by using a univariate GARCH specification $h_{it} = a + b\varepsilon_{it-1} + ch_{it-1}$ for i = 1,2,…..k.

The estimated standard deviations $\sqrt{h_{it}}$ are used to calculate the standardized residuals $u_{it} = {}^{\varepsilon_{it}}\big/_{\sqrt{h_{it}}}$. Moreover, the standard deviations are used to construct the k x k diagonal matrix $\boldsymbol{D}_t(\boldsymbol{\theta}_D)$ of time-varying standard deviations, where $\boldsymbol{\theta}_D$ refers to parameters a, b and c.

Following the estimation of the dynamic volatilities for each series, $\sqrt{h_{it}}$, the correlation matrix among the series is estimated. The time-varying conditional correlations are expressed as follows:

$$(3) \qquad \boldsymbol{Q}_t = (1 - \alpha - \beta)\overline{\boldsymbol{Q}} + \alpha \boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1} + \beta \boldsymbol{Q}_{t-1}$$

$$(4) \qquad \boldsymbol{R}_t(\boldsymbol{\theta}_R) = diag(\sqrt{q_{11t}}, \dots \dots \dots \sqrt{q_{kkt}})\boldsymbol{Q}_t diag(\sqrt{q_{11t}}, \dots \dots \dots \sqrt{q_{kkt}}\ )$$

where $\boldsymbol{Q}_t$ is a k x k dynamic covariance matrix of standardized residuals, $\overline{\boldsymbol{Q}} = E[u_t u'_t]$ is a k x k unconditional variance matrix of u_t, and α and β are non-negative parameters such that their estimate $(\alpha + \ \beta) < 1$.

Estimation of (3) provides a consistent but inefficient parameter of $\boldsymbol{\theta}_R$, which is the parameter set that specifies the dynamic conditional correlation (DCC) matrix $\boldsymbol{R}_t$. Full

[9] Given the large number of observations required to estimate each model according to the commodity's "time to maturity," estimating dynamic correlations among series with different time to maturity (as in the current method of MPP forecast, using static correlations) requires estimation of a very large number of parameters which is not feasible with the number of degrees of freedom available.

efficient estimates of $\boldsymbol{\theta}_D$ are obtained by a single optimizing iteration of the following log-likelihood function:

$$(5)\quad ll_t(\boldsymbol{\theta}_D) = -\frac{1}{2}\sum_{t=1}(n \cdot log(2\pi) + log\,|\boldsymbol{D}_t|^2 + \boldsymbol{\varepsilon}_t'\boldsymbol{D}_t^{-2}\boldsymbol{\varepsilon}_t) + [-\frac{1}{2}\sum_{t=1}(\log|\boldsymbol{R}_t| + \boldsymbol{u}_t'\boldsymbol{R}_t^{-1}\boldsymbol{u}_t - \boldsymbol{u}_t'\boldsymbol{u}_t)\,]$$

Efficient estimates of $\boldsymbol{D}_t$ and $\boldsymbol{R}_t$ are used to obtain $\boldsymbol{H}_t$ per (1).

We then estimate time-varying copulas, specifically using DCC models as function marginals applied to our series of price deviations. For this we make use of Patton (2006a and 2006b), who extended and proved the validity of Sklar's (copula) theorem (Sklar, 1959) under time-varying conditions. Copulas have been well documented in literature, including many applications in agricultural markets. A non-comprehensive list includes Power and Vedenov (2008), Tejeda and Goodwin (2008), Vedenov (2008), Woodard et al. (2011), Goodwin and Hungerford (2015).

Copulas are a useful tool for modeling the relationship among different variables without restricting the distribution of these variables. Sklar (1959) notes that any continuous multivariate distribution can be uniquely described by the variables' univariate marginals and a multivariate dependence structure, which is represented by a copula. Let $\boldsymbol{F}$ be an n-dimensional distribution function with marginals $F_1, \ldots \ldots F_n$; then there exists an n-dimensional copula $\boldsymbol{C}$ defined as a multivariate distribution function in the unit $[0,1]^n$ with uniform $U[0,1]$ marginal distributions such that for all x in $\mathcal{R}^n$:

$$(6)\qquad \boldsymbol{F}(x_1, \ldots \ldots \ldots \ldots., x_n) = \boldsymbol{C}(F_1(x_1), \ldots \ldots, F_1(x_n); \theta)$$

where θ is a vector of copula parameters called dependence parameters, measuring the dependence (relationship) between the marginals. The density function of a multivariate distribution defined by a copula function is obtained by differentiating the prior equation (6), resulting in:

$$(7)\qquad \boldsymbol{f}(x_1, \ldots \ldots.., x_n; \theta) = \boldsymbol{c}(F_1(x_1), \ldots \ldots. F_n(x_n)) \prod_{i=1}^{n} f_i(x_i)$$

where f_i represents the marginal density function of x_i and $\boldsymbol{c}$ is the density function of the copula function $\boldsymbol{C}$ in (6).

In particular, two common types of copulas used are the Gaussian and the Student-t, which belong to the elliptical class and both have radial symmetry (Nelsen, 1999). However, the Student-t copula has the flexibility advantage of identifying tail dependence among the variables, and the Gaussian does not. Either the Spearman or Kendall's Tau rank correlation is usually applied as measures of dependence, given that both measures of concordance are invariant to monotonic transformations.

Time-varying copulas have recently been used in financial fields (Patton, 2006b; Chollete, Heinen and Valdesogo, 2009; Ausin and Lopes, 2010); here we apply a DCC Gaussian copula and a DCC Student-t copula, and compare their results. Let $\boldsymbol{d}_t = d_{1t},\ldots\ldots d_{nt}$ be an n-dimensional random vector of price deviations which follow a copula GARCH model with joint distribution:

$$\text{(8)} \qquad \boldsymbol{F}(\boldsymbol{d}_t|\boldsymbol{u}_t,\boldsymbol{h}_t) = \boldsymbol{C}\big(F_1(d_{1t}|u_{1t},h_{1t}),\ldots\ldots\ldots F_n(d_{nt}|u_{nt},h_{nt})\big)$$

where F_i, $i = 1 \ldots.n$ is the conditional distribution of the ith marginal series density and $\boldsymbol{C}$ is the n-dimensional copula. The conditional mean is $E(d_{it}|\zeta_{t-1}) = u_{it}$, where $\zeta_{t-1}$is the σ-field generated by past realizations of $\boldsymbol{d}_t$. The conditional variance h_{it} follows a GARCH (1,1) such that $d_{it} = \sqrt{h_{it}}z_{it}$ and $h_{it} = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{it-1}$, where z_{it} are i.i.d. random variables which may follow a normal or a standardized skew Student-t distribution with shape and skew parameters ν and ξ, respectively. The dependence structure is then assumed to follow a copula with conditional correlation $\boldsymbol{R}_t$ and constant shape parameter η. The joint density at time t is as follows:

$$\text{(9)} \qquad f(\boldsymbol{d}_t|\boldsymbol{u}_t,\boldsymbol{h}_t,\boldsymbol{R}_t,\eta) = c_t(u_{1t},\ldots\ldots\ldots..,u_{nt}|\boldsymbol{R}_t,\eta)\prod_{i=1}^{n}\frac{1}{\sqrt{h_{it}}}f_{it}(z_{it}|\nu_i,\xi_i)$$

Estimation is in a two-stage process via maximum likelihood, where the DCC marginals are estimated first, followed by the copula estimation of the joint marginals. For this we make use of the R package “rmgarch” developed by Ghalanos (2014).

Results

The dynamic correlations obtained among the one-month price deviations of the commodities, for each time-to-maturity period, varied extensively per month and per years.[10] Figure 3 shows the (monthly) correlations among one-month price deviates of corn and soybean meal with “average” near-to-maturity six months. Large differences in correlations not only per month but through the years are observable and are expected, given the substantial different yearly yields of corn during this period.[11] Likewise, Figure

[10] Class III and Class IV (milk), corn, and soybean meal are the commodities futures prices whose (one-month) deviations are modeled by taking into account time to maturity of one month up to 15 months (i.e., 15 different models). The data considered here is from the 28th or 30th of September in order to obtain a valid comparison of forecasts with respect to the web-tool projections dated the same days.
[11] These and other variable dynamic correlations are subjects of different studies.

4 shows the dynamic correlations of Class III milk and corn one-month price deviates with "average" near-to-maturity of 15 months,[12] and arrive at similar results (i.e., there is sizeable monthly and yearly variability of the correlations among these series.)

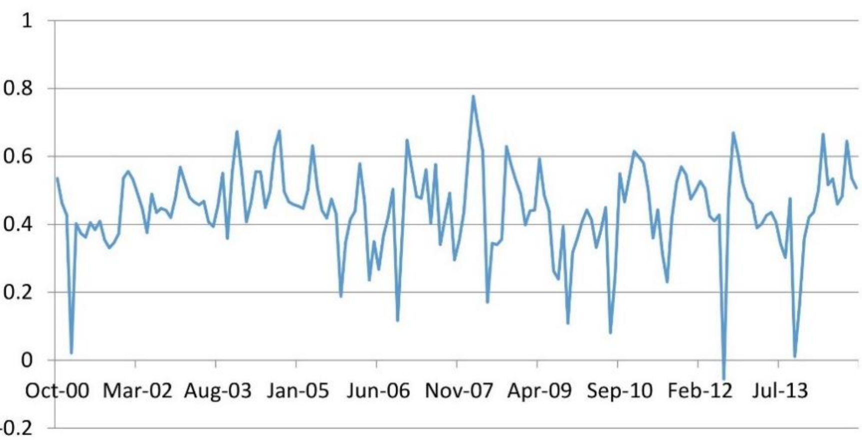

Figure 3: Dynamic Monthly correlations between one-month price deviates of the futures contracts of Corn with 'average' near-to-maturity of 6 months, and futures contract of Soybean Meal with 'average' near-to-maturity of 6 months.

We then applied a DCC Gaussian copula and a DCC Student-t copula to each of our joint series, separated by time to maturity. That is, for the first month to maturity, we estimated the two DCC copulas considering the marginals of the four commodities' price deviations previously modeled as DCC. We then calculated the two copulas considering price deviations with two months-to-maturity and repeated these copula estimations for the joint series up until 15 months-to-maturity. Once we modeled the two DCC copulas for each time to maturity, we opted for the Student-t since it arrived at a slightly lower Akaike Information Criterion and Bayesian Information Criterion (AIC and BIC) coefficient.

[12] We considered the weighted average of futures prices from contracts of nearby months for those months where the commodities had no contracts.

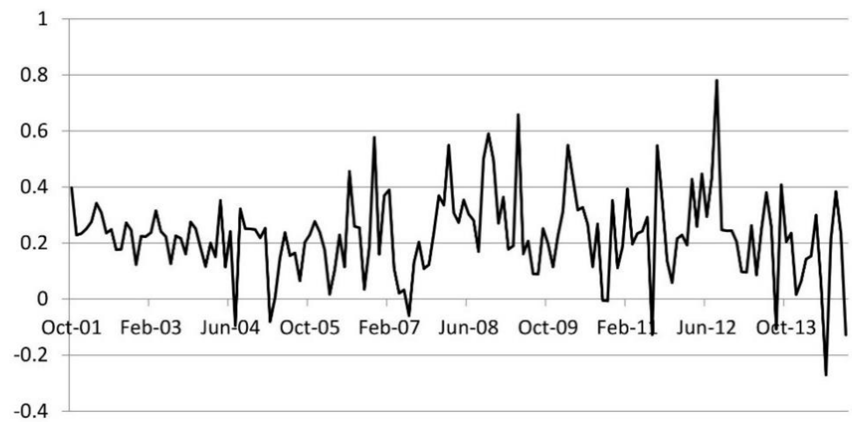

Figure 4: Dynamic Monthly correlations between one-month price deviates of the futures contracts for Milk Class III with 15 months' time to maturity, and futures contract of corn with 'average' near-to-maturity of 15 months.

We simulated 5,000 different one-month futures price deviates for each of the 15 modeled copulas and added these to each corresponding expected futures price according to their time to maturity (e.g., the simulated one-month price deviates of the dynamic correlations of commodities having four months' time to maturity was added to the expected futures prices with four months of time to maturity and so on). Thus we obtained 5,000 simulated futures prices for each commodity, depending on the time to maturity horizon, resulting in each commodity having 15 different sets of 5,000 simulated prices (one for each month to maturity, from October to December of the following year). With these futures prices, we then calculated our estimated cash prices by using the parameters from Table 2 of the appendix from Newton, Thraen, and Bozic (2015).

As mentioned previously (and in footnote 10), we applied our method to a data set that considered prices at the end of a month (similar to that of the projections in the web tool method) i.e., September, 28 or 30, in order to obtain forecasts that can directly be compared with those from the web tool. Moreover, as mentioned, this is the latest projection that producers can obtain before their deadline for signing a contract covering the coming year. We acknowledge that actual enrollment deadlines in 2014 and 2015 being delayed into November or December (footnote 4) may cause an interest in the estimation of margin projections beginning at that later date of the year. While this is an appealing endeavor as projections would have changed with the arrival of more recent information and most likely increased their accuracy, it would impede the study's purpose of comparing forecasts and its effects between this new proposed method and the current method. The study of these "later dated" projections, however, may be addressed in future work.

To compare the forecasts from the current web tool and those from our approach, we take into account different metrics broadly used in the literature aimed at capturing various aspects of the forecasting ability of these two methods. We apply the mean error (ME), the mean absolute error (MAE), and the root mean square error (RMSE). The ME, MAE, and RMSE are measured per equations (9), (10), (11), respectively:

(10) $$\text{ME} = \frac{1}{n}\sum_{i=1}^{n}(f_i - y_i)$$

(11) $$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|f_i - y_i|$$

(12) $$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f_i - y_i)^2}$$

where f_i is the forecasted value and y_i the actual value. Each of these metrics identifies the "closeness" of the forecast in regard to the actual value and, thus, enables us to compare the forecast error among the models. In particular, the ME is a (simple) calculation of the forecast bias; however, it permits positive and negative effects to balance out each other. The MAE controls for this positive/negative balance-out effect in a proportionate manner. The RMSE, likewise, controls for this effect but emphasizes large forecast errors. We apply these metrics to the forecasts made after the first three months (i.e., December) in order to gauge their initial accuracy, and then to the months in the following year where the program exercises its policy (i.e., February, April, June, August, October, and December).

Results of the forecasted margins taking into account the (forecast) error metrics are in Table 2 corresponding to the years 2008 and 2009, 2010 and 2011, 2012 and 2013, and 2014.[13] From Table 2 for years 2008 and 2009, we can see that, in some of the forecasted margins for the respective months, the error measurements indicate that the new forecasted method performs relatively similar or with improvements in regards to the actual method. This is the case for the latter months of 2008, but more so for 2009 where, from February onwards, there is a decrease in the values of forecasted errors

[13] Despite calculating our method with data available at the time—from September 2000 until June 2015—we present results up until 2014. Results from January 2015 to June 2015 are readily available and tend to resemble qualitative findings of previous years.

measurements in comparison to the current method, though both methods are off the mark. Results for 2010 and 2011 are mixed. Here we can see that, for 2010, the current method performs better; however, the measurements determine that the overall differences with respect to the actual margin values are rather small—in comparison to most other years—as seen in MAE and RMSE. Moreover, the actual margin is generally higher than projected, with this new method covering the down-side risk as seen in Figure 5. Conversely, for year 2011, measurements indicate the new method being better after the first month; however, for this year, both forecasts under-estimated the real margin value which, from February onwards, surpassed the minimum level of payout (i.e., producers had a better year than previously anticipated.)

Results are an improvement under the new method for the years 2012 and 2013, except for the first months of 2012 and last months of 2013, as seen in Table 2. In the case of 2012, the new method arrives at improved results over the current one after the first quarter of the year. So, for five of the six months where the policy is evaluated for payments, the new method shows improvement as seen in Figure 6. For 2013, the new method has improved or has similar results again in five of the six months where the policy is evaluated; yet in this case, it is the last quarter which does not show improvement. For 2014, the new method obtains less favorable results in comparison to the current method as seen in the measurements. However, similar to year 2011, the real margin values in 2014 were much higher than the anticipated projected margins and the minimum level for payment. It is relevant to note that, when comparing margin forecasts between the new method and the current method as illustrated in Figures 5 and 6, the new method may create non-smooth forecasts. This is attributable to the new method's calculation of price deviates which (as mentioned in the methodology section) considers one-month deviations of each futures price at a certain date, in contrast to the current method that accounts for differences of one-month, two-months, and up to 15 months of a futures price according to its corresponding terminal (expiration) price. In our method, these monthly price deviates may change quite a bit at certain months as information is becomes available.

Table 3. Net Expected Benefits per cwt.

	2008						**2009**					
	Premium if < 4 mil lbs.			Premium if > 4 mil lbs.			Premium if < 4 mil lbs.			Premium if > 4 mil lbs.		
Cover Level	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benfit	Current Net Benfit	New Net Benefit	Actual Net Benfit	Current Net Benfit	New Net Benefit	Actual Net Benefit	Current Net Benfit	New Net Benefit
4.00	0.000	0.000	0.000	0.000	0.000	0.000	2.537	0.000	0.000	2.537	0.000	0.000
4.50	(0.010)	(0.010)	(0.010)	(0.020)	(0.020)	(0.020)	4.527	(0.010)	(0.010)	4.517	(0.020)	(0.020)
5.00	(0.025)	(0.025)	(0.025)	(0.040)	(0.040)	(0.040)	6.512	(0.025)	(0.025)	6.497	(0.040)	(0.040)
5.50	(0.040)	(0.040)	(0.040)	(0.100)	(0.100)	(0.100)	8.497	(0.040)	(0.040)	8.437	(0.100)	(0.100)
6.00	(0.055)	(0.055)	(0.055)	(0.155)	(0.155)	(0.155)	10.597	(0.055)	(0.055)	10.497	(0.155)	(0.155)
6.50	(0.090)	(0.090)	(0.090)	(0.290)	(0.290)	(0.290)	13.062	(0.090)	(0.002)	12.862	(0.290)	(0.202)
7.00	(0.217)	(0.217)	(0.217)	(0.830)	(0.830)	(0.830)	15.435	(0.217)	1.875	14.822	(0.830)	1.262
7.50	(0.300)	(0.300)	(0.300)	(1.030)	(1.030)	(1.030)	17.852	0.897	4.792	17.122	0.167	4.062
8.00	(0.135)	(0.475)	(0.475)	(1.020)	(1.360)	(1.360)	20.177	2.533	7.617	19.292	1.648	6.732

	2010						**2011**					
	Premium if < 4 mil lbs.			Premium if > 4 mil lbs.			Premium if < 4 mil lbs.			Premium if > 4 mil lbs.		
Cover Level	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benfit	Current Net Benfit	New Net Benefit	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benefit	Current Net Benfit	New Net Benefit
4.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.50	(0.010)	(0.010)	(0.010)	(0.020)	(0.020)	(0.020)	(0.010)	(0.010)	(0.010)	(0.020)	(0.020)	(0.020)
5.00	(0.025)	(0.025)	(0.025)	(0.040)	(0.040)	(0.040)	(0.025)	(0.025)	(0.025)	(0.040)	(0.040)	(0.040)
5.50	(0.040)	(0.040)	(0.040)	(0.100)	(0.100)	(0.100)	(0.040)	(0.040)	(0.040)	(0.100)	(0.100)	(0.100)
6.00	(0.055)	(0.055)	(0.055)	(0.155)	(0.155)	(0.155)	(0.055)	(0.055)	(0.055)	(0.155)	(0.155)	(0.155)
6.50	(0.090)	(0.090)	(0.090)	(0.290)	(0.290)	(0.290)	(0.090)	0.177	(0.090)	(0.290)	(0.023)	(0.290)
7.00	(0.217)	(0.217)	(0.217)	(0.830)	(0.830)	(0.830)	(0.217)	1.513	0.057	(0.830)	0.900	(0.556)
7.50	(0.135)	(0.300)	(0.057)	(0.865)	(1.030)	(0.787)	(0.300)	3.344	1.837	(1.030)	2.614	1.107
8.00	0.511	(0.432)	0.960	(0.374)	(1.317)	0.075	(0.310)	6.000	4.421	(1.195)	5.115	3.536

	2012						**2013**					
	Premium if < 4 mil lbs			Premium if > 4 mil lbs			Premium if < 4 mil lbs			Premium if > 4 mil lbs		
Cover Level	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benfit	Current Net Benfit	New Net Benefit	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benfit	Current Net Benfit	New Net Benefit
4.00	1.698	0.000	0.000	1.698	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.50	2.688	(0.010)	(0.010)	2.678	(0.020)	(0.020)	(0.010)	(0.010)	0.025	(0.020)	(0.020)	0.015
5.00	4.082	(0.025)	(0.025)	4.067	(0.040)	(0.040)	(0.025)	(0.025)	0.510	(0.040)	(0.040)	0.495
5.50	5.567	(0.040)	0.302	5.507	(0.100)	0.242	(0.040)	(0.040)	1.483	(0.100)	(0.100)	1.423
6.00	7.052	(0.055)	2.725	6.952	(0.155)	2.625	0.841	(0.055)	3.723	0.741	(0.155)	3.623
6.50	8.517	(0.090)	5.661	8.317	(0.290)	5.461	2.750	0.231	6.251	2.550	0.031	6.051
7.00	10.558	(0.217)	8.534	9.945	(0.830)	7.921	4.623	1.441	9.124	4.010	0.828	8.511
7.50	12.975	0.710	11.451	12.245	(0.020)	10.721	6.540	3.834	12.041	5.810	3.104	11.311
8.00	15.658	2.570	14.276	14.773	1.685	13.391	8.365	6.659	14.866	7.480	5.774	13.981

	2014					
	Premium if < 4 mil lbs.			Premium if > 4 mil lbs.		
Cover Level	Actual Net Benfit	Current Net Benefit	New Net Benefit	Actual Net Benefit	Current Net Benefit	New Net Benefit
4.00	0.000	0.000	0.000	0.000	0.000	0.000
4.50	(0.010)	(0.010)	(0.010)	(0.020)	(0.020)	(0.020)
5.00	(0.025)	(0.025)	(0.025)	(0.040)	(0.040)	(0.040)
5.50	(0.040)	(0.040)	(0.040)	(0.100)	(0.100)	(0.100)
6.00	(0.055)	(0.055)	0.364	(0.155)	(0.155)	0.264
6.50	(0.090)	(0.090)	1.782	(0.290)	(0.290)	1.582
7.00	(0.217)	(0.217)	3.819	(0.830)	(0.830)	3.206
7.50	(0.300)	(0.300)	6.236	(1.030)	(1.030)	5.506
8.00	(0.475)	(0.475)	8.730	(1.360)	(1.360)	7.845

We also compute the expected net benefits that would result from our (new) method and compare them with the expected net benefits from the current method. These results are presented in Table 3. Table 3 leaves out the $100 fee covering catastrophic or CAT level (i.e., 4 $/cwt) and considers premium costs ($/cwt) for each margin threshold, in the case of supplementary coverage (i.e., 4.50 to 8.00 $/cwt). Moreover, the table partitions for marginal net benefits ($/cwt) received if production coverage is up to 4 million lbs./year or, in the case of the amount of insured production above that mark. The equations used to calculate the net expected benefits are:

$$(13) \qquad \pi_{m,i} = \max(0, y_{m,i}) - c_{2,i}$$

where $\pi_{m,i}$ is the net expected benefit from buy-up MPP protection (per unit of production; i.e., $/cwt), m indicates the different coverage levels selected (i.e., from 4.50 $/cwt to 8.00 $/cwt), i indicates the amount of production history coverage (i.e., < or > 4 million lbs.) affecting the premium rate,[14] $y_{m,i}$ is the indemnity payment to the producer: $y_{m,i} = \bar{y}_{m,i} - (p - c_1)$, $\bar{y}_{m,i}$ is the selected margin coverage under the amount of production covered i, $p - c_1$ is the national margin, and $c_{2,i}$ is the MPP premium as f(selected coverage level, amount of production covered).

The table indicates the net expected benefits (per cwt) for each level of coverage, and considering either a premium for up to 4 million lbs. of insured annual production or in excess of that. As seen for year 2008, neither the "current" forecasting method nor the "new" forecasting methods provide positive indemnity at the 8.00 $/cwt level in contrast to results observed in the "actual" level. For 2009, the new method provides positive indemnities after the 6.50 $/cwt level (i.e., at the 7 $/cwt) before the current method does at the 7.50 $/cwt. However, both methods are far off since the results from actual levels reach positive indemnity starting at the 4 $/cwt mark.

For 2010, the new method provides positive indemnity at the 8 $/cwt level just like the resulting actual margin does; however, the current method does not. If the premium considers more than 4 million lbs. insured, then the case is reversed. For 2011, the actual results do not provide any positive indemnities; however, the new method would provide positive net benefits at the 7.00 $/cwt or 7.50 $/cwt level in case of insuring less than or more than 4 million lbs, respectively. The current method would provide positive indemnity for 2011 before that, at the 6.50 $/cwt or 7.00 $/cwt, respectively. For 2012, actual positive net benefits are received from the 4 $/cwt level and upward. Here the new method would provide positive net benefits at the 5.50 $/cwt level and upward in contrast

[14] In the case of insuring over 4 million lbs./year, the higher premium is applied only to the difference between the insured production amount and 4 million lbs.

to the current level that provides positive net benefits only at 7.50 $/cwt or 8.00 $/cwt in case of insuring less than 4 million lbs. or more, respectively. For 2013, actual positive net benefits are received from 6.00 $/cwt and upwards. Here the new method begins providing positive net benefits from the 4.50 $/cwt level, and the current method does so from the 6.50 $/cwt level. For 2014, the new method would provide net benefits from the 6.00 $/cwt level upward; however, there are no positive net benefits from actual results nor from the current method.

Thus, as mentioned, results are somewhat favorable in regards to forecasting accuracy in some years, though not in others. This is also the case for expected net positive benefits considering coverage of (historical) production since there are years where the new method is closer to being in line to actual results, and other years where the current method is. We believe the new method achieves the purpose of offering an additional forecasting tool that provides pertinent stakeholders, i.e., dairy producers, risk managers, and policy makers with an additional perspective of future values to consider when selecting their level of insurance coverage. In this case, this forecasting tool may serve as a complement to the existing one.

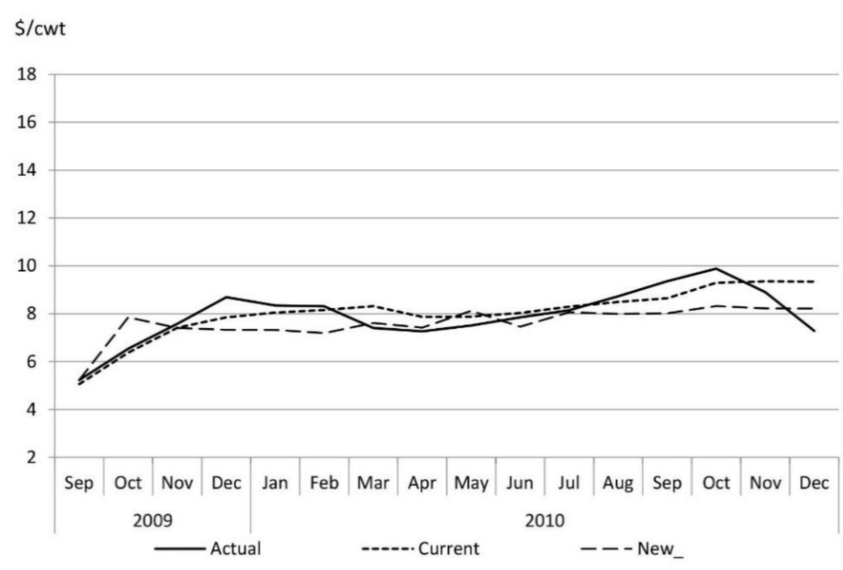

Figure 5: Dairy Margin Forecasts from September 30, 2009, for year 2010.

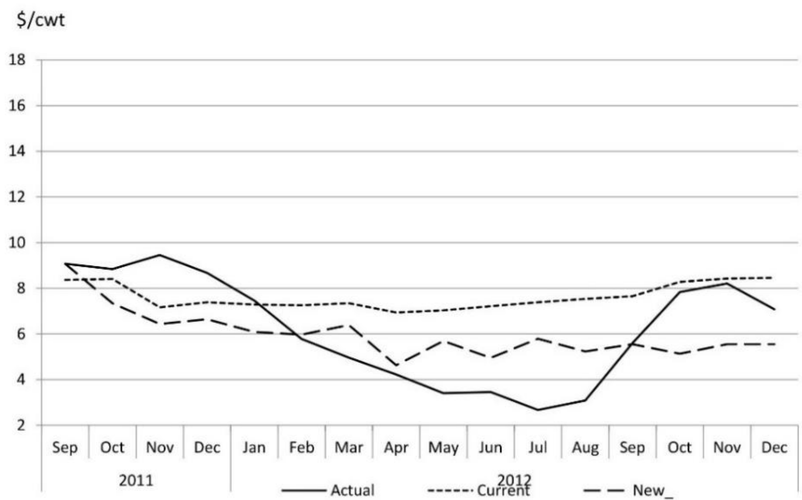

Figure 6: Dairy Margin Forecasts from September 30, 2011, for year 2012.

Conclusions

This study compares the (forecasting) performance of two methods that predict the dairy margin from the new dairy MPP in the 2014 Farm Bill. The margin consists of the difference between milk and feed costs, the latter involving prices of corn, soybean meal, and alfalfa. The current method forecasts the margin by applying a method that maintains the estimated static (rank) correlation among shocks to the futures prices of these commodities according to their time to maturity (excluding alfalfa which does not have futures contracts). The new, proposed method accounts for two differences: (1) it considers the dynamic correlations among shocks to the futures prices, accounting for the different relationships among the commodities at distinct periods of the year; and (2) it considers one-month shocks to the futures prices, and distinguishes among each commodity's shocks according to their initial time to maturity.

Results indicate a relative improvement in monthly forecasts for several of the years estimated, in accordance with forecast error measurements. Periods where there may not have been improvements were usually characterized by having relatively minor differences in regard to the actual margin or where the actual margin was much higher than anticipated, thus not impacting the payout application of the policy. In addition, a few of the monthly forecasts of the new method arrived at values which improved covering the downside risk in comparison to the current (present) model's values.

This new method of forecasting the dairy margin provides a dynamic approach in obtaining forecasted dairy margin values. It may serve as a useful comparison and complement to the current method used, and provide beneficial information to parties having a stake in the dairy MPP. Going forward, more data (degrees of freedom) will be available which should improve the precision of this proposed procedure. Likewise, incorporating the relationship of alfalfa prices and the closing futures prices of the other commodities, as well as considering these relationships in estimating the commodities' cash prices, may assist in improving the outcome. An additional avenue for comparing the present and proposed methods of forecasting may be conducted by pricing the private Revenue-Over-Feed products, as offered by the Ag Hedge Desk (http://www.aghedgedesk.com/?page_id=27). This would provide additional insight into the differences produced by accounting for static or time-varying correlations among the respective price deviations of each method. This is another interesting, related topic left for future research.

References

Ausin, M.C., and H.F. Lopes. (2010). "Time-varying joint distribution through copulas", *Computational Statistics and Data Analysis*. 54:2383-2399.

Bozic, M., J. Newton, C.S. Thraen, and B.W. Gould. (2014). "Tails Curtailed: Accounting for Non-Linear Dependence in Pricing Margin Insurance for Dairy Farmers." *American Journal of Agricultural Economics* 96, 4:1117-1135.

Chollete, L., A. Heinen, and A. Valdesogo. (2009). "Modeling international financial returns with multivariate regime-switching copula. *Journal of Financial Econometrics* 7, 4:437-480.

Cox, J.C., S.A. Ross, and M. Rubinstein. (1979). "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7:229-263.

Engle, R.F. (2002). "Dynamic conditional correlation —a simple class of multivariate GARCH models." *Journal of Business and Economic Statistics* 20:339-350.

Ghalanos, A. (2014). "The rmgarch models: Background and properties. (version1.2-6)." Available at http://cran.r-project.org/web/packages/rmgarch/index.html.

Goodwin, B.K., and A. Hungerford. (2015). "Copula-Based Models of Systemic Risk in U.S. Agriculture: Implications for Crop Insurance and Reinsurance Contracts." *American Journal of Agricultural Economics* 97, 3:879-896.

Gould, B. (2015). "Understanding Dairy Markets – Your Source for Market Information and Price Risk Management Principles." Available at http://future.aae.wisc.edu/.

Iman, R.L., and W.J. Conover. (1982). "A Distribution-Free Approach to Inducing Rank Correlation Among Input Variables." *Communications in Statistics - Simulation and Computation* 11:311-334.

Irwin, S., and D. Good. (2015, February 4). "Long-Term Corn, Soybeans, and Wheat Price Forecasts and the Farm Bill Program choice." *farmdoc daily* (5):20, Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign.

Jesse, E., and R. Cropp. (2001). "Federal milk marketing order reform (continued)." Marketing and Policy Briefing Paper No. 73. Madison (WI): Department of Agricultural and Applied Economics, University of Wisconsin–Madison.

Mildenhall, S.J. (2006). "Correlation and Aggregate Loss Distributions with an Emphasis on the Iman-Conover Method." Report of the Research Working Party on Correlations and Dependencies among All Risk Sources. 2005. Casualty Actuarial Society; Winter Forum. Available at http://www.casact.org/pubs/forum/06wforum/06w105.pdf.

Miranda, M.J., and P.L. Fackler. (2002). Applied Computational Economics and Finance. Cambridge: MIT Press.

Nelsen, R.B. (1999). An Introduction to Copulas, Lecture Notes in Statistics, 139. New York:Springer-Verlag.

Newton, J., C.S. Thraen, and M. Bozic. (2015) "Evaluating Policy Design Choices for the Margin Protection Program for Dairy Producers: An Expected Indemnity Approach." *Applied Economic Perspective and Policy*. doi:10.1093/aepp/ppv033.

Newton, J., and C. Nicholson. (2014). "Milk, Feed and MPP Margin Forecasting." Program on Dairy Markets and Policy. Available at http://dairymarkets.org/MPP/PowerPoint/TtT%20Newton%20Nicholson.pptx.

Patton, A.J. (2006a). "Estimation of multivariate models for time series of possibly different lengths." *Journal of Applied Econometrics* 21:147-173.

Patton, A.J. (2006b). Modeling asymmetric exchange rate dependence. *International Economic Review* 47, 2:527-556.

Phoon, K., S.T. Quek, and H. Huang. (2004). "Simulation of Non-Gaussian Processes Using Fractile Correlation." *Probabilistic Engineering Mechanics* 19:287-292.

Power, G.J., and D.V. Vedenov. (2008, April 21). "The shape of the optimal hedge ratio: Modeling joint spot-futures prices using an empirical copula-GARCH model." In Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. St Louis, Mo.

Sklar, A. (1959). Fonctions de repartition a n dimensions et leur marges. *Publications de l'Institut de Statistique de l'Universite de Paris*, 8:229-231.

Tejeda, H.A., and B.K. Goodwin. (2008, July 28). "Modeling Crop prices through a Burr distribution and Analysis of Correlation between Crop Prices and Yields using Copula method." Selected Paper Presentation at American Agricultural Economics Association Annual Meeting, Orlando, Fla.

U.S. Department of Agriculture, Notice MPP-1, 2014: Agricultural Act of 2014 (2014 Farm Bill), Title I Information About the Margin Protection Program for Dairy Producers (MPP-Dairy.) Available at http://dairymarkets.org/MPP/ResourceMaterial/mpp_1.pdf.

Vedenov, D. (2008). "Application of copulas to estimation of joint crop yield distributions." Selected Paper, American Agricultural Economics Association Annual Meeting, Orlando, Fla.

Westhoff, P. (2015, February 12). "Price Projections and Farm Bill Program Choices: Adding FAPRI-MU Projections to the Mix." *farmdoc daily (5)*:26, Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign.

Woodard, J.D., N.D. Paulson, D. Vedenov, and G.J. Power. (2011). "Impact of copula choice on the modeling of crop yield basis risk." *Agricultural Economics,* 42:101-112.