SELECTION OF A FARM MACHINERY REPLACEMENT CRITERION USING SIMULATION

Darrel Kletke*

Little research has focused on developing a model which farmers can use to make yearly machinery replacement decisions. This paper contains an optimizing replacement criterion and then demonstrates the results of alternative rules of thumb used to implement the criterion in a real world situation.

THE REPLACEMENT CRITERION

The economic life of a machine is here defined as the interval of time during which that machine reaches its minimum average yearly cost. If a machine is replaced by an exact duplicate with the same annual costs, replacement occurs when the currently owned machine attains its economic life. When average cost reaches its minimum, marginal cost and average cost are equal. This is the same as saying that when economic life is reached, the actual yearly cost (marginal cost) is equal to the average yearly cost of the machine. Theoretically, replacement should occur when marginal cost first crosses average cost from below.

In a timeless environment, the replacement criterion operates satisfactorily. To consider time in the model, it is necessary to restate the replacement criterion: Replacement should occur when yearly costs (marginal cost) first crosses the minimum amortized average cost of the proposed replacement. Amortized average cost for any year T is found in two steps. First, the total present value of all costs from year one to T is computed, then, the total present value is amortized for T years just as a mortgage is amortized.

In actual replacement situations, use of economic life as a criterion for replacement breaks down. First of all, farmers very seldom purchase duplicate machines. Second, actual yearly costs do not behave in such an orderly manner that replacement should occur whenever the marginal cost first exceeds the minimum average cost. The problem of farmers not purchasing duplicate machines is handled elsewhere.[1] Methods of handling the second problem, behavior of yearly costs, will be discussed in this paper.

In the real world, yearly machine costs fluctuate, therefore, some rule of thumb must be used to implement the theoretical replacement criterion. There are several alternative rules which may be suggested. First, the machine can be replaced at the theoretical optimum replacement interval. Second, the farmer may replace when some average of marginal costs exceeds the minimum average cost of the proposed replacement. Third, replacement may occur when marginal costs in any year reach a specified level. The size of repair cost required will be discussed later.

DISTRIBUTION OF REPAIR COSTS

A simulation procedure is used to determine which of the rules of thumb provides the lowest average cost over time. The most unpredictable farm tractor cost is repairs. Before simulation can take place, a distribution must be constructed from which yearly repair costs can be drawn at random. Because repair costs fluctuate widely, collection of a large number of observations is necessary to determine with some degree of confidence the distribution's shape. Data collection poses a problem since it is difficult to obtain data from a large number of tractors which are the same age, size, and which are used the same amount. This problem was overcome by constructing a generalized distribution. Repair cost data were collected on tractors of various sizes, ages, and use levels. Given the size, age, and use, a repair cost equation was used to

* Darrel Kletke is an assistant professor, Department of Agricultural Economics, Oklahoma State University.

[1] Darrel D. Kletke, "Dynamic Analysis of Farm Machinery Replacement," unpublished thesis, Oklahoma State University, 1968.

determine expected repair costs for that tractor.[2] Each repair cost observation was then divided by the repair cost expected for that machine. The ratios found were then tabulated giving a frequency distribution of actual repairs as a percent of expected repairs. The expected value of the frequency distribution should be one.

The data used to find the distribution, using the above procedure, are the same data used to construct the repair equation. Since the tractors surveyed varied in age and there has been a large amount of inflation since many of them were purchased, the index of prices paid by farmers was used to inflate the tractor prices to a 1966 equivalent. The data used in finding the repair cost distribution were collected on tractors between two and twenty-six years old with at least a $3,000 inflated purchase price and used a minimum of 400 hours per year.

Observations on 475 tractors were used in the construction of the repair cost distribution illustrated in Figure 1. After slight adjustment, the expected value of the repair cost distribution is .996. The adjustments alter the distribution towards a normal curve, but it is still significantly skewed. Because repair costs tend to occur in lumps every two or three years, the mode of the distribution is considerably less than the expected value. The adjusted distribution allows repair costs to vary from 5 to 495 percent of the expected value. If expected repair costs for a year were $100, then the possible range of repair costs would be from $5 to $495. As expected costs increase, the possible range of repair costs increase also. If expected repair costs were $200, then the possible range of repair costs would be from $10 to $990. Using the repair equation, it can be found that the expected repair cost for a thirty year old tractor is above $600. Using the repair cost distribution, the highest possible repair cost in year thirty is above $3,000. Clearly, repair costs of this magnitude are not conceivable in normal everyday operations and available data do not indicate that they would ever be that high. Since the distribution gives unsatisfactory results when expected repair costs are high, an arbitrary limit of $1,300 is placed on the repair cost size which could occur in any year.

The high percentage of low costs indicate that the distribution is skewed. Over 50 percent of the time, simulated repair costs will be less than 65 percent of the expected (average) value of repair costs. About 64 percent of the time, simulated repair costs will be less than their expected value. On the other end of the distribution, only 10 percent of the repair costs will be more than 2.45 times the expected cost.

In the simulation procedure, the repair cost density function is used to determine yearly repair costs. Random numbers are used to select from a cumulative of the distribution illustrated in Figure 1 the proportion of expected repair costs to be used for the year. The repair cost proportion obtained is then multiplied times the expected repair cost computed from the repair equation to procure the simulated repair charge.

THE SIMULATION PROCEDURE

The simulation procedure used for replacement criteria evaluation is as follows: First, the minimum amortized average cost of the proposed replacement is found. The replacement's minimum amortized average cost is the pivotal variable in trading decisions. Except for the repair portion, marginal costs are computed for the existing machine by using appropriate cost equations.[3] A sample simulation procedure is given in Table 1. The procedure is applied to a $6,100 machine with an expected minimum amortized average cost of $2,644.98. A random number is used to select a repair cost proportion from a cumulative of the distribution illustrated in Figure 1. Simulated repair costs are found by multiplying expected repair costs by the appropriate proportion of expected repair costs. Yearly simulated costs are equal to expected costs plus the difference between simulated and expected repairs.

Once the simulated yearly cost is obtained, the procedure used to implement the replacement criterion is applied. For expositional purposes, the replacement

[2] The repair cost equation is:

$$R_i = W_i - W_{i-1}$$

where R_i = repair cost in year i,

$W_i = .00000913\ C(tD)^{1.5}$,

C = tractor list price,

t = age of machine in years,

and D = yearly use in hours.

W_i is an altered form of an equation constructed by W.E. Larsen and W. Bowers, "Engineering Analysis of Machinery Costs," presented at the 1965 meeting of the American Society of Agricultural Engineers, Appendix p. 2, June, 1965.

[3] The equations for cost components other than repairs were taken from several sources and are summarized in: Darrel D. Kletke, "Dynamic Analysis of Farm Machinery Replacement," unpublished thesis, Oklahoma State University, 1968. In this analysis, marginal costs (yearly costs) includes all costs for a given year including overhead.

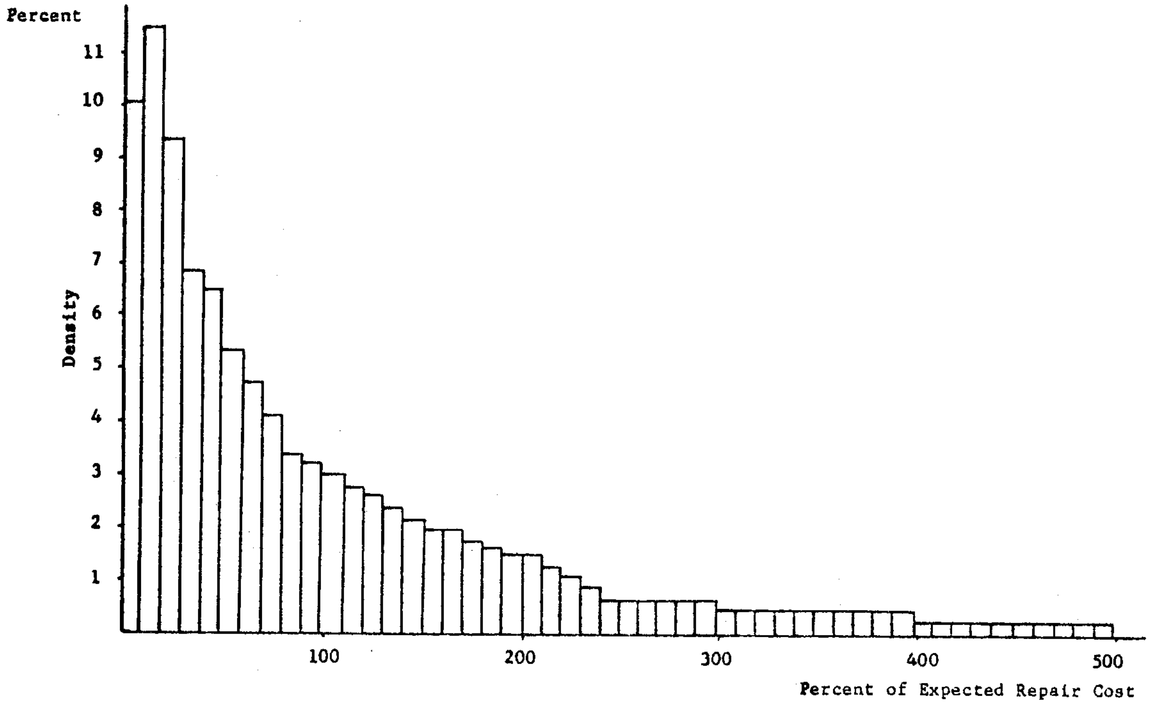

FIGURE 1. DENSITY DISTRIBUTIONS OF REPAIR COST AS PERCENT OF EXPECTED REPAIR COST FOR A $6,100 MACHINE

procedure used in Table 1 is a three-year average of marginal (annual actual) costs. As explained earlier, an average of marginal costs may be used to implement the replacement criterion because of marginal (annual) cost variability. By using an average of marginal costs, it is hoped that premature replacement due to one large repair cost can be prevented. When the three-year average of marginal cost exceeds the minimum expected average cost of the proposed replacement, it is time to trade. Other replacement procedures will be considered and evaluated later but the analytic procedure is the same as for the three-year average. It was previously pointed out that only when marginal costs are rising is the replacement model relevant. In Table 1, expected yearly costs begin rising in year 2. Thus, not until year 4 is it possible to have a three-year average which can be tested against the minimum amortized average cost ($2,644.98) of the proposed replacement.

In Table 1, all that is done for the first 3 years of tractor life is to find the simulated yearly cost. In year 4, a three-year average of marginal costs is found. This average is checked against the minimum amortized average cost of the proposed replacement. If the three-year average is larger, the tractor is traded. Otherwise, the tractor is kept and the simulation of year five begun.

The procedure outlined above continues until the tractor is replaced. In Table 1, two tractor lives are simulated. One machine is kept 7 years; the next is kept 19. The way in which the simulation procedure is used to evaluate various replacement criteria is the topic of the following section.

EVALUATION OF REPLACEMENT PROCEDURES

The purpose of simulating tractor ownership intervals is to have some means of evaluating alternative replacement procedures. In theory, there is no problem - - as soon as marginal cost exceeds the minimum average cost of the proposed replacement, it is time to trade. Also, when marginal cost exceeds the minimum average cost, it is necessary that it remain above average cost. This condition will not be met in real life as yearly costs fluctuate considerably, especially the repairs component. When large repair costs occur early in machine life, the farmer may either trade or keep the machine. If he follows the theory directly, he will trade. If he trades, he may forego the subsequent low marginal costs on the current machine for the relatively high average yearly cost of the replacement.

The objective of effective tractor management is the minimization of long-run costs. Therefore, average cost is the norm chosen to compare alternative replacement strategies. The simulation procedure presented provides a means of determining with a reasonable degree of accuracy the average costs associated with each procedure. A large number of tractor lives are simulated using a given rule of thumb for deter-

TABLE 1. ILLUSTRATION OF SIMULATION PROCEDURE USING THREE-YEAR-AVERAGE CRITERION ON A $6,100 MACHINE WHICH HAS A MINIMUM AMORTIZED AVERAGE COST OF $2,644.98

Tractor Age	Random Number	Repair Cost Proportion	Expected Repair	Simulated Repair	Expected Yearly Cost	Simulated Yearly Cost	3 Yr. Avg. of Simulated Cost	Is Replacement Criteria Met?
(Years)			(----------	----------	Dollars	----------	----------)	
1	42,365	.45	81.85	36.83	4,298.71	4,253.69		
2	92,667	2.85	149.66	426.63	2.358.36	2,635.33		
3	22,746	.25	193.80	48.45	2,350.24	2,234.89		
4	29,222	.45	229.50	103.27	2,396.84	2,270.61	2,380.27	no
5	98,762	4.45	260.32	1,158.42	2,411.51	3,309.61	2,605.03	no
6	20,159	.15	287.84	43.18	2,425.65	2,180.99	2,588.90	no
7	95,497	3.45	312.92	1,079.57	2,439.94	3,206.59	2,899.06	yes
1	88,460	2.25	81.85	184.16	4,298.71	4,401.02		
2	47,195	.55	149.66	82.31	2,358.36	2,291.01		
3	53,963	.75	193.80	145.35	2,380.24	2,331.79		
4	68,423	1.15	229.50	263.93	2,396.84	2,431.27	2,351.35	no
5	43,590	.55	260.32	143.18	2,411.51	2,294.37	2,352.47	no
6	39,020	.45	287.84	129.53	2,425.65	2,267.34	2,370.99	no
7	30,866	.35	312.95	109.53	2,439.94	2,236.52	2,266.07	no
8	18,813	.15	336.18	50.43	2,454.76	2,169.01	2,224.29	no
9	29,888	.25	357.90	89.48	2,470.31	2,201.89	2,202.47	no
10	19,141	.15	378.32	56.76	2,486.70	2,165.08	2,178.66	no
11	67,205	1.15	397.81	457.48	2,503.99	2,563.66	2,310.21	no
12	74,732	1.45	416.33	603.68	2,522.19	2,709.54	2,479.42	no
13	53,695	.75	434.06	325.55	2,541.30	2,422.79	2,568.66	no
14	15,578	.15	451.09	67.66	2,561.29	2,177.86	2,440.06	no
15	56,432	.75	467.50	350.62	2,582.14	2,465.21	2,538.63	no
16	15,578	.45	483.35	217.51	2,603.81	2,337.97	2,327.03	no
17	56,432	.85	498.71	423.90	2,626.26	2,551.45	2,451.56	no
18	80,571	1.75	513.60	898.80	2,649.45	3,034.65	2,642.12	no
19	91,216	2.65	528.07	1,300.00	2,673.34	3,445.27	4,010.45	yes

mining when to replace. The total costs associated with each tractor can then be summed and divided by the number of years to give an average cost over time. The replacement procedure offering the lowest average cost over time is the most economical choice.

In this simulation of tractor lives, it is assumed that the farmer can correctly anticipate costs for the following year. Using a three-year average cost rule of thumb, the simulation results presented in Table 1 imply that the first tractor would actually be traded in 6 years. The high repair cost in year 7 would have been anticipated and the farmer would have traded machines before the cost occurred.

As mentioned earlier, procedures proposed for implementing the replacement criteria fall into three groups. The first requires keeping each tractor its economic life and then trading. For a $6,100 machine, the expected minimum amortized average cost is $2,644.98 and the corresponding economic life 17 years. This is based on single-valued, expected annual costs with no provision for cost variability.

The second rule of thumb involves averages of marginal cost. Two, three, four, five, seven, nine, and twelve-year averages are considered. If a twelve-year average of marginal costs is used, it means that no machine could be replaced before year 13. Therefore, an alteration is made in the average cost criteria. In year 4, a three-year marginal cost average is tested against the minimum average cost of the proposed replacement. In year 5, a four-year average is used. The averaging process is continued until a maximum twelve-year average is found. Thus, replacement based on (say) twelve-year averages can occur as early as year 4.

The third rule of thumb is based on the occurrence of a very large repair cost. Required to cause replacement is a repair cost which, when added to the sum of marginal costs between the large cost year and the expected optimal year, would yield an average of marginal costs greater than the minimum average cost of the proposed replacement. Also considered in the simulation analysis were combinations of the large cost replacement rule and the average of marginal costs rule.

Table 2 gives the simulation results. The procedures marked with asterisks offer the lowest average costs over time. The large cost criterion, averaged over 1,000 trials, offers an average cost over time $20 per year less than other methods tested. The average replacement interval, using the large cost method, is 13.7 years; whereas, the economic life of the machine is 17 years. The expected simple average cost of owning a $6,100 tractor 17 years is $2,592. The averages in Table 2 and $2,592 are comparable figures. Several of the procedures have average costs above $2,592, which indicates that trading in a set pattern of every 17 years would be preferred to using such methods.

The large cost procedure provides a saving of about $50 per year over the arbitrary decision rule of trading every 17 years. The large cost method, used in conjunction with the average of marginal costs, provide lower costs than the average of marginal costs criterion used alone.

The simulation results indicate that over a long period of time the various replacement procedures tested offer small cost reductions compared to trading every 17 years. However, a long period of time is many times the farmer's age. Therefore, it may be argued that during a farmer's lifespan utilization of rules two and three may be very important. If the rule of trading every 17 years were followed for a $6,100 tractor, the typical farmer would own no more than 3 tractors during his life. Using rules two and three may not always save much, but, if a "lemon" were purchased, savings could be considerable.

DISTRIBUTION OF REPLACEMENT INTERVALS

Once the optimum replacement procedure is selected, it is possible to construct a replacement interval distribution based on the chosen method. The density distribution of replacement intervals for the large cost procedure is given in Figure 2. The data used for construction of this distribution were obtained from the simulation results. Each time a tractor life was simulated, the replacement year was recorded. Figure 2 is based on the results of 1,000 simulated tractor lives. The expected value of the distribution is 13.74. In the simulation, no machines were replaced before year 8 because the cost equations used made it impossible to have a sufficiently large cost.

SUMMARY

In this paper, a simulation routine was devised for evaluating alternative rules of thumb which could be used to implement the theoretical replacement criterion. The replacement criterion is the equating of current machine marginal cost and the proposed replacement's minimum amortized average cost. In a real world situation costs do not behave in an orderly manner, causing application of the theoretical model to lead to costly replacement decisions.

Rules of thumb tested using simulation were: First, trading only when expected economic life expires. Second, trading when a selected average of marginal costs is greater than the minimum average cost of the proposed replacement. Third, trading when a sufficiently large cost occurs.

TABLE 2. EXPECTED VALUE OF REPLACEMENT INTERVALS AND AVERAGE COSTS FOR ALTERNATIVE REPLACEMENT PROCEDURE, 1,000 TRACTOR LIVES SIMULATED USING EACH PROCEDURE[a]

Criteria	Average Cost	Expected Replacement Interval
	(Dollars)	(Years)
Large Cost	2,540.96[b]	13.7
2-year-average	2,620.63	11.3
3-year-average	2,591.54	14.7
5-year-average	2,603.54	17.3
9-year-average	2,617.75	21.3
12-year-average	2,614.50	24.0
2-year-average + Large Cost	2,595.69	10.6
3-year-average + Large Cost	2,572.42	12.0
4-year-average + Large Cost	2,562.59[b]	12.8
5-year-average + Large Cost	2,564.79	12.9
7-year-average + Large Cost	2,567.97	13.2
9-year-average + Large Cost	2,566.96	13.4
12-year-average + Large Cost	2,568.99	13.8

[a] The minimum amortized average cost of the proposed replacement, $2,644.98, is equal to a simple average cost in year 17 of $2,592. The difference between $2,592 and the average costs above are measures of the savings per year.

[b] Denotes replacement rule of thumb with lowest average cost overtime.

Simulation results indicate that over the lives of a number of tractors, use of economic life as the replacement procedure offers nearly as low an average cost as any other rule of thumb. However, other replacement rules offer advantages to farmers who own few tractors in a lifetime.

Other replacement procedures might be proposed and evaluated using simulation. Although a $6,100 tractor was used in the simulation analysis, any tractor size could be used. In addition, it is not necessary that the proposed replacement be a duplicate of the existing machine.

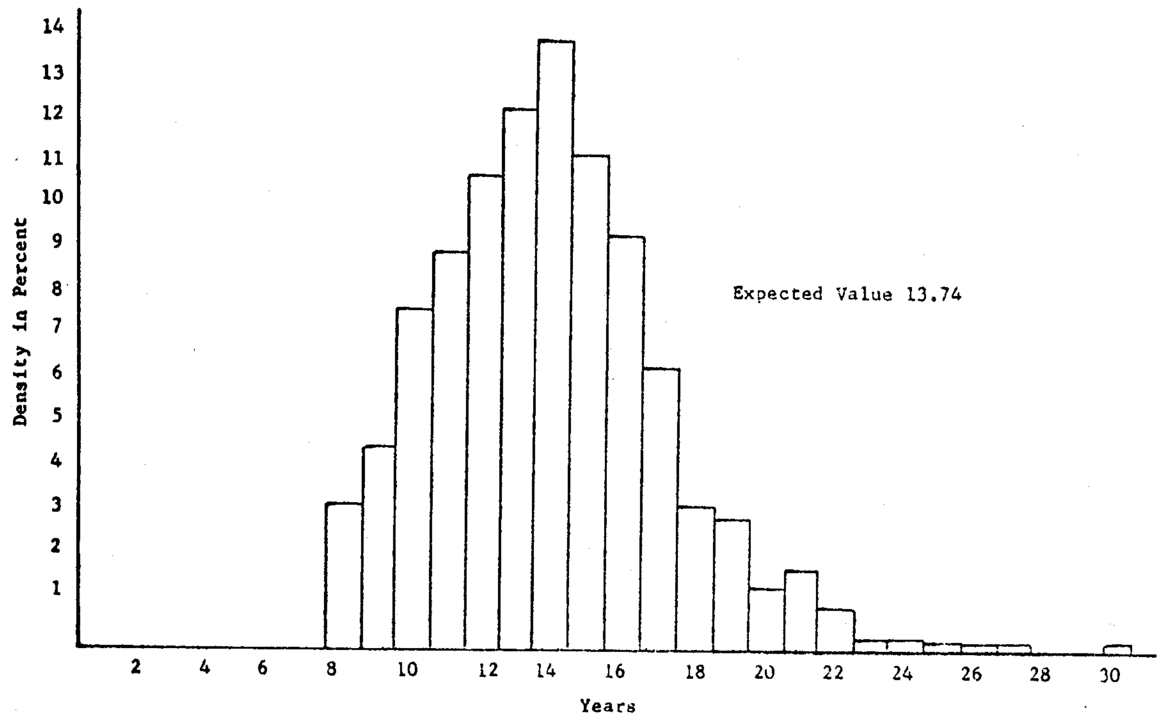

FIGURE 2. TRACTOR REPLACEMENT INTERVAL DISTRIBUTION FOUND USING LARGE COST CRITERIA AND 1,000 REPLICATIONS