IRREVERSIBLE IMPORT SHARES FOR FROZEN CONCENTRATED ORANGE JUICE IN CANADA

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Canada is the most important U.S. export market for frozen concentrated orange juice, accounting for over 8 million gallons of exports each of the past 9 years. Brazil and the U.S. are the dominant suppliers of orange juice in Canada. Prior to 1975, the U.S. held more than 60 percent of the Canadian market. Brazil's share of the market has grown dramatically in the past ten years, capturing more than a 50 percent share in 1978 (see Table 1). Brazil's success in gaining a dominant market share can be partially traced to freezes in Florida in 1977, 1981, and 1982 that severely curtailed U.S. production of oranges for processing. As a result of these shocks and concern about the decrease in the U.S. market share, competitive advertising strategies have been adopted by the Florida Department of Citrus. In this paper we examine U.S. and Brazilian market-share relationships in Canada to test the hypothesis that the relationships are irreversible.

Graphically, the nature of the hypothesized relationship is shown in Figure 1. Initially, the price level is P_1 and the U.S. market share is MS_1 (point A). Prices increase to P_2 and the U.S. market share is reduced to MS_2 (point B). When prices return to their original level, the U.S. market share increases to MS_3, which is less than MS_1.

Irreversibilities have long been recognized in applied economic models. Irreversible relationships have been modeled by using distributed lags (Nerlove), lagged dependent variables (Houthaker and Taylor), segmented independent variables (Goodwin et al.; Tweeten and Quance; Houck), and a variety of other techniques, including time-varying parameters (Ward and Tilley). Each of these approaches has been used for a variety of reasons. The reasons for hypothesizing irreversibility can generally be classified as: (1) psychological, (2) technological, and (3) institutional (Nerlove, p. 1).

In the Canadian orange juice market, technological factors are a major source of irreversibility because of the form in which the product is imported from the U.S. and Brazil.

Nearly 90 percent of U.S. exports to Canada are already packaged, with the rest delivered as high-density bulk juice, either in drums or tanker trucks. All Brazilian juice arrives in 55-gallon drums and is packaged in Canada. Frozen concentrated orange juice (FCOJ) in 6-, 12-, and 16-ounce cans is the dominant form of orange juice consumption in Canada (Surveys International, Limited); thus, to use Brazilian FCOJ,

Table 1. Canadian OJ Imports, 1974 through 1982

Year	U. S.		Brazil		Other		Total
	Vol.[a]	%	Vol.[a]	%	Vol.[a]	%	Vol.[a]
1974	8.32	64.6	2.02	15.7	2.52	19.6	12.86
1975	9.11	53.4	7.10	41.5	.87	5.1	17.08
1976	10.02	59.1	5.98	35.3	.96	5.6	16.96
1977	9.29	54.4	6.28	34.8	1.83	10.8	17.40
1978	8.97	40.1	12.38	55.4	1.00	4.5	22.35
1979	10.24	44.4	12.05	52.3	.77	3.3	23.06
1980	11.98	51.2	11.32	47.0	.41	1.8	23.71
1981	10.61	37.5	16.93	59.8	.75	2.7	28.29
1982	8.38	33.3	16.14	64.0	.68	2.7	25.20

Source columns (U. S., Brazil, Other) are grouped under "Source".

[a]Million gallons 42° Brix equivalent.
Source: *Statistics Canada*.

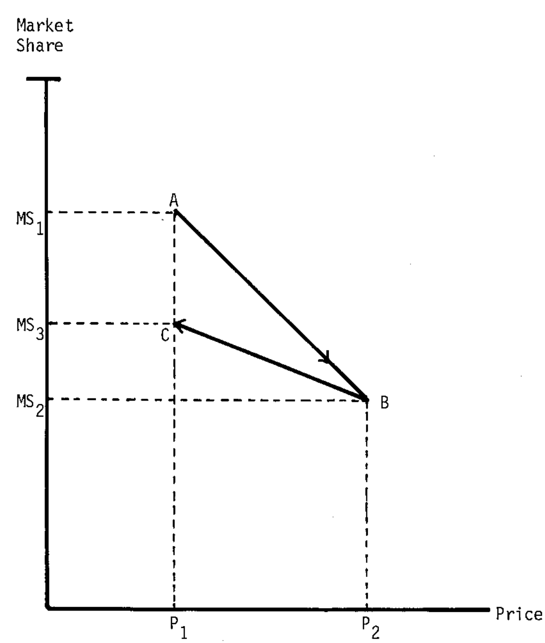

Figure 1. Irreversible Market Share Relationship.

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firms acquired the expertise as well as equipment to package FCOJ or reconstitute FCOJ for production of chilled orange juice. In order to market the products, several brands of Canadian-packed FCOJ were established. The existence of the facilities and brands for Canadian-packed Brazilian FCOJ reduces the prospect for future domination of the Canadian market by U.S. suppliers in the short run. Brands are thought to promote loyalty and consumption habits which cause irreversibility. The hypothesis is that once price shocks cause old habits to be broken and new brands are adopted, returning to previous brands may be resisted.

EMPIRICAL MODEL

The conceptual model is designed to test for own-price effect irreversibilities as well as substitute-price effect irreversibilities. The method of segmenting variables as described by Nelson is used.[1] This method was chosen because the changes we are attempting to measure are governed by discrete decisions made by large firms. Both distributed-lag and lagged-dependent variable models assume smooth adjustment processes.

The U.S. share of total Canadian orange juice imports during time period t (USS_t) is considered a function of the import price of orange juice from the U.S. (PUS_t) and other exogenous variables (X_t). To account for the irreversibilities, the price slope for positive price changes above $PUS_{t,max}$, that is, the maximum U.S. import price attained from time $t-n$ up to and including t, is hypothesized to be different from the one for negative and positive price changes below $PUS_{t,max}$. Thus, the model (see Appendix for derivation) is specified as:

$$(1)\quad USS_t = C_1 + a_{11}\,PUS_t^+ + a_{21}\,PUS_t^* + b_1X_t + e_{1t}$$

where

$$C_1 = a_0W_1 + a_{00}(1-W_1)$$

$$PUS_t^+ = \sum_{S=2}^{t} W_s\,(PUS_S - PUS_{S-1}) + W_1\,PUS_1$$

$$PUS_t^* = \sum_{S=2}^{t} (1-W_s)\,(PUS_S - PUS_{S-1}) + (1-W_1)\,PUS_1$$

$$W_t = 1 \text{ if } PUS_t \geqslant PUS_{t,max}$$
$$= 0 \text{ otherwise.}$$

The prices of substitutes for United States orange juice imports, that is, the price of imported Brazilian juice (PBR_t) and the price of orange juice imported from countries other than the United States and Brazil ($POTH_t$), can be included in equation (1) using a similar derivation. That is

$$(2)\quad USS_t = C_1^* + a_{11}PUS_t^+ + a_{21}PUS_t^* + a_{31}PBR_t^+ + a_{41}PBR_t^* + a_{51}POTH_t^+ + a_{61}POTH_t^* + b_1X_t + e_{1t}$$

where

$$(3)\quad C_1^* = a_0W_1 + a_{00}(1-W_1) + a_0U_1 + a_{00}(1-U_1) + a_0V_1 + a_{00}(1-V_1)$$

$$PBR_t^+ = \sum_{s=2}^{t} U_s(PBR_s - PBR_{s-1}) + U_1PBR_1$$

$$PBR_t^* = \sum_{s=2}^{t} (1-U_s)\,(PBR_s - PBR_{s-1}) + (1-U_1)PBR_1$$

$$POTH_t^+ = \sum_{s=2}^{t} V_s(POTH_s - POTH_{s-1}) + V_1POTH_1$$

$$POTH_t^* = \sum_{s=2}^{t} (1-V_s)\,(POTH_s - POTH_{s-1}) + (1-V_1)POTH_1$$

$$U_s = 1 \quad \text{if } PBR_s \geqslant PBR_{s,max}$$
$$= 0 \quad \text{otherwise}$$
$$V_s = 1 \quad \text{if } POTH_s \geqslant POTH_{s,max}$$
$$= 0 \quad \text{otherwise.}$$

$PBR_{s,max}$ and $POTH_{s,max}$ are the maximum Brazilian import price and maximum import price for orange juice from other countries attained from time $t-n$ up to and including t, respectively.

A similar relationship is hypothesized for the Brazilian market share (BRS). Since all imports from Brazil were in bulk form rather than retail packages, the import price of bulk U.S. FCOJ was used in lieu of PUS.

$$(4)\quad BRS_t = C_2 + a_{12}PBUS_t^+ + a_{22}PBUS_t^* + a_{32}PBR_t^+ + a_{42}PBR_t^* + a_{52}POTH_t^+ + a_{62}POTH_t^* + b_2X_t + e_{2t}$$

where

$$PBUS_t^+ = \sum_{s=2}^{t} W_s(PBUS_s - PBUS_{s-1}) + W_1PBUS_1$$

[1] See Wolffram for reasons for using the following model derivation instead of the approach suggested by Tweeten and Quance. The following approach is developed by Nelson and is similar to those used by Traill et al., and Young.

$$PBUS_t^* = \sum_{s=2}^{t} (1-W_s)(PBUS_s - PBUS_{s-1}) + (1-W_1)PBUS_1$$

$$W_s = 1 \quad \text{if } PBUS_s \geqslant PBUS_{s,max}$$
$$= 0 \quad \text{otherwise}$$

$PBUS_s$ is the import price of bulk U.S. FCOJ during period s, while $PBUS_{s,max}$ is the maximum import price of U.S. bulk FCOJ attained from $s-n$ up to and including s. The definitions of other variables are similar to those defined in (3). Other exogenous variables included in the market share equations under X_t are Florida Department of Citrus (FDOC) advertising expenditures in Canada (ADV), a zero-one dummy variable (D_t) to capture the impacts of the 1977 freeze, three zero-one dummy variables (Q1, Q2, Q3) to show seasonal variations in the market shares of FCOJ imported during the year, and an income variable (INC) to capture the composite effect of the time trend and the income effect.

Seemingly unrelated regression was used to estimate equations (2) and (4).[2] A procedure proposed by Kmenta and Gilbert (pp. 187–88) was used to correct for first order auto-correlation in equation (2). Different values of n were tried in order to select $P_{t,max}$. Based on the expected signs of the coefficients and the weighted mean square error for system, $n=2$ was chosen, that is, $P_{t,max}$ is defined as the maximum import price of the FCOJ from a particular origin attained from time $t-2$ up to and including t.[3] The results are presented in Table 2. All price variables are deflated by the Canadian wholesale price index and the income variable is deflated by the Canadian consumer price index. The time period used in this study is from the first quarter of 1972 through the second quarter of 1981.

RESULTS

Table 2 shows the estimated parameters along with the estimated standard errors in parentheses. In general, the signs of the price slopes are consistent with *a priori* expectations. In those cases where the signs were not as expected, the estimated coefficients were not statistically different from zero. The estimated composite effect of income and time (measured by the coefficient of the income variable) shows that these factors had a negative impact on the U. S. share of the Canadian FCOJ import market. The impact on Brazil's share was not evident.

The advertising variables used in this study indicate a decreasing rate of return from advertising. The estimated coefficients of the advertising variables show that Florida orange juice advertising in Canada had a significant positive effect on the U. S. market share in

Table 2. Regression results [a]

Independent Variable[b]	Market Share	
	U. S.	Brazil
Intercept	2.7704*	1.0016
	(.6526)	(.8359)
PUS_t^+	-.8612*	--
	(.3117)	
PUS_t^*	-1.1954*	--
	(.1838)	
$PBUS_t^+$	--	-.0998
		(.1257)
$PBUS_t^*$	--	.7566*
		(.3555)
PBR_t^+	.1301	-.0952
	(.1863)	(.4559)
PBR_t^*	.7865*	-.9869**
	(.2059)	(.6077)
$POTH_t^+$	.4750**	.1884
	(.3351)	(.5107)
$POTH_t^*$	.1894	.0043
	(.1866)	(.4782)
INC_t	-.0004	-.0002
	(.0001)	(.0003)
ADV_t	.0004*	-.00002
	(.0002)	(.00045)
ADV_{t-1}	.0003**	-.0005**
	(.0002)	(.0004)
D_t	-.1317*	.3077*
	(.6453)	(.1399)
$Q1_t$	-.1189*	.1139*
	(.0412)	(.0608)
$Q2_t$	.0808*	-.0622
	(.0360)	(.0911)
$Q3_t$	-.0151	.0198
	(.0452)	(.0741)

[a]Numbers in parentheses are estimated standard errors. The weighted mean square error for system is 1.5043 with 44 degrees of freedom. '*' indicates the estimate is statistically different from zero at $\alpha = .05$ level and '**' indicates the estimate is statistically different from zero at $\alpha = .10$ level.

[b]Variable definitions:

PUS_t^+, PUS_t^*:	Average Canadian import price of U.S. FCOJ during quarter t in U.S. dollars per SSE gallon (U.S. Department of Commerce).
$PBUS_t^+$, $PBUS_t^*$:	Average Canadian import price of U.S. bulk FCOJ during quarter t in U.S. dollars per SSE gallon (U.S. Department of Commerce).
$PBRT_t^+$, PBR_t^*:	Average Canadian import price of Brazilian FCOJ during quarter t in U.S. dollars per SSE gallon (Statistics Canada).
$POTH_t^+$, $POTH_t^*$:	Average Canadian import price of FCOJ from countries other than Brazil and the U.S. in U.S. dollars per SSE gallon (Statistics Canada).
INC_t:	Private consumption per capita in U.S. dollars (International Monetary Funds).
ADV_t:	Florida generic advertising expenditures in Canada deflated by the index per thousand households reached (CPM) in U.S. dollars (Florida Department of Citrus).
D_t:	Zero-one dummy variable. $D_t = 1$ if t is after the fourth quarter of 1977. Otherwise $D_t = 0$.
$Q1_t, Q2_t, Q3_t$:	Zero-one dummy variables. $Q1_t = 1$ if t is the second quarter of a year, otherwise $Q1_t = 0$; $Q2_t = 1$ if t is the third quarter of a year, otherwise $Q2_t = 0$, and so on.

[2] Under the linear form specification, the fact that market shares must sum up to unity implies that the sum of all intercepts in the market share equations must equal unity and the sum of the estimated coefficients for each variable must equal zero. Therefore, the market share equation for other imports can be derived from estimates for equations (2) and (4). Since all shares sum to unity, the disturbances across the three-share equations will add up to zero for each observation; therefore, a seemingly unrelated regression approach can be applied to only two of the three equations.

[3] n = 1, 2, 3, 4 were tried.

Canada for one quarter beyond the quarter during which the advertising actually occurred. The initial impact of advertising was greater than the subsequent impact a quarter later. The estimated coefficients of the advertising variables in Brazil's market share equation show that Florida orange juice advertising efforts had a negative impact on the Brazilian market share in Canada. The result shows that the impact was not evident during the quarter when advertising occurred, but the lagged impact was both negative and statistically different from zero.

The estimated coefficients of the zero-one dummy variable, D_t, indicate that the freeze in Florida during 1977 increased Brazil's market share at the expense of the U. S. market share. The estimated coefficients for the seasonal dummy variables show that the U. S. market share was bigger in the third quarter than in the first quarter and smaller in the second quarter than in the first quarter, while Brazil's market share is bigger in the second quarter than in the first quarter.[4]

The estimated impact of the price changes on market shares and the tests carried out for the irreversibility hypothesis are summarized in Table 3. The estimated impact of price on the U. S. share in Canada is presented in the upper half of Table 3, and the estimated impact of price on Brazil's market share is shown in the lower half of the same table.

The estimates of equation (2) show that if the average import price of U. S. FCOJ is increased by 10 cents per single strength equivalent (SSE) gallon, the U. S. market share would be decreased by 8.61 percent, given other variables held constant. On the other hand, if the average import price of U. S. FCOJ is decreased by 10 cents per SSE gallon, the U. S. market share would be increased by 11.95 percent. The difference between the changes in U. S. market shares in response to PUS_t^+ and PUS_t^* is not significant (calculated t-statistic is 1.0603). The reason is probably that 95 percent of the FCOJ imported from the U. S. is in retail and institutional packages, which cannot be readily replaced by Brazilian juices that require reprocessing; that is, it is difficult for importers to replace their U. S. juice with imported juice from Brazil since retail-packed product is not available from Brazilian exporters.

Table 3. Estimated Impact of a 10-Cent Change in Prices on Market Shares

Price	10¢/SSE gal. Price Change	Change in Market Share	t-statistic for Irreversibility
		U. S. Market Share	
U. S. Retail (PUS)	Increase	-8.61[a]	1.0603
	Decrease	11.95[a]	
Brazil (PBR)	Increase	1.30	3.5332[a]
	Decrease	-7.87[a]	
Other (POTH)	Increase	4.75[b]	0.8821
	Decrease	-1.89	
		Brazil's Market Share	
U. S. Bulk (PBUS)	Increase	-1.00	2.4686[a]
	Decrease	-7.57[a]	
Brazil (PBR)	Increase	-0.95	1.6720[a]
	Decrease	9.87[b]	
Other (POTH)	Increase	1.88	0.3765
	Decrease	-0.04	

[a]Statistically different from zero at α = .05 level.
[b]Statistically different from zero at α = .10 level.

The estimated parameter for PBR^+ indicates that when the Brazilian FCOJ price is increased, the U. S. market share will increase. However, the parameter is very small relative to its estimated standard error. The reason may be that Brazilian FCOJ has been priced well below U. S. FCOJ. Thus, even with a moderate price increase in Brazilian FCOJ, the price of U. S. FCOJ would probably still be higher than the price of Brazilian FCOJ. On the other hand, when the Brazilian FCOJ price declines, the U. S. market share also declines. In this case, the decrease is statistically significant. In addition, the difference between the changes in the U. S. market share resulting from Brazilian FCOJ price increases versus decreases is statistically significant (the calculated t-statistic is 3.5332). Thus, a decrease in the Brazilian FCOJ price had a bigger impact on the U. S. market share than an increase in the Brazilian FCOJ price.

The estimated parameter for $POTH^+$ (that is, the impact of price changes in FCOJ imports from countries other than the U. S. and Brazil on the U. S. market share) indicates that if POTH is increased by 10 cents per SSE gallon, the U. S. market share would be increased by 4.75 percent, which is statistically significant; however, the impact resulting in a decrease in the U. S. market share was not significant, as indicated by the estimates for $POTH^*$.

The estimate of $PBUS^+$ shows that increases in U. S. bulk FCOJ export prices had no significant impact on Brazil's market share, while the estimate of $PBUS^*$ indicates that for every 10-cent decrease in the U. S. bulk FCOJ export price, Brazil's market share would be decreased by 7.57 percent. The difference between the parameters of $PBUS^+$ and $PBUS^*$ is statistically significant (the calculated t-statistic is 2.4686), which indicates irreversible relationship.

The estimate of PBR^+ shows that when the import price of Brazilian FCOJ increases, Brazilian market share decreases; however, the decrease was not statistically significant. This may occur because Brazilian price increases generally follow U. S. price increases and Brazilian juice becomes relatively less expensive because the magnitude of their change is generally smaller.

The coefficient estimate of PBR^* indicates that for every 10-cent decrease in the import price of Brazilian FCOJ, Brazilian market share would be increased by 9.87 percent, and the increase is statistically significant. The difference between the parameters of PBR^+ and PBR^* is statistically significant (the calculated t-statistic is 1.6720), thus indicating that the relationship is irreversible.

The estimates of $POTH^+$ and $POTH^*$ show that the import prices of FCOJ from countries other than the

[4] These seasonality patterns can be attributed to production differences in the two countries and the desire of their processing industries to avoid holding large inventories of processed juice.

United States and Brazil do not have significant effects on the Brazilian market share.

CONCLUSIONS

Estimates of irreversible functions based on the segmented-variables approach provide explanations for shifts in Canadian FCOJ market share by alternative suppliers. The U. S. market share was found to have symmetric own-price effects, but the Brazilian price effects were asymmetric. An increase in the Brazilian price resulted in an insignificant increase in the U. S. maket share, while a decrease in the Brazilian price was estimated to have a large and significantly negative impact on the U. S. share. For the Brazilian share equation, a Brazilian price increase had only one-tenth of the impact of a Brazilian price decrease. The effect of U. S. bulk FCOJ prices on Brazilian market share were asymmetric.

The results support the general conclusion that entry of new suppliers into markets may cause asymmetric price effects. This hypothesis has special merit in markets subject to dramatic shocks like the freezes that occur in the Florida citrus industry. Thus, we feel the approach used in this paper can be applied to other commodities subject to increased competition when prices increase dramatically. Segmenting variables is a valid method of estimating equations when asymmetric responses are expected.

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APPENDIX

Derivation of Equation (1)

Following Nelson, the statistical model is

$$\text{(A.1)}\quad USS_t = a_0 + a_{11}PUS_t + b_1X_t + e_{1t} \quad \text{if } PUS_t \geqslant PUS_{t,max}$$

$$= a_{00} + a_{21}PUS_t + b_1X_t + e_{1t} \quad \text{if } PUS_t < PUS_{t,max}$$

where e_{1t} is the error term. Lagging equation (A.1) and subtracting it from (A.1) yields[1]

$$\text{(A.2)}\quad USS_t - USS_{t-1} = a_{11}(PUS_t - PUS_{t-1}) + b_1(X_t - X_{t-1}) + e_{1t} - e_{1,t-1} \quad \text{if } PUS_t \geqslant PUS_{t,max}$$

$$USS_t - USS_{t-1} = a_{21}(PUS_t - PUS_{t-1}) + b_1(X_t - X_{t-1}) + e_{1t} - e_{1,t-1} \quad \text{if } PUS_t < PUS_{t,max}$$

Equation (A.2) can be rearranged into

$$(A.3)\quad USS_t - USS_{t-1} = a_{11}W_t(PUS_t - PUS_{t-1}) + a_{21}(1 - W_t)(PUS_t - PUS_{t-1}) + b_1(X_t - X_{t-1}) + e_{1t} - e_{1,t-1}$$

where $W_t = 1$ if $PUS_t \geqslant PUS_{t,max}$
$= 0$ otherwise.

Moving USS_{t-1} to the right-hand side and making successive substitutions yields

$$(A.4)\quad USS_t = USS_1 + a_{11}[\sum_{s=2}^{t} W_s(PUS_s - PUS_{s-1})] + a_{21}[\sum_{s=2}^{t}(1-W_s)(PUS_s - PUS_{s-1})] + b_1[\sum_{s=2}^{t}(X_s - X_{s-1})] + \sum_{s=2}^{t}(e_{1s} - e_{1,s-1})$$

Assume that

$$(A.5)\quad USS_1 = a_0 + a_{11}PUS_1 + b_1X_1 + e_{11} \quad \text{if } PUS_1 \geqslant PUS_0$$

$$a_{00} + a_{21}PUS_1 + b_1X_1 + e_{11} \quad \text{if } PUS_1 < PUS_0$$

Substituting (A.5) into (A.4) and combining terms yields

$$(A.6)\quad USS_t = a_0W_1 + a_{00}(1-W_1) + a_{11}[\sum_{s=2}^{t} W_s(PUS_s - PUS_{s-1}) + W_1PUS_1] + a_{21}[\sum_{s=2}^{t}(1-W_s)(PUS_s - PUS_{s-1}) + (1-W_1)PUS_1] + b_1X_t + e_{1t}$$

Note that X_t is no longer transformed, and the disturbance term has the usual desirable properties, the equation can be estimated by ordinary least squares (OLS). The statistical test of whether the relationship is irreversible is merely the usual t-test of the null hypothesis that the two parameters (a_{11} and a_{21}) are equal. In order to simplify notation let

$$C_1 = a_0W_1 + a_{00}(1-W_1)$$

$$PUS_t^{\dagger} = \sum_{s=2}^{t} W_s(PUS_s - PUS_{s-1}) + W_1PUS_1$$

$$PUS_t^{*} = \sum_{s=2}^{t}(1-W_s)(PUS_s - PUS_{s-1}) + (1-W_1)PUS_1$$

Then (A.6) can be written as

$$(A.7)\quad USS_t = C_1 + a_{11}PUS_t^{\dagger} + a_{21}PUS_t^{*} + b_1X_t + e_{1t}.$$