AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

OPTIMAL IRRIGATION PIVOT LOCATION ON IRREGULARLY SHAPED FIELDS

L. Upton Hatch, William E. Hardy, Jr., Eugene W. Rochester, and Gregory C. Johnson

Abstract

Although annual rainfall in the Southeast is adequate, its distribution is a potential constraint to agricultural production. Farmers require production information concerning efficient use of irrigation technology adapted to regional growing conditions. Selection of optimal position, size, and number of pivots in center pivot irrigation systems poses special problems on small, irregularly shaped fields. In the southeastern United States, field size and shape are often varied and irregular. A mixed integer programming model was constructed to assist in irrigation investment decisions. The model is illustrated using irrigated peanut production in southeast Alabama. Results indicate the importance of economic engineering considerations.

Key words: mixed integer programming, supplemental irrigation, economic engineering.

Selection of the optimal position, size, and number of pivots in a center pivot irrigation system poses special problems on small, irregularly shaped fields. In the southeastern United States, field size and shape are often varied and irregular. Irrigation technology is rapidly being adopted; however, most research on center pivot irrigation systems has focused on large regularly shaped fields. Farmers in the Southeast require production information concerning the efficient use of irrigation technology adapted to regional growing conditions. This paper applies a new methodology for selection of the optimal number, size, and position of irrigation pivots that may be used with any field size or shape.

Most irrigation research has focused on arid regions because of the obvious importance of water in these areas (Ruttan; Burt and Stauber). However, irrigation economics is receiving increased attention in the Southeast, particularly in Alabama, Florida, Georgia, and Mississippi (Curtis; Boggess et al.; McClelland et al.; Salassi et al.). This increased research effort has resulted from the realization that the need for irrigation is dependent upon the distribution of rainfall over the growing season. Annual rainfall in the region is adequate; however, its distribution is a potential constraint to agricultural production (Getz and Owsley). Thus, annual rainfall data can provide a misleading impression of the usefulness of irrigation technology.

Several techniques have been developed to optimize irrigation resource allocation. Trava et al. used a linear programming model to determine the date and quantity of water to apply to agricultural crops. Integer programming was incorporated to specify the decision to irrigate. Udeh and Busch incorporated Bayesian decision theory into an optimal irrigation management strategy model to address stochastic, probabilistic, and risk elements. In addition, optimal irrigation water use from probability distributions of evapotranspiration and benefit-cost analyses of irrigation systems has been estimated (Khanjani and Busch, 1982). Khanjani and Busch (1983) also developed a method to determine optimal size and location of farm irrigation reservoirs.

A technique has been developed that describes a field as a series of grid points and attempts to analyze field coverage by center pivot irrigation systems (Rochester). An integer programming analysis was utilized to determine the optimal locations and number of center pivots that would maximize coverage of the field (Anderson et al.). Solutions were obtained which examined the effect of two different irrigation strategies: one per-

L. Upton Hatch and William E. Hardy, Jr. are Assistant Professor and Professor, respectively, Department of Agricultural Economics, Auburn University. Eugene W. Rochester and Gregory C. Johnson are Associate Professor and Research Associate, respectively, Department of Agricultural Engineering, Auburn University.
Alabama Agricultural Experiment Station Journal Article No. 1-85920.

mitted coverage areas of the individual pivots to overlap and the second did not allow an overlap. The engineering techniques developed by Anderson et al. to simulate field coverage served as a framework for addressing both the economic incentive to irrigate and the optimal irrigation investment decision questions evaluated in this paper.

MODEL

A mixed integer linear programming model was constructed to determine the optimal number, size, and location of center pivots using profit maximization as the objective. Data for peanut production in southeast Alabama were used to illustrate the technique. The field was represented by a series of grid points and mixed integer programming was used to optimize field coverage. A grid point refers to a potential pivot location and a grid area is the portion of the field associated with a particular grid point.

Several decision variables were included in the model. They were: selection of pivot size from among three alternative sizes (96 acres, 138 acres, and 188 acres)[1]; selection of the location for each pivot; whether to produce peanuts without irrigation; whether to rent out available land for other uses; and the selling of peanuts at quota, contract, and world prices.

The mixed integer programming model used to solve the questions of how many, what size, and what locations for center pivot irrigation systems may be expressed mathematically as follows:

$$(1)\quad \text{Maximize} \sum_{k=1}^{3} P_k Y_k + TA - \sum_{i=1}^{N} \sum_{j=1}^{3} C_j V_{ij} - XS$$

Subject to:

$$(2)\quad \sum_{j=1}^{3} V_{ij} \leq 1 \text{ for all } i\ ;$$

$$(3)\quad \sum_{i=1}^{N} \sum_{j=1}^{3} R_j V_{ij} - \sum_{h=1}^{M} DW_l - \sum_{k=1}^{3} Y_k + US = 0;$$

$$(4)\quad \left(\sum_{i=1}^{N} \sum_{j=1}^{3} B_j V_{ij} - \sum_{h=1}^{M} W_h\right)Z + S + A \leq F;$$

$$(5)\quad Y_1 \leq Q; \text{ and}$$

$$(6)\quad Y_2 \leq G;$$

where:

P = price received per pound of peanuts;
B = area irrigated by a pivot;
Z = conversion factor from grid area to acreage;
Y = total pounds of peanuts sold;
A = number of acres rented out;
T = rent received per acre;
N = number of potential pivot locations;
C = annual cost of a center pivot of size j (\$/pivot) plus the cost of producing irrigated peanuts for the area covered;
X = cost per acre of producing non-irrigated peanuts;
S = number of acres of non-irrigated peanuts;
V = zero-one integer variable indicating whether a center pivot system of size j is placed at point i;
M = number of grid areas in the field;
R = peanut yield expected for a center pivot of size j;
D = peanut yield expected for each grid area if it is irrigated;
W = overwater variable which prohibits multiple yields if a single grid area is watered more than once;
U = yield per acre for non-irrigated peanuts;
F = total land available;
Q = peanut quota;
G = peanut volume that may be sold at the contract price;
i = pivot locations (i = 1, 2,..., N);
j = pivot sizes (j = 1, 2, 3);
k = peanut price levels (quota = 1, contract = 2, world = 3); and
h = grid areas (h = 1, 2,..., m).

[1] Center pivot irrigation technology permits the selection of numerous system sizes. Only three were used in this analysis since the authors felt that enough alternatives would be provided to illustrate the procedure. Any number of alternative sizes could be easily included for an actual analysis.

restricted to partial circles so that overwatering would not occur. Overlapping is considered in the programming model.

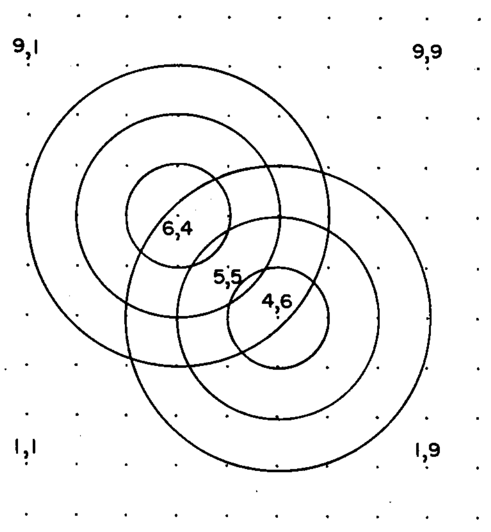

Figure 1. Illustration of Grid System Used to Simulate Field Coverage for Pivot Irrigation Systems of Various Sizes and Different Locations.

Equation (1) is the objective function representing the difference in revenues (the value of peanut production plus income from land rental) and costs (for producing both irrigated and non-irrigated peanuts). Equation (2) assures that only one size pivot may be located at a given point. The relationship expressed in equation (3) accounts for the peanut production and selling alternatives and requires that all production be sold. Equation (4) controls total land use, while equations (5) and (6) limit the quantity that can be sold at quota and contract prices.

Figure 1 illustrates a nine-by-nine grid field with irrigation coverage that could be expected from placing either of three size center pivot systems at two of several potential locations. For example, if the medium sized center pivot system were placed at point 6,4, the following grid areas would be covered: 4,4; 5,3; 5,4; 5,5; 6,2; 6,3; 6,4; 6,5; 6,6; 7,3; 7,4; 7,5; and 8,4. Obviously, portions of other grids would be covered and some area associated with the specified grid areas would not be covered. The error associated with simulating the exact area covered is not large as long as the grid size is kept relatively small.

As is illustrated in Figure 1, overlapping coverage could result from the selection of certain sizes and locations of pivots. In actual field applications, irrigation units could be restricted to partial circles so that overwatering would not occur. Overlapping is considered in the programming model.

Partial budgets giving the ownership, operating, and additional production costs for the specified sizes of irrigation systems were developed (Boutwell and Curtis). These cost estimates were combined with production budgets obtained from the Alabama Cooperative Extension Service, experimental crop response data (Rochester et al.), and peanut prices (quota, non-quota, and world) to obtain profit. Thus, the model is able to select the optimal grid points at which to locate a center pivot and the optimal size of that pivot. Also, the profit associated with that irrigation system is estimated. The model permits determination of the number of acres that should be grown without irrigation and the number of acres that should be rented out for a given field.[2]

Cost data from partial budgets for small, intermediate, and large pivot systems are summarized in Table 1. A typical water supply system was specified using Soil Conservation Service and Extension Service data (Boutwell and Curtis). Economies of size are evident in ownership and operating costs. The effect of pivot size is particularly dramatic on ownership costs. Pivot size limitations are imposed by field size and shape. Operating cost per acre differ by only 1.3

TABLE 1. COSTS AND BREAKEVEN PRICES FOR SELECTED CENTER PIVOT SIZES FOR IRRIGATION SYSTEMS ON PEANUTS IN SOUTHEAST ALABAMA, 1984

	Irrigation system size[a]		
Item	Small (96 acres)	Intermediate (138 acres)	Large (188 acres)
Ownership cost (dollars/acre)	86.96	72.51	65.94
Operating cost[b] (dollars/acre)	30.33	30.08	29.93
Additional production cost (dollars/acre)	14.04	14.04	14.04
Total annual cost (dollars/acre)	131.33	116.63	109.91
Breakeven price[c] (dollars/pound)	.253	.225	.211

[a] Acreage reflects radius and grid size selected for the model.

[b] Operating costs are estimated for 8 acre-inches of irrigation water. This was the mean level of annual application in the crop response experiment, 1976-1981 (Rochester et al.).

[c] Based on average response of 520 lb. of peanuts per acre on class 1 soils.

[2] The "rented out" option was included to reflect potential allocation of the land resource to other uses.

TABLE 2. ILLUSTRATION OF A LINEAR PROGRAMMING MATRIX USED TO DETERMINE THE OPTIMAL NUMBER, LOCATION AND SIZE OF CENTER PIVOT IRRIGATION SYSTEMS

	Decision variables																			
Item[a]	Pivot 1 at 5,5	Pivot 2 at 5,5	Pivot 3 at 5,5	...W2,5	...W3,3	W3,4	W3,5	W3,6	W3,7	...W4,3	W4,4	W4,5	W4,6	W4,7.	Sel 11	Sel 12	Sel 13	Rent	Dry	Constraint values
Objective	–A	–B	–C												P1	P2	P3	R	–K	
Pivot 5,5	1	1	1																	≤1
G2,5			1	–1																≤1
⋮																				⋮
G3,3			1		–1															≤1
G3,4			1			–1														≤1
G3,5		1	1				–1													≤1
G3,6			1					–1												≤1
G3,7			1						≤1											
⋮																				⋮
G5,2			1																	≤1
G5,3		1	1																	≤1
G5,4	1	1	1																	≤1
G5,5	1	1	1																	≤1
G5,6	1	1	1																	≤1
G5,7		1	1																	≤1
G5,8			1																	≤1
⋮				...	...					...										⋮
G8,5			1																	≤1
Yield	D	E	F	–H	–H	–H	–H	–H	–H	–H	–H	–H	–H	–H	–1	–1	–1		V	=0
Land	X	Y	Z															1	1	≤L
Quota															1					≤Q_1
Contract																1				≤Q_2

[a] Alphabetic characters in the matrix represent specific coefficients used in the analysis: A, B, and C are the annual costs of each size of center pivot system (including peanut production costs); D, E, and F are the total yields of peanuts that would be expected from the land covered by each pivot; H is the irrigated yield increase for each grid area; K is the cost/acre of non-irrigated peanuts; L is the total land available; P1, P2, and P3 are the quota, contract, and world prices for peanuts, respectively; Q_1 is the quota available for the field being considered; Q_2 is the total that can be sold at contract price; R is the rent per acre; V is the yield per acre for non-irrigated peanuts; and X, Y, and Z are the acres covered by each pivot irrigation system.

percent among sizes. Additional production costs were estimated as \$.027 per pound and were invariant with respect to system size. The variation in breakeven price for the three pivot sizes was largely a result of economies of size in ownership costs.

Table 2 illustrates an abbreviated matrix of the form used to solve the problem. Integer (0-1) decision variables are included which illustrate locating a center pivot at point 5,5 which is one of many possible locations illustrated in Figure 1. The complete model included variables for all feasible locations. Pivot 1 is the smallest system considered and would cover only five grid areas; 4,5; 5,4; 5,5; 5,6; and 6,5. Obviously, larger systems would cover more grid areas—the intermediate size covers 13 and the largest size covers 29 grid areas.

The "W" decision variables account for any overwatering that might occur if center pivots should overlap in the field. The "Sell" activities represent selling peanuts at quota, contract, and world prices, respectively. The "Rent" activity permits land to be rented out and the "Dry" activity permits the production of non-irrigated peanuts.

RESULTS

Dependent upon quota poundage and the contract selected, the marginal price received will be either the quota price, contract price for non-quota peanuts, or world price. Profit resulting from this price is determined for dry and irrigated peanuts and compared to the rental value or land value in its best alternative enterprise. If irrigated peanuts are the most profitable of the three alternatives, the grid system technique will select the optimal size and location for center pivots. The model follows existing peanut marketing practices by first selling as much as possible at the quota price. After the quota level is completely filled, peanuts are sold at the contract non-quota price and finally, any additional peanuts are sold at the world price. This procedure in effect negates consideration of the peanut prices as blended prices.

In a more simplistic farm management enterprise selection linear programming model, the conditions for inclusion of an enterprise in the optimal solution could be developed in a rather straightforward comparison of relative profitability. The points of indifference between pairs of enterprises would be represented mathematically as:

$$(7)\quad (P-C)\,Y_I = R;$$

$$(8)\quad (P-C)\,Y_I = (P-C)\,Y_D;\ \text{and}$$

$$(9)\quad (P-C)\,Y_D = R;$$

where:

P = marginal price of peanuts;
C = cost per unit of peanut production;
Y = yield (I = irrigated; D = dry); and
R = rental value.

The model would choose irrigated peanuts over land rental (eq. 7), if the irrigated peanut profit margin times the irrigated yield exceeded rental value. Irrigated peanuts would be included instead of dry production (eq. 8), if the irrigated profit margin times irrigated yield were greater than dry profit margin times dry yield. Dry peanut production would be selected in the place of rental acreage (eq. 9), if the dry profit margin times dry yield exceeded rental value.

The peanut poundage quota system and engineering considerations concerning field size and shape complicate the relative profitability conditions (equations (7)-(9)). These conditions are still relevant but must be interpreted carefully. Under the quota system, the price of the last peanut sold is the appropriate price to use for relative profitability calculations. Thus, a marginal price is being used.

On first inspection, it might be concluded from equation (7) that a higher land rental value will necessarily decrease irrigated peanut acreage. However, increased land rental acreage will reduce acreage available to fulfill the peanut quota and can thereby change the marginal price.

In addition, the conditions might suggest the greater relative profitability of irrigation with a certain pivot size, but field size and shape may not allow positioning of the pivot without excessive overlap. This possibility is illustrated in Figure 2 by the absence of the intermediate sized pivot even though its relative profitability is clearly higher than the small sized pivot.

As would be expected, when the value of land rental alternatives increase, land rental acreage increases. Also, higher rental values result in a higher marginal price at which peanuts would be produced with more irrigated acreage. Thus, the existence of higher valued alternative crops causes irrigation and the more intensive use of land to be more profitable.

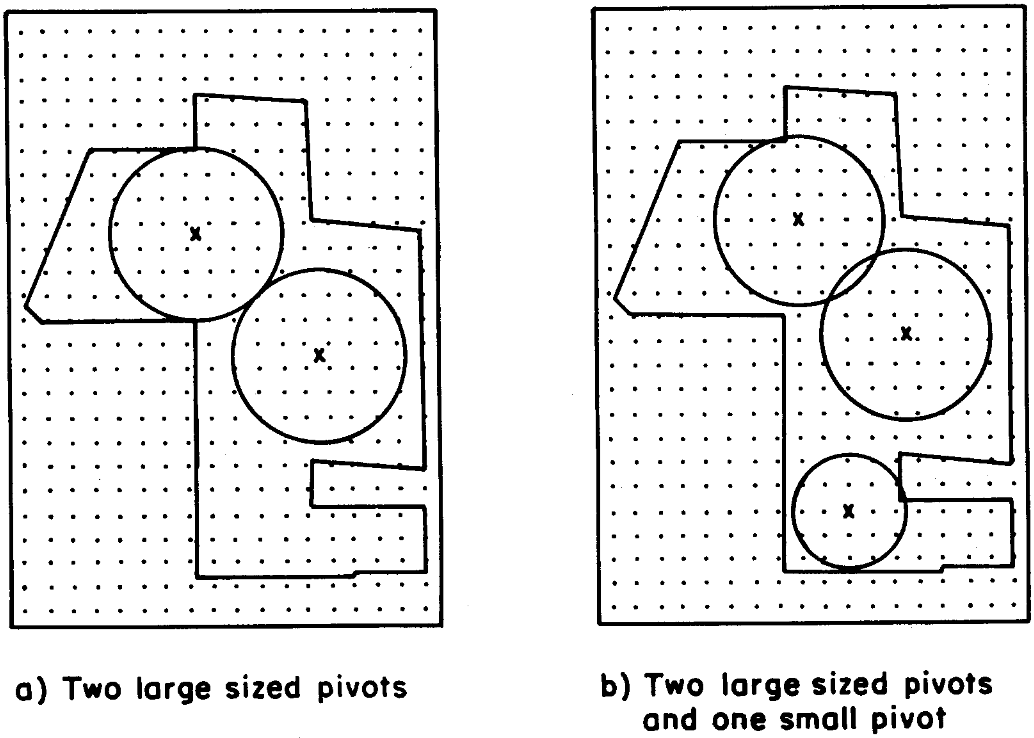

Figure 2. Optimal Size, Number, and Location of Center Pivot Irrigation Systems for a Hypothetical Irregularly Shaped Field.

World peanut price was varied within the relevant historic range (United Nations) to illustrate the possibility of irrigating peanuts sold at the world price. Two large center pivots were selected by the model when the world price was raised to 22 cents per pound and using a land rental value of $40 per acre.

A larger peanut quota clearly results in more irrigation and less land rental, Table 3. As the peanut quota increases, production of peanuts at the world price is reduced. World price was increased to 18 cents and 22 cents for 2,400 poundage quota as was done with the 2,100 poundage quota. No peanuts were produced at the world price at the higher quota level. If marginal price exceeds breakeven price for a particular pivot size, a larger quota will result in more irrigated acreage. Even with a marginal price in excess of breakeven for the intermediate sized pivot, increased quota will not result in positioning an intermediate sized pivot.

The interrelationship between engineering and economic considerations is clearly demonstrated. First, price must sufficiently exceed breakeven for a particular sized pivot and, second, the shape and size of the field must allow appropriate positioning of that particular sized pivot.

Figure 2 illustrates the optimal sizes and locations for center pivots using the grid system in Figure 1 and cost and return data in Table 1. When the marginal price exceeds 21 cents, two large pivots are selected and can be located in multiple positions in the field. The small pivot in conjunction with the two large pivots will be employed if marginal price of peanuts exceeds 25 cents. The intermediate pivot does not appear in the solution because of field size and shape.

SUMMARY AND CONCLUSIONS

Irrigation in humid areas is used to supplement water availability from rainfall, thus providing protection from droughts that frequently occur in crucial stages of the plant growing cycle. The small and irregularly shaped fields that are often found in the Southeast have hindered the adoption of efficient high technology center pivot irrigation systems. The mixed integer linear programming model presented in this paper

TABLE 3. SUMMARY RESULTS FOR SELECTED WORLD PEANUT PRICES, QUOTA POUNDAGE FOR PEANUTS, AND LAND RENTAL VALUE, SOUTHEAST ALABAMA, 1984

World price (dollar/pound)	Marginal price of peanuts[a]	Land rental value[b] (dollar/acre)	Irrigated peanuts (acres)	Dryland peanuts[c] (acres)	Land rented (acres)
Peanut quota = 2,100 pounds/acre:					
.14	CP	40	0	898	61
.14	CP	200	376	456	126
.14	QP	250	376	233	349
.14	QP	300	472	120	366
.18	WP	40	0	959	0
.18	CP	60	0	898	61
.22	WP	40	376	582	0
Peanut quota = 2,400 pounds/acre:					
.14	CP	40	376	582	0
.14	QP	250	376	330	253
.14	QP	300	472	217	270

[a] Quota price = QP = $.30 per pound; contract price = CP = $.235 per pound; and WP = world price.
[b] Rental value is the profit made on land in the best alternative enterprise.
[c] Two large pivots are represented by 376 irrigated acres while two large pivots and one small pivot are represented by 472 irrigated acres.

is a relatively easy to use procedure for evaluating the economics of irrigation and it could lead to greater utilization of center pivot irrigation systems in the Southeast. The model permits an evaluation of the overall profitability of irrigation. If the option is profitable, the optimal number, size, and locations for the irrigation units are determined.

Parametric analysis was undertaken to demonstrate the effect of selected variables on size, number, and location of center pivots. Peanut production in the Wiregrass region of southeast Alabama was chosen for analysis and a land rental activity was added to allow selection of the best alternative enterprise. The importance of center pivot size economies, pivot location, marginal price of peanuts, peanut poundage quota, and returns to alternative enterprises was indicated. The smallest pivot considered (96 acres) was chosen only when all production was sold at quota price and rental value was far in excess of existing land rental prices. Location considerations negated the use of the intermediate sized pivot (138 acres) even when the peanut price exceeded its breakeven point. Sufficient increases in land rental values, up to the point where irrigated returns per acre exceeded irrigated returns, resulted in more irrigated acreage. The model illustrates the interrelationship between engineering and economic considerations that influence irrigation investment decisions.

The example analysis presented for peanut production illustrates factors that influence the decision to irrigate. Field size and shape, the size and cost of alternative irrigation systems, product prices, and the availability of other alternative uses for the land are all important variables which should be included in the model for an actual irrigation profitability analysis.

REFERENCES

Anderson, Clark, Eugene W. Rochester, and William E. Hardy. "Position Selection of Center Pivot Irrigation Systems Using Linear Programming," ASAE Paper No. 84-2102; Knoxville, Tennessee, 1984.

Boggess, W. G., G. D. Lynne, J. W. Jones, and D. P. Swaney. "Risk-Return Assessment of Irrigation Decisions in Humid Regions," *So. J. Agr. Econ.*, 15,1(1983): 135-43.

Boutwell, John L. and Larry M. Curtis. "Irrigation System Cost Analysis," Alabama Cooperative Extension Service, Circular ANR-17; Auburn, Alabama; 1983.

Burt, Oscar and M. S. Stauber. "Economic Analysis of Irrigation in Subhumid Climate," *Amer. J. Agr. Econ.*, 53,1(1971): 33-46.

Curtis, Larry M. 1982 Alabama Irrigation Survey, Alabama Cooperative Extension Service, Auburn University, Alabama; 1983.

Getz, Rodger R. and Larry M. Owsley. "Precipitation Probabilities and Statistics for Alabama," Environmental Studies Service Center Special Report, Weather Series No. 18, Alabama Agricultural Experiment Station; Auburn, Alabama; 1979.

Khanjani, M. J. and J. R. Busch. "Optimal Irrigation Distribution Systems with Internal Storage," *Transactions of the ASAE,* 26,3(1983): 743-7.

Khanjani, M. J. and J. R. Busch. "Optimal Irrigation Water Use from Probability and Cost-Benefit Analyses," *Transactions of the ASAE,* 25,4(1982): 961-5.

McClelland, J. W., W. N. Musser, and R. W. Dubman. "Output Risk and Fertilization under Irrigation." Paper presented at American Agricultural Economics Association; July, 1983.

Rochester, Eugene W. "Irrigation Application Analysis of Irregularly Shaped Fields," ASAE paper No. 83-2163; American Society of Agricultural Engineers; 1983.

Rochester, Eugene W., Paul A. Backman, John A. McGuire, Larry M. Curtis, Jim Starling, and Henry Ivey. "Irrigation Schedules for Peanut Production," Alabama Agricultural Experiment Station, Auburn University, Bulletin 536; 1984.

Ruttan, Vernon W. *The Economic Demand for Irrigated Acreage*. Resources for the Future; Baltimore, Maryland: The Johns Hopkins Press, 1965.

Salassi, M. E., J. A. Musick, L. G. Heatherly, and J. G. Hamill. "An Economic Analysis of Soybean Yield Response to Irrigation of Mississippi River Delta Soils," Mississippi Agricultural & Forestry Experiment Station, Bulletin 928, Mississippi State, Mississippi; 1984.

Trava, J., D. F. Heerman, and J. W. Labadie. "Optimal On-Farm Allocations of Irrigation Water," *Transactions of the ASAE,* 20,1(1977): 85-8.

Udeh, C. N. and J. R. Busch. "Optimal Irrigation Management Using Probabilistic Hydrology and Irrigation Efficiency Parameters," *Transactions of the ASAE,* 25,4(1982): 954-60.

United Nations, Department of International Economic and Social Affairs, "Monthly Bulletin of Statistics." January, 1985 and earlier issues.