APPLICATION OF LINEAR GOAL PROGRAMMING TO FOREST HARVEST SCHEDULING

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Forest management and planning is complex, involving the application of many scarce and diverse resources to the production and maintenance of a multitude of products and services from the forest over a relatively long period of time. The forest manager hopes to produce a balanced mix of products and services, with the mixture depending upon the landowner's objectives. Although many objectives are complementary in nature, others are competitive, with some mutually exclusive. As a result, allocating the resource manager's scarce and diverse resources among the alternative and possibly competitive products and services becomes a complex problem.

Timber management planning is normally an integral part of managing a forest, and two traditional tasks of timber management planning are establishing harvest schedules (cutting budgets) and developing a regulated forest. The harvest-scheduling problem involves determining what, where, when, and how much to cut in order to ensure a smooth transition from an unregulated to a regulated forest structure, while at the same time meeting short-term requirements, objectives, and constraints. A regulated forest is a forest with age and size classes represented in such a proportion that a stable periodic yield of products and services may be obtained over time (Davis). The regulation problem involves selecting and developing a long-term, steady-state forest structure, the regulated forest (Dress).

There are, in general, many ways to manipulate existing and future forest stands to solve the regulation and harvest-scheduling problems. As a result, many forest product companies and public forest management agencies have adopted advanced planning techniques for developing harvest schedules and determining long-term, steady-state forest structures. This has been encouraged by increased competition for available stumpage, anticipated increases in stumpage costs, interest in a stable wood supply, and the potential for increased financial returns from fee and leased lands.

Advanced forest management planning techniques developed over the last two decades have incorporated operations research methodologies, with linear programming (LP) the methodology most commonly used. Early linear programming applications to timber management planning (e.g., Theiler; Loucks; Kidd et al.; Ware and Clutter; and Navon) were developed, in general, to aid in the systematic selection of optimal sets of forest-stand treatments and harvest schedules. Present net worth maximization, harvest volume maximization, and cost minimization, all over relatively long periods of time, have all been specified in objective functions.

Economic analyses incorporating linear programming are often based, in part, on the implied assumption that economic man has but one objective. Indeed, the classical theory of the firm postulates "rational" economic man as an optimizer (Henderson and Quandt), whether it be output maximization subject to a cost constraint, cost minimization subject to an output constraint, or profit maximization. This view of economic man has been questioned, however (Arrow; Cyert and March; Lane; Margolis; Simon). Firms do not seek to satisfy a unidimensional goal, but rather seek to satisfy a multidimensional goal set.

A problem associated with using linear programming for solving multiobjective problems is that it requires that all incommensurable goals be transformed into a common unit of measure (usually dollars), and this may often be difficult or impossible to achieve. The commonly used approach for resolving this problem has been to select one goal for specification in the objective function, while all other goals are assigned minimal or maximal desired levels of achievement and placed in the constraint set. Since these latter goals are not optimized, however, conflicts may arise between advocates of different goals or objectives. Given the development and availability of multiple objective linear programming (MOLP) procedures, the use of single-objective LP procedures may no longer be appropriate for resolving multiobjective problems.

Goal programming, a MOLP procedure, has been introduced as an alternative to linear programming for public forest management planning models incorporating multiobjective planning (Dress; Field, Dress, and Fortson; Schuler and Meadows). It is possible to determine simultaneous solutions to systems of multiple, incompatible, and incommensurable goals, rather than being limited to solutions resulting from models incorporating only a single decision criterion. Goal programming neither restricts nor limits the number of objectives specified. Further, goals need not be defined or specified in the same unit of measure. Multiple goals may be specified, for example, in terms of board feet of timber, dollars of present net worth, number of cattle, and number of recreation user days, to name a few.

Another valuable asset of goal programming is that

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goal trade-offs can be studied more readily by changing the weights. All solutions produced on the production possibilities curve are noninferior, and thus all are potentially preferred solutions. The trade-offs associated with different goal programming solution sets are more readily apparent than those employing numerous linear programs, each with a different objective function.

This paper presents a harvest-scheduling model employing goal programming developed for a small pulp and paper company to determine the species composition, age, and volume of stands thinned and final-harvested by period.

THE GOAL PROGRAMMING MODEL

Goal programming is a variant of linear programming; the major differences lie in the formulation of the objective function and the use of deviation variables in the goal-constraint equations. The term "goal programming" was first coined by Charnes and Cooper, who originally developed the mathematical model to address the problem of infeasibilities caused by incompatible constraints.

The general form of the linear, cardinally weighted goal-programming model can be expressed as

$$\text{Min } Z = w^{+}d^{+} + w^{-}d^{-}$$

subject to

$$Ax - d^{+} + d^{-} = g$$

$$Bx \leqslant b$$

$$d^{+}d^{-} = 0$$

$$x, d^{+}, d^{-} = 0$$

where x is a vector of activities or decision variables; g is a vector of goal target levels; A and B are matrices of input-output coefficients relating the system constraints and goal target levels, respectively, to the decision variables; d^{+} and d^{-} are vectors of positive and negative deviations from the goal target levels (g); and w^{+} and w^{-} are vectors of weights associated with the positive and negative goal deviations.

Although a multiobjective decision model, linear goal programming is converted to the traditional single-objective linear programming model by minimizing Z, the sum of weighted deviations from specified goal target levels. Decision variables are not generally found in the objective function.

The system constraints, b, represents resource limitations and output flow restrictions. The goal constraints, g, are desired levels of goal achievement. Under-achievement and over-achievement, individually or both, can be minimized, depending upon the formulation of the objective function. Where maximization of goals individually would be desired, as in most economic optimization analyses, only the minimization of under-achievement, d^{-}, in desired target levels would be of interest since most decision-makers would be indifferent to exceeding the prespecified targets. This might be true for cash flow, profit, and volume goals, but not for minimization of cost goals. The latter would require minimization of over-achievement, d^{+}. Various forms of the objective function can be found in goal programming textbooks (Ignizio; Ijiri; Lee).

STUDY AREA AND DATA

A harvest-scheduling model employing cardinally weighted linear goal programming was developed using the goals, constraints, management regimes, and forest structure of a small pulp and paper company (The Company). The Company, located in the southeastern United States, owns approximately 300,000 acres of fee timberlands, including pine flats, uplands, bottomlands, and swamps. A management area comprising approximately 84,000 acres was used to construct the model.

Initial forest-stand conditions on the 84,000 acre forest are summarized in Table 1. A stand is defined as a contiguous arrangement of trees occupying a specific area that is relatively uniform in species composition, age, structure, and site quality. Over one-half of the forest was in plantations (stands hand or machine planted at a specified tree spacing); the major cover type (primary tree cover) was pine; and the predominant site index (a measure of site productivity and defined as the average height of dominant and codominant trees at some base age) was 60, base age 25. The age-class distribution tended toward younger stands, with over 50 percent of the stands ranging in ages from 6 to 20 years old.

The management units used in the harvest scheduling model were "stand classes," with a stand class comprising all stands in an area having the same age, cover type, stand classification (natural stand or plantation), and site index. Over 2,000 individual stands were aggregated into 163 stand classes.

MODEL SPECIFICATION

The Company wanted a first-generation harvest-scheduling model developed for its timberlands that would provide stand-specific results. Output was to indicate what stands to cut and treat by period or, alternatively, for a given stand in what periods it ought to be site-prepared, planted, thinned, and final-harvested.

The primary goals established by The Company decision-makers were total volume harvested, total undiscounted cash flow, total discounted cash flow, and total discounted cost. Maximizing total volume over a planning period has probably been used most often in harvest-scheduling models employing LP because of the biological nature of forest management. Total discounted cash flow (net revenue) is possibly used most

Table 1. Initial Forest Stand Conditions, By Age Class, Cover Type, Stand Classification, and Site Index.[a]

Age Class	%	Cover Type	%	Site Index	%	Stand Classification	%
0–10	20.7	Pine	79.1	50	0.8	Plantation	53.1
11–20	38.2	Pine-hardwood	8.9	55	15.5	Natural	46.9
21–30	14.1	Hardwood-pine	4.5	60	75.1		
31–40	8.5	Hardwood	7.1	65	7.0		
41–50	7.9	Cutover	0.4	70	1.6		
51–60	6.2						
61+	4.5						

[a] %'s indicate percent of the 84,735 acre area

often in industrial harvest-scheduling models. Some companies are currently placing greater emphasis on maximizing short-term cash flows, reflecting a need to maintain positions of corporate solvency.

The planning horizon was limited to 90 years. It was subdivided into 18 periods of 5 years each to reduce the size of the input-output matrices, A and B.

The decision variables, x_{ij}, used in the harvest-scheduling model were defined as the acres of stand class i managed under management regime j. Each management regime defined represented a particular sequence of managerial, silvicultural, and harvesting treatments, including the period in which each treatment was accomplished. For example, an imaginary stand class might be potentially thinned and final-harvested under a number of regimes over the first 50-year period (Table 2). All site preparation and planting was assumed to be done immediately after final harvesting.

Table 2. Sample of Alternate Harvesting Strategies for An Imaginary Stand.[a]

Management regime	Periods[b] 1	2	3	4	5	6	7	8	9	10
1	H				T			H		
2	H							H		
3		H				T			H	
4		H							H	
5	T		H				T			H
6	T		H							H
7			H					T		H
8			H							H

[a] T (thin) and H (final harvest) indicate periods in which a harvest is conducted.
[b] Five years per period.

Restrictions were placed on the harvesting strategies specified for currently existing natural stands and plantations. Current natural stands under 30 years old had to be final-harvested between the ages of 30 and 50 years old. No thinning of natural stands was allowed. Current plantations less than 30 years old had to be final-harvested between the ages of 30 and 40 years old. Finally, all stands greater than 30 years old, both natural and plantations, had to be final-harvested within the next 15 years. Thinnings were allowed in current plantations only if age, site index, volume, and basal area (area occupied by trees, usually expressed in square feet and on a per acre basis) criteria were met. These conditions were imposed to assure an orderly transition from an unregulated to a regulated forest structure, while at the same time providing flexibility in the harvesting options. Also they ensured that all current stands, regardless of initial condition, would be harvested and converted to plantations in the 90-year planning horizon.

Specification of future stand-management strategies was based on optimal loblolly pine thinning and final harvesting regimes developed by Broderick. All future stands had to be plantations, and only one harvesting strategy was specified for stand classes having the same site index. Only stand classes with a site index of 60 or above were thinned, and this in the fifth period (20–25 years old). All stands, regardless of site index, were harvested in the seventh period (30–35 years old).

Three classes of "real" constraints were defined: timber-class acreage constraints, harvesting constraints, and economic constraints. The timber-class acreage constraints restricted the total acres managed of a particular stand class to the total acres available. Harvest constraints restricted periodic harvests to a specified range, limiting both periodic acreage and volume harvested. These constraints were placed on the harvest-scheduling model to ensure that at the end of the planning period the forest structure approximated a regulated forest structure and that the periodic stumpage supply was reasonably stable. Economic constraints were specified to assure a minimum periodic level of net returns (cash flow).

Goal constraints were formed by relating desired goal achievement levels to the decision variables and adding positive or negative deviation variables, d^+ and d^-, to the functions. Only the negative deviational variable was included in the goal functions for volume harvested, undiscounted cash flow, and discounted cash flow goals since decision-makers would only be concerned with under-achievement of these. The reverse was true for the discounted cost goal.

An unstructured approach developed by Hotvedt et al. was used to find the weights, w^+ and w^-, associated with the four goals. The weights themselves have no intuitive meaning or interpretation and, consequently, cannot be specified on an *a priori* basis. In the proposed procedure, the set of cardinal weights, w^+ and w^-, are varied in a number of goal-programming runs, each with a widely different weight structure, to produce points on a noninferior trade-off surface, a production possibilities curve. Management decision-makers assess the trade-offs associated with the various goal-programming runs and choose the most preferred solution set. An optimal solution would rarely result from this procedure since not all infinitely possible solution sets on the production possibilities curve are generated and analyzed. Furthermore, there is no one optimal solution since different decision-makers would be willing to accept different sets of trade-offs.

The general harvest-scheduling model employing goal programming can be represented by

$$\text{Min } Z = w_v^- d_v^- + w_m^- d_m^- + w_p^- d_p^- + w_c^+ d_c^+$$

such that

$$\sum_t\sum_i\sum_j v_{ijt} x_{ij} + d_v^- - d_v^+ = TV$$

$$\sum_t\sum_i\sum_j m_{ijt} x_{ij} + d_m^- - d_m^+ = UCF$$

$$\sum_t\sum_i\sum_j P_{ijt} x_{ij} + d_p^- - d_p^+ = DCF$$

$$\sum_t\sum_i\sum_j c_{ijt} x_{ij} + d_c^- - d_c^+ = TDC$$

$$\sum_j x_{ij} \leq X_i$$

$$\sum_i\sum_j x_{ij} \gtreqless A_t$$

$$\sum_i\sum_j v_{ijt} x_{ij} \gtreqless V_t$$

$$\sum_i\sum_j m_{ijt} x_{ij} \geq M_t$$

$$d_v^+ d_v^- = 0$$

$$d_m^+ d_m^- = 0$$

$$d_p^+ d_p^- = 0$$

$$d_c^+ d_c^- = 0$$

$$x_{ij}, d_v^\pm, d_m^\pm, d_p^\pm, d_c^\pm \geq 0$$

where TV, UCF, DCF, and TDC are total volume, total undiscounted cash flow, total discounted cash flow, and total discounted cost goals, respectively; X_i is the total acreage in stand class i constraint; A_t is the maximum or minimum (or both) acres harvested in period t; V_t is the maximum or minimum (or both) volume harvested in period t; M_t is the minimum cash flow in period t; v_{ijt} is the per acre volume harvested from stand class i from management regime j in period t; m_{ijt} is the per acre undiscounted cash flow associated with harvesting stand class i under management regime j in period t; p_{ijt} is the per acre discounted cash flow associated with harvesting stand class i under management regime j in period t; c_{ijt} is the per acre discounted cost associated with stand class i managed by management regime j in period t; $d_g^\pm$ and $w_g^\pm$ are deviational variables and their respective weights associated with the four goals; and x_{ij} and Z are previously defined.

RESULTS

Goal target levels for the goal-programming model were determined by running four linear programs, a procedure recommended by Field et al. The primary goals—total volume harvested (TV), total discounted cash flow (DCF), total undiscounted cash flow (UCF), and total discounted costs (TDC)—were each optimized. These programs ensured that target levels specified in the goal constraints were realistic, representing the optimum levels possible, given no constraints imposed on achievement of the other goals. Determining the target levels in this way also assured that subsequent goal-programming solutions would be noninferior.

Results of the four linear programs are presented in Table 3. The starred values (*) in Table 3 became the goal target levels in the subsequent goal-programming model. The DCF goal appears to be most affected by changes in the LP objective function. For example, specifying TV in the objective function resulted in a 26.2 percent decrease in the maximum DCF possible. However, specifying DCF in the objective function reflects only a 2.0 percent decrease in the maximum TV over the 90-year planning period.

Table 3 also presents the results of two goal-programming runs, a constained run (the "preferred" solution) and an unconstrained run, both with the same set of weights. The unconstrained run indicates the achievement values of the various goals when no periodic harvest or economic constraints are specified and when a regulated forest is of no concern. Differences in the respective goal achievement levels between the unconstrained and preferred solutions for the two higher priority goals, TV and DCF, are not significant. Indeed, the DCF goal is not at all affected by imposing constraints, while TV is decreased by only 2.5 percent. TDC is the most affected and increased by 15.2 percent.

Periodic volumes harvested (Figure 1) were severely affected. Periodic harvests under the unconstrained goal-programming run fluctuated considerably, ranging from 95,000 cords in the first period to 980,000 cords in the third period. This is unacceptable

Table 3. Linear and Goal Programming Solutions to Four Goals. [a]

	Achievement Levels			
	TV	DCF	UCF	TDC
Programs	MCD[b]	M$[c]	M$[c]	M$[c]
LP Programs				
Maximize TV	6945.5*	11,014	1,036,746	4378
Maximize DCF	6800.1	14,858*	1,027,731	4547
Maximize UCF	6833.0	11,690	1,065,411*	4267
Minimize TDC	6818.7	11,488	1,029,320	3943*
GP Programs				
Preferred	6801.6	14,857	1,028,246	4545
Unconstrained[d]	6974.4	14,859	1,065,412	3943

[a] TV = total volume
DCF = total discounted cash flow
UCF = total undiscounted cash flow
TDC = total discounted costs
[b] MCDS = thousand cords
[c] M$ = thousand dollars
[d] "Preferred" goal programming solution set with no periodic wood flow constraints
* Optimal solution values

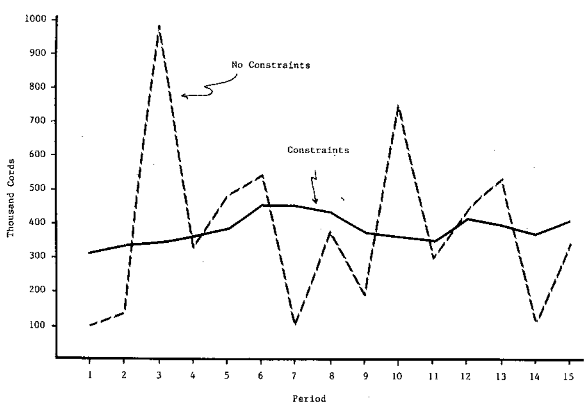

Figure 1. Periodic Harvest Volume from Preferred Solution, Constraints vs. No Constraints.

to forest managers who require a more stable flow in acreages site-prepared, planted, and harvested.

Age-class distributions of the 84,735-acre area at the beginning and end of the planning horizon are presented in Table 4. Age-class distributions are used in the work with plantations to determine whether regulated forest structures have been approximated. Acreages in the various age-class distributions prior to implementation of the harvest-scheduling program (the preferred solution) ranged from 200 acres (or .2 percent of the total acreage) in the 70- to 75-year-old stands to 21,508 acres (or 25.4 percent of the total acreage) in the 5- to 10-year-old stands at the beginning of the first period.

A regulated forest was reasonably achieved by the end of the planning horizon, with the distribution ranging from 9,824 acres in the 5- to 10-year age class to 14,422 acres in the 25- to 30-year age class. These represent 11.6 and 17.0 percent of the total acreage. Acreages of future stands are shown under only seven age classes since all future stands had to be cut between the ages of 30 and 35 (seven periods). No future stands were allowed to exceed this age class.

Table 4. Initial and Final Age Class Distributions.

	Distribution	
	Initial	Final
Age Class[a]	Acres	Acres
0	320	
1	4561	9824
2	12588	11557
3	21508	13963
4	10810	11827
5	6724	11142
6	5293	14422
7	3797	11999
8	3401	
9	3189	
10	3465	
11	3033	
12	2209	
13	1201	
14	1162	
15	200	
16+	1275	

[a] By five-year period.

SUMMARY AND CONCLUSIONS

Prior to development of the harvest-scheduling model, The Company decision-makers had no advanced forest management planning models to aid them in developing cutting budgets. Whatever strategy was used, the forest managers had to "feed the mill," while at the same time meeting long- and short-term objectives and constraints.

The harvest-scheduling model developed reasonably achieved The Company's long- and short-term requirements. A highly irregular forest structure was converted to a regulated forest structure by the end of the tenth period (50 years). Further, this was achieved in an orderly manner, with periodic harvest volumes ranging from 300 to 446 thousand cords. An unconstrained run resulted in similar solution values for the four decision criteria but had periodic wood flows ranging from 95 to 980 thousand cords. Thus, the opportunity costs associated with imposing the periodic harvest constraints appear to have been minimized.

Employing goal programming permitted the incorporation of multi-decision criteria in the harvest-scheduling model. Using this methodology should help reduce conflicts among The Company decision-makers over what goals and objectives should have priority; more specifically, problems between production-oriented and finance-oriented decision-makers should be minimized.

Indeed, The Company decision-makers were not aware of the potential trade-offs associated with placing greater weights or priorities on different goals. In the past, wood procurement (total volume) had always been the highest priority. The goal program illustrated that losses in total volume harvested over the planning period were small when greater weight was placed on financial goals. The reverse was not true, however. Concentrating on total volume resulted in significant losses in discounted cash flows.

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