Quantifying Gains To Risk Diversification Using Certainty Equivalence In A Mean-Variance Model: An Application To Florida Citrus

Allen M. Featherstone and Charles B. Moss

Abstract

The marginal benefit and cost of diversification for Florida orange producers is analyzed using certainty equivalents. Results indicate that for moderate and high levels of risk aversion, diversification into strawberry, grapefruit, or additional orange production is not optimal. However, moderately risk averse Florida orange producers can gain by diversifying into grapefruit production if the annual amortized fixed costs can be reduced by as little as 10 percent.

Key words: certainty equivalence, mean-variance, diversification, Florida orange production

Increased variability of farm income and asset values during the 1970s and 1980s has increased interest in risk management. Risk can be managed using several instruments ranging from forward contracting and other marketing strategies to adaptive control models for irrigation systems. One popular risk management technique is enterprise diversification. At the firm level, the manager tries to control production and price risk by producing a combination or portfolio of enterprises.

A common approach used to evaluate diversification opportunities involves the mean-variance efficiency criterion. This criterion states that an asset is inefficient or dominated if another asset can produce the same or higher rate of return for a lower variance of return (Markowitz; Anderson *et al.*). In diversification, a single asset is constructed by combining two or more individual assets. Several studies have shown diversification to be a useful tool in managing risk (Heady; Jones; Freund).

However, past applications of the mean-variance criterion have often failed to consider the marginal costs and marginal benefits of additional diversification (Adams *et al.*; Schurle and Erven). In the agricultural finance literature, the typical crop diversification model emphasizes a set of crops that can be grown from the same initial set of fixed resources. Gross margins are often used to calculate the optimal set of crops. However, diversification often will not occur unless there is an increase in at least some fixed resources. For example, a corn farmer would find it necessary to obtain a different header for the combine before diversification into soybeans could occur. The traditional method used for diversification studies does not account for these additional costs, including investment in specialized equipment or the extra managerial ability required to operate a diversified enterprise.

The first objective of this paper is to examine diversification opportunities for a Florida orange producer. The second objective is to illustrate how a broader interpretation of results from a mean-variance optimization model can be useful in making decisions.

CERTAINTY EQUIVALENCE AND THE MEAN-VARIANCE CRITERION

Under certain assumptions, the mean-variance criterion is related to the expected utility hypothesis. This linkage can be exploited to derive the certainty equivalent of an investment opportunity. The following derivation is based on the results of Robison and Barry, where the objective of a mean-variance model can be interpreted as the certainty equivalent. This derivation formalizes the assumptions necessary for this linkage to hold in empirical work.

At the basic level, the mean-variance criterion has a limited theoretical basis. The mean-variance criterion reduces a set of all possible investments to a smaller set of risk-efficient investments. Without additional assumptions, there is little or no guarantee that this efficient set of investments contains the utility-maximizing choice. The usual assumption required for equivalence between the mean-variance set and the utility-maximizing set of investments is

Allen Featherstone is an Assistant Professor of Agricultural Economics, Kansas State University, and Charles Moss is an Assistant Professor of Food and Resource Economics, University of Florida. The authors would like to acknowledge the helpful suggestions of Barry Goodwin, Ted Schroeder, and three anonymous reviewers.
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that returns are distributed normally. To calculate the certainty equivalents, Freunds' more stringent assumptions that returns are distributed multivariate normal and that the agent's utility function is a negative exponential are also required.

Formally, a negative exponential utility function can be specified as

$$(1) \quad U[W(X)] = -\exp[-\theta W(X)]; \quad X \varepsilon I_0,$$

where wealth (W) is a function of an investment bundle (X), θ is the Pratt-Arrow absolute risk aversion coefficient, and I_0 is the set of feasible investment bundles. Wealth is generated by investing in the feasible bundle X, and if the returns on X are multivariate normal, then $W(X) \sim N[\mu(X), \sigma^2(X)]$

Bussey has shown that under this specification, the expected utility of the negative exponential is equivalent to

$$(2) \quad E\{U[W(X)]\} = -\exp.\{-\theta[\mu(X) - \frac{\theta}{2}\sigma^2(X)]\}.$$

Choosing the vector of activities, X, to maximize expected utility in (2) yields the same solution as choosing X to maximize

$$(3) \quad Z = \mu(X) - \frac{\theta}{2}\sigma^2(X)$$

because (2) ia a monotonic transformation of (3).

In addition to yielding the same maximum, thereby simplifying the process of finding the utility maximizing portfolio, (2) also allows calculation of the certainty equivalent for a risky investment. The certainty equivalent is simply the certain level of wealth for which the decision-maker is indifferent with respect to a risky alternative. To compute the certainty equivalent for a risky opportunity, an expenditure function or inverse utility function is set equal to expected utility. Specifically, we are interested in determining the certainty equivalent, $W^*(X)$, that yields the same level of utility as $E\{U[W(X)]\}$. Substituting $W^*(X)$ for $W(X)$ in (1) and solving for $W^*(X)$ yields the certainty equivalent

$$(4) \quad W^*(X) = \frac{1}{\theta}\ln(E\{U[W(X)]\}).$$

Substituting (2) into (4) and simplifying, the certainty equivalent is

$$(5) \quad W^*(X) = \mu(X) - \frac{\theta}{2}\sigma^2(X).$$

The certainty equivalent of a risky investment is equal to the objective function, Z. The above derivation also has a heuristic explanation. By definition, a certainty equivalent has no variance, otherwise it would not be certain. To find the certainty equivalent, a utility function is set equal to the level of expected utility of a risky alternative. Because a certain outcome has no variance, $\mu(X)$ is equal to expected utility (Z). For any set of assumptions in which expected utility is maximized by maximizing (3), Z defines the certainty equivalent.

THE MARGINAL BENEFIT AND COST OF ADDITIONAL DIVERSIFICATION

A change in the feasible set can be used to derive the marginal benefit and cost from additional diversification opportunities. Once the risky investment opportunity is expressed in terms of a certainty equivalent, standard concepts of deterministic consumer behavior become applicable. For example, given that preferences are monotonically increasing in wealth, a consumer will always prefer more wealth. Therefore, the consumer will prefer an alternative with a higher certainty equivalent. The certainty equivalent includes an adjustment for risk preferences. Hence, the agent, in choosing an investment with the greater certainty equivalent, is considering his or her risk preference. If a consumer is faced with two risky alternatives and the certainty equivalent of the first is greater than the certainty equivalent of the second, the agent will prefer the first. Further, the maximum price that agents will pay for the first, given that they already have the second, is the difference in the certainty equivalents. Because the marginal benefit can be defined as the most an agent is willing to pay for an item, the marginal benefit of the additional diversification opportunity is the change in certainty equivalents.

Mean-variance studies typically have examined diversification based on gross margins (returns minus variable costs, Adams *et al.*). Incremental fixed costs play an important role in determining the desirability of diversification. These incremental fixed costs constitute the marginal costs of diversification, which are often not considered. The marginal costs of diversification can be determined by calculating the net present value of the incremental fixed costs and amortizing those costs over the life of the investment. The amortized fixed costs can be either subtracted from the mean return (μ) in (2) or compared directly with the marginal benefit defined above. If the marginal incremental fixed costs are subtracted from (2), then the investment would be desirable when the marginal benefit is positive.

It may not be appropriate to subtract the fixed costs of diversification from the returns above variable costs given the lumpiness of an investment. A solution for a risk programming problem often involves a fraction of an activity. However, the costs of obtaining fixed facilities are often not proportional. For example, if an investment requires specialized equipment, the average costs of obtaining the equipment for the first acre may be different from the cost for multiple acres. Extrapolating the results may yield incorrect diversification recommendations. Thus, whether or not the investment is considered divisible helps to determine whether or not fixed costs can be subtracted from variable costs.

APPLICATIONS

In the late 1980s, orange juice production in Florida appeared profitable in comparison with many other agricultural enterprises. However, memories of devastating freezes and increased imports from Brazil indicated that significant risk existed in orange production. Several alternatives were available for Florida orange producers considering expansion. This application assumed that a Florida orange producer currently had 150 acres of oranges and that three expansion opportunities were available: producing 10 acres of strawberries, 50 acres of grapefruit, or another 50 acres of oranges.[1] Each expansion opportunity required roughly the same managerial ability to operate.[2]

The income information for orange production was derived from state seasonal yields and cash prices for oranges marketed as frozen concentrated orange juice (FCOJ) for the period 1973-1987. Three orange harvesting periods were chosen: December, February, and April. The Florida Department of Citrus provided FCOJ prices in dollars per pound solid. The yield, in pounds of solids per acre, for each marketing period was derived from the state average, measured in boxes of oranges per acre, for early and midseason oranges in the December and February marketing periods, and Valencia oranges in the April marketing period (Florida Agricultural Statistics, 1988a). The yield variability of FCOJ depends not only on tree yields, but also on the quality of the oranges. Quality of oranges is measured by the gallons of juice that can be obtained from a box. The variety of the orange and weather are primary factors in determining this quality.

Average yields for white grapefruit and on-tree prices for Florida white grapefruit between 1973 and 1987 were obtained from Florida Agricultural Statistics (1988a). The variable cost of producing one acre of oranges or grapefruit was assumed to be $748.15 (Murraro), and all returns were deflated using the personal consumption expenditure component of the implicit GNP deflator. The marginal cost of diversification, which is the rental rate for an acre of oranges or grapefruit, was $630 (Hunt).

The returns to strawberries were computed based on state average prices and yields (Florida Agricultural Statistics, 1988b). The variable cost of production for strawberries was assumed to be $11,710.54 per acre (Taylor and Smith). The marginal cost of diversification into strawberries included $260 per year per acre for land rental and a one-time cost of $22,000 for additional equipment investment (Hewit). The $22,000 of additional equipment was amortized into equal annual payments for 10 acres assuming a 10-year equipment life and a 12.5 percent interest rate. Amortization resulted in an annual charge of $3,974 for the additional investment in equipment. Thus, the total annual cost of diversification into 10 acres of strawberries was $6,574.

Gross revenues less variable costs expressed in 1987 dollars, mean returns, and standard deviation of returns for strawberries, grapefruit and each marketing period for oranges are reported in Table 1 on a per acre basis. Strawberries had the highest mean return per acre. December-produced oranges had the lowest mean return from 1973-1987. Strawberries also had the highest standard deviation. April-produced Valencia oranges had the lowest standard deviation per acre. The correlation matrix of returns for oranges, strawberries, and grapefruit is reported in Table 2. The returns from oranges harvested during different periods were highly correlated. Grapefruit and strawberry returns were less correlated with oranges.

A mean-variance model was constructed using the means, variances, and covariances. The objective of the mean-variance model was to maximize (3) subject to the constraint that total acres of oranges raised were less than or equal to 150. Six Pratt-Arrow coefficients of absolute risk aversion ranging from zero to 0.0001 were used. Individuals with a zero

[1] This study assumed that the average variable cost curve was flat for the additional expansion opportunities considered. Also, if an individual producer was interested in more than one of the expansion activities these could be put into one programming model. However, given the large increase in managerial expertise required (roughly one-third), it is unlikely that more than one addition would be considered at a time.

[2] It is unlikely that some additional education may be required for a producer to manage the expansion. The return for this additional education could be determined by comparing the marginal benefit with the marginal cost of expansion . The difference could be considered as the return to education. The producer could determine whether the return was high enough to warant additional education followed by the expansion.

Table 1. Gross Revenue Less Variable Costs in Dollars per Acre for Oranges in Three Harvest Periods, Strawberries ,and Grapefruit, 1973 through 1987

	Oranges				
Year	December[a]	February	April	Strawberries	Grapefruit
1973	259.90	468.21	387.25	2,868.30	1,269.54
1974	326.39	512.99	256.82	83.40	780.93
1975	192.46	235.57	250.74	2,913.20	627.76
1976	421.77	683.32	619.85	733.40	386.33
1977	-90.58	-149.62	222.32	-615.10	553.33
1978	1,198.82	1,518.40	1,317.43	2,809.90	397.25
1979	1,282.93	1,832.44	1,309.49	3,107.30	831.65
1980	1,354.17	1,562.99	1,092.69	4,457.80	1,350.73
1981	756.30	1,064.82	819.27	-279.80	1,185.12
1982	749.51	754.22	387.40	1,149.80	231.03
1983	791.69	814.40	1,099.59	-255.60	-155.55
1984	486.82	1,022.50	664.39	-3,079.70	146.70
1985	1,731.98	1,417.93	1,130.23	971.30	662.12
1986	711.21	841.22	668.53	-815.70	1,065.17
1987	708.64	1,124.19	902.59	2,441.10	1,577.37
Mean	725.47	913.57	741.91	1,099.31	727.30
Standard Deviation	494.60	535.04	388.33	1,990.17	491.01

[a] December of the previous year.

risk aversion coefficient were profit maximizers. Farmers with a risk aversion coefficient of 0.000005 were slightly risk averse. Producers with a risk aversion coefficient of 0.00001 or 0.00002 were moderately risk averse, while producers with a coefficient of 0.00005 or 0.0001 were strongly risk averse. The risk aversion levels were chosen based on Raskin and Cochran and on the certainty equivalent. When the certainty equivalent drops below the lowest observed outcome, the risk aversion coefficient is likely too high. The most risk averse coefficient examined (0.0001) was the only one in which the certainty equivalent was less than the lowest outcome, suggesting this as an upper limit on risk aversion.

The base scenario results are presented in the top block of Table 3. The base scenario provides the orange producer with the optimal plan for the current 150 acres. For the producer with risk aversion coefficients less than or equal to .00001, midseason (February) maturing oranges should have been raised on the 150 acres. Farmers with a risk aversion coefficient of 0.00002 should have raised both midseason oranges and Valencia (April) oranges.

Table 2. Correlation Coefficients of Returns for Oranges in Three Harvest Periods , Strawberries, and Grapefruit, 1973 thorugh 1987

	Oranges				
	December	February	April	Strawberries	Grapefruit
Oranges					
December	1.000				
February	.903*	1.000			
April	.861*	.902*	1.000		
Strawberries	.356	.347	.299	1.000	
Grapefruit	.093	.180	.029	.464	1.000

* Significantly different from zero at the 5 percent level of significance.

Table 3. Optimal Portfolios of Oranges, Grapefruit and Oranges, and Strawberries and Oranges for Various Risk Aversion Levels

Risk Aversion Coefficient	Certainty Equivalent	Marginal Benefit of Expansion	Marginal Cost of Expansion	Acres of Oranges			Potential Expansion Activity
				Dec.	.Feb.	Apr.	
			Base Plan				
0.0	137,036	—	—	—	150	—	—
0.000005	120,933	—	—	—	150	—	—
0.00001	104,830	—	—	—	150	—	—
0.00002	78,007	—	—	—	50	100	—
0.00005	26,461	—	—	—	—	150	—
0.0001	-58,364	—	—	—	0	150	—
			Diversification into Strawberries				
0.0	148,029	10,993	6,574	—	150	—	10
0.000005	128,163	7,230	6,574	—	150	—	10
0.00001	108,297	3,467	6,574	—	150	—	10[a]
0.00002	79,394	1,387	6,574	—	42	108	4[a]
0.00005	26,461	0	6,574	—	—	150	0[a]
0.0001	-58,364	0	6,574	—	—	150	0[a]
			Diversification into Grapefruit				
0.0	173,401	36,365	31,500	—	150	—	50
0.000005	154,018	33,085	31,500	—	150	—	50
0.00001	134,635	29,815	31,500	—	150	—	50[a]
0.00002	107,030	29,023	31,500	—	16	134	50[a]
0.00005	45,685	19,224	31,500	—	—	150	50[a]
0.0001	-49,753	8,011	31,500	—	—	150	27[a]
			Diversification into Oranges				
0.0	182,714	45,678	31,500	—	200	—	50
0.000005	154,087	33,154	31,500	—	200	—	50
0.00001	126,012	21,182	31,500	—	158	42	50[a]
0.00002	88,324	10,317	31,500	—	20	180	50[a]
0.00005	26,461	0	31,500	—	—	150	0[a]
0.0001	-58,364	0	31,500	—	—	150	0[a]

[a] Expansion would not occur because the marginal cost of expansion exceeds the marginal benefit of expansion. The marginal cost and the marginal benefit of expansion are zero in these cases in actuality because diversification does not take place.

Farmers who had a risk aversion coefficient greater than 0.00005 should have grown only Valencia oranges.

Oranges and Strawberries

The results from the risk programming model for diversification into strawberries are presented in the second block of Table 3. The objective function was altered to allow for the addition of strawberry production. An additional constraint was added, restricting the number of acres of strawberries to less than 10. Strawberries offered potential for increasing the expected utility of the farmer if the farmer's risk aversion coefficient was less than or equal to 0.00002. The marginal benefit of diversification (the difference in certainty equivalents between the base plan and augmented plan) into strawberries was $10,993 for the zero risk aversion coefficient, $7,230 for 0.000005, $3,467 for 0.00001, and $1,387 for 0.00002. The annual amortized fixed cost of diversifying into strawberries was $6,574. Therefore, only those producers who were risk-neutral or those with a Pratt-Arrow absolute risk aversion coefficient of 0.000005 should have diversified into strawberries. The marginal costs of diversifying would have to be cut by nearly 50 percent before the producer with a risk aversion coefficient of 0.00001 would have been willing to grow strawberries. Those producers who were more risk averse would not have wanted to diversify into strawberries.

Oranges and Grapefruit

The solution for the risk diversification problem with oranges and grapefruit is given in the third block of Table 3. The objective function used in the base model was altered to allow for the addition of grapefruit production. An additional constraint was added restricting the number of acres of grapefruit to less than 50. Before considering the marginal costs of diversification, for every level of risk aversion except the highest, grapefruit was raised on all 50 acres. However, the marginal benefit for leasing the grapefruit only exceeded the marginal cost of $31,500 for Pratt-Arrow risk aversion coefficients less than or equal to 0.000005. The orange producer who has a risk aversion coefficient of 0.00001 (0.00002) would have been willing to raise grapefruit if the annual rent on land were reduced by 5 percent (8 percent). Currently, the more risk averse managers would not have rented the grapefruit grove and more risk-neutral managers would have. However, in the current scenario no one would have rented the grapefruit grove without planting and maintaining all 50 acres.

Oranges and Oranges

The solution for the risk diversification into production of additional oranges is given in the bottom block of Table 3. The base model objective was used with the constraint on acreage grown increased from 150 to 200 acres. The marginal benefit for leasing the extra acreage of oranges exceeded the marginal cost for the two smallest risk-aversion coefficients. The producer who had a risk-aversion coefficient of 0.00001 would have rented the 50 acre grove of oranges, if the rent had been reduced by 33 percent (the marginal cost would have needed to be less than $21,182).

SUMMARY

The above analysis indicated that only the orange producer with a risk-aversion coefficient of 0.000005 or less would have expanded his enterprise after considering the marginal costs of diversification. Although producers who were more risk-averse would want to diversify based on the returns over variable costs, the marginal benefits did not outweigh the marginal costs. However, the more risk-averse producer might have wanted to consider grapefruit, because the marginal benefit minus the marginal cost of diversification would have been positive, if the marginal costs could have been reduced by between 5 and 10 percent. It should be noted that the above analysis was based upon statewide information and that the individual producer is likely to have faced yields that were more variable.

CONCLUSION

This study used the certainty equivalent of a risky investment derived from the objective function to evaluate the marginal benefits and costs of diversification opportunities. Specifically, this paper recognized that the objective value from a popular form of a quadratic risk (mean-variance) programming problem is equal to the certainty equivalent under Freund's assumptions. The change in certainty equivalent between two mean-variance solutions, one without and one with an additional diversification opportunity, was shown to be the marginal benefit of the diversification opportunity. This marginal benefit can be compared with the marginal cost of the opportunity to determine the economic efficiency of additional diversification.

Using this framework, three investment opportunities available to Florida orange producers were evaluated: strawberry, grapefruit, and additional orange production. The results indicated that the marginal benefit of diversification into any of the enterprises was exceeded by the cost for moderate and high levels of risk aversion. The marginal benefit to additional investment was greater than the marginal cost of diversification for all three enterprises for the profit maximizer and the individual with a Pratt-Arrow risk-aversion coefficient less than 0.00001. For the moderately risk-averse producer, the marginal benefit of grapefruit production would have been greater than the marginal cost, if the costs had been reduced by as little as 10 percent.

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