ESTIMATING U.S. DEMAND FOR MEAT WITH A FLEXIBLE FUNCTIONAL FORM

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One of the fundamental problems in applied econometric work is the choice of functional form. Economic theory is insufficient in suggesting the functional forms appropriate to the specification of economic relationships. Conventionally, the functional form of the regression equation is assumed *a priori* and parameter estimates are obtained according to certain desirable criteria, such as least squares. A wide variety of functional forms have been investigated empirically with respect to the demand function for food, yet no single functional form has been generally accepted among economists [2, 6, 7].

Chang [3] employed a technique suggested by Box and Cox [1] and Zarembka [9] to determine and test for the correct functional form of the demand for meat in the U.S. On the basis of the findings, Chang suggests that empirical studies on the demand for food which uses linear or double-log forms should be reexamined.

The purpose of this article is to present a more flexible model using the same data to reestimate the demand for meat in the U.S. Most significantly, the empirical results obtained from the alternative specifications of functional form are compared. The author shows that by applying the Box-Cox transformation procedure the proper functional forms of the hypothesized relation are outcomes of the estimating process so that alternative forms can be discriminated through the use of the likelihood ratio test.

THE MODEL AND PROCEDURE

First, Chang's model is reproduced:

$$(1)\quad Q^*_t = B_1 + B_2Y^*_t + B_3\,(P_{mt}/P_{ft})^* + U_t,$$

where

$$Q^*_t = (Q^\lambda_t - 1)/\lambda,$$
$$Y^*_t = (Y^\lambda_t - 1)/\lambda, \text{ and}$$
$$(P_{mt}/P_{ft})^* = [(P_{mt}/P_{ft})^\lambda - 1]/\lambda.$$

Q_t represents the per capita consumption of meat in period t, Y_t is the per capita real income, and P_{mt} is the price of meat. P_{ft} is the price of food and λ is a transformation parameter to be determined. Note that the same power transformation, λ, has been applied to both the dependent and independent variables.[1] In the following equation, a more general formulation is proposed inasmuch as it allows different power transformations for the dependent and independent variables. Therefore, equation 1 is rewritten into a more generalized functional form:[2]

$$(2)\quad Q^*_t = B'_1 + B'_2Y'_t + B'_3\,(P_{mt}/P_{ft})' + U'_t,$$

where

$$Q^*_t = (Q^\lambda_t - 1)/\lambda,$$
$$Y'_t = (Y^\mu_t - 1)/\mu, \text{ and}$$
$$(P_{mt}/P_{ft})' = [(P_{mt}/P_{ft})^\mu - 1]/\mu.$$

μ is another transformation parameter to be determined in addition to λ. It is apparent that different values of λ and μ imply different specifications of the functional form. Specifically, equation 1 can be shown as a special case of equation 2, when $\lambda = \mu$.

The parameters of equation 2 can be estimated by using maximum likelihood estimators. The maximization of the likelihood function of equation 2 can be accomplished by iterated least squares as discussed by Zarembka. In practice, the likelihood function is maximized by a two-dimensional search over a grid

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[1]The effect of a transformation is to increase the degree of approximation to which some desirable properties for statistical analysis hold. Specifically, transformation of variables may lead to a more linear model, may reduce the error variability and lead to homoskedasticity of the error term, and/or may lead to a model for which the error distribution is symmetrical and likely nearly normal. A transformation, of course, may increase the degree of approximation to two or three of these properties simultaneously.

[2]See Zarembka [9] for the derivation and properties of the general form. This formulation, however, does not exhaust the investigation of functional forms. Theoretically, it is possible to assign a different transformation parameter to each variable that is included in the model. Such a generalization would be extremely expensive in terms of computer time and programming burden. The gains in the sense of better estimates statistically and economically from such specification may be relatively limited in comparison with the proposed formulation. If different parameters with the same range and intervals are applied to each variable, the total number of iterated regressions required will be X^n, where X is the number of times that each transformation parameter will vary and n is the number of transformation parameters to be included in the model.

TABLE 1. PARAMETER ESTIMATES AND VALUES OF MAXIMUM LIKELIHOOD FUNCTION FOR SELECTED λ AND μ, AND RELATED STATISTICAL RESULTS

Equation	B_1	B_2	B_3	λ	μ	$\bar{R}^2$	D.W.	$L_{max}(\lambda,\mu)$
(1) Maximum likelihood estimates with both λ and μ free	-182.821	135.879 (3.885)[a]	-41.035 (4.577)	0.4	-.48	.977	1.48	-37.447
(2) Semi-log function	-12.412	77.307 (2.332)	-89.402 (10.860)	1.0	0.0	.975	1.37	-39.222
(3) Chang's estimate with $\lambda = \mu$	-2.92	3.84 (0.11)	-0.40 (0.05)	-.84	-.84	.976	1.51	-39.396
(4) Log-inverse function	-797.987	865.434 (27.067)	-62.529 (7.339)	0.0	-1.0	.974	1.33	-40.463
(5) Double-log function	3.776	0.493 (0.017)	-0.527 (0.080)	0.0	0.0	.967	1.08	-44.439
(6) Inverse function	-123803.299	134167.104 (5103.713)	-10233.453 (133.746)	1.0	-1.0	.961	0.97	-46.629
(7) Linear function	150.792	0.040 (0.002)	-0.704 (0.170)	1.0	1.0	.934	0.72	-55.451

[a] Estimated standard errors.

of various values of λ and μ to find a combination of λ and μ where the maximum likelihood surface over the entire parameter space has peaked.[3]

RESULTS AND DISCUSSION

To allow direct comparison with Chang's results, equation 2 was estimated with the same data. Estimated results for selected values of λ and μ are given in Table 1. The maximum likelihood function is maximized at $\lambda = .4$ and $\mu = -.48$. All parameter estimates are of the correct *a priori* sign. Note that the $\bar{R}^2$ (coefficient of determination adjusted for degrees of freedom) in Table 1 is very high for each specification of the functional form and thus high $\bar{R}^2$ does not necessarily suggest appropriate functional form. Nevertheless, the proposed specification does provide the highest $\bar{R}^2$ among the alternative regression equations. The results thus conform with Granger and Newbold's [4] theorem that under the assumption of normality, the "correct" model from a set of alternative specifications involving different transformations of the same dependent variable is the formulation for which $\bar{R}^2$ is highest.

To test a null hypothesis of a specific functional form, a joint $(1-\alpha)$ percent confidence region for λ and μ can be constructed around the maximum likelihood estimates with $\lambda = 0.4$ and $\mu = -.48$. By use of the likelihood ratio test, the null hypothesis of a significant difference between these functions can be tested. It is known that under general conditions $-2\ln L$ is distributed approximately as $\chi^2_\alpha(f)$, where L is the ratio of the two likelihood functions, and f is the degrees of freedom equal to the number of transformation parameters. The results of the likelihood ratio tests are given in Table 2.

TABLE 2. RESULTS OF LIKELIHOOD RATIO TEST

Equation	Value of λ	Value of μ	$-2\ln L$[a]	Reference about the null hypothesis
(1)	0.4	-0.48		
(2)	1.0	0.0	3.55	Cannot reject at the .05 significance level
(3)	-0.84	-0.84	3.898	Cannot reject at the .05 significance level
(4)	0.0	-1.0	6.032	Reject at the .05 but not at the .01 significance level
(5)	0.0	0.0	13.984	Reject at the .01 significance level
(6)	1.0	-1.0	18.36	Reject at the .01 significance level
(7)	1.0	1.0	36.008	Reject at the .01 significance level

[a] $-2\ln L \sim \chi^2(2)$ in this case; $\chi^2_{.95}(2) = 5.991$; $\chi^2_{.99}(2) = 9.210$.

The resulting functional forms for demand equations are ranked in Table 2 in ascending order of level of significance of the χ^2 test.

The resulting χ^2 tests indicate significant differences among the different models. The proposed model is found to be significantly different from the double-log, inverse, and linear models. It is not significantly different from the semi-log model and Chang's version. However, no definite conclusion can be drawn about the log-inverse model. The log-inverse specification cannot be rejected at the .01 significance level, but it can be rejected at the .05

[3]For a description of the computer program that was used to estimate equation 2, see Huang, Moon, and Chang [5].

significance level. If the resulting function were used as a discrimination tool, the semi-log functional form would be ranked the best among the alternative specifications tested.[4] In practice, the semi-log implies a relatively simple functional form and statistically it does not differ significantly from the maximum likelihood estimates. However, the appropriate functional form or the appropriate degree of nonlinearity of the regression equations may be different with respect to different commodities or sample observations.

The estimated elasticities are very important information particularly for the projection of food consumption and the understanding of food-consumption-related behavior. The implications of the author's study are most evident in the measures of demand elasticities which resulted from different specifications of functional form. The elasticities of demand for meat with respect to income and price for the selected years are listed in Tables 3 and 4, respectively. It is noteworthy that the elasticities computed at the mean values appear to be insensitive to the alternative specifications of the functional form for which the likelihood ratio tests indicate no significant differences between these functions at the .05 significance level. For income elasticity, the difference between the lowest and highest estimate at the means is about 0.02; for price elasticity, the estimates are within a range of 0.06. In comparison, the range is 0.05 and 0.21 for income elasticity and price elasticity, respectively, when all the functional forms presented in Table 1 are included. In fact, the extreme estimates of the demand elasticities are all associated with the functional forms that are ranked fifth or lower in Table 2. Prais and Houthakker [7] suggest that the estimated income elasticities can vary by 50 percent or more among different specifications of functional forms.

TABLE 3. INCOME ELASTICITY OF DEMAND FOR MEAT BASED ON ALTERNATIVE FUNCTIONAL FORMS, SELECTED YEARS

Year	Elasticity estimated by equation[a]						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1935	0.721	0.658	0.647	0.836	0.493	1.104	0.353
1940	0.608	0.543	0.579	0.687	0.493	0.738	0.354
1950	0.531	0.535	0.492	0.527	0.493	0.564	0.455
1955	0.486	0.475	0.506	0.482	0.493	0.459	0.441
1960	0.477	0.480	0.487	0.460	0.493	0.443	0.468
1965	0.432	0.463	0.468	0.387	0.493	0.359	0.536
1970	0.385	0.415	0.403	0.332	0.493	0.276	0.560
1974	0.368	0.413	0.402	0.304	0.493	0.252	0.608
At the means	0.477	0.488	0.493	0.454	0.493	0.444	0.481

[a] The income elasticity is defined as $E_y = b_y (Y^{\mu}/Q^{\lambda})$, where b_y is the regression coefficient of the income variable; Y and Q are the per capita real income and meat consumption, respectively.

TABLE 4. PRICE ELASTICITY OF DEMAND FOR MEAT BASED ON ALTERNATIVE FUNCTIONAL FORMS, SELECTED YEARS

Year	Elasticity estimated by equation[a]						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1935	−0.698	−0.762	−0.493	−0.683	−0.527	−0.953	−0.549
1940	−0.654	−0.628	−0.593	−0.701	−0.527	−0.806	−0.441
1950	−0.594	−0.618	−0.512	−0.580	−0.527	−0.657	−0.525
1955	−0.603	−0.549	−0.632	−0.662	−0.527	−0.665	−0.409
1960	−0.592	−0.556	−0.601	−0.631	−0.527	−0.642	−0.434
1965	−0.582	−0.535	−0.618	−0.629	−0.527	−0.616	−0.419
1970	−0.550	−0.480	−0.662	−0.611	−0.527	−0.537	−0.387
1974	−0.551	−0.477	−0.669	−0.616	−0.527	−0.538	−0.381
At the means	−0.595	−0.564	−0.620	−0.629	−0.527	−0.650	−0.441

[a] The price elasticity is defined as $E_p = b_p (P^{\mu}/Q^{\lambda})$, where b_p is the regression coefficient of the price variable; P and Q are the real price of meat and the per capita meat consumption, respectively.

[4] The semi-log form was not included in Chang's specification because he imposed the restriction of $\lambda = \mu$ on the transformation parameters.

Results of the author's investigation obviously suggest that the wide range of variations of estimated elasticities may be attributed to the incorrect specification of the regression equation.

The differences between the estimates are, of course, greater when calculated at any point away from the mean. It is evident that the estimated elasticities are far from constant and vary over the time period. Interestingly, the income elasticities of demand for meat obtained by the proposed formulation decrease monotonically over the sample period from a high in 1935 of 0.72 to a low of 0.37 in 1974. This behavioral property is consistent with both theoretical considerations and empirical evidence. Except for the double-log and the linear equations, other functional forms investigated exhibit a general decline in income elasticities over the sample period.

In addition, Tomek [8] has presented evidence to support the argument that demand for meat has become more price inelastic over time. It is argued that as income elasticity decreases, the own price elasticity must be consistent with the cross- and income elasticities and hence must become smaller in absolute value. The estimated price elasticities as shown in Table 4, excluding Chang's model and the double-log model, seem to be supportive of and in accord with the argument that price elasticities for meat are becoming more inelastic. Though the double-log form assumes *a priori* that elasticities of demand for meat are constant at any price level over the sample period, Chang's model implies that the price elasticity of meat is rising. This implication is inconsistent with theoretical considerations and empirical evidence.

Although the author's study confirms and supports Chang's findings that empirical studies on demand for meat using linear or double-log forms are unacceptable, the last observation indicates a significant difference between the proposed model and Chang's model. Specifically, it suggests an important implication which is most relevant to applied econometric studies. The author's study does not discriminate Chang's model from the appropriate functional form of demand for meat on the statistical ground through the use of likelihood ratio test. However, it is evident that, on the basis of the author's findings, Chang's model should be rejected as an appropriate functional form on the basis of economic judgment. Thus, it is imperative to note that although the statistical procedure may provide additional informaton for identifying the feasible sets of functions, its usefulness is a complement to but not a substitute for sound theoretical considerations.

CONCLUSION

The author demonstrates that *a priori* specification of functional form may impose undesirable behavioral properties which are not subject to substantiation by economic theory. It is suggested that a more flexible functional form can be best determined and tested by applying the Box-Cox transformation and the maximum likelihood method to sample data without *a priori* restrictions about the mathematical form of the regression equation. When economic theory is insufficient to suggest *a priori* restrictions with respect to the appropriate specification of economic relationships, the least restrictive formulation combined with careful exercise of economic judgments is a feasible approach which is least likely to lead to erroneous results and undesirable implications.

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