OPTIMIZING SOIL MOISTURE CONSERVATION BY MULCH OF HYDROPHOBIC AGGREGATES USING SIMULATION AND NUMERICAL SEARCH PROCEDURES*

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INTRODUCTION

Reducing soil water losses due to evaporation, runoff or weeds, is a primary concern in the management of agriculture in arid and semiarid regions. A method has recently been proposed to promote infiltration and retard evaporation and weed infestation by waterproofing surface-zone clods with chemical agents to form a mulch of loose, dry clods [3, 4]. The effect of this treatment is to stabilize the clods against breakdown and consequent water and wind erosion. Rain or irrigation water can trickle off individual clods and flow downward through open spaces between them, thus penetrating directly into the deeper soil layers. In this way, the intake of water is enhanced and the hazard of runoff (entailing erosion) is reduced.

The effect of clod hydrophobization becomes even more important during dry spells which generally follow rains in semiarid and arid regions. The evaporation phase normally begins when the soil top layer is in a state of near-saturation, so radiation and wind can cause the extraction of a considerable fraction of the moisture contributed by the preceding rain. However, when a top layer of waterproofed dry clods is present, upward capillary conduction of soil moisture is inhibited, and the dry mulch constitutes a barrier through which soil moisture can escape only by the relatively slow process of vapor diffusion. Hydrophobization of top-layer clods can also inhbit the germination of weeds, which might otherwise compete with crop plants for nutrients, space and light, as well as for soil moisture.

Experimental evidence of these effects was reported by Hillel and Berliner [4]. Integrated physical effects of waterproofing surface clods on the field water regime have been analyzed theoretically by means of a computer-based mechanistic simulation model [6]. While results of waterproofing surface clods depend on a number of factors (including clod sizes, basic soil properties and climatic variables), once an optimal range of clod sizes is established, it is the depth of the mulch which constitutes the decisive controllable variable to be optimized for any specific set of conditions.

THE ECONOMIC PROBLEM

So far, insufficient attention has been given economic aspects of this proposed method of soil treatment, still at the stage of a scientific innovation and not yet a proven practical method. The specific machinery for applying it on a large scale does not yet exist, and the best formulation of the material has not been identified specifically or made available commercially.

However, the need already exists for developing a conceptual and methodological framework looking toward the economic evaluation of alternative soil treatments with variable machinery and price structures. More specifically, the question is: Given a certain predetermined soil-machinery system, how thick should the hydrophobic mulch be to maximize net benefit resulting from making additional water available in the soil for a particular crop season? Since there is a practical limit to the number of

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experimental tests which can be made for different possible combinations of soil, cropping and environmental conditions, we ought to infuse some *a priori* economic considerations and criteria into the design of such experiments, which are in themselves rather expensive to conduct.

At the present stage, it is impossible to give a fully realistic answer to the question regarding optimal mulch thickness, since sufficiently precise data on empirical crop yield response and cost of applying the treatment are not yet available. However, by assuming a set of data parameters based on theoretical considerations, we may develop a general framework for analysis, which can later be adapted to specific real systems. Such a framework can properly be constructed by means of a computer simulation model incorporating basic processes and requirements. It can be capable of performing an overall economic evaluation of cost-benefit relations.

The basic objective of this study was to construct an economic optimizing procedure capable of calculating *a priori* the thickness of the mulch which might provide maximum profit to the operator, subject to physical and dynamic characteristics of the soil and climiate, as well as to the functional dependence of soil treatment costs upon thickness of the mulch to be formed.

In this paper, we shall summarize the conceptual and computerized model of soil water dynamics under variable mulch thickness conditions. This will be followed by an explanation of the economic evaluation procedure. Finally, the application of a numerical search algorithm for the maximum profit combination is illustrated. Sample results will be presented and interpreted.

OUTLINE OF THE SOIL MOISTURE DYNAMICS MODEL

A mechanistic numerical model, derived from basic physical principles and written in IBM 360 CSMP language [7], was designed to compute the dynamic balance and storage of water in a fallow soil through repeated cycles of infiltration and evaporation as a function of depth of a hydrophobic-clod mulch [6]. The necessary inputs are: hydraulic characteristics of the soil and of the surface crust or mulch layer, duration and intensity characteristics of rainstorms or irrigations, and the potential evaporation rate as it varies diurnally and from day to day. The output provides time-dependent rates and cumulative quantities of infiltration, runoff, surface retention, evaporation, internal drainage and changes in water content of different layers and of the profile as a whole. Computations were reported for a four-day simulation (including two rainstorms and four evaporation cycles), illustrating the use of the model for a uniform unmulched soil and for a soil covered with various thicknesses of mulch. The results of these computations indicate that the presence of a mulch of hydrophobic aggregates, several centimeters thick, can greatly increase the quantity of water absorbed and retained in the root zone. Computations carried out for ten- and 15-day periods were in accord with the above. These findings corroborate experimental results and confirm that the hydrophobic mulch concept is indeed a promising approach to soil management for water conservation in dryland and irrigated farming.

This paper, with the procedure it describes, is an addendum to the above model, from which it derives the relation between thickness of mulch and amount of available water stored in the soil.

ECONOMIC OPTIMIZATION

We assume the ultimate objective of farm operators, if they are to apply the hydrophobic mulch method, will be to maximize the net benefit possible for a given crop season or crop rotation. Stated formally:

$$\max \Pi = R(M) - C(M) \text{ for all feasible } M \quad (1)$$

where

M = the thickness of the mulch layer (in meters)

Π = the net benefit function

$R(M)$ = the gross benefit resulting from the additional available soil water, as a function of M

$C(M)$ = the soil treatment costs as a function of M

In the absence of detailed information about either the benefit or cost functions,[1] we introduce the following hypothetical functions:

$$C(M) = P_1 ML + P_2 M^{\alpha} \quad (2)$$

$$R(M) = P_3 N \left(W(M)\right)^{\beta} \quad (3)$$

where

P_1 = purchasing price of the hydrophobic material

[1] Field experiments are needed to get the explicit functional form and the corresponding parameters.

L = amount of material needed for one acre per unit (meter) treatment depth

P_2 = cost per unit (meter) depth of applying the treatment

P_3 = net revenue per unit of water

$W(M)$ = water added. Water retained and mulch M minus water retained without mulch (for the simulation period FINTIM)

N = crop season duration divided by FINTIM (where FINTIM is the characteristic period simulated in the physical model cited above)

α = empirical parameter ($\alpha \geq 1$) responsible for the slope increase of the total application cost as M increases

β = empirical parameter responsible for the efficiency of crop response to additional water.

While these functions are hypothetical, they are nevertheless plausible. The amount of chemical material to be applied is simply proportional to the mass of clods to be treated, the latter expressible in terms of depth of treatment. Hence, the first term on the right-hand of equation (2) is linear.

On the other hand, costs of energy, labor and equipment necessary to break up the soil surface into treatable clods of the desired size range, increase disproportionately to the depth of treatment. Evidence of this is available from earlier studies of soil tillage [2]. Hence, we assigned a value greater than unity to the exponential parameter β of the second term in equation (2). Similar reasoning is applicable to the crop response function, expressed in equation (3).

As a first approximation, we might consider the incremental income derived from the increased yield to be proportional to additional amount of available soil moisture. Thus, we might assume a linear form of equation (3) with β=1. However, in arid climates, a relatively small increment of water can often result in a disproportionate increased in yield [5] and might even spell the difference between crop failure and success. Hence, β might have a value greater than unity.

The value of this analysis is not limited to the somewhat arbitrary choice of functions and parameters elucidated above. These values are illustrative rather than universal. The particular relationships with which the model is illustrated may or may not be realistic in any specific location. Yet, in principle, the same general analysis ought to apply even if appropriate functions and parameters are somewhat different.

Necessary conditions for maximizing (1) with respect to M are:

$$\frac{d\Pi}{dM} = 0 \text{ and } \frac{d^2\Pi}{dM^2} < 0 \qquad (4)$$

W(M) is not known explicitly; however, it is a continuous and single valued function which can be evaluated numerically by the physical simulation system mentioned above. Unfortunately, it can be expensive to evaluate W(M) for a large number of values.

The Fibonacci numerical search procedure was chosen as the method for maximizing Π in equation (1). This method facilitates efficient convergence toward the optimal value of M without requiring prior knowledge of the explicit form of the function Π versus M.

OPTIMAL MULCH THICKNESS BY A FIBONACCI SEARCH

Of the various methods possible for maximization of Π by a direct search method, the Fibonacci method requires the least number of repeated evaluations of Π. For more details see [9] and [1] or [8]. This property makes the Fibonacci method particularly attractive, in view of the fact that each evaluation requires a complete simulation run with a specific value of the parameter M.

For the Fibonacci Search to be applicable, the function Π must be unimodal and possess a maximum in the finite interval $a_o \leq M \leq b_o$. Both a_o and b_o are chosen by judgment so that the initial interval of uncertainty (a_o,b_o) is not too wide. The total number of test evaluation points must also be specified in advance. This number n can be predetermined by the accuracy criterion chosen. Figure 1 may assist in understanding this procedure. Suppose that after k (k=0,1, ..., n-1) steps, the interval of uncertainty is reduced to (a_k,b_k), then the two points M_k and $M_k{}^*$ are chosen by:

$$M_k = \frac{F_{n-1-k}}{F_{n+1-k}} \quad (b_k - a_k) + a_k \qquad (5)$$

$$M_k{}^* = \frac{F_{n-k}}{F_{n+1-k}} \quad (b_k - a_k) + a_k \qquad (6)$$

where

$$F_k = F_{k-1} + F_{k-2} \qquad (7)$$

$$F_o = F_1 = 1 \qquad (8)$$

Equation (7) is the so called Fibonacci Series.

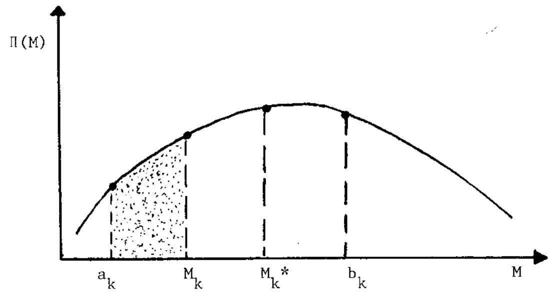

FIGURE 1. FIBONACCI SEARCH FOR MAXIMA

If $\Pi(M_k) \geq \Pi(M_k^*)$, then the next interval of uncertainty is chosen as $(a_{k+1}, b_{k+1}) = (a_k, M_k^*)$; if $\Pi(M_k) \leq \Pi(M_k^*)$, then $(a_{k+1}, b_{k+1}) = (M_k, b_k)$.

In Figure 1, the shaded area will be dropped for the next step, resetting $a_{k+1} = M_k$, $b_{k+1} = b_k$, $M_{k+1} = M_k^*$, then perform equation (6) to calculate M_{k+1}^*, evaluate $\Pi(M_{k+1}^*)$ and so on. The final interval of uncertainty is $[a_{n-1}, b_{n-1}]$.

Clearly, one of the test points always lies in the interior of the reduced interval and serves as one of the trial points for the next iteration. Thus, after the first, only one test point is required per iteration. Size of the last interval is given by:

$$b_{n-1} - a_{n-1} = \frac{1}{F_{n+1}} (b_o - a_o) \tag{9}$$

Thus, after n evaluations, the initial interval of uncertainty is reduced by a factor of $1/F_{n+1}$. For example, a final interval of uncertainty, which is less than three percent of the initial one, requires eight Fibonacci search steps, since $F_8 = 34$.

Coupling this approach with the simulation model by CSMP [7] presents a few problems which are discussed in footnote 4.

Let us now turn to the results obtained when this model is applied in a hypothetical case.

RESULTS

To demonstrate the program's capability, a computer run was made with the following arbitrarily assigned values: $n=8$, $P_1=6.0$, $P_2=1.0$, $P_3=18.0$, $\alpha=1.15$, $a_o=0.0$, $b_o=0.20$ meter, $L=0.443$, $\beta=1.05$, $N=18$.

When the search was completed, the final interval of uncertainty was (0.0,0.02885) with a midpoint of 0.01442 meter, for the thickness of the mulch. Costs reached \$43.9 per acre and benefits \$95.8 per acre, or a net benefit of \$51.9 per acre.[2]

To check the soundness of the model, a selective sensitivity analysis was carried out for some economical and physical parameters. All values used were hypothetical.

Figure 2 shows net benefit curves as a function of different thicknesses of mulch for three levels of P_3 as defined above.[3] As expected, optimal mulch thickness increased as water value increased.

Figure 3 illustrates the effect of reduction in purchasing price of the hydrophobic material used (P_1). As the price decreased, optimal thickness increased.

Figure 4 illustrates the need to increase M if the crop's efficient utilization (β) of additional water saved by the mulch is higher, and vice versa. Notice that β in this model was taken as a constant for simplicity; however, treating β as a variable depending on W(M) should not cause undue difficulty, because solutions are obtained numerically.

It now remains for this method to be applied to actual dry farming practices in semiarid areas in the U.S. and elsewhere, and to be tested with realistic prices and other parameters mentioned above.

SUMMARY AND DISCUSSION

A computer simulation system of soil moisture dynamics under different thicknesses of hydrophobic

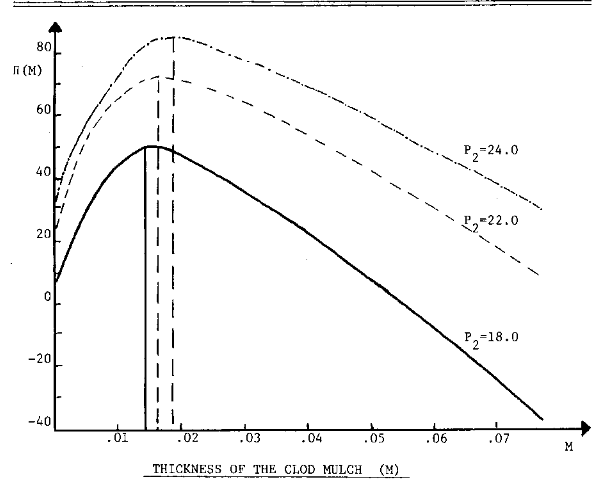

FIGURE 2. NET BENEFIT VS. THICKNESS OF MULCH WITH THREE PRICE LEVELS OF WATER (P_3 = 18.0, 22.0, 24.0)

[2]A better approximation of the maximum point could be computed by a quadratic approximation using the last point calculated and $\Pi(a_k)$ and $\Pi(b_k)$ points. With these values, the optimum is found by calculus to be $M_{opt}=0.0139$.

[3]The solid line in all figures represents the "basic" situation with initial values mentioned above. Other lines reflect the net benefit response of modifying only the single parameter specified for each figure.

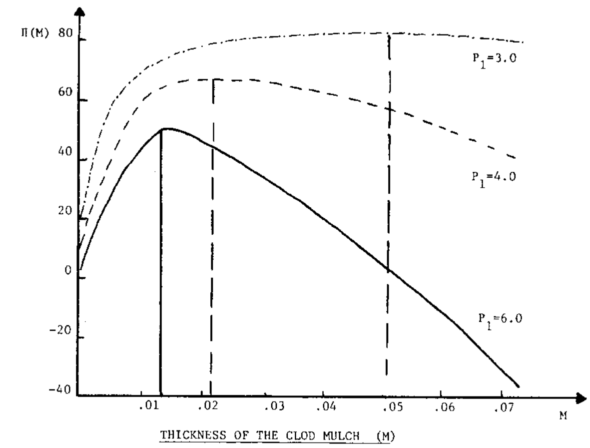

FIGURE 3. NET BENEFIT VS. THICKNESS OF MULCH WITH THREE PRICE LEVELS OF HYDROPHOBIC MATERIAL (P_1 = 6.0, 4.0, 3.0)

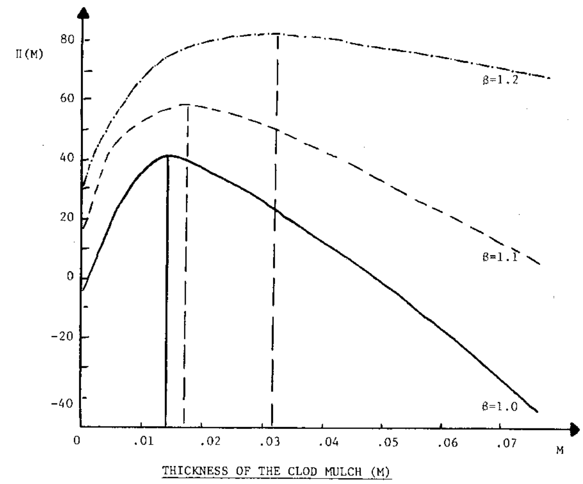

FIGURE 4. NET BENEFIT VS. THICKNESS OF MULCH WITH THREE LEVELS OF CROP RESPONSE COEFFICIENT (β = 1.0, 1.1, 1.2)

mulch has been described. An optimizing algorithm, based on the numerical Fibonacci Search, was used to calculate the optimum mulch thickness which can maximize the operator's net benefit, subject to soil-water dynamics, market prices and the state-of-the-art of this particular soil treatment.

Application of this system could be useful in two major ways: first, as a tool for assessing physical or economic effects of the hydrophobic mulch treatment as it interacts with other components of the system such as machinery, cropping practices, irrigation methods, et cetera; secondly, once in use, it can guide the farmer to efficient treatment for soil control under actual environmental conditions.

Much additional research is needed before this practice can become established. Field experiments are necessary to determine necessary physical parameters and to validate this and future studies of its type. The machinery to be designed for performing such soil control treatments must be related to cost-benefit analysis, which can be performed by simulation methods.

APPENDIX I
THE CSMP TERMINAL SECTION[4] (EXECUTING THE FIBONACCI SEARCH)

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TERMINAL

SORT

TIMER FINTIM=432000.,OUTDEL=86400.,DELMIN=.1E-5,PRDEL=86400.,DELT=1800.
PRINT CUMINF, TMULCH, CUMEVP, CUMPET, CUMDRN, CUMWTR, DETAIN, BALANS
METHOD RKS

NOSORT

      GO TO (9991,9992,9993,9994,9995),IPOINT
9991  CONTINUE

*     FIBONACCI SEARCH

      IF (TMULCH) 600,600,650
 600  WATERO = CUMWTR
      X(1) = 0.0
      X(2) = 0.0
      TMULCH = TMLCMX
      CALL RERUN
      IPOINT = 2
      GO TO 9999
9992  CONTINUE
 650  IF (TMULCH .NE. TMLCMX) GO TO 700
      W = CUMWTR - WATERO
      Z(1) = TMULCH
      Z(2) = GETBEN (TMULCH, W. PM. PW. C1, C2, C3, NF, FINTIM, PT)
      A = X(1)
      B = Z(1)

 700  CONTINUE
      NF = NF -1
      IF (NF - 2) 800,800,710
 710  CONTINUE
      IF (Z(2) - X(2)) 770,720,720

 720  IF (Z(1) - X(1)) 750,750,730
 730  B = Z(1)
      EL = B - A
      DELTL = EL * FIB (NF-2) / FIB(NF)
      Z(1) = A + DELTL
 740  TMULCH = Z(1)
      CALL RERUN
      IPOINT = 3
      GO to 9999

9993  CONTINUE
      W = CUMWTR - WATERO
      Z(2) = GETBEN (TMULCH, W, PM, C1, C2, C3, NF, FINTIM, PT)
      WRITE (6,9) A,B,Z(1),X(1),Z(2),TMULCH,FIB(NF),DELTL,EL
   9  FORMAT (///10F13.5)
      GO TO 700

 750  A = Z(1)
      EL = B - A
      DELTL = EL * FIB(NF-2) / FIB(NF)
      Z(1) = B - DELTL
      GO TO 740
```

[4] Notice that, in this program, a combination of computed GO TO statements with preassignments has been introduced to force execution of the RERUN to be carried out in various places of the program (note that the RERUN does not occur until the program sequence reaches the END card).

```
 770   IF (Z(1) - X(1)) 790,790,780

 780   A = X(1)
       EL = B - A
       DELTL = EL * FIB(NF-2) / FIB(NF)
       X(1) = B -DELTL
 785   TMULCH = X(1)
       CALL RERUN
       IPOINT = 4
       GO TO 9999
9994   CONTINUE
       W = CUMWTR - WATERO
       X(2) = GETBEN (TMULCH, W, PM, PW, C1, C2, C3, NF, FINTIM, PT)
       WRITE (6,9), A,B,Z(1),X(1),Z(2),X(2),TMULCH,FIB(NF),DELTL,EL
       GO TO 700
 790   B = X(1)
       EL = B - A
       DELTL = EL * FIB(NF-2) / FIB(NF)
       X(1) = A + DELTL
       GO TO 785
 800   CONTINUE
9995   CONTINUE
       W = CUMWTR - WATERO
       WRITE (6,803)
 803   FORMAT (////' THE OPTIMUM VALUE IS '//)
       V = GETBEN (TMULCH, W, PM, PW, C1, C2, C3, NF,FINTIM, PT)
       WRITE (6,9) A,B,Z,(1),X(1),Z(2),X(2),TMULCH, FIB(NF),DELTL,EL
*      END OF OPTIMIZATION
9999   CONTINUE
       END
       STOP
       FUNCTION GETBEN (TMULCH, W, PM, PW, C1, C2, C3, NF, FINTIM, PT)
       COST = PM*(C1*TMULCH*100.0) + PT*(TMULCH*100.0)**C2
       BENEFIT = PW * 18.0*(864000./FINTIM)*C3*W
       Q = COST - BENEFT
       GETBEN = Q
       WRITE (6,1) TMULCH,W,COST,BENEFT,Q,NF
     1 FORMAT (//' TMULCH WATER COST BENEFT NETBEN', 5F14.4, 19//)
       RETURN
       END
```

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