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## A DISCRETE STOCHASTIC PROGRAMMING MODEL TO ESTIMATE OPTIMAL BURNING SCHEDULES ON RANGELAND

L. Garoian, J. R. Conner, and C. J. Scifres

### Abstract

Macartney rose is a range management problem on 500,000 acres of rangeland in Texas. Roller chopping followed by burning is an effective method of improving infested rangeland. However, uncertainty associated with implementing effective burns adversely affects economic feasibility of the treatment sequence. Discrete stochastic programming is used to determine optimal burning schedules under uncertainty. Optimal schedules and expected net returns vary with changes in the probability of a successful burn.

**Key words:** Macartney rose, discrete stochastic programming, prescribed burning, rangeland.

Macartney rose (*Rosa bracteata* Wendl.) is a range management problem on approximately 500,000 acres of highly productive rangeland in southeast Texas (Scifres). Macartney rose spreads rapidly and may develop canopy covers exceeding 75 percent on the Coastal Prairies of Texas, forming impenetrable thickets which severely limit forage production on rangelands and tame pastures (Gordon and Scifres).

Economic evaluation of selected multiple-treatment systems indicates that roller chopping followed by prescribed burning is the most economically effective treatment alternative (Garoian et al.). Roller chopping in the fall of the treatment year removes the woody canopy and releases fine fuels (readily combustible organic materials) for burning during the subsequent late winter-early spring.

Roller chopping kills relatively few, if any, of the Macartney rose plants. Canes severed from the parent plant and pressed into the soil surface by roller chopping may take root. The Macartney rose stand density may actually increase with roller chopping compared to pre-treatment levels if not followed promptly by burning. Prescribed burns must be applied regularly to prevent Macartney rose regrowth from nullifying the positive effects of previous treatments.

Scheduling and implementing burns is a critical consideration of the roller-chopping treatment. Burning too frequently results in excessive effort and cost. Infrequent burning can bring about decreased range productivity. However, burning is not always successful, and, in fact, its success is determined by stochastic factors. A successful burn must be uniform and of sufficient intensity and duration to kill the Macartney rose plants. Determinants of an effective burn are topography; wind direction, velocity, and gustiness; fuel quantity, continuity, and moisture content; and relative humidity (White). As the time of a scheduled burn approaches, these factors are revealed and the advisability of burning determined. Based on research experience of the third author during a 10-year study period, the probability of an effective burn during any particular winter on the Coastal Prairie is estimated at between 60 and 80 percent.

Since uncertainty associated with implementation of effective prescribed burns is an essential element of the roller-chopping treatment, the objective of this study is to determine an optimal burning policy. An optimal burning schedule is derived for each year of the planning horizon, given the outcomes of previous decisions.

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## METHODOLOGY

Dynamic programming is a widely used approach for optimizing sequential decision problems. This procedure has received considerable theoretical and applied attention. Dynamic programming's usefulness is due to: 1) the diversity of problems that may be formulated in a multi-stage manner; 2) the ease with which integer restrictions and uncertainty may be included, and 3) the efficiency of solution algorithms. Limitations of dynamic programming are: 1) problem size and 2) the lack of a general algorithm (Budnick et al.).

Discrete stochastic programming was developed to solve sequential decision problems with uncertain outcomes (Dantzig). Theoretical considerations have extended the applicability of the technique (Cocks; Rae, 1971b). Tutorial efforts have provided assistance to the researcher (Anderson et al.; Hansotia; Rae, 1971a; Apland and Kaiser). Despite sharing many of the positive characteristics of dynamic programming, applications are not overly abundant (Klemme; Tice; Leatham; O'Brien; Gebremeskel and Shumway; Yaron and Horowitz). This method enables solution of multi-stage problems where the objective function coefficients, input-output coefficients, or resource endowments (RHS values) are subject to uncertainty (Cocks). An advantage of discrete stochastic programming is that problems are formulated in a linear programming framework. Accessibility of linear programming algorithms is the primary reason discrete stochastic programming is utilized to determine an optimal burning policy.

Similar to dynamic programming, discrete stochastic programming suffers from the "curse of dimensionality." Formulation requires including activities for all possible outcomes. Simple problems can generate large matrices with even a limited number of stages and states.

Results from Macartney rose field experiments established rules for determining feasible burning schedules. Feasible schedules provide reasonable beef production parameters while reducing the infestation. Forming schedules based on the following rules considerably reduced the dimensionality problem.

1. A burn will not be attempted in year 2 unless there is an unsuccessful burn in year 1.
2. Burning in two consecutive years will not be attempted except under the following conditions.

- a. If the burn in year 1 is unsuccessful, then it will be necessary to burn in years 2 and 3. When Macartney rose is roller chopped in the fall it is recommended that the initial burn be applied in the winter. Waiting until the winter of the second year will reduce the effectiveness by about 50 percent when measured by stocking rates. A second fire in year 3 is necessary to obtain the equivalent benefit of a fire in year 1. With burns in years 2 and 3, stocking rate improvements occur two years later than with a successful burn in year 1.
- b. Three years have elapsed since the last successful burn and less than three burns have been applied. Longer periods between burns allow Macartney rose canopy cover to develop to the extent that fine fuel is inadequate to carry a fire through the regrowth. But, if three successful burns have been applied since roller chopping, it is possible to apply a single effective burn. The earlier burning pressure will reduce regrowth to a level where fine fuel will be adequate to carry a fire.
3. It is possible to terminate burning at anytime.

Production parameters for feasible burning schedules are determined from research results and best estimates of the third author based on experience gained during a 10-year study period. Parameters that vary with respect to schedules are stocking rates, weaning percentages, and weaning weights.

Embedded in a discrete stochastic programming model is an underlying decision tree. The three-stage decision tree shown in Figure 1 demonstrates the procedure for determining optimal burning schedules for Macartney rose. Initially it may be determined that burning in year 1 is desirable. The decision is made before the prevailing state of nature is revealed and preparation must be undertaken before the uncertainty is resolved. Resolution of the uncertainty results in good weather and bad weather with probabilities  $P_1$  and  $P_2$ , respectively. Depending on this outcome a return,  $R_{ijk}$  occurring in the  $i^{\text{th}}$  stage, emanating from the  $j^{\text{th}}$  set of state  $i-1$  activities, and under the  $k^{\text{th}}$  state of nature is determined. If "good" weather prevails, the burn is applied and the return  $R_{111}$  is achieved. At  $R_{111}$ , it is known that the burn

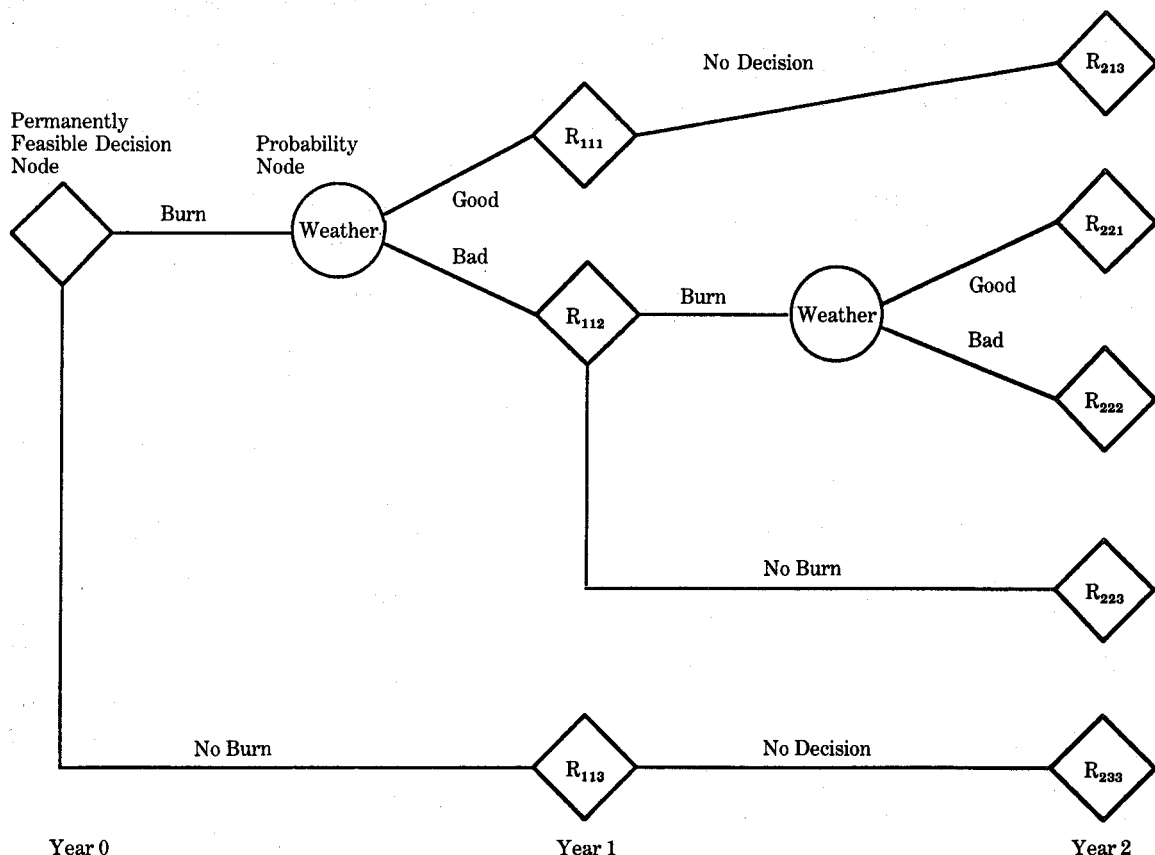


Figure 1: Three Stages of Macartney Rose Decision Tree.

was successful and that a burn in year 2 should not be applied. The return in year 2,  $R_{213}$ , is obtained with probability  $P_3=1$ , or complete certainty. The joint probability associated with  $R_{111}$  and  $R_{213}$  is  $P_1 \times P_3$ . If "bad" weather prevails, the initial burn is not successful and the return  $R_{112}$  is obtained. At  $R_{112}$ , the decision to burn is again considered. Deciding to burn and "good" weather produce the return at  $R_{221}$ . Deciding to burn and "bad" weather produce the return at  $R_{222}$ . The joint probability associated with  $R_{112}$  and  $R_{221}$  is  $P_2 \times P_1$ ; similarly, the probability of  $R_{112}$  and  $R_{222}$  is  $P_2 \times P_2$ . Deciding not to burn at any point results in obtaining the corresponding return with certainty.

This process demonstrates the adaptive nature of the problem. Each decision is made with the knowledge of the past. But for the most part, the future remains uncertain.

The decision tree for the 10-year planning horizon based on experimental rules con-

tained 51 different burning schedules and 381 branches. Many of the branches represent the same burning schedule. However, because they come about by different decisions and outcomes, there are different probabilities associated with the various returns. An unrestricted decision tree with a 10-year planning horizon and three possible outcomes for each decision would have produced  $10^3$  branches. Eliminating infeasible schedules resulted in a more manageable problem.

A general representation of the unrestricted discrete stochastic programming model for Macartney rose is presented for two years (3 stages) of the planning horizon. This representation can be extended without difficulty, but for compactness will not be shown. The model is as follows:

maximize

$$C_{00} X_{00} + P'R_{11} X_{11} + P_1 P'R_{21} X_{21} + P_2 P'R_{22} X_{22} + P_3 P'R_{23} X_{23},$$

subject to:

$$\begin{bmatrix} A_{000} & 0 & 0 & 0 & 0 \\ A_{100} & A_{111} & : & : & : \\ A_{200} & A_{211} & : & : & : \\ A_{300} & A_{311} & 0 & : & : \\ 0 & A_{411} & A_{421} & 0 & : \\ 0 & A_{511} & 0 & A_{522} & 0 \\ 0 & A_{611} & 0 & 0 & A_{623} \end{bmatrix} \begin{bmatrix} X_{00} \\ X_{11} \\ X_{21} \\ X_{22} \\ X_{23} \end{bmatrix} \leq \begin{bmatrix} b_{00} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and  $X_{ij} \geq 0$   $i=0,1,2$   $j=0,1,2,3$ ;  
where

$$C_{00} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$$

= a vector of cost parameters at stage 0;

$$R_{ij} = \begin{bmatrix} R_{ij1} & 0 & 0 \\ 0 & R_{ij2} & 0 \\ 0 & 0 & R_{ij3} \end{bmatrix}$$

= a diagonal matrix of income parameters received at stage  $i=1,2$ ; resulting from decision node  $j$  at stage  $i-1$ , where  $j=1$  if  $i=1$  and  $j=1,2,3$  if  $i=2$ ; and under state of nature  $k$ , with  $k=1$  if good,  $k=2$  if bad, and  $k=3$  if certain;

$$X_{00} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

= a vector of activities at stage 0;

$$X_{ij} = \begin{bmatrix} X_{ij1} \\ X_{ij2} \\ X_{ij3} \end{bmatrix}$$

= a vector of activities at stage  $i$ , resulting from decision node  $j$  at stage  $i-1$ , and under state of nature  $k$ ;

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

= a vector of probabilities ( $P_1$ =probability of good state of nature,  $P_2$ =probability of bad state, and  $P_3=1$  is a certain state, with  $P_1+P_2=1$ ); and

$A_{hij}$  = matrices of input-output coefficients ( $h=0, \dots, 6$ ).

A detailed description of the restricted model used in this study is shown in Figure 2 in a linear programming format. In this formulation all uncertainty has been transferred to the objective function by multiplying discounted net returns,  $R_{ijk}$ 's, by the probability of obtaining the returns. These values are an aggregation of all activities associated with a cow-calf operation on a 500-acre pasture. The returns included in the  $R_{ijk}$ 's are primarily dependent on stocking rates, weaning weights, and weaning percentages offered by a particular burning schedule. Weaned calves are sold for \$70/cwt. Against this contribution are variable costs of \$105/cow unit and a 5 percent return to a fixed livestock investment of \$35/cow unit. Additionally, livestock herd size adjustments are accounted for by including costs and returns from buying and selling livestock in response to stocking rate changes. Salvage values for improved pastures are included in the returns of the final year. These values are measures of productivity above the roller-chopping treatment without burns and beyond the 10-year planning period. Costs and returns are expressed in constant 1983 dollars, and a 5 percent discount rate is used in determining net present value. The objective function represents expected net present value of annual returns to land, capital (excluding livestock), management, and overhead.

Pasture is the only resource constraint in the model. The pasture constraint allocates one pasture (500 acres) between burn and no burn activities at stage 0. The remaining constraints link the stage  $i$  activities with the stage  $i+1$  activities. A characteristic of this formulation is that the sum of the probabilities associated with returns from activities that come into solution equal unity in each stage. In stage 1, either the permanently feasible activity  $X_{113}$  or both  $X_{111}$  and  $X_{112}$  will come into solution. Similarly, in stage 2, three possible solution combinations exist—the permanently feasible activity  $X_{233}$ ,  $X_{213}$  and  $X_{223}$  or  $X_{213}$ ,  $X_{221}$  and  $X_{222}$ . It is this feature that permits determination of the best course of action given a particular decision node.

## RESULTS

Optimal burning schedules for 60, 70, and 80 percent probabilities of implementing successful burns are shown in Table 1. Non-optimal schedules are also obtained by constraining the model to meet specific assumptions concerning the timing of the second burn.

Obj. Function Coefficient	$C_1$	$C_2$	$P_1^* R_{111}$	$P_2^* R_{112}$	$P_3^* R_{113}$	$P_1^* P_3^* R_{213}$	$P_2^* P_1^* R_{221}$	$P_2^* P_2^* R_{222}$	$P_2^* P_3^* R_{223}$	$P_3^* P_3^* R_{233}$	RHS
s. t.											
Pasture	$A_{000}$ 1 1										$\leq 1$
$G_{011}$	$A_{100}$ - 1		$A_{111}$ 1								$\leq 0$
$B_{012}$	$A_{200}$ - 1		$A_{211}$ 1								$\leq 0$
$N_{013}$	$A_{300}$ - 1		$A_{311}$ 1								$\leq 0$
$N_{121}$			$A_{411}$ - 1		$A_{421}$ 1						$\leq 0$
$G_{122}$			$A_{511}$ - 1		$A_{521}$ 1 1						$\leq 0$
$B_{123}$			$A_{611}$ - 1		$A_{623}$ 1 1						$\leq 0$
$N_{124}$			$A_{811}$ - 1		$A_{823}$ 1						$\leq 0$
Stage 0			Stage 1			Stage 2					

Key for Objective Function:

$P_k$  = probability of state  $k$ ; good weather if  $k=1$ , bad weather if  $k=2$ , and certainty if  $k=3$ ;

$R_{ijk}$  = discounted net returns from 500-acre pasture in stage  $i$ , decision node  $j$ , and state of nature  $k$ ; and

$C_m$  = initial allocation of pasture burn if  $m=1$  and don't burn if  $m=2$ .

Key for constraints: G = good weather, B = bad weather, and N = no burn; the first two digits indicate transfers from stage  $i$  to stage  $i+1$ ; and the last digit refers to the number of transfers between stage  $i$  and stage  $i+1$ .

Figure 2: Three Stages of Macartney Rose Linear Programming Matrix.

The optimal schedule under a 60 percent probability is to burn in years 1, 3, 5, 8, and 10. Expected net returns associated with this schedule are \$3,960.87. Delaying the second burn until year 4 resulted in scheduling burns for years 1, 4, 7, and 10. This delay reduced income by only \$124.93. Attempting the second burn in year 5 and following with burns in 6, 8, and 10 reduced income by \$2,061.44. In fact, following this schedule produced a return substantially lower than not burning. With the exception of burning in years 1, 5, 6, 8 and 10, the variation between alternative schedules is relatively small.

At a probability level between 60 and 70 percent, burning schedules converge. Burning in years 1, 4, 7, and 10 is optimal under 70 and 80 percent probabilities with returns of \$4,670.79 and \$5,781.15, respectively. Scheduling the second burn one year earlier results in a modest reduction of \$123.84 under a 70 per-

cent probability. A relatively large reduction of \$515.26 occurs from scheduling the second burn in year 3 under an 80 percent probability. Applying the second burn in year 5 provides large reductions of returns under both 70 and 80 percent probabilities.

These results demonstrate the relationship between the probability of a successful burn and expected returns from effective burning schedules. As the probability of a successful burn increases, expected returns increase at an increasing rate for non-optimal burning schedules. This is also true for optimal burning schedules where an increase in probability of a successful burn from 60 to 80 percent brought about a 46 percent increase in expected returns. Substantial benefits are obtained from maintaining a burning schedule, but weather conditions play an important role in realization of benefits.

Also, higher probabilities of success produce

optimal burning schedules that have fewer burns and longer periods between burns. Obtaining the benefits of burning with more certainty reduces the necessity of frequent burning.

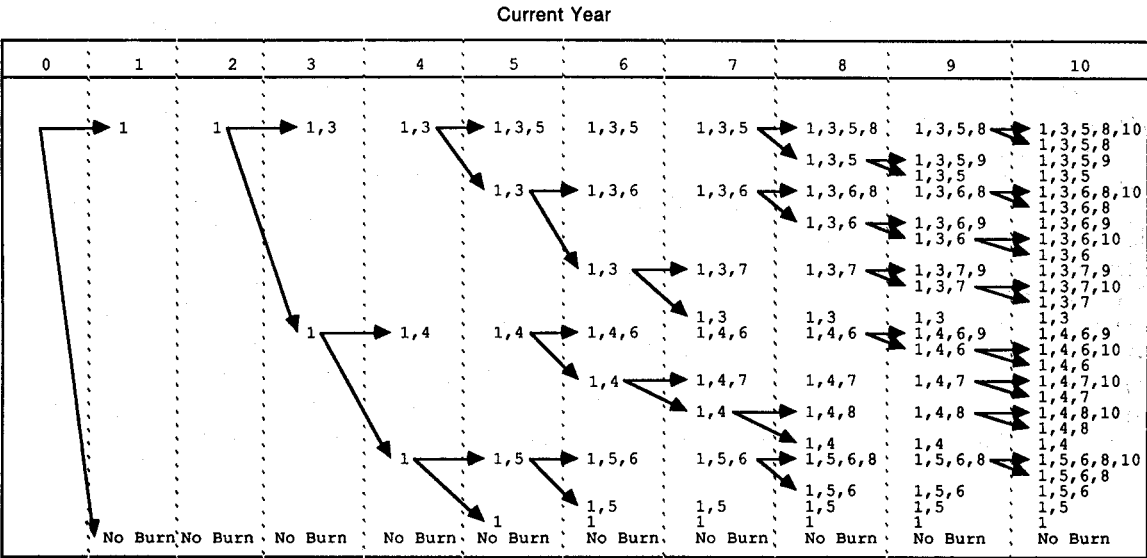
Since it may not always be possible to follow the optimal burning schedule, alternative schedules are of interest. The discrete stochastic programming model provides burning strategies given the outcomes of previous burning attempts as shown in Figures 3 and 4. Recommended burning schedules given the current year and the years that successful burns have been applied are found by moving horizontally from year to year. The decision rule following an unsuccessful burn is found by moving diagonally. Illustrating determination of burning schedules, Figure 4 shows that if in year 7 burns have been applied in years 1 and 4, then a burn should be attempted in year

TABLE 1: ALTERNATE BURNING SCHEDULES AND EXPECTED NET PRESENT VALUES ASSOCIATED WITH PROBABILITIES OF SUCCESSFULLY BURNING MACARTNEY ROSE

Year of Second Burn	Burning Schedule (Years)	Probability of Successful Burn		
		.60	.70	.80
		Dollars Per 500-Acre Pasture		
3	1,3,5,8,10	3960.87 <sup>a</sup>	4546.95	5265.89
4	1,4,7,10	3835.94	4670.79 <sup>a</sup>	5781.15 <sup>a</sup>
5	1,5,6,8,10	1899.43	1854.07	1888.77
No Burn	No Burns	3467.46	3467.46	3467.46

<sup>a</sup>Optimal burning schedule for given probability of successful burn.

8. The optimal burning schedule from that time is burning again in year 10. If the burn in year 8 is unsuccessful, the best course is not to attempt any other burns. Macartney rose re-growth would be too substantial for an economically-effective fire.



<sup>a</sup>Schedules refer to the years that successful burns have been applied.  
 Note: Horizontal movement between years provides recommended burning schedule given the current year and burning history. Diagonal movement provides optimal schedules after an unsuccessful burn.

Figure 3: Determination of Alternate Schedules Given the Year and Burning History<sup>a</sup> Under 60 Percent Probability of Successful Burning of Macartney Rose.





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