THE DAWNING OF THE AGE OF DYNAMIC THEORY: ITS IMPLICATIONS FOR AGRICULTURAL ECONOMICS RESEARCH AND TEACHING

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The opportunity to present a presidential address provides a rare and unique opportunity. It is perhaps the only time one gets to speak to the profession without your material either being reviewed and corrected before its presentation, or reviewed and corrected by a discussant after your presentation. Indeed the freedom that a presidential address offers takes a little getting used to, but it provides a wonderful opportunity to express one's biases. To you, the members of the profession who took the risk to allow me this opportunity, let me say thank you. I have chosen to use this opportunity to address a topic that I think provides one of the most exciting and potentially productive challenges our profession will face in our lifetimes, that is "The Dawning of the Age of Dynamic Theory."

I would like to begin my presentation by expressing one of my most fundamental biases, that is that the single equation static production function that we have all studied in our traditional production economics classes is an obsolete research tool. Why we continue to teach and use the traditional production function must be looked upon in a new light. That new light is being provided by the dawning of the age of dynamic theory. This age is bringing us far-superior tools with which to model input/output relations and to determine optimal resource allocation and management strategies.

Before I am misinterpreted, let me hasten to add that there is still a very important need for the concepts of static production function theory. Let me emphasize the word "concept." The principles learned in mastering the profit maximization concepts of static production theory are still conceptually useful for explaining the responses of individual producers and the agricultural sector in general to input and output price changes, technological change, resource and policy constraints, taxation, subsidization, and so forth. Indeed the basic concepts of static production theory are not lost in dynamic theory, but are assimilated and expanded. But as a stand-alone quantitative tool of economics, the production function should be, and is being, replaced. If you read the literature of today, as opposed to that of a decade or two ago, I believe you will see that the production function is being replaced by dynamic models of processes, simulation models if you will. The most advanced of our research is now optimizing these dynamic process models using optimal control theory.

The awakening process during the dawning of the age of dynamic theory has not been uniform throughout our profession. Inevitably, I believe, all of our profession will have to struggle through the waking moments of this new day of dynamic stochastic theory and become prepared to go forth into it. It is this waking process I wish to address. It will require new curriculum in our graduate programs, the dispelling of some old ways of thinking, new types of data, and new working relations with our peers who understand dynamic physical processes.

My focus in this presentation will be upon dynamic theory and its use in studying physical resource allocation problems. More specifically, I will tend to focus many of my remarks toward examples of the use of control theory in the area of livestock production and management problems. However, I would hasten to add that the attributes of dynamic theory are general in nature and are likely to

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be just as applicable for many other areas, such as the dynamic social interactions in the political process and the price discovery process. Indeed, optimal control theory has been used frequently in policy analysis in the past. A major area of use of dynamic theory is in the field of resource management. The basic question of how renewable resources, such as forests, should be managed versus nonrenewable resources, such as oil and minerals, is inherently a dynamic problem. Closely linked to the study of dynamic biological production processes for major agricultural commodities would appear to come a new perspective of dynamic supply response, and hence a new perspective on the market equilibrium seeking process. Dynamic theory is in general well suited for any problem involving the accumulation or growth of some entity over time, such as an animal. However the same basic accumulation process is apparent in capital theory, in the firm growth process, or the economic growth process in general. Indeed our world is basically a dynamic one, which makes dynamic theory a very natural tool to seek, but alas it has not been an easy one for our profession to implement.

My focus will also tend more toward considering the impacts of injecting dynamic frameworks into our research, as opposed to injecting stochastic properties into our research. However, I believe dynamics and stochastics go hand-in-hand, just as statics and perfect knowledge do. I do not believe we will go very far forward in dynamic theory without further consideration of questions of imperfect knowledge and stochasticness.

I will attempt to make five basic points during the course of this address. First, static theory poorly defines economic processes. By economic process I mean the day-to-day sequence of production events managers must understand and deal with to implement their production plans. Second, simulation modeling provides an effective tool for modeling economic processes that static theory fails to adequately deal with. Third, optimal control theory provides the conceptual basis for extending static production theory to dynamic optimization theory. Fourth, dynamic programing, nonlinear programing, and an array of heuristic search optimizing methods as used by engineers provide an "artful," somewhat untractable, but nevertheless feasible method of solution to the dynamic optimization problem. Last, the skillful combining of well-designed simulation models of processes, coupled with the artful selection and implementation of known solution processes, can lead us into a new age of dynamic theory—an age which I believe will be very productive for our profession.

DISPELLING OLD IDEAS ABOUT PRODUCTION THEORY

One of the reasons I stated my bias against the use of production functions as a quantitative research tool in the introduction of this address is that I believe our profession still has "hang-ups" about the use of production functions and about static economic theory in general. These hang-ups, while subtle, are devastating to our research because of the way they lead us to think. Warren Samuels of Michigan State University recently addressed some of these "hang-ups" in a paper titled "Determinant Solutions and Valuational Processes: Overcoming the Foreclosure of Process." Samuels argues that economists emphasize the need to find a determinant solution to a problem so much that it causes them to overlook the "process." More specifically, economists must quantify a process so as to find a maximum determinant solution. Samuels argues that there is much to be learned by just studying processes. It is my contention that single equation production functions are a poor representation of the production process and that their continued use is primarily based upon habit, their convenience of mathematical manipulation, and their ease of statistical application and validation. Animal scientists and agronomists generally gaze in amusement when economists confront them with production functions which do not describe the dynamic sequential processes they understand as plant and animal growth.

An example may serve to emphasize here what I mean by a "process-oriented model" versus a static model. A production function, as opposed to a dynamic process model, is like a shopping list versus a recipe. The shopping list tells you all the ingredients you need to prepare a dish, but it contains none of the process instructions present in a recipe (i.e., what to mix together; whether to stir, chop, or blend; whether the items should be heated, cooled, etc.). Admittedly, the production function gives you a very precise shopping list of exact quantities and assures us it is the least-cost list for producing an optimal amount, but it does not tell us about the process of using these ingredients. A similar analogy I recall from my chemistry class is that a production function is like a chemical formula, while a process model is like the set of instructions given to carry out an experiment. One cannot produce alcohol by just knowing the chemical formula for it. Physical scientists tend to view our production functions much as I have just described shopping lists and chemical formulas. They may help in budgeting and costing, but they say nothing about the process.

Another way to state the problem of the production function is that what is missing is management. Knowing the production function and its optimal solution leaves us far short of being able to implement and manage the production process.

Another issue to be raised is that static production functions may not be giving us the truly optimal input shopping list and production target. There is evidence that dynamic theory gives us different answers than static theory to the same basic question of what is optimal in a given case. We may not have yet resolved which is right, but I have my own suspicion.

MODELING DYNAMIC PROCESSES

So what is available to model the processes that production functions fail to adequately deal with? Perhaps Samuels would say, "There you go again trying to model and find determinant answers." I would beg to differ. Simulation modeling is a technique that focuses on processes and structures, not on optimization and determinant solutions. In dealing with plant and animal production, the simulation modeling approach to describing the production process is the growth model. There are several examples of such models that have been used by our profession, of which the most familiar may be the National Research Council's model (NRC model) of beef growth. These growth models incorporate a multiequation, dynamic process-oriented structure that our colleagues in plant and animal science readily identify with. With the injection of such models into our research, we open up the potential for what I believe is an exciting and fruitful basis for dialogue with our physical scientist counterparts.

To some degree simulation has been shunned by economists because of its non-determinant, non-optimizing nature, and hence its ability and tendency to use non-statistically based parameters. Boggess has stated that "fundamentally, all biophysical simulation does is generate the production response surface which is necessary for all empirical production economics research...[they are however] mathematically less tractable than neoclassical production functions. As a result, simulation analysis coupled with various search algorithms are normally used to analyze decision alternatives, rather than [for] analytical derivation of the 'optimal' input level." Musser and Tew comment that simulation "does not propose to identify 'optimal' plans for firm managers. Rather it proposes to provide information which most likely has qualitative value for farm managers." Samuels would likely endorse "analyzing decision alternatives" and providing information that has "qualitative value." But to the traditional economist, still bound to determinant solutions, maximization, and mathematical rigor, these activities have a hollow ring.

In my opinion, simulation has been undervalued by some in our profession in two ways. First, I believe it has considerable stand-alone merit as a nonoptimizing method for studying processes, particularly processes involving stochasticity. But an even greater potential for the use of simulation is in combination with optimal control theory to model and find determinant, optimal solutions for dynamic models of processes. Together with Walker, I make this point in a paper presented at a Beef Cattle Production Systems Simulation Conference in 1985. We envisioned that progress in integrating ongoing work in animal growth modeling with optimal control theory and the calculus of variation could lead to a "new theory of dynamic production." I believe that the evidence is even stronger today that this linkage is possible and is occurring.

Indeed, our profession has been optimizing simulation models rather frequently for some time, but not with the rigor and insight that I believe optimal control theory permits. This is the major point of my address. Simulation and optimal control are naturally complementary tools that need to be wedded by our profession to make a significant advance in the age of dynamic theory. I will comment in more detail on this point presently. Before doing so, I want to build a background perspective of the promising potential I see in this wedding. I shall do this by looking at the "roots of dynamic theory" in our profession and by defining what I mean by optimal control theory.

THE ROOTS OF DYNAMIC THEORY

The roots of dynamic theory trace back to two classical works. The first is Bellman's work in 1957 titled "Dynamic Programming"

(DP) in which he presented his "principle of optimality" and thus established a somewhat tractable way to solve dynamic optimization problems. Indeed, it is debatable whether any solution method provides a very tractable way to obtain a solution to the dynamic optimization problem. By tractable I mean a generalized method that can be readily applied in a routine manner, rather than having to be customized for each modeling case as seems to be the current situation for optimal control solution methods in general, including dynamic programing.

The second classical work is in the 1962 work of four Russian mathematicians named Pontryagin, Boltyanskii, Gamkrelidze, and Mishchenko titled "The Mathematical Theory of Optimal Processes." This work presented what is commonly known as Pontryagin's maximum principle. The body of work evolving from Pontryagin's maximum principle is generally known as optimal control theory. Pontryagin and Bellman's principles are essentially rooted in the same concepts but do have different focuses. I shall approach my discussion here from the perspective of the optimal control theorist. I do this primarily because I believe it is a much more natural extension of static economic theory than DP and because it focuses more on the properties associated with the maximum being obtained than on the solution process itself. I hasten to add, however, that the lack of a tractable solution process has been more the downfall of optimal control theory than DP. Because of its tractability in obtaining a solution, DP has to date received wider use in the agricultural economics profession than optimal control. In a sense, optimal control theory has been to DP what static production theory has been to linear programing. It has provided the theoretical base, while DP has provided the bulk of the meaningful applied work.

The concept of optimal control theory seems to have first entered the economics profession in a noticeable way in 1969 with an article by Dorfman published in the *American Economic Review.* The article was titled "An Economic Interpretation of Optimal Control Theory." Dorfman argues "that optimal control theory is formally identical with capital theory, and that its main insights can be attained by strictly economic reasoning." With regard to static capital theory and optimal control theory, Dorfman stated what I have attempted to state for production functions verses biological growth models: "A mode of analysis that is confined to a distant, ultimate position is poorly suited to the understanding of accumulation and growth..." The article proceeds to discuss the mathematics of optimal control theory in economic terms using mathematics that is essentially the same as that used in static production theory. I recommend it as useful introductory reading. An even better treatment of the same fundamental information is available in a research report from North Carolina State University by Thomas Johnson titled "Growth and Harvest Without Cultivation: An Introduction to Dynamic Optimization." For those interested in pursuing the subject further, other useful texts include Kamien and Schwartz's text titled "Dynamic Optimization: The Calculus of Variation and Optimal Control in Economics and Management"; Bryson and Ho's text titled "Applied Optimal Control"; Cannon, Cullum and Polak's text titled "Theory of Optimal Control and Mathematical Programming"; and Chow's text titled "Analysis and Control of Dynamic Economic Systems: Economic Prediction and Control." There are other good texts, but these are cited the most frequently in the agricultural economics literature.

AN OVERVIEW OF THE CONCEPTS OF OPTIMAL CONTROL THEORY

Optimal control theory explicitly considers time, while static production theory does not. Because of this, optimal control simultaneously considers two problems that static theory has attempted to deal with independently. Optimal control theory simultaneously considers the problem of optimal input use and optimal replacement policy. To see this point, consider that the ration an animal is fed determines its rate of growth and eventually its marketing date and weight. Tradeoffs exist between the speed of growth, cost of growth, and optimal marketing weight. Likewise, if fixed assets, such as feedlot facilities, are involved in the growth/production process, there is a tradeoff between continuing to use the fixed assets for this animal's growth versus a replacement animal's growth.

Optimal control solutions contain a time path of optimal input use rates as well as an optimizing termination date and weight. Static solution sets do not specify a terminal date and give only the optimal values for total input use and optimal output. Because of the differences in the solutions they provide, the solution processes for static and dynamic problems are different. There is, however, another more fundamental difference between the dynamic theory case and the static theory case. The fundamental physical relationship in the dynamic problem is what is referred to as the "equation of motion," rather than the production function. One way to conceptualize an equation of motion is to think of it as a biological growth function. The equation of motion describes how a state variable, such as an animal's weight or a plant's biomass, changes over time as different inputs and/or controls are used. The equation of motion is generally written as a differential equation and is recursive in nature. That is to say, growth in period t+1 is a function of the current weight of the animal as well as the inputs and controls applied during the upcoming period.

Dynamic theory specified that the net present value of the flow of profits over time is to be maximized, subject to the constraint of the equation of motion and any other existing constraints, such as feedlot capacity.

(1) $\text{Max } F = \int_{t=0}^{\infty} \pi e^{-rt} dt,$

subject to

(2) $G_t = f(Y_t, x_t, t)$, and

(3) $h(Y_t, x_t, t) \leqq 0,$

where π is the profit at time t, G_t is the rate of change or growth in the state variable Y_t at time t, and x_t is the input rate at time t. The function h denotes the possibility of constraints to the system, such as input availability, terminal date restrictions, etc.

Application of the calculus of variation indicates that the net present value of profits over time, as expressed in equation (1), can be maximized by maximizing the undiscounted "Hamiltonian" function:

(4) $H_t = CNR_t + \lambda G_t + \gamma_t C_t.$

The Hamiltonian (H_t) is defined for each point in time and contains three basic terms. The first term describes the current period net revenue conditions (CNR_t). The second term defines the value of the net addition to the state variable (animal weight) during the current period. It is the product of G_t, the "growth" during the period, and the value associated with each unit of growth during the period, λ_t. λ_t is also often referred to in the optimal control literature as the costate variable and is analogous to a shadow price as defined in programing literature. The third term describes any constraints placed on the system, such as feedlot capacity, contracted terminal date, etc. In the discussion that follows, I will ignore the existence of any constraints to the system.

Three types of first order conditions, as represented in the three equations below, are involved in maximizing the Hamiltonian. The conditions are derived from taking first derivatives of the Hamiltonian with respect to inputs (x_t), the state variable (Y_t), which in this case can be thought of as animal weight, and the costate variable (λ_t). These three conditions are named the Hamiltonian or Optimality Condition, the Adjoint Condition, and the Equation of Motion Constraint.

Optimality/Hamiltonian Condition

(5) $\partial H_t/\partial x_t \rightarrow -\partial CNR_t/\partial x_t = \lambda_t{*}(\partial G_t/\partial x_t).$

Adjoint Condition

(6) $\partial H_t/\partial Y_t \rightarrow \lambda_t = \lambda_{t+1} + (\partial H_{t+1}/\partial Y_{t+1}),$

where

(6a) $\partial H_{t+1}/\partial Y_{t+1} = \partial CNR_{t+1}/\partial Y_{t+1} + \lambda_{t+1}{*}(\partial G_{t+1}/\partial Y_{t+1}).$

Equation of Motion Constraint

(7) $\partial H_t/\partial \lambda_t \rightarrow Y_{t+1} = Y_t + G_t.$

The Optimality/Hamiltonian condition gives essentially the same first order conditions as static production theory. In the case of slaughter animals, that is that the value of the growth achieved from one unit of input must equal the cost of that unit of input. The difference here is that the value of a unit of growth is not exogenously determined, but is instead the costate variable which is defined by the Adjoint Condition.

Let me elaborate a bit on the dynamic aspects of the Optimality Condition. In the case of meat animals, the first derivative of the growth function with respect to feed input is positive, while the second derivative is negative. Hence the growth rate increases with more feed being fed, but at a decreasing rate. However, as the animal grows, this response changes. Heavier animals are not as efficient at growth because their bodies require more feed for maintenance of past growth. Properly modeled, the growth function will capture this phenomenon. Thus for the Optimality Conditions to hold over time, either the feeding intensity, the cost of feed, or the value placed on growth by the costate variable must change. In actuality, feed costs are usually stable, hence feeding intensity changes as does the value of the animal's growth.

The preceding discussion leads us to focus our attention on the costate variable, λ_t. The Adjoint Condition explicitly focuses upon defining the costate variable. The costate variable is the heart of dynamic theory. It is wedded with the equation of motion, or growth function, and has no counterpart in static theory. It is critical to note that the costate variable is not the average price of the accumulated growth (i.e., the state variable) but is the price per unit of the marginal addition or growth of the state variable during the period in question. Thus the distinction is, an animal's slaughter price is the average value per pound of accumulated growth, but it is not necessarily the value of the last pound of growth occurring. Indeed, it is this distinction that leads to the optimal terminal weight condition. For the optimal terminal weight to have been reached, the shadow price given by the costate variable must equal the slaughter price. This condition is embedded in the Adjoint Condition, but is also specifically referred to as the Transversality Condition. It is the Transversality Condition that determines the optimal replacement policy.

Changes in the costate variable over time are determined by the changes in the Hamiltonian caused by accumulated growth (i.e., animal weight). Positive contributions to the Hamiltonian by growth cause the costate variable to decline over time. This may seem counter-intuitive, but it is reality. Consider the fact that productive assets are worth more per unit when they are new, young, or small. An asset that is going to contribute a great deal of income and accumulated stocks tomorrow is worth more today because of that expected contribution. As an animal grows or ages, it becomes less productive and hence worth less per unit.

Thus the Adjoint Condition defines how growth should be valued in each period over time. It provides the value that is critical to the Optimality Condition in equation (5) that enables resources to be used correctly in each period. It also assures that the resource use level in a given period is correct in relation to all other periods so that the eventual slaughter weight and replacement or termination date are optimal.

The third, and last, maximizing condition is the Equation of Motion Constraint. When the Hamiltonian is differentiated over time with respect to the costate variable, the result is that the optimal solution set is restricted by the equation of motion. This is analogous to a similar condition that can be proven in static production theory (i.e., the optimal solution must satisfy the production function).

Combined, these three first order conditions provide the essence of what is known as "Pontryagin's Principle of Maximization." That principle basically proves that there is no other path, or set of control variables, that will result in a greater value for the dynamic objective function specified. Application of this principle as just described constitutes what is generally known as open-loop, deterministic, optimal control.

THE EVOLUTION OF DYNAMIC THEORY IN AGRICULTURAL ECONOMICS

Having completed this divergence into an intuitive explanation of optimal control theory, let me return to my literature review. My approach to reviewing the literature will be to select a few key articles that reflect the evolution of dynamic theory and give a perspective of where it stands in the profession today. The works of Bellman and Pontryagin have been noted. The first application of Bellman's principle of optimality in the agricultural economics profession was in 1963 by Burt and Allison in their article titled "Farm Management Decisions with Dynamic Programming." Other articles have improved upon the efficiency of the model solution process and the complexity of the dynamic model dealt with. But in my opinion, none have made a significant breakthrough in the basic methodology.

Turning to the literature on simulation and optimal control theory, the first noted mention of simulation in agricultural economics was a short note by Babb and French in the 1963 *Journal of Farm Economics* titled "Use of Simulation Procedures." Early simulation work did not attempt to maximize simulation models. When efforts did begin to maximize simulation models, two fundamental approaches were used. The less frequently used of the two methods was that of optimal control and Pontryagin's principle. The efforts using this approach were predicated by Dorfman's work, which I have already made mention of, and a session at the 1969 winter meetings of the American Agricultural Association dealing with optimal control. The first effort I am aware of in the agricultural economics literature to directly use Pontryagin's principle of optimality was an article by Hochman, Regev, and Ward in 1974 titled "Optimal Advertising Signals in the Florida Citrus Industry: A Research Application." For the most part, there were few direct applications of Pontryagin's principle during the '70s and early '80s in conjunction with simulation models. When Pontryagin's principle was used, it was generally in conjunction with relatively small simulation models. The more frequent use of simulation analysis was to build larger, more-complex models that were either not maximized at all or which used a direct approach to maximization of the objective function that avoided specification of the Hamiltonian function and its first order conditions. Such approaches generally involved the use of heuristic search methods, including gradient search methods and response surface techniques. Indeed, some linked themselves to dynamic programing as a solution process. These modeling efforts served a useful and productive purpose and still have a place in agricultural economics research. Technically they can claim to be a part of the dynamic optimization literature because they deal with maximization over time. However, they lose much of the power and knowledge available through optimal control theory that is associated with knowing the Hamiltonian first order conditions and how they are being satisfied. The direct approach to maximizing the dynamic objective function of a simulation model is somewhat like solving the static production theory profit function by a heuristic search procedure. It is feasible, and perhaps with today's computers and optimization algorithms, even reasonably efficient. But, it lacks the depth of information that consideration of the first order conditions can provide.

To further emphasize this point, I would ask the question, "Would we understand as much as we do about static theory today if we had a black box solution method that somehow gave us the new optimal solution every time we changed a price or production parameter?" Perhaps I overstate my point, but I believe that is where we are today with regard to understanding dynamic processes with simulation models that use direct solution processes and ignore the Hamiltonian first order condition. To a lesser extent I have some of the same reservations about dynamic programing.

I am encouraged by the fact that over the last several years articles that make use of simulation and Pontryagin's principle have been appearing in the agricultural economics literature with more frequency. Four of these articles have been in the area of animal growth. They include the work of Chavas, Kliebenstein, and Chrenshaw in 1985 dealing with swine growth. This article appears to be the first application of optimal control theory to animal growth in the agricultural economics literature. Chavas and his co-authors clearly lay out an optimal control model and develop a swine growth function to fit the theory. They find a number of very insightful results of the nature described in presenting the theory of optimal control, such as the influence upon optimal marketing weight and date of input prices and output prices. The shortcoming of this study in my opinion is the simplicity of the growth function estimated. It is a single differential equation with four variables. As simple as it is, it provides a substantial improvement over a four-variable production function, simply because of its conceptual orientation. Herein lies the dilemma of control theory; mathematical solution of the control problem is not easy if the equation of motion is complex. But the realism, and hence potential usefulness of any dynamic model, is tied to increasing the complexity of the equation of motion.

Three articles have followed Chavas, Kliebenstein, and Crenshaw's article. One of these articles was by myself in 1986. It dealt with beef breeding herd replacement problems. In the article I essentially specified a multiequation beef herd growth model as the equation of motion. A more recent comment by McClelland and Wetzstein in the 1988 *American Journal of Agricultural Economics* and my response to their comment in the same issue of the *Journal* help tie my original article more closely to control theory.

The third of the four recent articles to apply control theory to animal growth was also published in 1986 and was by Chavas and Klemme. It dealt with milk production and supply response. This article is unique in that it links optimal control theory to supply response function specification.

The most recent, and perhaps the most advanced, article dealing with animal growth and using optimal control theory is by Hertzler in the 1988 *Western Journal of Agricultural Economics*. It deals with optimal feed rations and feeding rates over time. Hertzler uses a six-equation growth model as his equation of motion which incorporated much of the state-of-the-art knowledge in animal science about modeling animal growth.

These four efforts show the profession is achieving an enhanced ability to consider larger, more complex, and more realistic simulation models using optimal control theory. They also appear to reflect progress in using solution techniques which allow more natural specifications of the model structure. I would warn, however, that the explicit nature of the solution processes used in these articles is not well defined or standardized in the articles.

At this point I should make specific reference to a useful set of papers presented at the 1982 Western Agricultural Economics meetings by Burt, Talpaz, Howitt, and Zilberman, which were also published in the *Western Journal of Agricultural Economics*. These papers deal with the state of the art in dynamic programing and optimal control theory at that time. In the session, Howitt expressed great faith that "given the long history of Pontryagin-based control applications in engineering and operations research" there should be some answer as to how to obtain more tractable solutions to dynamic optimization problems using optimal control theory. Until such tractable solution processes are found, Howitt indicated there will continue to be very little applied use of optimal control theory in our profession. Burt made similar comments regarding improving the tractability of dynamic programing procedures. Burt went on to comment more about why DP has not been more widely used some 20 years after its introduction into the profession. He concluded that there are several reasons, the greatest of which "seems to be the conceptualization difficulties, i.e., understanding how to formulate an empirical situation as a DP model. The conceptual and computational difficulties compound one another because the most intuitive and direct way to structure the model is often infeasible computationally, or at least cumbersome and expensive in computer resources." Secondly, Burt states that DP is difficult to teach, probably because of a lack of background experience by both teachers and students.

BRIDGING STATIC THEORY AND DYNAMIC THEORY

Three notable publications occurred outside the field of dynamic theory that have contributed to bridging the gap between static theory and dynamic theory. In a fundamental sense they are not as advanced as the Bellman and Pontryagin's principles developed nearly a decade or more before them. But they provided a useful link to extend what the agricultural economics profession knows about applied static problem solving to dynamic problem solving. The first of these works was by Dillion in 1968 and was titled "The Analysis of Response in Crop and Livestock Production." In this work, Dillion recognized and classified the effects of time upon the production process and classified it into four types of influences. He also classified the influence of time upon prices into three categories. Another useful contribution of Dillion was to classify production processes in terms of the dynamics of the input and output patterns.

A second author who helped to conceptually bridge the gap from static production theory and control theory was Fawcett. In his 1973 article titled "Toward a Dynamic Production Function," Fawcett argues in favor of using a differential (difference) equation to characterize the production process. He cited several advantages to the approach. First, it allows for the use of nutritional information in the specification of the biological growth process. Second, it appears to be better suited than Dillion's approach for discussing the influence of changes in input use over time. Indeed, the differential equation approach to modeling growth as suggested by Fawcett is totally compatible with the state equation and equation of motion concepts in optimal control theory.

A third author in this area is Antle. His 1983 work on "Sequential Decision Making in Production Models" formulates the production model as a sequence of interlinked production steps. Each step can be defined as an individual production function whose output feeds forward as an input into the next production stage. Antle characterizes the difference between single equation static production theory and dynamic/sequential decision procedures. One of the essential points that can be deduced from Antle's characterization is that when the assumption of perfect knowledge is made, many, but not all, of the advantages of dynamic versus static theory are lost.

CHALLENGES OF THE AGE OF DYNAMIC THEORY

Let me conclude by summarizing and then stating the challenges to our profession of the age of dynamic theory. I am personally intrigued with optimal control theory and its potential to advance agricultural economics' research. Perhaps my intrigue is because of optimal control theory's theoretical eloquence. Perhaps it is because of my hope that optimal control theory will eventually accomplish what dynamic programing has begun, but has not successfully finished—that is, to lead our profession into the widespread use of dynamic theory. However, to date, optimal control theory has proven to be less tractable in application than dynamic programing. It has instead served primarily as the conceptual basis for dynamic programing, much as production theory has served as the conceptual basis for linear programing. It would seem, however, that optimal control and the dynamic process orientation of simulation are such powerfully compatible tools that their joint development should not, and cannot, be ignored by our profession. Furthermore, the apparent success of engineering and operations research to design customized solution procedures based on Pontryagin's maximization principle would seem to indicate that economic problems can be solved in a similar way. But alas, we are not engineers, and our problems, while dynamic, are not engineering problems. Because of this, and because of the apparent individualistic nature of dynamic optimization problems and their solution, we may be forced to develop our own body of dynamic theory, rather than borrow our tools as easily and directly as we have in the past for other theoretical developments. What I am suggesting is contrary to what Burt and Howitt suggested in 1982. They hypothesized that the widespread adoption of dynamic theory in agricultural economics would be dependent upon the development of tractable solution procedures for dynamic problems. I would contend that if there were tractable solution procedures for dynamic optimization problems, we would have found them by now. What is likely to be necessary to achieve widespread use of dynamic theory in agricultural economics is not more tractable solution procedures but more tractable agricultural economists. Perhaps by making this statement I am calling for a more quantitative profession. To some degree that is true. But I am also calling for a profession with more insight in conceptualizing problems as they exist in reality, as dynamic problems. I believe the conceptual challenges of understanding dynamic processes are as great as the quantitative challenges. Let me close by highlighting what I believe are the three basic challenges to our profession to brighten the current dawning of the age of dynamic theory.

The Challenge of Understanding Dynamic Processes

The most basic of these three challenges is multidisciplinary interaction to find the data and knowledge with which to develop our equation of motion models of dynamic economic processes. Equation of motion models require much different data and knowledge than our traditional static models. As a result, I believe, we will have to work closely with biological scientists, not only to obtain the data we need, but to understand the processes we are modeling. I firmly believe, and it is my experience, that biological scientists and physical scientists in general are much more responsive to economists who define their research approach in terms of dynamic process models, as opposed to traditional static economic models. This concept and way of thinking are much more compatible to them. What we find, however, as we venture into the world of the biological scientist is that many of the key parameters essential to defining a good biological process model, for the purposes of economic analysis, are not firmly established. One cause of this is the biological scientists' tendency to focus upon different levels of aggregation than economists. One of the results of the past lack of appropriate parameters for constructing economic process models is the emergence of simulation models with logically sound structures, but non-data supported parameters. One might include in this group the emergence of expert systems models as they are being viewed and developed by some.

The Challenge of Being a More Tractable Economist

A second major challenge of the age of dynamic theory is to improve our ability to both construct and solve complex multiequation models of motion, hence to become more analytically tractable economists. I am not optimistic that these methods will be made significantly more general and easier to use in the near future, if ever. Thus, I believe we will have to become more adept at being our own builders of customized solution procedures.

While we may continue to hope for more tractable solution methods, we cannot hope for outside help in formulating our dynamic models. I see the skills required for formulating dynamic problems to be as formidable as the solution processes themselves, if not even more so. To a large degree, model construction and model solution cannot be separated. Until we understand dynamic theory, we will not understand the nature of the solution process required by it, nor will we fully comprehend the key elements required in the structuring of an adequate dynamic model. When I *used* to estimate production functions, I knew the results I was looking for because I knew what influenced the solutions to the static profit maximization problem. We must obtain the same joint insight into constructing dynamic equation of motion models and solving them.

The Challenge of Teaching a New Generation of Dynamic Ag Economists

The third challenge of the age of dynamic theory is teaching. We must teach ourselves and our students to better understand dynamic theory. This involves learning the calculus of variation and all its applications. I believe every traditional production economics course should be extended, or perhaps followed by a course, to teach dynamic theory. Hand in hand with this material should come another course dealing with stochastic theory and measurement. As I stated previously, this is a subject in itself that I have chosen not to address here but which is likely of equal importance. In addition to these new concepts, our students need formal training beyond econometrics and linear programing in modeling. The processes and techniques of systems analysis and simulation as taught in engineering go hand in hand with optimal control. Without formal training in simulation, our students will not have the insight to develop good equation of motion models. Lastly, study in the area of solution to nonlinear systems of equations, nonlinear programing, and optimization methods in general, including gradient search methods as used in the fields of engineering, needs to be included in our graduate curriculum.

Closing Statement

I should note in closing, as I did in opening, that dynamic theory and its principles are general in nature. Although I see great opportunities for its use in biological production problems and firm level management problems, it is my hope and belief that others will see many applications in other areas. Indeed, our whole world is dynamic in nature. We have at hand the theory and tools to analyze our world as it really is. We are at the dawning moments of a truly new era in economics, one that I believe will make our profession more productive as well as more compatible with other professions. Let us meet the challenges of this new age quickly by enhancing our own professional growth functions and thereby raising our current costate values.

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