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Spatially Smoothed Crop Yield Density Estimation: Physical Distance versus Climate Similarity

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Many crop insurance studies have pointed out that considering spatial yield similarity can help provide more precise premium rating. We use Bayesian Kriging for spatial smoothing to consider such similarities when estimating crop yield densities. This article's innovation is that the spatial smoothing is based on climate space, which is composed of climatological measures. We compare the climate-space smoothing with a general physical space (longitude–latitude space) smoothing. The test results are favorable to the proposed climate-smoothing method. Climate smoothing performs particularly well in states that have many missing counties and varied climate due to varying topography.

Key words: Bayesian hierarchical structure, Bayesian Kriging, Bayesian spatial smoothing, corn, crop insurance, rainfall, spatial dependence, temperature

Introduction

Crop yield densities are spatially similar due to nearby locations having similar soil and weather. As previous literature (Annan et al., 2014; Du et al., 2015; Goodwin and Hungerford, 2015; Ker, Tolhurst, and Liu, 2016; Park, Brorsen, and Harri, 2019) has found, considering data from nearby locations when estimating crop yield densities could help provide more accurate crop insurance premium rates and thus reduce adverse selection. Since U.S. crop insurance is subsidized, more accurate densities can also reduce the political problem of some locations receiving more subsidies than others.

Several statistical methods have been used to include spatial similarity when estimating crop yield density. Goodwin and Ker (1998) use pooled observations from surrounding counties. Ozaki, Goodwin, and Shirota (2008) use a spatial weighting matrix and impose uniform weights on parameter estimates from surrounding counties with no weight on other counties. Ozaki and Silva (2009) also impose weights of 0 beyond surrounding counties. Current area-based crop insurance programs are rated with a model suggested by Harri et al. (2011). To reflect the spatial structure, their model imposes a district level restriction on county level parameters. Studies based on Bayesian model averaging (BMA) estimate a conditional yield density for a target county and then use Bayesian averaging of its own conditional density and densities from other counties (Ker, Tolhurst, and Liu, 2016; Ker and Liu, 2017; Liu and Ker, 2020). Zhang (2017) shrinks an individual county's density toward the density of a pooled model, which is similar to the applications of BMA in that it does not use a measure of distance. Park, Brorsen, and Harri (2019) suggest Bayesian Kriging as

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a method for spatial smoothing. They produce spatially smoothed density parameter estimates that vary smoothly over longitude–latitude space (i.e., physical space).

These past methods measured distance by contiguity or physical distance or—in the case of BMA—did not consider distance at all. We extend previous literature by using climatological coordinates, which is a climate space, to measure distance. Climate space potentially offers greater precision when physical distance is not a good measure of climate similarity and when there are many locations with inadequate data. We follow Park, Brorsen, and Harri (2019) by using the Bayesian Kriging method for spatial smoothing. The climate space uses temperature and precipitation as coordinates rather than latitude and longitude. We use two climate data sources as well as various climatological measures to define two- and three-dimensional climate spaces. Our focus is on how the distribution of crop yield varies over different types of spaces (physical and climate spaces). We compare the ability of the approaches to provide premiums that give the same expected relative payout across space using out-of-sample tests.

We choose corn as a crop for evaluating the ability of our approach to produce accurately rated crop insurance premiums. We use annual county-level yield data from the U.S. Department of Agriculture National Agricultural Statistics Service (NASS) for Iowa, Illinois, Nebraska, Minnesota, Indiana, and Colorado. Colorado was chosen because of its unique topography and climate, while the others were chosen to represent major corn-producing states. Climate-space smoothing should offer little advantage where climate varies gradually across space, but it should be more accurate than using physical distance in Colorado, Nebraska, or other states with multiple geographical features or many missing counties.

Bayesian Kriging

Bayesian Kriging for spatial smoothing is a geostatistical spatial interpolation method used in a broad variety of disciplines. The method assumes that the similarity of probability density functions smoothly decreases with the distance between locations.

Overview of the Bayesian Hierarchical Structure

Our Kriging method has a Bayesian hierarchical structure. With a Bayesian hierarchical model, the Bayesian modeling structure is written in multiple levels (i.e., hierarchies). In a Bayesian hierarchical framework, a prior distribution of the general Bayesian model can be structured as additional prior parameters, called hyperpriors.

We consider two specifications of the process layer: a general Gaussian spatial process (GP) and an autoregressive with Gaussian spatial process (AR).[1] GP only considers a spatial structure, whereas AR has both spatial and temporal structure (spatio-temporal). The Bayesian hierarchical structure has three layers: likelihood layer, process layer, and prior layer. In the likelihood layer, the crop yield distribution for each county is a normal distribution in which the random effects are taken as given. The process layer contains the distribution of the spatial random effects as well as deterministic mean effects. The spatial smoothing is in the process layer. The third layer of the hierarchy consists of the priors. This hierarchy is

$$\begin{aligned} \boldsymbol{Y}|\boldsymbol{\mu},\boldsymbol{\Theta} &\sim p_1(\boldsymbol{Y}|\boldsymbol{\mu},\boldsymbol{\Theta}); \\ \boldsymbol{\mu}|\boldsymbol{\Theta} &\sim p_2(\boldsymbol{\mu}|\boldsymbol{\Theta}); \\ \boldsymbol{\Theta} &\sim p_3(\boldsymbol{\Theta}); \end{aligned} \tag{1}$$

[1] The Gaussian spatial process is a stochastic process in which any finite subcollection of random variables (i.e., mean parameters of any set of counties) is multivariate normally distributed. The covariance of those random variables between any two locations (i.e., covariance between μ_i and μ_j) is determined by a Euclidean distance between two locations and spatial covariance matrix Σ.

where p_1, p_2, and p_3 are the densities associated with each layer of the hierarchy, likelihood layer, process layer, and prior layer; $\boldsymbol{Y}$ is a matrix of crop yield observations that spans all counties $(n = 1, \dots, N)$ and all years $(t = 1, \dots, T)$; $\boldsymbol{\mu}$ is a matrix (or vector for GP model) of random parameters; and $\boldsymbol{\Theta}$ is a vector of hyperparameters, where $\boldsymbol{\Theta} = \left[\beta_1, \dots, \beta_K, \omega, \theta, \rho, \sigma^2\right]'$. By Bayes' theorem, the joint posterior distribution is proportional to the multiplication of the three layers:

$$p(\boldsymbol{\mu}, \boldsymbol{\Theta}|\boldsymbol{Y}) \propto p_1(\boldsymbol{Y}|\boldsymbol{\mu}, \boldsymbol{\Theta})\, p_2(\boldsymbol{\mu}|\boldsymbol{\Theta})\, p_3(\boldsymbol{\Theta}). \tag{2}$$

Likelihood Layer

A likelihood function of the crop yield distribution forms the first layer of the model. Let y_{it} be the crop yield of county i at year t, where $i = 1, \dots, N$ and $t = 1, \dots, T$. We assume that y_{it} is decomposed as $y_{it} = \mu_{it} + \varepsilon_{it}$, where μ_{it} is a county-specific mean of the county i at year t, and ε_{it} is an independently identically distributed error $\varepsilon_{it} \sim n\left(0, \sigma^2\right)$. Both GP and AR assume normality of crop yield distributions. The likelihood layer is

$$p_1(\boldsymbol{Y}|\boldsymbol{\mu}, \boldsymbol{\Theta}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{(\boldsymbol{y}_t - \boldsymbol{\mu}_t)'(\boldsymbol{y}_t - \boldsymbol{\mu}_t)}{2\sigma^2}, \tag{3}$$

where $\boldsymbol{y}_t$ denotes a vector of crop yield at year t that spans all counties, $\boldsymbol{y}_t = [y_{1t}, \dots, y_{Nt}]'$, and $\boldsymbol{Y} = [\boldsymbol{y}_1, \dots, \boldsymbol{y}_T]$; $\boldsymbol{\mu}_t$ is a column of $\boldsymbol{\mu}$ so $\boldsymbol{\mu}_t = [\mu_{1t}, \dots, \mu_{Nt}]'$; and $\boldsymbol{\Theta}$ is a vector of hyperparameters, $\boldsymbol{\Theta} = \left[\beta_1, \dots, \beta_K, \omega, \theta, \rho, \sigma^2\right]'$.

Process Layer

The second layer provides the spatial processes. The spatial correlation is in the mean for each location and not in the error term. The GP has spatial random effects. The AR has spatial random effects and deterministic temporal effects. Since we assume a Gaussian spatial process, the distributions are multivariate normal.[2] The spatial dependency is measured from a spatial weight matrix that depends on the Kriging parameters.

The autoregressive model with a Gaussian spatial process (AR) model is

$$\begin{aligned} \boldsymbol{\mu}_t &= \omega \boldsymbol{\mu}_{t-1} + \boldsymbol{\psi}_t + \boldsymbol{\eta}, \\ \boldsymbol{\psi}_t &= \boldsymbol{X}_t \boldsymbol{\beta}, \\ \boldsymbol{\eta} &\sim MVGP(\mathbf{0}, \Sigma), \\ \Sigma &= \begin{bmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & \ddots & \vdots \\ w_{N1} & \cdots & w_{NN} \end{bmatrix}, \end{aligned} \tag{4}$$

where $\boldsymbol{\mu}_t$ is a vector of mean parameters for counties at year t, $\boldsymbol{\mu}_t = [\mu_{1t}, \dots, \mu_{Nt}]'$; $\boldsymbol{\psi}_t$ is a deterministic part of the mean structure at year t (linear trend for example); $\boldsymbol{X}_t$ is a $N \times K$ matrix of explanatory variables at year t; $\boldsymbol{X}_t = [\boldsymbol{x}_{t1}, \dots, \boldsymbol{x}_{tK}]$; $\boldsymbol{\beta}$ is a $K \times 1$ vector of coefficients of explanatory variables, $\boldsymbol{\beta} = [\beta_1, \dots, \beta_K]'$; $\boldsymbol{\eta}$ is the time-independent Gaussian spatial process term that generates spatial smoothing, $\boldsymbol{\eta} = [\eta_1, \dots, \eta_N]'$, that is assumed to follow an exponential type spatial weight matrix Σ, where $\Sigma_{ij} = \rho e^{-D_{ij}/\theta}$, which is a function of distance, D_{ij}, between counties i and j on the

[2] Under the Bayesian framework, the vector of a stochastic part of mean parameters across the locations $\boldsymbol{\eta}$ is a vector of random variables. When the random vector $\boldsymbol{\eta} = (\eta_1, \dots \eta_N)^T$ follows a Gaussian spatial process, the mean parameters of the counties $\eta_1, \dots \eta_N$ are jointly normally distributed.

corresponding spaces (physical and climate), sill parameter ρ, and range parameter θ. The parameter of the first-order autoregressive process ω is assumed to be in the interval $-1 < \omega < 1$. When $\omega = 0$, AR is the same as the GP model. Maximum likelihood methods typically integrate out the random effects, $\boldsymbol{\eta}$, but the Bayesian approach estimates a distribution for them that is spatially correlated and thus the mean varies across space.

Then the process layer densities for the AR model can be specified as

$$(5) \qquad p_2(\boldsymbol{\mu}_t|\boldsymbol{\Theta}) = \frac{1}{\sqrt{(2\pi)^N |\Sigma|}} \exp\left[-\frac{1}{2}\left(\boldsymbol{\mu}_t - \omega\boldsymbol{\mu}_{t-1} - \boldsymbol{\psi}_t\right)' \Sigma^{-1}\left(\boldsymbol{\mu}_t - \omega\boldsymbol{\mu}_{t-1} - \boldsymbol{\psi}_t\right)\right],$$

where $\boldsymbol{\psi}_t$ is a vector of the deterministic part of the mean process at time t defined by equation (4) and $\boldsymbol{\Theta} = \left[\beta_1, \ldots, \beta_K, \theta, \rho, \sigma^2\right]'$ is a vector of parameters; $\boldsymbol{\Sigma}$ is spatial covariance matrix, where $\Sigma_{ij} = \rho e^{-D_{ij}/\theta}$. Since we are in the Bayesian framework, the parameters are treated as random variables. We impose independent priors for the hyperparameters $(\beta_1, \ldots, \beta_K, \theta,\ \rho,\ \sigma^2)$ in the following prior layer. As explained by Park, Brorsen, and Harri (2019), the GP process is defined similarly, except $\omega = 0$.

Prior Layer

The third layer of the hierarchy has the priors for the parameters $\boldsymbol{\Theta}$, which are coefficients for explanatory variables, Kriging parameters, and the variance parameter in the process layer. Since the model assumes the parameters in the prior layer are independent, multiplying the individual priors gives the joint density for the prior layer. For convenience, we group hyperparameters into three different types depending on their role in the process layer: coefficient, variance, and Kriging parameters. First, all coefficient parameters $\beta_1, \ldots, \beta_k$ are given $N\left(0, 10^4\right)$ priors. For the variance parameter σ^2, we impose general inverse gamma priors $IG(0.1, 0.1)$ as Ozaki, Goodwin, and Shirota (2008) did. However, imposing priors for the Kriging parameters (ρ, θ), which describe the spatial structure of the Gaussian spatial process, is more problematic. The Bayesian statistics literature (Berger, De Oliveira, and Sansó, 2001; Banerjee, Carlin, and Gelfand, 2004; Cooley, Nychka, and Naveau, 2007) regarding consistency of proper priors for the Kriging parameters argues that improper priors for such parameters may induce improper posteriors. Some statistics literature (Banerjee, Carlin, and Gelfand, 2004; Sahu, Gelfand, and Holland, 2010; Cooley, Nychka, and Naveau, 2007) suggests an empirical Bayes method where the Kriging priors are estimated from the data to give proper priors. We use the empirical information to find the priors of the Kriging parameters. We first estimate the mean parameter of each county using maximum likelihood. Then, using the estimated MLE parameters for each county, we fit the empirical variogram.[3] The empirical variogram is used to specify an inverse gamma prior for the sill parameter ρ since the value of the sill parameter determines the maximum of the variogram.

The next step is to find the prior distributions for the range parameter θ that determines the maximum distance of the spatial effect. We use empirical distance information to impose the prior of the range parameter. Two parameters of the gamma prior for the range parameter θ are imposed based on the previous empirical distance information and maximum likelihood estimation.

With the priors as above, the third layer in equation (2) can be expressed as

$$(6) \qquad p_3(\boldsymbol{\Theta}) = p(\beta_1)\cdots p(\beta_k)\, p(\omega)\, p(\rho)\, p(\theta)\, p\left(\sigma^2\right).$$

[3] We use the empirical variogram to impose proper priors for the sill parameter ρ. For MLE estimates of mean parameter $\hat{\mu}_i$ for county location $i = 1, \ldots, N$, the empirical variogram is

$$\hat{\gamma}(D_{ij}) = \frac{1}{2M} \sum_{(i,j)\in M} \left(\hat{\mu}_i - \hat{\mu}_j\right)^2,$$

where M is the number of all possible pairs of counties, D_{ij} is Euclidean distance between two counties i and j, and $\hat{\mu}_i$ and $\hat{\mu}_j$ are MLE estimates of the mean parameter of county i and j, respectively.

Joint Posterior Distribution

We now have densities for each hierarchy, $p_1(\boldsymbol{Y}|\boldsymbol{\mu}_t, \boldsymbol{\Theta})$, $p_2(\boldsymbol{\mu}_t|\boldsymbol{\Theta})$, and $p_3(\boldsymbol{\Theta})$ from the previous sections. The joint posterior distribution is obtained by multiplying these three layers. The logarithm of the joint posterior distributions of the AR is

$$\begin{aligned}\log p(\boldsymbol{\mu}, \boldsymbol{\Theta}|\boldsymbol{Y}) \propto & -\frac{NT}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(\boldsymbol{y}_t - \boldsymbol{\mu}_t)'(\boldsymbol{y}_t - \boldsymbol{\mu}_t) - \frac{\sum_{t=1}^{T}\log|\Sigma|}{2} \\ & -\frac{1}{2}\left(\boldsymbol{\mu}_t - \omega\boldsymbol{\mu}_{t-1} - \boldsymbol{\psi}_t\right)'\Sigma^{-1}\left(\boldsymbol{\mu}_t - \omega\boldsymbol{\mu}_{t-1} - \boldsymbol{\psi}_t\right) + \log\left(p_3(\boldsymbol{\Theta})\right). \end{aligned} \tag{7}$$

Data and Methods

Our study uses county-level yearly corn yield data from NASS. The data contain 1955–2014 annual yields (bushels per acre) for Iowa, Illinois, Nebraska, Minnesota, and Indiana and 1963–2009 annual yields for Colorado. Counties with missing observations are discarded. Therefore, the final dataset includes 99 counties for Iowa, 77 for Illinois and Nebraska, 68 for Minnesota, 75 for Indiana, and 18 counties for Colorado.

The states of Iowa, Illinois, and Nebraska have been the first, second and third largest corn producers in the United States. In 2015, Iowa produced around 2.5 billion bushels, Illinois produced 2.01 billion bushels, and Nebraska produced 1.7 billion bushels (U.S. Department of Agriculture, 2016). Minnesota and Indiana are the fourth and fifth largest corn-producing states. Colorado is the 14th largest corn producer in the United States. Colorado is included because it has more varying climatic conditions than the other states, such as mountainous terrain, vast plains, desert canyons, and mesas. We expect such climate diversity in Nebraska as well, which comprises two main parts of the geographical regions, from the till plains of the lowland (eastern part of the state) to the Great Plains (center of the state).

A coordinate of physical space consists of longitude and latitude of each county; thus, the spatial smoothing from physical space uses distance between the locations on the physical space (i.e., physical distance). The physical distance is calculated using SpTimer with the option geodetic:km, which gives the great circle distance. On the other hand, a coordinate of each point in climate space is given by its climatological quantities. Our climate-space smoothing uses the Euclidean distance between the locations on the climate space. Both the coordinates of the physical and the climate space are standardized to avoid the scale issue arising from the different units of the dimensions.

Several empirical studies have tried to determine the relationship between climate factors and crop yields. Schlenker and Roberts (2009) estimate the impact of climate change on agricultural output using panel data. They find that temperature above a threshold level has a negative impact on crop yields since it increases heat exposure and water stress. They use temperature and precipitation as the main climate variables for their research. Other, related studies (Hendricks and Peterson, 2012; Lobell et al., 2013; Dell, Jones, and Olken, 2014) employ temperature and precipitation as explanatory variables. In accordance with the previous literature, we choose temperature and precipitation at each location as the climate coordinates.

We use two data sources. Climate space I uses the Global Historical Climatology Network Database (GHCND) under the National Oceanic and Atmospheric Administration (NOAA), which has 18 meteorological variables. We use the average number of days in months (July and August) with maximum temperature greater than or equal to 90°F (*Max Temperature Days*) and average precipitation (mm) for the months from May to August (*Mean Precipitation*) from 1955 to 2014 as the climate coordinates. We standardize the coordinates to get equally weighted smoothing from the *Max Temperature Days* and *Mean Precipitation*. We average the climate quantities for counties with multiple weather stations. Counties with no weather stations (five counties in the dataset) are discarded.

The other data for climate space comes from the Agro-Climatic Data by County (ACDC), designed by Yun and Gramig (2019). Their data provide county-level growing degree days for 1°C intervals between −60 to +60°C and total precipitation for the growing season from 1981 to 2015. The climate variables are estimated using PRISM weather data. A noticeable advantage of the dataset is that all variables are calculated at the county level and include only data for land being used for agricultural uses based on the USGS NLCD (U.S. Geological Survey National Land Cover Database) Land Use and Land Cover (LULC) data.

Corn yields are often modeled as being reduced when temperatures are above a threshold, commonly suggested in agronomy studies to be 86°F (McMaster, 1997). Schlenker and Roberts (2009) assumed the temperature and precipitation effects on corn yields are cumulative over the growing season (March to August). Following Schlenker and Roberts, we use the temperature threshold and the growing season to define the climate variables. From the data source, we calculate the growing degree days for 32°F to 86°F (i.e., 0°C to 30°C) from March to August (*Medium Temperature*), the growing degree days above 86°F (i.e., 30°C) from March to August (*High Temperature*), and the total precipitation from March to August (*Total Precipitation*) and use them as the climate coordinates. We then create two- (*Medium Temperature* and *Total Precipitation*) and three- (*Medium Temperature*, *High Temperature*, and *Total Precipitation*) dimensional climate coordinates by using the standardized climate factors and define them as climate space II and III, respectively.

In climate space, counties with similar climate features are weighted more, even when they are physically distant. Since Colorado has diverse geographical features, we expect that county locations in Colorado on the two different spaces are substantially different. Figure 1 translates county locations in Colorado from physical space (longitude/latitude) to climate space II (*Medium Temperature* and *Total Precipitation*). The county locations in the physical and climate spaces tend to be clustered differently. For instance, Pueblo and Otero Counties are neighbors in physical space but distant in climate space. Also, Cheyenne County is adjacent to Kit Carson County, but the climate in Kit Carson County is more similar to that in Logan County than that in Cheyenne County. In contrast to Colorado, counties in corn-belt states such as Iowa or Illinois that are close in climate space are also close in physical space.

To highlight the difference between Colorado and Iowa, we calculate Pearson correlations between the distances from the types of space. Since our dataset includes 99 counties for Iowa and 18 counties for Colorado, there are 4,851 and 153 possible pairs of distances among the counties for each state, respectively. We first obtain the distances from each type of space (physical and climate). We then calculate the Pearson correlation coefficient between the two types of distance. Aligning with our expectation for the two states, the correlations between two types of distances are 0.51 in Iowa and 0.23 in Colorado.

Posterior Predictive Distribution

As suggested by Ozaki, Goodwin, and Shirota (2008), we compute premium rates from posterior predictive values. The posterior predictive distribution $p(\boldsymbol{y}^*|\boldsymbol{Y})$ is obtained by integrating over the parameters with respect to the joint posterior distributions. The posterior predictive distribution can be used to estimate densities of counties with insufficient historical observations (i.e., missing observation) or even no observations.

The prediction quality of the model is evaluated by calculating the predictive model choice criteria (PMCC) suggested by Gelfand (1998):

$$PMCC = \sum_{t=1}^{T} E\left[(\boldsymbol{y}_t^* - \boldsymbol{y}_t)'(\boldsymbol{y}_t^* - \boldsymbol{y}_t)\right] + var(\boldsymbol{y}_t^*), \tag{8}$$

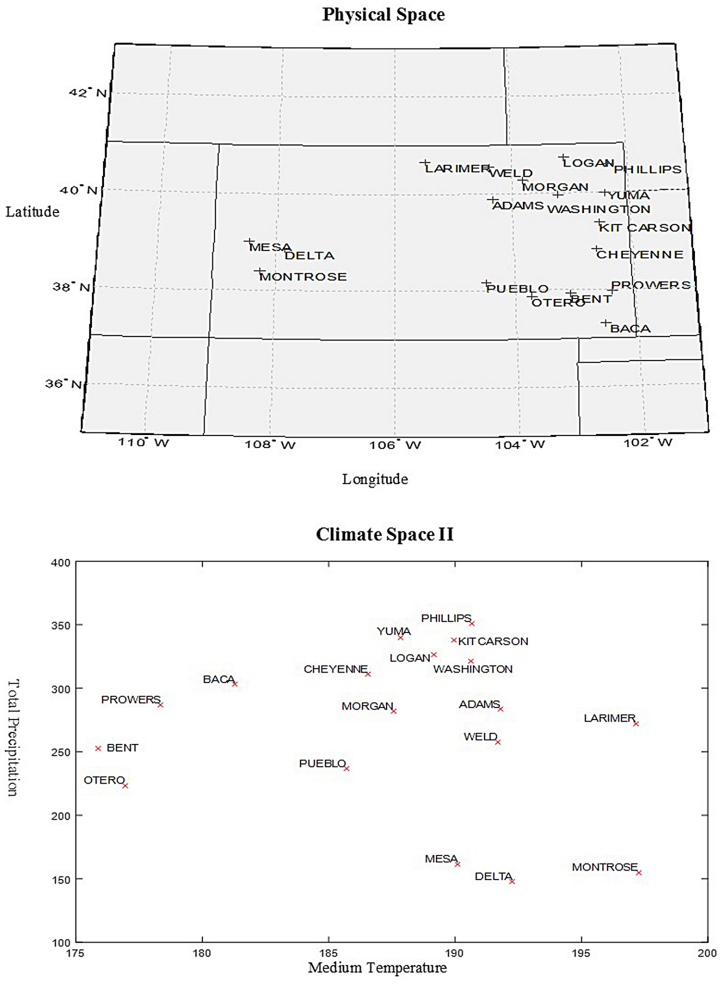

Figure 1. Translation of Counties of Colorado in Physical Space to Climate Space II

where $\boldsymbol{y}_t^*$ denotes a vector of predicted yield level from $p\left(\boldsymbol{y}^*|\boldsymbol{Y}\right)$ at year t and $\boldsymbol{y}_t$ is a vector of actual yields at year t. The first term of the PMCC represents the goodness of fit of the model, and the second term represents a penalty for model complexity. We calculate each state's *PMCC* using the predictions from 1985–2014. Several specifications for GP and AR models are tested as potential candidates for the empirical analysis. A model with lower PMCC is chosen as the preferred model. We test specifications for the process layer structure with a moving average of historical yields (tested for different lengths from the last 5 years to 10 years), simple linear trend, and quadratic linear trend. Among these alternatives, the AR model including 5 years of moving average and simple linear trend minimizes PMCC in all states in our dataset and thus is selected as our main model.

Computer Algorithm

Several R packages provide Markov chain Monte Carlo (MCMC) algorithms for Bayesian Kriging estimation. While our estimation cannot be completely done with any existing R package, we rely heavily on three existing packages: spTimer, spBayes, and SpatialExtremes. These packages are used to estimate posterior distributions, posterior predictive distributions, and PMCC of the models. All models are fit using a Metropolis–Hastings (MH) algorithm within Gibbs sampling algorithm.

We run 20,000 iterations for MCMC chains and burn in the first 5,000 observations. We check for all parameters the graphical diagnostics of convergence using trace plots and autocorrelation plots. All posterior densities achieve fast convergence with no substantial autocorrelation.

Results

Table 1 presents the averages and standard deviations of the posterior parameter values for four states in the dataset. (The results of Minnesota and Indiana are presented in Table S1 in the Online Supplement.) The slopes are similar across states for the GP model. The slopes of the time trends (β_2) differ across states for the AR models. Table 1 shows significant temporal correlations (ω) in Nebraska and Colorado (i.e., above 0.34 for Nebraska and 0.56 for Colorado in all models). This significant level of ω makes the trend parameter estimates β_2 in both Nebraska and Colorado far smaller than that in Iowa and Illinois.[4]

Out-of-Sample Performance

Providing actuarially sound premiums is a goal of the Risk Management Agency (RMA). Crop producers are less likely to purchase insurance contracts when premiums are overrated (Coble et al., 1996). Likewise, underrated premiums may induce insurance losses to agencies through adverse selection (Coble, Dismukes, and Glauber, 2007). The premium rate of crop insurance represents expected payouts as a proportion of total liability:

$$prem_i = \frac{P(y_i < \lambda \hat{y}_i)(\lambda \hat{y}_i - E[(y_i | y_i < \lambda \hat{y}_i)])}{\lambda \hat{y}_i}, \tag{9}$$

where λ is the coverage level, $0 < \lambda < 1$, $\hat{y}_i$ is an expected crop yield at county i, the function $P()$ denotes probability, and the function $E()$ denotes expectation.

We calculate the 90% coverage premium rates from the posterior predictive distributions ($p(\boldsymbol{y}^* | \boldsymbol{Y})$) and equation (9). The expected yield $\hat{y}_i$ in county i is the posterior mean of the predictive distribution for each county. Table 2 presents each state's average and standard deviation premium rates for all counties from 2000 to 2014 except Colorado (1994–2008 in Colorado). Consistent with our expectation, Colorado shows a notable difference in average premium rates between using the physical space and the climate space. Also, Colorado shows the most significant regional disparity when using the physical-space smoothing, but the disparity dramatically reduces when using the climate-space smoothing.

Figure 2 illustrates absolute differences between estimated premiums of Iowa and Colorado from the different types of spaces. We use the climate space II (*Medium Temperature* and *Total Precipitation*) premiums for comparison. The Iowa premium estimates show no substantial difference (all differences are below 0.6%), which reflects the fact that Iowa has similar spatial structure with both climate and physical distance. In contrast to Iowa, premiums in Colorado show meaningful spatial differences. Specifically, premiums in eastern-region counties (Cheyenne and Kit Carson) and northern-region counties (Adam and Larimer) increase when using climate-space smoothing. The differences in eastern region are 1.58% in Cheyenne and 1.42% in Kit Carson County. In the northern region, the differences are 1.21% in Adams County and 0.98% in Larimer County.

We use a loss ratio under the corresponding crop insurance program as a tool to evaluate out-of-sample performance of the models based on the two types of smoothing space:

$$lossratio_i = \frac{\sum_{t=1}^{T} \max[\lambda \hat{y}_{it} - y_{it}, 0]}{\sum_{t=1}^{T} prem_{it} \hat{y}_{it}}, \tag{10}$$

[4] Restricting both β_1 and ω to 0 gives even more similar trend parameter estimates for all states in the dataset.

Table 1. Average Posterior Parameter Values from Selected States

	Physical		Climate I		Climate II		Climate III	
State/Parameter	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**
Iowa								
β_1	1.04	0.94	1.01	1.02	1.05	0.94	1.05	0.94
	(0.04)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
β_2	1.91	1.72	1.76	1.78	1.91	1.72	1.76	1.78
	(0.01)	(0.03)	(0.02)	(0.02)	(0.01)	(0.03)	(0.02)	(0.02)
ω	–	0.10	–	0.10	–	0.10	–	0.10
		(0.00)		(0.00)		(0.00)		(0.00)
ρ	18.64	18.55	21.16	25.31	18.67	18.54	18.58	18.55
	(6.70)	(6.54)	(4.12)	(4.19)	(5.25)	(6.54)	(6.63)	(6.54)
θ	19.02	19.89	21.83	22.43	19.02	19.89	19.02	19.89
	(2.65)	(4.47)	(3.65)	(3.11)	(4.28)	(4.47)	(4.28)	(4.47)
Illinois								
β_1	1.03	1.01	1.02	1.01	1.04	0.94	1.01	1.02
	(0.03)	(0.03)	(0.02)	(0.02)	(0.04)	(0.03)	(0.02)	(0.03)
β_2	1.72	1.71	1.72	1.72	1.91	1.72	1.76	1.78
	(0.02)	(0.03)	(0.02)	(0.02)	(0.01)	(0.03)	(0.02)	(0.02)
ω	–	0.02	–	0.02	–	0.10	–	0.10
		(0.00)		(0.00)		(0.00)		(0.00)
ρ	18.95	18.29	22.18	23.23	20.24	20.25	20.26	20.26
	(3.95)	(3.99)	(5.22)	(5.23)	(8.74)	(6.54)	(8.75)	(8.79)
θ	19.38	19.22	24.13	24.15	18.98	20.14	18.97	20.14
	(3.26)	(3.15)	(4.82)	(5.11)	(4.18)	(4.48)	(4.23)	(4.48)
Nebraska								
β_1	0.78	0.63	0.99	0.63	0.99	0.65	0.99	0.66
	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
β_2	2.07	1.34	2.13	1.34	2.17	1.41	2.17	1.42
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)	(0.04)
ω	–	0.37	–	0.36	–	0.34	–	0.34
		(0.01)		(0.02)		(0.01)		(0.01)
ρ	21.15	21.32	30.17	31.17	17.92	16.81	17.92	16.81
	(5.51)	(5.52)	(5.23)	(5.19)	(7.24)	(6.46)	(7.25)	(6.47)
θ	15.12	14.21	20.69	20.66	19.09	19.97	19.09	19.97
	(1.11)	(1.12)	(2.18)	(2.26)	(4.41)	(4.45)	(4.41)	(4.52)
Colorado								
β_1	0.3	0.14	0.3	0.15	1.07	0.43	1.07	0.43
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.13)	(0.14)	(0.13)
β_2	1.83	0.74	1.81	0.74	1.97	0.65	1.97	0.65
	(0.05)	(0.01)	(0.04)	(0.01)	(0.01)	(0.02)	(0.10)	(0.15)
ω	–	0.56	–	0.58	–	0.63	–	0.63
		(0.01)		(0.01)		(0.06)		(0.06)
ρ	33.55	35.01	49.89	49.24	17.41	13.42	17.43	13.45
	(7.42)	(7.44)	(5.22)	(5.99)	(4.12)	(4.62)	(3.34)	(4.47)
θ	12.13	12.12	19.13	19.12	18.83	20.04	18.83	20.04
	(1.00)	(1.00)	(3.65)	(3.61)	(3.15)	(4.41)	(4.33)	(4.41)

Notes: GP and AR represent a general Gaussian spatial process and autoregressive Gaussian spatial process models, respectively. Numbers in parentheses are standard deviations. The symbols β_1, β_2, and ω are coefficients for the moving average, trend, and first-order autocorrelation parameters, and ρ and θ are sill and range parameters that indicate magnitude and distance of the spatial dependence. Climate I is a two-dimensional space (*Mean Precipitation* and *Max Temperature Days*). Climate II is a two-dimensional space (*Total Precipitation* and *Medium Temperature*). Climate III is a three-dimensional space (*Total Precipitation*, *Medium Temperature*, and *High Temperature*).

Table 2. Average of 90% Coverage Premium Rates across Counties (%)

	Physical		Climate I		Climate II		Climate III	
State	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**
Iowa	1.38	1.39	1.57	1.73	1.59	1.39	1.68	1.39
	(0.51)	(0.52)	(0.32)	(0.36)	(0.23)	(0.28)	(0.25)	(0.28)
Illinois	1.52	1.54	1.54	1.52	1.47	1.59	1.48	1.50
	(0.40)	(0.42)	(0.42)	(0.47)	(0.38)	(0.38)	(0.38)	(0.39)
Nebraska	1.38	1.27	1.28	1.38	1.27	1.29	1.27	1.30
	(0.42)	(0.43)	(0.52)	(0.52)	(0.48)	(0.37)	(0.43)	(0.34)
Minnesota	1.59	1.57	1.63	1.59	1.37	1.36	1.38	1.37
	(0.57)	(0.58)	(0.47)	(0.47)	(0.43)	(0.43)	(0.43)	(0.43)
Indiana	1.65	1.69	1.64	1.65	1.25	1.24	1.25	1.23
	(0.42)	(0.43)	(0.38)	(0.39)	(0.23)	(0.23)	(0.23)	(0.23)
Colorado	1.48	1.62	2.41	1.68	2.4	1.95	2.41	1.93
	(0.67)	(0.97)	(0.44)	(0.82)	(0.19)	(0.37)	(0.16)	(0.36)

Notes: GP and AR represent a general Gaussian spatial process and an autoregressive Gaussian spatial process models, respectively. Numbers in parentheses are standard deviations. Climate I is a two-dimensional space (*Mean Precipitation* and *Max Temperature Days*). Climate II is a two-dimensional space (*Total Precipitation* and *Medium Temperature*). Climate III is a three-dimensional space (*Total Precipitation*, *Medium Temperature*, and *High Temperature*).

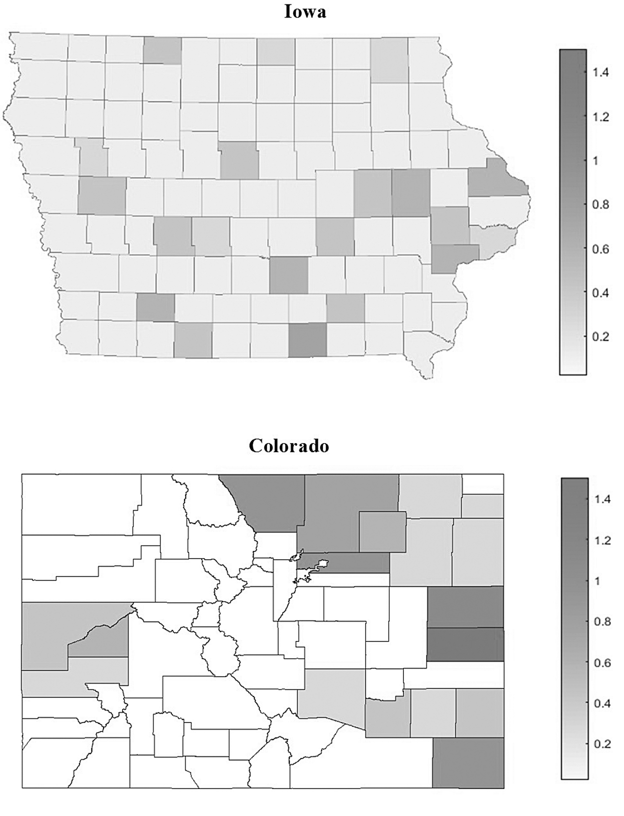

Figure 2. Absolute Differences (%) of Two Types of Spatial Smoothing Premiums in Iowa and Colorado

Notes: Blank (white-colored) counties in Colorado had insufficient yield history and were discarded from the analysis..

Table 3. Estimated Loss Ratio under the Physical Space and the Climate Space

State/Smoothing Space	Physical	Climate I	Climate II	Climate III
Iowa				
Median	1.11	1.01	0.87	0.89
Std. dev.	1.25	1.41	0.96	0.89
1st quartile	0.56	0.36	0.29	0.32
3rd quartile	2.03	2.43	1.38	1.21
Illinois				
Median	1.98	1.96	1.59	1.58
Std. dev.	1.07	1.04	0.83	0.95
1st quartile	1.29	1.26	0.8	0.81
3rd quartile	2.88	2.92	2.17	2.23
Nebraska				
Median	2.34	1.97	0.48	0.49
Std. dev.	5.99	4.31	1.42	1.41
1st quartile	0.35	0.55	0.00	0.00
3rd quartile	4.51	3.24	1.43	1.43
Minnesota				
Median	0.37	0.22	0.48	0.47
Std. dev.	1.17	0.44	1.14	0.89
1st quartile	0.00	0.03	0.00	0.00
3rd quartile	1.26	0.75	1.44	1.19
Indiana				
Median	2.90	2.25	2.25	2.25
Std. dev.	1.79	1.63	1.52	1.53
1st quartile	1.61	1.20	1.40	1.39
3rd quartile	4.33	4.15	3.80	3.80
Colorado				
Median	3.70	2.08	1.8	1.80
Std. dev.	3.62	3.96	2.78	2.91
1st quartile	1.96	1.34	1.14	1.01
3rd quartile	7.44	7.35	5.68	6.64

Notes: GP and AR represent a general Gaussian spatial process and an autoregressive Gaussian spatial process models, respectively. Climate I is a two-dimensional space (*Mean Precipitation* and *Max Temperature Days*). Climate II is a two-dimensional space (*Total Precipitation* and *Medium Temperature*). Climate III is a three-dimensional space (*Total Precipitation*, *Medium Temperature*, and *High Temperature*). An actuarially fair loss ratio is equal to 1.

where λ is coverage level, $\hat{y}_{it}$ is predicted yield of county i at year t, y_{it} is actual yield, and $prem_{it}$ is the premium rate, obtained from equation (9).

The premium gains and indemnity losses from equation (10) are obtained using actual yields and premium estimates from 2000 to 2014 except for Colorado (1994–2008 in Colorado). Since calculated loss ratios are bounded at 0 and have outliers that could distort the robustness of measures, we report median, standard deviation, and first/third empirical quantiles of the loss ratio measures across counties for the six states in Table 3. The loss ratio of *fairly rated* crop insurance should equal 1. Thus, a model with a median loss ratio close to 1 and with a small standard deviation and narrow inter-quartile ranges across counties might be a preferred model.

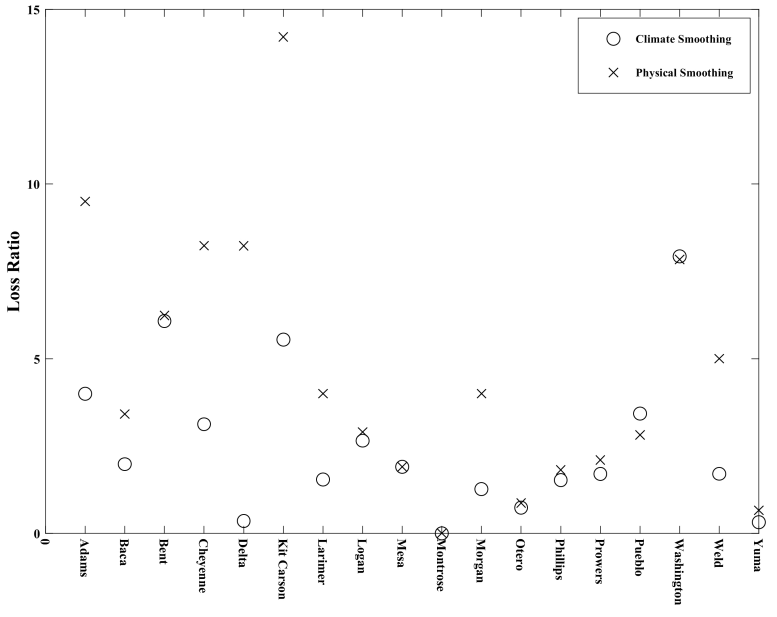

Figure 3. Loss Ratios from the Physical Smoothing and the Climate Smoothing in Colorado

Notes: The loss ratio is calculated as total indemnity divided by total premium. The loss ratio should be one under the actuarially fair insurance product.

The median loss ratios calculated from the climate-space smoothing tend to be closer to 1 than the loss ratios from the physical-space smoothing in every state in our dataset. Our results also show that all types of climate space have a smaller variation of loss ratio across counties. The standard deviation of the loss ratio falls from 3.62 to 2.78 in Colorado and from 5.99 to 1.41 in Nebraska.

Figure 3 plots county-level loss ratios from the physical space and climate space II in Colorado. The loss ratios become closer to 1 when using climate-space smoothing for the counties with high loss ratios under physical smoothing, such as Adam, Cheyenne, Delta, and Kit Carson Counties. Our results demonstrate that climate-space smoothing more pertinently describes the spatial structure of the crop yield density compared to physical-space smoothing, especially for a state like Colorado, where the climate is diverse. Based on the results above, however, whether the climate smoothing statistically outperforms the physical smoothing is not definitive since the empirical evidence from those criteria of the median and the standard deviation from our dataset would not guarantee its statistical significance.

In this regard, we conduct two different premium rating tests. One is based on a state average loss ratio across counties for each year, and the other is based on a county-level loss ratio across years. For the first test (Test I), each state has 15 observations of year specific state average loss ratios across counties. On the other hand, with the second test (Test II), each state has N county-specific loss ratios calculated over 15 years, where N is the number of counties of each state. We choose climate space II (*Medium Temperature* and *Total Precipitation*) as the primary comparison model since it provides premiums with more accuracy and less regional inequalities than other types of smoothing space in terms of the loss ratio calculation.[5]

Test I evaluates out-of-sample performance by simulating a premium rating game between a representative private insurance company and the federal government agency. The representative insurance company is assumed to choose whether to retain or cede policies under the Standard

[5] The three-dimensional smoothing space, which is climate space III (*Medium Temperature*, *High Temperature*, and *Total Precipitation*), also provides almost identical premiums with climate space II. However, we choose climate space II as our primary model due to its simplicity. The Pearson correlation coefficient between Euclidean distances calculated from climate space II and III is 0.96.

Reinsurance Agreement (Harri et al., 2011; Ker, Tolhurst, and Liu, 2016; Park, Brorsen, and Harri, 2019). The out-of-sample simulation is generated over the 15 years from 2000 to 2014 (1994–2008 in Colorado). We employ the index I suggested by Ker, Tolhurst, and Liu (2016) to reflect an advantage of the private insurance company that can make decisions after identifying premiums proposed by the federal agency. The index I can be defined as

$$I = \frac{LR^C_{\text{Cede}}/LR^C_{\text{Retain}}}{LR^P_{\text{Cede}}/LR^P_{\text{Retain}}}, \tag{11}$$

where LR^C_{Cede} and LR^C_{Retain} are the ceded/retained loss ratios that the private insurance company make decisions based on the premiums from the climate smoothing against the physical smoothing premiums, and LR^P_{Cede} and LR^P_{Retain} are the loss ratios that the private insurane company selects their cede/retain decisions based on the premiums from the physical smoothing against the premiums from the climate smoothing. The index I represents the relative accuracy of the two methods. If $I > 1$, then the climate-smoothing method outperforms the physical smoothing method, and vice versa.

We conduct a statistical test under the null hypothesis that premiums from the two types of smoothing methods are identically accurate. We first calculate the county-level index I over 15 years and obtain a state average I for each year. We then define a random variable, I^*, that indicates the number of I greater than 1 in the 15 years for each state. The random variable I^* follows a binomial distribution, where $I^* \sim \text{binomial}(0.5, 15)$.

The second type of premium rating test (Test II) more directly evaluates the performance of the models. We define an index D_i such that

$$D_i = \frac{\left|LR^C_i - 1\right|}{\left|LR^P_i - 1\right|}, \tag{12}$$

where $\left|LR^C_i - 1\right|$ and $\left|LR^P_i - 1\right|$ are absolute deviations of county i's loss ratio from 1 under the climate/physical spatial smoothing. The index D_i indicates a relative accuracy of two methods. If $D_i < 1$, then the climate-smoothing method provides more fairly rated premiums at county i than that of the physical smoothing method, and vice versa.

Like Test I, the null hypothesis of Test II is that premiums from the two types of smoothing methods are identically accurate. We calculate each state's county-level index D_i over 15 years. Therefore, each state has N number of D_i, where N is the number of counties. We next define a random variable, D^*, that represents the number of counties in each state in which $D_i < 1$. The random variable D^* follows a binomial distribution, where $D^* \sim \text{binomial}(0.5, N)$.

Tables 4 and 5 present the estimated p-values from the two tests. A low p-value suggests that the climate-smoothing premiums are more accurate, whereas a large p-value corresponds to the physical-smoothing premiums being more accurate. Based on results from Test I (Table 4), climate-space smoothing is significantly (p-value < 0.1) better than physical-space smoothing in all state/coverage combinations, except Iowa at the 90% coverage level.

Test II results (Table 5) are generally favorable to the climate-smoothing method in all state-coverage combinations (p-value < 0.5), except Illinois at the 70% coverage level; 2 cases out of 12 state/coverage combinations are statistically significant (p-value < 0.01). Both test results favor density smoothing based on climate similarity rather than on physical closeness.

Conclusion

Providing accurate premiums for insurance contracts is a key responsibility of RMA. Bayesian Kriging with spatial distance is more accurate than the current RMA model and as good as alternatives such as BMA (Park, Brorsen, and Harri, 2019). Our results show that Bayesian

Table 4. Out-of-Sample Rating Game Results with Test I (climate smoothing vs. physical smoothing)

State	70% Coverage p-Value (no. of years, I^*)	90% Coverage p-Value (no. of years, I^*)
Iowa	0.0592	0.1509
	(10)	(9)
Illinois	0.0592	0.0175
	(10)	(11)
Nebraska	0.0037	0.0037
	(12)	(12)
Minnesota	0.0005	0.0037
	(13)	(12)
Indiana	0.0176	0.0176
	(11)	(11)
Colorado	0.0000	0.0004
	(14)	(13)

Notes: A p-value smaller than 0.5 corresponds to the climate smoothing dominates the physical smoothing, and vice versa. The random variable I^* follows a binomial distribution.

Table 5. Out-of-Sample Rating Game Results with Test II (climate smoothing vs. physical smoothing)

State	No. of Counties (N)	70% Coverage p-Value (no. of years, D^*)	90% Coverage p-Value (no. of years, D^*)
Iowa	99	0.3439	0.1574
		(51)	(54)
Illinois	82	0.7094	0.4561
		(38.00)	(41)
Nebraska	77	0.2472	0.1272
		(41.00)	(43)
Minnesota	57	0.3957	0.2135
		(29)	(31)
Indiana	60	0.4487	0.0002
		(30)	(43)
Colorado	18	0.0000	0.1189
		(15)	(11)

Notes: A p-value smaller than 0.5 corresponds to the climate smoothing dominates the physical smoothing, and vice versa. The random variable D^* follows a binomial distribution, where $D^* \sim \text{binomial}(0.5, N)$, where N is the number of counties in each state.

Kriging model with climate-space smoothing provides more accurate crop insurance premiums than Bayesian Kriging with physical distance. The advantages are largest for Nebraska and Colorado, where there is more geographic diversity and numerous counties with missing data.

There are only a few examples of Bayesian Kriging models in the agricultural economics literature, and this article is the first to use climate-space smoothing. Future research could consider additional measures of similarity such as soil type and slope of agricultural land.

The Kriging method could be adopted in any research area that involves spatial dependence. A prominent extension of the model, for example, would be for precision agriculture. Today the availability of accurate and abundant yield monitoring data allows the development of a crop yield response model in which parameters are smoothed by site-specific agrological characteristics (e.g., soil type, water). This application could provide better site-specific variable rate application (VRA) fertilizer prescriptions.

A great strength of Kriging is that it can estimate densities for counties with no or limited historical yield data. In this regard, the method suggests a useful way to produce density measures for counties with no yield reported. The Kriging method for climate-space smoothing proposed here clearly has the potential to offer significant efficiency gains in crop yield density estimation, where historical observations are limited and under varying climate conditions.

Because we are focusing on introducing and comparing the performance of the climate-space smoothing method compared to the physical distance smoothing, our model treats crop yield density in a simple manner using the normality assumption. The normality assumption simplifies the MCMC structure so that a trend variable is easily included, which is something that Park, Brorsen, and Harri (2019) had to consider using a two-step method. While it may require developing techniques that are not yet available in the statistics literature, future research should attempt to relax this distributional assumption.

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Online Supplement: Spatially Smoothed Crop Yield Density Estimation: Physical Distance versus Climate Similarity

Eunchun Park, B. Wade Brorsen, and Ardian Harri

Table S1. Average Posterior Parameter Values from Selected States

	Physical		Climate I		Climate II		Climate III	
State/Parameter	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**	**GP**	**AR**
Indiana								
β_1	1.03	1.12	1.01	1.11	1.03	1.12	1.04	1.12
	(0.04)	(0.04)	(0.02)	(0.04)	(0.01)	(0.03)	(0.01)	(0.04)
β_2	1.68	1.82	1.65	1.83	1.67	1.81	1.68	1.82
	(0.02)	(0.03)	(0.03)	(0.05)	(0.04)	(0.03)	(0.02)	(0.03)
ω	–	−0.09	–	−0.04	–	−0.08	–	−0.09
		(0.02)		(0.01)		(0.00)		(0.00)
ρ	17.19	18.41	19.82	18.40	18.34	18.40	18.46	18.40
	(3.12)	(2.79)	(2.18)	(2.83)	(2.84)	(2.83)	(2.85)	(2.83)
θ	18.96	20.12	19.49	21.11	18.94	20.11	18.95	20.12
	(4.42)	(4.31)	(4.37)	(5.32)	(4.37)	(4.31)	(4.38)	(4.31)
Minnesota								
β_1	0.99	0.90	0.98	0.91	0.99	0.90	1.00	0.89
	(0.02)	(0.03)	(0.01)	(0.05)	(0.02)	(0.03)	(0.02)	(0.03)
β_2	2.08	1.87	2.12	1.88	2.08	1.87	2.09	1.87
	(0.02)	(0.04)	(0.03)	(0.03)	(0.02)	(0.04)	(0.02)	(0.04)
ω	–	0.10	–	0.09	–	0.10	–	0.10
		(0.02)		(0.03)		(0.02)		(0.02)
ρ	19.05	20.03	18.25	20.18	19.05	20.03	19.03	20.08
	(4.47)	(4.62)	(4.12)	(4.31)	(4.48)	(4.62)	(4.41)	(4.60)
θ	16.93	16.84	16.81	16.87	16.93	16.84	16.87	16.84
	(2.63)	(2.63)	(2.61)	(2.61)	(2.55)	(2.15)	(2.43)	(2.64)

Notes: GP and AR represent a general Gaussian spatial process and autoregressive Gaussian spatial process models. The numbers in parentheses are standard deviations. β_1, β_2, and ω are the moving average, trend, and first-order autocorrelation parameters, and ρ and θ are sill and range parameters that indicate magnitude and distance of the spatial dependence. Climate I is a two-dimensional space (*Mean Precipitation* and *Max Temperature Days*). Climate II is a two-dimensional space (*Total Precipitation* and *Medium Temperature*). Climate III is a three-dimensional space (*Total Precipitation*, *Medium Temperature*, and *High Temperature*).

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