A STOCHASTIC PROGRAMMING ANALYSIS OF THE FARM LEVEL IMPLICATIONS OF SOIL EROSION CONTROL

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Abstract

This paper analyzes the effects of uncertain soil loss in farm planning models. A disaggregated approach was used because of an interest in examining the impact of probabilistic soil loss constraints on farm level decisionmaking. A stochastic programming model was used to consider different levels of probability of soil loss. Traditional methods of analysis are shown to consistently overestimate net returns.

Key words: soil conservation, stochastic programming, uncertainty.

Corresponding to the growing national interest in reducing soil erosion and maintaining soil productivity, there has been an increase in research on the economics of soil conservation. Many studies have relied on mathematical programming models. Typically, optimization models are solved subject to constraints on allowable soil loss as computed by the Universal Soil Loss Equation (USLE) (Wade and Heady; Walker and Timmons; Kramer et al.). The USLE is used to compute mean values of soil erosion rates associated with alternative farm practices. Because the resulting erosion rates are mean values, they are actually erosion levels which would be higher 50 percent of the time (assuming a symmetric distribution).

This paper demonstrates the effects of considering the probability distribution of soil loss, rather than just its mean value in farm planning models. This is important in policy analysis because many policies are short run in nature. For example, because of the 4-year cycle of major farm legislation, a cross-compliance requirement between commodity programs and soil conservation would be legislated for no more than 4 years. Thus, policymakers might want to be assured of a high probability of meeting conservation goals in the short run, rather than rely on long-run averages implied by the USLE mean values.

A stochastic, farm level programming model was used to analyze the impacts of probabilistic soil loss constraints. The objective of this paper is to determine how net returns and the combinations of production activities are affected when levels of probability of soil loss are varied for a representative farm in South-Central Virginia.

STOCHASTIC PROGRAMMING MODEL OF SOIL CONSERVATION DECISIONMAKING

Traditional formulations of linear programming problems assume that the technical coefficients of the A matrix (a_{ij}'s) are known and constant. In the context of soil conservation analysis, population means of soil erosion levels derived from samples of different cropping practices are used to determine the soil loss associated with these practices. Soil loss estimates calculated using the USLE become the technical coefficients in the programming model and are rarely changed even though variability between years is known to exist.

The USLE estimates of soil loss should be regarded as stochastic because the USLE is a function of a random variable, rainfall. It would, therefore, be of interest to account for the probabilistic nature of soil loss in mathematical programming models of soil loss control in order to assess the effects, if any, of uncertainty in erosion levels on optimal farm plans.

A typical mathematical programming model may be stated as follows:

(1)   optimize $Z = C'X$

(2) subject to $AX \leq b$

(3) and $X \geq 0$,

where: Z is the objective function to be optimized; C is a vector of costs and returns; X represents the activities; A is the matrix of technical coefficients; and b is a vector of constraint coefficients.

In many applications, the elements of C, A, and b are all assumed to be known and constant in solving the problem. As discussed, however, this assumption is not valid for the technical coefficients in matrix A which represent soil loss from the different crop rotations. The a_{ij}'s of A corresponding to soil loss from a particular crop rotation, j, have probability density functions (pdf's) associated with them. If soil loss for the jth crop rotation is assumed to be normally distributed (this assumption will be discussed), the pdf can be summarized as $a_{ij} \sim N(\mu_{ij}, \sigma^2_{ij})$. It is also assumed that the a_{ij}'s are independently distributed; that is, they are not related to each other across rotations. This is a reasonable assumption since soil loss from various rotations are independent events.

Let the ith row of A be the soil loss constraint. This can be written as:

(4) $a_{i1}X_1 + a_{i2}X_2 + \ldots + a_{in}X_n \leq b_i$,

where:

b_i = the soil loss constraint level,
a_{ij} = the soil loss implied by the jth rotation, and
X_j = the acres in the jth rotation.

For convenience, the inequality in equation (4) can be regarded as an equality and written as:

(5) $b_i = \sum_j a_{ij}X_j$.

As noted by Rahman and Bender, this is a linear function in which b_i is stochastic because the a_{ij}'s are stochastic, while the X_j's are deterministic. The mean and variance of this linear function can be summarized as follows:

mean: $\mu b_i = E(b_i) = \sum_j X_j E(a_{ij})$,

(6) $\mu b_i = \sum_j X_j \mu_{ij}$,

where: μ_{ij} is the mean of a_{ij};
variance: $\sigma^2_{b_i} = E[b_i - E(b_i)^2]$,

(7) $\sigma^2_{b_i} = \sum_j \sigma^2_{ij}X^2_j + \sum_j \sum_k \sigma_{ij,k}X_jX_k$.
$ \qquad \qquad \qquad \qquad j \neq k$

Since the soil losses associated with the different crop rotations are assumed to be independent of each other, the second term in equation (7) is zero which reduces the equation to:

(8) $\sigma^2_{b_i} = \sum_j \sigma^2_{ij}X^2_j$.

This is the quadratic expression for the variance of soil loss.

Given equations (6) and (8), if it is desirable to satisfy the ith constraint a proportion (D%) of the time, equation (4) can be written as:

(9) $\sum_j X_j \mu_{ij} + D^{\bullet}(\sum_j \sigma^2_{ij}X^2_j)^{\frac{1}{2}} \leq b_i$,

where $D^{\bullet}$ is the standarized normal value associated with a D% probability.

In order to account for the variability of soil loss and still use available linear programming algorithms, equation (9) must be linearized.

Consider the following relation:

(10) $\sigma^{\bullet}_{b_i} = \sum_j \sigma_{ij}X_j$.

By squaring both sides, the following is obtained:

$\sigma^{\bullet 2}_{b_i} = \sum_j \sigma^2_{ij}X^2_j + \sum_j \sum_k \sigma_{ij}\sigma_{ik}X_jX_k$,
$ j \neq k$

or

(11) $\sigma^{\bullet 2}_{b_i} = \sigma^2_{b_i} + \sum_j \sum_k \sigma_{ij}\sigma_{ik}X_jX_k$.

This differs from equation (8) by the second term, which (being a sum of positive cross-products) is positive. Therefore:

(12) $\quad \sigma_{b_i}^{*2} > \sigma_{b_i}^{2}$.

If σ_{b_i} is approximted by $\sigma_{b_i}^{*}$, the result, as a consequence of relation (12), would be biased. The bias is, however, in a known direction, as depicted by equation (12) (Nott and Combs; Rahman and Bender). The practical consequence of this result is that the actual probability of meeting the soil loss would generally be more than the specified value. Equation (10) is therefore an acceptable linear approximation for the purpose of this study.

By substituting equation(10) into equation (9), the following equation is obatained:

$$\sum_j X_j \mu_{ij} + D^* \sum_j \sigma_{ij} X_j \leq b_i$$

or

$$(13) \quad \sum_j X_j \, (\mu_{ij} + D^* \, \sigma_{ij}) \leq b_i$$

which represents the soil loss constraint of the problem.

This revised problem can be described as follows. The model is solved taking into account equation (13) for a particular b_i.[1] From this solution, with activity levels, X^*, which maximize the objective function, Z^*, there is a D% probability that soil loss will be lower than b_i.

DESCRIPTION OF THE MODEL AND DATA

The Soil Conservation Service (SCS) of the United States Department of Agriculture has determined that the soils of the 14-county Piedmont Bright Leaf Area of South-Central Virginia are among the most severely eroded in the nation.[2] Average annual soil loss on the crop land is 18 tons per acre. This rate is over twice the state average and three and one-half times greater than the soil loss tolerance for the soils common in the area (SCS, 1983). The most common soils in the area are: Appling, Cecil, and Cullen.

Soil conservation in this region has become an important policy issue due to soil erosion's impacts on long-term agricultural productivity and degradation of water quality. The area has been selected to receive targeted technical and financial assistance from the SCS and the Agricultural Stabilization and Conservation Service.

Using data from the 1982 Census of Agricultural, a representative farm with 174 acres of cropland was developed for the Piedmont Bright Leaf Area. The major crops grown in the area are: tobacco, corn, soybeans, and small grains. These crops were introduced in the model.

The model included production activities for various rotations, selling activities for each crop, and labor activities for the months of May, June, September, and October. It was assumed that the farmer would either provide his own labor or hire labor in these critical months; that is, if his opportunity cost was higher than the wage rate, he would hire all the necessary labor.

The production activities in representative farm models are typically considered as acres in the production of a specific crop. However, since this study was concerned with the soil erosion caused by crop rotations, crop rotations involving one or more crops were used as the production activities. Sixteen crop rotations were considered. The following crops which form part of the rotations are defined as: CT − conventional tillage corn; CNO − no-till corn; W − wheat; BA − barley; G − grass; S − soybeans; TB − tobacco; DWS − wheat-soybean double crop; and TB/c − tobacco with a cover crop. The rotations considered (the numbers in parentheses represent the number of years in the rotations) were: CTW (2); CNOW (2); CTBA (2); CNOBA (2); TB/cTBBAG (4); TB/cTBWG (4); TB (1); TBW (2); TBBA (2); CTWS (3); CNOWS (3); CTBAS (3); CNOBAS (3); CTDWS (2); CNODWS (2); and G (1). For example, CTW (2) means a 2-year rotation with conventional tillage corn followed by wheat.

The objective function maximized net returns to land, risk, overhead, and management. The objective function reflected: (a) prices for each crop for 1983, (b) variable costs for each rotation for 1983, and (c) a wage rate prevalent in the study area, $3.50 per hour. The constraints included: (a) a land constraint; (b) a soil loss constraint; (c) monthly labor requirements for the months of May, June, September, and October; and (d) an assumed tobacco allotment of 37,800 pounds (Forbes and Marshall). The modified

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[1]b_i in the framework of this analysis will represent the maximum level of soil loss allowed. Also, the programming model is solved for $(\mu_{ij} + D^* \, \sigma_{ij})$ in order to minimize the risk of underestimating erosion if this is solved at mean levels of soil loss.

[2]Bright Leaf refers to a popular variety of tobacco, the leading income producer for farmers in the area.

TABLE 1. PROGRAMMING MODEL TABLEAU FOR A REPRESENTATIVE FARM IN THE PIEDMONT BRIGHT LEAF AREA OF SOUTH-CENTRAL VIRGINIA

Item	Production activities																Selling activities						Labor activities				RHS
	CT W	CNO W	CT BA	CNO BA	TB/c TB BA G	TB/c TB W G	TB	TB W	TB BA	CT W S	CNO W S	CT BA S	CNO BA S	CT DWS	CNO DWS	G	C	S	W	TB	BA	G	MAY	JUNE	SEPT	OCTL	
RLORM	-C[a]	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	-C	P[b]	P	P	P	P	P	-L[c]	-L	-L	-L	≤ 174
LAND[d]	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1											= 0
TCORN[e]	A	A	A	A						A	A	A	A	A	A		-1										= 0
TSOYBEAN[f]										S	S	S	S	S	S			-1									= 0
TWHEAT[g]	W	W				W		W		W	W			W	W				-1								= 0
TTOBACCO[h]					T	T	T	T	T											-1							= 0
TBARLEY[i]			B	B	B				B			B	B								-1						= 0
TGRASS[j]					G	G										G						-1					= 0
TMAYL[k]	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M								-1				≥ 0
TJUNEL[l]	J	J	J	J	J	J	J	J	J	J	J	J	J	J	J									-1			≥ 0
TSEPTL[m]					R	R	R	R	R																-1		≥ 0
TOCTL[n]	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O											-1	≥ 0
TOBQUOTA[o]					Q	Q	Q	Q	Q																		≤ 37800
TSOIL[p]	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL	SL												≤ L*

[a] C= variable cost of rotation. [b] P= price of crop. [c] L= wage rate. [d] Land constraint. [e], [f], [g], [h], [i], [j] Transfer rows of yields. [k], [l], [m], [n] Monthly labor requirements. [o] Tobacco quota. [p] Soil loss constraint at different levels of soil loss L*. The SL values were varied according to Table 3 for different upper bounds of soil loss.

enterprise budgets used in the model were adapted using the Soil Conservation Service Crop Budget System (SCS, 1977). Table 1 depicts the programming model tableau.

The soil loss coefficients of the model were based on soil loss estimates given by the USLE. The USLE is an equation that predicts gross soil loss per acre as the inner-product of various erosion related factors. Specifically, the USLE is:

(14) $A = RKLSCP$,

where: A = tons of soil loss per acre per year; R = rainfall and runoff factor; K = soil erodibility factor; L = slope-length factor; S = slope-steepness factor; C = cover and management factor; and P = support practice factor (Wischmeier and Smith).

Through consultations with soil scientists and SCS personnel (Anderson, Googins, Smith), the following parameters of the USLE equation for the representative farm in the Piedmont Bright Leaf Area were developed: K = 0.32 and LS = 0.928 (corresponding to L = 300 feet and S = 5 percent slope). A P factor equal to one was assumed due to a lack of information to determine, on average, how many acres were under strip cropping, terracing, etc. The C factor depicted in Table 2 was obtained for the different crop rotations analyzed. The rainfall and runoff factor, R, was found to follow a lognormal probability distribution (Wischmeier and Smith). For the sake of simplicity in determining the R value in the linear programming model, this lognormal probability distribution was approximated as a normal probability distribution by utilizing standard statistical procedures. The estimated mean and standard deviation were 178.66 and 77.24, respectively.

Table 2 shows the mean and standard deviation of soil loss associated with the different crop rotations analyzed. These soil losses associated with the different crop rotations were assumed to be normally distributed. Thus, in equation (14) for a particular rotation, the factors K, L, S, C, and P are constant and given that $R \sim N(\mu_r, \sigma_r^2)$, this implies that $A \sim N(\mu_a, \sigma_a^2)$ for that particular rotation. By using information in Table 2 and statistical tables for a normal probability distribution, upper bounds of soil loss at different probability levels were constructed for the crop rotations analyzed. These upper bound levels are depicted in Table 3.

TABLE 2. MEAN AND STANDARD DEVIATION OF TONS OF SOIL LOSS FOR DIFFERENT CROP ROTATIONS: PIEDMONT BRIGHT LEAF AREA, VIRGINIA

Crop rotation and years in rotation	C factor[a]	Mean	Standard deviation
		······ tons/acre/year ······	
CTW (2)	0.20	10.611	4.587
CNOW (2)	0.13	6.897	2.982
CTBA (2)	0.20	10.611	4.587
CNOBA (2)	0.13	6.897	2.982
TB/cTBBAG (4)	0.15	7.958	3.441
TB/cTBWG (4)	0.15	7.958	3.441
TB (1)	0.42	22.283	9.634
TBW (2)	0.25	13.264	5.734
TBBA (2)	0.25	13.264	5.734
CTWS (3)	0.32	16.978	7.340
CNOWS (3)	0.15	7.958	3.441
CTBAS (3)	0.32	16.978	7.340
CNOBAS (3)	0.15	7.958	3.441
CTDWS (2)	0.17	9.019	3.899
CNODWS (2)	0.09	4.775	2.064
G (1)	0.04	2.120	0.917

[a]C factors were obtained from SCS personnel. Means and standard deviations were computed by the authors.

Information in Table 3 can be interpreted as follows: evaluating rotation CTW, there is a 50 percent probability that less than 10.611 tons of soil per acre per year will be lost if this rotations is undertaken.

RESULTS

As pointed out before, the model can be solved by considering the upper bound of the confidence interval of the soil loss constraint, equation (13), for the different probability levels shown in Table 3. Five different soil loss constraints were considered; that is, five different b_i values were employed for each one of the four probability levels (including the mean values) in Table 3. One of these b_i's was set "free" at an arbitrarily high value in order to solve the model without imposing a constraint on soil loss. The other four soil loss constraint levels were 10, 8, 6, and 5 tons of soil loss per acre per year. This last value of 5 is the recommended "T" or tolerance level for the soils of the area.

After the different models were solved, only four rotations of the sixteen considered appeared in the alternative optimal solutions. These were: TB/cTBWG, TB, CNODWS, and G. Thus, the rest of the rotations considered were omitted in presenting the results.

A particular crop's acreage in an optimal solution can be obtained as a proportion of the acreage of a rotation in the solution to the number of crops grown under that ro-

TABLE 3. UPPER BOUND OF SOIL LOSS FOR DIFFERENT CROP ROTATIONS, PIEDMONT BRIGHT LEAF AREA, VIRGINIA

Crop rotation	Probability			
	Mean value[a]	.80	.90	.95
	Tons of soil loss/acre/year			
CTW	10.611	14.464	16.482	18.180
CNOW	6.897	9.402	10.714	11.817
CTBA	10.611	14.464	16.482	18.180
CNOBA	6.897	9.402	10.714	11.817
TB/cTBBAG	7.958	10.848	12.362	13.636
TB/cTBWG	7.958	10.848	12.362	13.636
TB	22.283	30.376	34.615	38.179
TBW	13.264	18.081	20.604	22.725
TBBA	13.264	18.081	20.604	22.725
CTWS	16.978	23.144	26.373	29.089
CNOWS	7.958	10.848	12.362	13.636
CTBAS	16.978	23.144	26.373	29.089
CNOBAS	7.958	10.848	12.362	13.636
CTDWS	9.019	12.294	14.010	15.452
CNODWS	4.775	6.509	7.417	8.181
G	2.120	2.890	3.294	3.633

[a]Probability = 0.50.

tation. For example, in Section I of Table 4 for the mean values of soil loss under a free soil loss constraint, 18 acres of tobacco and 156 acres under the CNODWS rotation will be grown. Since CNODWS is a 2-year rotation, 78 acres will involve production of no-till corn and 78 acres will involve double cropped wheat-soybeans. Thus, the specific acreage devoted to produce a particular crop under any of the model solutions can be obtained by following similar procedures.

The first section of Table 4, corresponding to the mean values of soil loss, is the tradi-

tional solution to LP models of soil conservation. Net returns would be $45,733 to the point at which soil loss is constrained to 8 tons per acre per year. At the other two constraint levels of 6 and 5 tons of soil loss per acre per year, net returns would decrease to $43,900 and $34,436, respectively. Notice that in these last two optimal solutions, some switching of land from the more erosive crop rotations to less erosive ones occurs in order to fulfill the soil loss constraint. In the last solution, 28.41 acres of midland grass, a crop which has low soil loss, is required

TABLE 4. OPTIMAL SOLUTIONS OBTAINED FOR THE DIFFERENT PROBABILITY LEVELS OF SOIL LOSS, PIEDMONT BRIGHT LEAF AREA, VIRGINIA, 1983

Probability level of soil loss	b_i[a]	Acres in rotation				Soil loss T/A[f]	Net returns $
		TB/cTBWG[b]	TB[c]	CNODWS[d]	G[e]		
Section I:							
50 percent	Free		18.00	156.00		6.586	45,733
(mean values)	10 T/A		18.00	156.00		6.586	45,733
	8 T/A		18.00	156.00		6.586	45,733
	6 T/A	18.32	8.84	146.84		6.0	43,900
	5 T/A	36.00		109.58	28.41	5.0	34,436
Section II:							
80 percent	Free		18.00	156.00		8.978	45,733
	10 T/A		18.00	156.00		8.978	45,733
	8 T/A	22.42	6.79	144.79		8.0	43,490
	6 T/A	36.00		70.36	67.73	6.0	23,816
	5 T/A	36.00		22.28	115.71	5.0	10,784
Section III:							
90 percent	Free		18.00	156.00		10.231	45,733
	10 T/A	4.63	15.68	153.68		10.0	45,269
	8 T/A	36.00		119.42	18.57	8.0	37,101
	6 T/A	36.00		35.02	102.97	6.0	14,247
	5 T/A	36.00			138.00	5.0	2,137
Section IV:							
95 percent	Free		18.00	156.00		11.284	45,733
	10 T/A	23.41	6.29	144.29		10.0	43,389
	8 T/A	36.00		87.89	50.10	8.0	28,563
	6 T/A	36.00		11.37	126.62	6.0	7,779
	5 T/A	23.77			150.22	5.0	−4,740

[a]Constraint of soil loss, tons of soil loss per acre per year. [b]TB/cTBWG = 4-year rotation with tobacco and a cover crop, tobacco, wheat, and grass. [c]TB = 1-year rotation with continuous tobacco. [d]CNODWS = 2-year rotation with no-till corn and double cropped wheat and soybeans. [e]G = grass. [f]T/A = tons of soil loss per acre per year.

to satisfy the stringent constraint of 5 tons of soil loss per acre per year.

Using the traditional analysis, it would be concluded that soil erosion could be reduced to the recommended "T" value (5 tons per acre per year) in the Piedmont Bright Leaf Area by following production recommendations given by the last optimal solution in Section I of Table 4. As noted earlier, however, soil loss follows a probability distribution which should be taken into consideration in analysis of soil conservation policy. It is, therefore, of interest to examine what happens to the production decisions, if the assumption of nonrandom soil loss is relaxed.

The following discussion is based on the optimal solutions depicted in Table 4 for the other probability levels of soil loss (sections II, III, and IV). In order to illustrate the information contained in those sections, consider the results in Section II obtained for the upper bound of 80 percent probability of soil loss. There is an 80 percent probability that soil loss will be 8 tons per acre per year or less if 22.42 acres of TB/cTBWG, 144.79 acres of CNODWS, and 6.79 acres of tobacco are produced. The net returns associated with this production plan would be $43,490.

At higher probability levels of soil loss, net returns tend to decrease. At the 95 percent probability level, reducing soil loss to the recommended "T" level of 5 tons of soil loss per acre per year, would result in 23.77 acres devoted to the production of the TB/cTBWG rotation and 150.22 acres devoted to midland grass. With these production levels, there is a 95 percent probability that soil loss for a farm in the Bright Leaf Area will be 5 tons per acre per year or less. However, it is important to note that the level of net returns in this case would be negative; that is, the representative farm would lose $4,740. This optimal solution is very different from the "traditional" one that would have been obtained if the mean values of soil loss were used.

CONCLUSIONS AND LIMITATIONS

This paper has illustrated that the probabilistic nature of soil loss can be incorporated

into a linear programming framework and that it can be an important factor in policy analysis. Those who are in a position of making policy recommendations and farm level conservation decisions, therefore, should be aware of the differences in results obtained from the "traditional" method of finding optimal soil saving strategies and those obtained by introducing the notion of a probability distribution of soil erosion. Stochastic programming analysis seems particularly appropriate for analysis of shortrun policies, such as cross-compliance, in order to be reasonably sure of achieving shortrun conservation objectives.

It is recognized that this study has several limitations. First, this analysis did not consider the possibility of a farmer having a choice of participating in commodity programs which require set-aside acreage. Under these programs, one would expect the final results to be somewhat different from the ones found because set-aside acreage is required to be devoted to conservation uses.

Another limitation of the analysis is that crop yield variability was not considered. It is reasonable to expect a positive relationship between rainfall and crop yields and thus, to find a relationship between annual soil loss and yields. Also, variability in the cover and management factor, C factor, of the USLE due to rainfall was not taken into consideration. These issues are left to future studies.

In spite of these limitations, the paper provides insights into the tradeoff between soil loss and income resulting from increasing the probability of meeting a desired conservation goal. Also, it might be particularly important for those individuals working on erosion related non-point source pollution to account for the probabilistic nature of soil loss. In general, the lower the probability of the soil loss constraint, the more likely it is that non-point source pollution will, from time-to-time, exceed the planned levels. The result might be that problems, such as sediment deposition and adverse impacts on fisheries, could be more severe than anticipated if analysts use models which do not recognize the probabilistic nature of erosion.

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