DEMAND FOR PLANT NUTRIENTS IN TENNESSEE DISAGGREGATED BY MIXED FERTILIZERS AND DIRECT APPLICATION MATERIALS

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Abstract

When obtaining nitrogen (N), phosphate (P) and potash (K), purchasing decisions concerning the quantity and the form of each plant nutrient must be made. Logit models are estimated for the choice-of-form decision by considering those variables influencing the probability that plant nutrients will be purchased in Tennessee as part of a mixed fertilizer or as direct application materials. Parameter and elasticity estimates can be used by fertilizer manufacturers and distributors to anticipate changes in the composition of demand for plant nutrients in Tennessee.

Key words: plant nutrients, fertilizer, demand, logit model, inputs.

Decisions concerning the quantity and the form of each plant nutrient must be made when fertilizers are purchased. Studies which provide national, regional, or state estimates of demands for individual plant nutrients—nitrogen (N), phosphate (P) and potash (K) (Heady and Yeh; Carman; Roberts; Roberts and Heady)—address the former decision but are of limited value for anticipating changes in demands for alternative forms of plant nutrients. The former decision for Tennessee was addressed by Roberts who estimated demand functions for N, P, and K. The objective of this study is to concentrate on the latter decision by estimating logit models which consider those variables influencing the probability that plant nutrients will be purchased as part of a mixed fertilizer or as direct application materials.

Among previous studies, only Gyawu et al. recognize the usefulness of disaggregating the demand for major plant nutrients by form. As part of their econometric model of the U.S. fertilizer industry, demand functions are estimated for direct and mixed forms of each major plant nutrient. The current study differs from theirs in that it distinguishes between the aforementioned fertilizer decisions, concentrating on the latter, and accounts for cross-price effects between direct and mixed forms of application.[1]

THE MODEL

Demand for the ith plant nutrient, A_i, has at least two components as expressed by

(1) $A_i = A_{im} + A_{id}$,

where A_{im} is the amount of plant nutrient i purchased in mixture and A_{id} is the amount purchased as direct application materials. Conditional demand functions for each form of the ith plant nutrient are defined as

(2) $A_{ij} = f_{ij}(x, A_i), \quad j = m, d,$

where x is a vector of the price of mixed fertilizer, the price of direct application materials for the ith plant nutrient[2] and prices of crops and other inputs. Specification of these demand functions is based on the theory of the

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[1]In the context of this paper, the word "forms" refers to whether a plant nutrient is purchased as part of a mixed fertilizer or as a direct application material. In other contexts, "forms" may refer to whether fertilizers are purchased as dry bulk, dry bagged, or liquid materials. These forms are not considered in this paper.

[2]If there was only one grade of mixed fertilizer, prices representing direct application materials for the other two plant nutrients would also be included. By excluding prices of direct application materials for the other two major plant nutrients, it is recognized that there are many grades of mixed fertilizers from which to choose. It is assumed that changes in the form of one plant nutrient in response to changes in its own price can be accommodated by changing grades of fertilizers without affecting the proportions of the other two plant nutrients in mixture.

firm (Henderson and Quandt, pp. 69–70), except that they contain A_i. Hence, they are conditional demand functions defined for changes in each form of the ith plant nutrient, conditional upon total quantity of the ith plant nutrient being known. By including A_i in equation (2) it is implicitly assumed that the choice of plant nutrient quantity and the choice of form are made recursively and that their relationships can be estimated independently. This assumption is discussed further in a later section of this paper.[3]

The share of the ith plant nutrient purchased in mixtures is

(3) $k_{im} = A_{im}/A_i$.

Thus, k_{im} can be thought of as the probability that a unit of plant nutrient i will be purchased in mixture. Further, if

(4) $f_{ij} = e^{g_{ij}(x, A_i)}$, $j = m, d$,

then k_{im} is defined by a dichotomous universal logit function (Amemiya, pp. 1502, 1523),

(5) $k_{im} = e^{g_{im}}/\Sigma_j e^{g_{ij}}$, $j = m, d$,

and estimates of A_{im} can be obtained from

$\ln(k_{im}/k_{id}) = g_{im} - g_{id}$, or equivalently,

(6) $\ln(A_{im}/A_{id}) = g_{im} - g_{id} = h_{im}(x, A_i)$.

If h_{im} is assumed to be stochastic and linear in its arguments,[4] then equation (6) can be estimated by standard regression techniques. This provides estimates of the share of the ith plant nutrient purchased in mixtures or the probability that a unit of plant nutrient i will be purchased as part of a mixed fertilizer,

(7) $k_{im} = e^{h_{im}}/(1 + e^{h_{im}})$,

and from equations (1) and (3) it follows that

(8) $k_{id} = 1 - k_{im}$.

Conditional demand elasticities of A_{im} with respect to x can be obtained by solving equation (7) for A_{im}, differentiating with respect to x, and multiplying the result by $x \mid A_{im}$ giving,[5]

(9) $E(A_{im},x \mid A_i) = x(1-k_{im})\partial h_{im} \mid \partial x$.

Similarly, the conditional elasticity of A_{id} with respect to x is,

(10) $E(A_{id},x \mid A_i) = x(1-k_{id})\partial h_{id}/\partial x$,

where $h_{id} = g_{id} - g_{im}$.

These elasticities have several important properties: 1) as k_{ij} approaches unity, $E(A_{ij},x \mid A_i)$ approaches zero, suggesting that as the choice becomes limited to only one alternative, that alternative cannot change in the short run because A_{ij} would equal A_i, and A_i is assumed to be fixed; 2) since $\partial h_{ij}/\partial x = \partial g_{ij}/\partial x - \partial g_{il}/\partial x$ (j not equal to 1), the sign and magnitude of $E(A_{ij},x \mid A_i)$ is determined by the relative marginal responses to x of the alternative forms; and 3) the weighted sum of the conditional elasticities for the ith plant nutrient is equal to zero, where the weights are the shares of each form of the ith plant nutrient, implying that in the short run the quantity of a particular plant nutrient purchased in mixture cannot be increased (decreased) without decreasing (increasing) the quantity of that plant nutrient purchased as a direct application material (Garrod and Roberts).

If A_i is allowed to vary, then the elasticity of the jth form of the ith plant nutrient with respect to A_i can be calculated as,

(11) $E(A_{ij},A_i) = A_i(1-k_{ij})\partial h_{ij}/\partial A_i + 1$,

where the weighted sum ($j = d, m$) of these elasticities for the ith plant nutrient is unity.

EMPIRICAL MODEL

The following relationships based upon the logit model described in equation (6) are estimated for Tennessee from annual time series data for 1965–84 as follows:

(12) $Ln(N_m/N_d) = a_1 + b_1PN + c_1PM + d_1PR + e_1N + u_1$,

(13) $Ln(P_m/P_d) = a_2 + b_2PP + c_2PM + d_2PR + e_2P + u_2$, and

(14) $Ln(K_m/K_d) = a_3 + b_3PK + c_3PM + d_3PR + e_3K + u_3$,

[3] A similar assumption is made by Miklius et al. in their logit analysis of the demand for freight transport services. They assume that the decisions of whether or not to purchase a particular commodity, the size of shipment, and the origin of shipment are made independently of the transport mode decision.

[4] Linearization of h_{im} represents a Taylor series approximation (Chiang, pp. 255–60) of equation (6), which is derived from the more complex and unknown functional form of the production function. Although less accurate for an individual firm, this approximation may be more accurate for the aggregation of firms and is necessary to facilitate estimation which would otherwise be complex.

[5] The conditional elasticity of A_{ij} with respect to x is easily obtained from the logit model. By definition of the logit model, $k_{ij} = A_{ij}/A_i = e^{h_{ij}}/(1+e^{h_{ij}})$, where h_{ij} is assumed to be a function of x. Solving for A_{ij} gives $A_{ij} = A_ie^{h_{ij}}/(1+e^{h_{ij}})$. Taking the partial derivative of A_{ij} with respect to x yields $\partial A_{ij}/\partial x = h_{ij}'k_{ij}(1-k_{ij})A_i$, where h_{ij}' is $\partial h_{ij}/\partial x$, which assumes of course that $\partial A_i/\partial x = 0$. Multiplying both sides by x/A_{ij} gives the conditional elasticity of A_{ij} with respect to x, $E(A_{ij},x \mid A_i) = x(1-k_{ij})h_{ij}'$, because $A_i/A_{ij} = 1/k_{ij}$.

The elasticity of A_{ij} with respect to A_i, presented in equation (11), is similarly obtained by differentiating A_{ij} with respect to A_i and multiplying the result by A_i/A_{ij}.

where N, P, and K are quantities of nitrogen, phosphate, and potash purchased in Tennessee (1,000 tons), respectively; subscripts m and d refer to mixed and direct application forms of plant nutrients, respectively; PN, PP, and PK are prices of urea, concentrated superphosphate, and muriate of potash in Tennessee ($/ton), respectively; PM is the price of 6-12-12 mixed fertilizer in Tennessee ($/ton); PR is an index of crop prices received by farmers in Tennessee (1977=100), lagged one year; u_i's ($i = 1, 2, 3$) are error terms, and a_i, b_i, c_i, d_i and e_i ($i = 1, 2, 3$) are parameters to be estimated. All prices are deflated by the U. S. producer (wholesale) price index (1977=1.00), and time subscripts are suppressed for convenience.[6]

Data are from *Agricultural Prices, Annual Summary* (U. S. Department of Agriculture, 1964–84), *Agricultural Statistics* (U. S. Department of Agriculture, 1978–85), *Fertilizer Summary Data* (Tennessee Valley Authority), and *Tennessee Agricultural Statistics* (Tennessee Department of Agriculture). Fertilizer price data are for April 15 of each year until 1976, after which they are for May 15. Also, beginning with 1977, fertilizer prices are averages over the East South Central Region which includes Tennessee, Kentucky, Alabama, and Mississippi.

Quantity data for N, P, and K represent all reported commercial fertilizer sold or shipped in Tennessee (Tennessee Valley Authority, 1984, p. 6). Reporting firms do not necessarily sell directly to consumers. For example, the Tennessee Farmers Cooperative distributes to local farmers cooperatives about 60 percent of all fertilizer materials sold in Tennessee. To avoid double counting, sales by local cooperatives are not reported. Therefore, the data employed in this paper represent quantities demanded at the reporting level of the Tennessee fertilizer industry.

Prices of urea, concentrated superphosphate, and muriate of potash are used for prices of N, P, and K purchased as direct application materials because they represent dominant direct application materials in Tennessee for their respective nutrients (Tennessee Valley Authority). The price of 6-12-12 mixed fertilizer enters the model as a proxy for the price of mixed fertilizers because in Tennessee it has been consistently one of the two most used mixed fertilizers[7] (Tennessee Valley Authority). The lagged index of crop prices received by Tennessee farmers is used to represent crop price expectations.

The coefficients of equations (12)–(14) are expected to have the following signs: $b_i > 0$, $c_i < 0$, $d_i > 0$, and $e_i <=> 0$ ($i = 1, 2, 3$). Hypotheses regarding b_i and c_i result from the theory of the firm which allows only negative own-price effects and from the fact that there are only two nutrient forms (mixed and direct application) being considered. Therefore, if one form decreases (increases) because of an increase (decrease) in its own price, the other must increase (decrease), holding total quantity constant. Signs for d_i are hypothesized to be positive because farmers may apply plant nutrients more selectively when crop prices are expected to be lower than average. Alternatively, in years when crop prices are expected to be higher than average they may attempt to build overall soil fertility by applying a more balanced mixture of plant nutrients. A specific sign for e_i is not hypothesized because there is no *a priori* reason to believe that an increase in the total quantity of a plant nutrient would favor the choice of one form over the other.

ESTIMATION AND RESULTS

Ordinary least squares is an appropriate regression technique when 1) the choice of plant nutrient quantity decision is made recursively with the choice of form decision, and 2) the errors across equations (12)–(14) are uncorrelated. Roberts addresses the choice of quantity decision by estimating demand functions with quantities of N, P, and K as dependent variables. In this paper, quantities of N, P, and K are explanatory variables. If errors between equation (3) estimated by Roberts and equation (12) estimated in this paper are correlated, then the two equations are not recursive for N and ordinary least squares estimation of equation (12) would yield coefficients with simultaneous

[6]No index of prices paid by farmers or producer (wholesale) price index is available specifically for Tennessee, and the Tennessee consumer price index is considered inappropriate in the context of fertilizer demand. Therefore, the U. S. producer (wholesale) price index is used to control inflation.

Prices of other inputs, including those used in applying or otherwise handling direct application materials and mixed fertilizers, have been excluded from the empirical model for practical reasons (multicollinearity). If other input prices affected demands for the two plant nutrient forms equally, their exclusion would not bias the coefficients of the remaining variables.

[7]Although in recent years diammonium phosphate (18-46-0) has been the most used mixed fertilizer in Tennessee, its price is not used because a complete time series could not be obtained.

equations bias. The result would be similar for P and K. Simple correlation coefficients between the ordinary least squares residuals of equations (3)-(5) in Roberts and of equations (12)-(14) in this paper were tested for significance. The highest correlation found between pairs of equations was for K. The correlation between the residuals of equation (5) in Roberts and equation (14) in this study was estimated to be 0.346, which is not significantly different from zero at the 10 percent level (Johnston, pp. 41-42). Therefore, N, P, and K can be treated as predetermined variables (Theil, pp. 460-461), allowing equations (12)-(14) to be estimated independently of those estimated by Roberts.

If errors across equations (12)-(14) are correlated, as Roberts and Heady suggest is possible in plant nutrient demand models, seemingly unrelated regression is a more efficient estimation method than ordinary least squares (Johnston, p. 338). The correlation matrix of ordinary least squares residuals for equations (12)-(14) was tested against the identity matrix (Bartlett). The hypothesis that the error correlation matrix is a unit matrix could not be rejected at the 5 percent level, suggesting that seemingly unrelated regression would provide little gain in efficiency.[8]

Table 1 contains ordinary least squares parameter estimates for equations (12)-(14). All coefficients are significant at the 10 percent level except the coefficient for PN in equation (12),[9] and signs of all coefficients conform with expectations. The positive coefficients for PR support the hypothesis that larger proportions of plant nutrients are purchased as mixed fertilizers when crop prices are expected to be higher than average than when they are expected to be lower than average. Negative coefficients for N, P, and K indicate that as total demand for a plant nutrient increases, the proportion of the total purchased as mixed fertilizers decreases relative to the proportion purchased as direct application materials.

Estimated conditional demand elasticities for plant nutrients purchased in mixtures and as direct application materials, evaluated at the 1965-84 means, are presented in Table 2 These conditional price elasticities are not elasticities of demand in the traditional sense and should be interpreted differently from elasticities estimated in previous fertilizer and plant nutrient demand studies. They represent the estimated impacts of one percent changes in explanatory variables on the quantity of a plant nutrient purchased either as direct application materials or as mixed fer-

Table 1. Estimated Regression Coefficients for Equations (12)-(14): Choice Between Purchasing Plant Nutrients in Mixed Fertilizers or as Direct Application Materials, Tennessee, 1965-84

Explanatory variable	Dependent variable and equation number: 12 $Ln(N_m/N_d)$	13 $Ln(P_m/P_d)$	14 $Ln(K_m/K_d)$
PN	0.0004		
	(0.3657)[a]		
PP		0.0152	
		(2.85)	
PK			0.0282
			(1.48)
PM	−0.0165	−0.0574	−0.0848
	(−3.68)	(−5.62)	(−6.36)
PR	0.0116	0.0221	0.0622
	(3.03)	(2.96)	(4.95)
N	−0.0076		
	(−3.60)		
P		−0.0145	
		(−3.62)	
K			−0.0433
			(−6.72)
Intercept	0.8504	4.8066	5.0897
	(2.52)	(7.21)	(2.43)
R^2	0.78	0.79	0.90
DW[b]	2.36	2.40	1.78

[a]Numbers in parentheses are t-statistics. All t-statistics, except this one, are sufficiently large to suggest significance at the 10 percent level or better using a t-test with 15 degrees of freedom. A one-tailed test is used for coefficients of PN, PP, PK, and PM. Two-tailed tests are used for PR, N, P, and K.

[b]Durbin-Watson statistics are in the inconclusive range for all equations.

[8]The test statistic proposed by Bartlett is $\chi^2 = -B \log_e[R]$, where B is equal to $(N-1) - (2p+5)/6$, R is the sample correlation matrix, N is the number of observations, and p is the dimension of R. The test statistic has an approximate chi-squared distribution with $p(p-1)/2$ degrees of freedom. For the problem at hand, the calculated chi-square is 6.966. This is less than 7.81, which is the tabular value of the chi-square distribution with three degrees of freedom and a five percent significance level. Therefore, the hypothesis that the error correlation matrix is an identity matrix cannot be rejected.

[9]The low t-statistic for the coefficient of PN in equation (12) prompted investigation into whether multicollinearity was the cause. A multicollinearity diagnostics procedure suggested by Belsley et al. indicated that the coefficient of PN in equation (12) was harmfully degraded by multicollinearity. Therefore, the test that fails to reject the hypothesis that PN has no effect on the proportion of N applied in mixture is inconclusive (Belsley et al., pp. 172-73).

tilizers, given that total quantity demanded of the plant nutrient is constant. For example, estimated conditional elasticities for P purchased in mixtures and as direct application materials with respect to the mixed fertilizer price (PM) are −0.70 and 4.95, respectively. A one percent increase in the price of mixed fertilizer results in a decrease in the quantity of P purchased in mixtures, and because the total quantity of P is held constant, it requires an equal increase in the quantity of P purchased as direct application materials. These equal absolute changes translate into a 0.70 percent decrease and a 4.94 percent increase in the quantities of P purchased in mixtures and as direct application materials, respectively. If the shares of P purchased in mixtures and as direct application materials were equal, then the elasticities would be equal in absolute value. Conditional elasticities for P purchased as direct application materials are relatively large because the share of P purchased as direct application materials is relatively small (1965–84 mean is 0.12). Relative magnitudes of conditional elasticities for N and K are also determined by the relative shares of mixed and direct forms.

Conditional own-price elasticities of demand for nutrients purchased in mixtures are estimated to be inelastic for P (−0.70), elastic for K (−3.42), and about unity for N (−1.02). Directly applied N is least responsive to its own price with an estimated conditional own-price elasticity close to zero (−0.03). Similar elasticities for directly applied P and K are estimated to be greater than unity at −2.05 and −1.78, respectively.

Conditional cross-price elasticities for N, P, and K purchased in mixtures with respect to direct application material prices are estimated to be 0.04, 0.29, and 1.24, respectively. Cross-price elasticities are considerably larger for N, P, and K purchased as direct application materials with respect to the mixed fertilizer price. They are estimated to be 0.61, 4.95, and 4.94 for directly applied N, P, and K, respectively. These elasticities suggest that the allocation of plant nutrients by form is more responsive to the mixed fertilizer price than to prices of direct application materials.

Conditional elasticities with respect to the index of crop prices received by farmers are estimated to be inelastic for N purchased in mixture (0.66) and as direct application materials (−0.39), and for the P purchased in mixture (0.25). For P purchased as direct application materials (−1.74) and K purchased in mixture (2.29) or as direct application materials (−3.31), conditional elasticities with respect to the crop price index are estimated to be greater than unity. As expected, these crop price elasticities indicate that changes in crop prices positively affect the proportion of plant nutrients purchased in mixed fertilizers as opposed to direct application materials.

Elasticities of plant nutrients purchased as direct application materials with respect to total quantities of plant nutrients, $E(A_{id},A_i)$, are also shown in Table 2. They are all greater than unity, indicating that the proportion of plant nutrients purchased as direct application materials increases relative to purchase in mixed form as total quantity increases. Similar elasticities for plant nutrients purchased in mixture are all less than unity because the weighted sum of the alternatives for each plant nutrient must equal unity.

TABLE 2. ESTIMATED CONDITIONAL DEMAND ELASTICITIES FOR N, P, AND K PURCHASED IN MIXTURE AND AS DIRECT APPLICATION MATERIALS, EVALUATED AT THE MEANS OF THE DATA, TENNESSEE, 1965–84

	Plant nutrient					
	N		P		K	
Explanatory variable	Mixed (0.37)[a]	Direct (0.63)	Mixed (0.88)	Direct (0.12)	Mixed (0.59)	Direct (0.41)
PN (166.7)	0.04	−0.03				
PP (153.3)			0.29	−2.05		
PK (107.1)					1.24	−1.78
PM (98.6)	−1.02	0.61	−0.70	4.95	−3.42	4.94
PR (90.1)	0.66	−0.39	0.25	−1.74	2.29	−3.31
N[b] (111.6)	0.47	1.32				
P[b] (97.1)			0.83	2.23		
K[b] (108.4)					−0.92	3.78

[a]Numbers in parentheses beneath or to the right of a variable are sample means of the corresponding variable.

[b]Numbers in these rows are estimates of $E(A_{ij},A_i)$ given by equation (11). Other numbers in this table are estimates of $E(A_{ij},x|A_i)$ given by equations (9) and (10).

CONCLUSIONS

Parameter and elasticity estimates presented in this paper and the methodology employed are important to the fertilizer industry when total quantities of plant nutrients are either known, assumed, or estimated. For example, the equations estimated by Roberts deal only with demands for plant nutrients, irrespective of form. Those equations, in conjunction with the equations presented in this paper, could be used by the Tennessee Farmers Cooperative and others who report fertilizer sales in Tennessee to anticipate changes in demand for plant nutrients as direct application materials and in mixed fertilizers.

Similar estimates at the national level also would be useful. Various issues of *Inputs Outlook and Situation Report* (e.g., U. S. Department of Agriculture, 1984, p. 32) provide national projections of changes in plant nutrient quantities and prices. These projections, when combined with estimates for the United States analogous to those presented in this paper, could be helpful in anticipating aggregate changes in quantities of N, P, and K demanded as direct application materials and in mixture. Such information in turn could be used for planning as raw materials, fertilizer production capacity, and other resources are allocated to the production and marketing of mixed fertilizers versus direct application materials in anticipation of changes in demand.

These equations could be used, along with those estimated by Roberts and similar equations estimated for other regions, as part of an econometric model of the U. S. fertilizer industry in which Tennessee would be a separate region. Regional demands might be linked at the national level through direct application material and mixed fertilizer prices. Such linkages would allow interrelationships between N, P, and K demands, which are lacking in previous models (e.g., Gyawu et al.), to be captured through endogenously determined mixed fertilizer prices. Such a model would be useful to policymakers in evaluating the effects of crop, energy, and trade policies on the U. S. fertilizer industry or on specific fertilizer regions.

Finally, the methodology presented in this paper could be extended to address more complex choice-of-form situations requiring multinominal logit specifications. For example, conditional demand functions could be estimated to allocate the quantity of N purchased as direct application materials among various nitrogenous compounds such as ammonium nitrate, anhydrous ammonia, and urea.

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