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Articles in the field of agricultural economics, suitable for publication in the journal, will be welcomed.

Articles should have a maximum length of 10 folio pages (including tables, graphs, etc.), typed in double spacing. Contributions, in the language preferred by the writer, should be submitted in triplicate to the Editor, c/o Department of Agricultural Economics and Marketing, Pretoria, and should reach him at least one month prior to date of publication.

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Contents

The utilisation of operations research techniques in the planning of agricultural undertakings*

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1. INTRODUCTION

In the place of the more familiar planning techniques, as for example the budget and gross margin method, stress is beginning to be laid in overseas countries on the utilisation of operations research techniques in agriculture. Linear programming in particular, is already being used there on a large scale and in the recent past this technique has also had its impact in the South African agricultural sector. In this article attention is being given to linear and quadratic programming, while particular emphasis is laid on the use of the Monte Carlo method.

2. LINEAR PROGRAMMING

As is known, this technique comprises the optimising of a linear function, subject to linear restraints. Among the assumptions made in the application of this technique is, inter alia, that of single valued expectations. It is assumed, therefore, that the future is predictable with great certainty. This assumption admittedly sounds farfetched, but it must nevertheless be borne in mind that the same assumption has to be made with any other planning technique - the future must always be estimated in advance, irrespective of the technique used.

Of more importance is the fact that agricultural economists, when applying a technique such as linear programming, invariably attempt to maximise gross profit, subject of course to the usual resource restrictions. This means it is assumed tacitly that a firm obtains maximum satisfaction of its needs, simply by realising as great a profit as possible - economics is, however, defined as the science in which it is endeavoured to apply scarce means in such a manner as to obtain maximum satisfaction of needs. Few entrepreneurs, however, aim at attaining maximum profit as the sole objective, and especially in respect of agricultural entrepreneurs, subjective preferences also play a role. In addition, very few, if any, business undertakings will blindly endeavour to realise large profits without consideration of risk factors.

In order to render the said two assumptions less limiting, modified simplex methods were developed in the course of time, in which the assumption of single valued expectations became less limiting and it also became possible to take into consideration the risk factors in formulating and solving a simplex model.

2.1 Methods for the handling of risk factors

In practice it is usually found that an enterprise or activity rendering a relatively stable income over a period and thus by implication can be considered as less risky, cannot compete with enterprises which may possibly render high profits, judged according to potential profitability alone. Here the entrepreneur has to choose between an enterprise which he is fairly certain will render a stable income and an enterprise which may render a high income, but which is more subject to income fluctuations. In an ordinary standard simplex model, the less risky enterprise will always play a subordinate role, since the simplex algorithm is of such a nature that only the most profitable enterprises are incorporated in the production plan - irrespective of the stability of the income.

However, an entrepreneur may prefer, for risk purposes, to incorporate in his organisation an enterprise rendering a stable income over a period at the expense of another which may possibly render a high but more risky income. In order to act rationally, the entrepreneur must combine these two enterprises in such a manner as to derive the maximum marginal benefit from each of them.

This combination can be obtained in at least three ways by means of linear programming:

(i) Enterprises rendering a smaller but more stable income are forced into the programme at a minimum level by adding an extra row to the restraints. The minimum level is raised gradually until a stage is reached where the entrepreneur is no longer prepared to decrease his expected gross profit in favour of greater stability in income.

(ii) Maximum limits can be placed on the more risky activities and these limits can be increased or decreased until a solution is obtained where a proper balance exists between the risky and the less risky activities in the production plan.

(iii) The entrepreneur could decide in advance on the scale of an activity which should serve as a buffer against risk. The resource requirements for this particular activity are then simply deducted from the available total and only the remaining activities and recources are taken into consideration for further planning.

* Based on M.Sc. thesis, University of Stellenbosch

2.2 Parametric linear programming

The optimum plan obtained for a given situation rests entirely on the restraints, input-output coefficients and gross margins used for the mathematical formulation of the problem. Although prices and other data can be estimated in advance and with considerable care, the entrepreneur always bears in mind that the actual value may deviate one or other way from the calculated value. It is much easier to define a series within which an estimated value will fall and to select from such a series a specific value which would most likely materialise.

In order to incorporate this approach in a linear programming problem and to obviate the limitation of single valued expectations, so called modified simplex methods were developed in the course of time. These methods become important when resource supplies, input-output coefficients or gross margins cannot be defined with precision. The method used to indicate how the solution of a linear programming problem varies if one or more of the coefficients vary, is known as parametric linear programming. By using this method, it is possible to determine how the optimal solution varies as a result of a change in one or more of the following:

(a) Limitations;
(b) the object function; and
(c) input-output coefficients.

3. QUADRATIC PROGRAMMING

The object here is to minimise a quadratic function, subject to linear limitations. By using this technique, the enterprise combination in respect of which the variance in income for a specified income is a minimum, can be obtained.

It is possible to indicate as follows the variance in total income as a quadratic function of the variances and covariances of all enterprises taken into consideration:

$$V(I) = \sum_i \sum_j \sigma ij \quad Xi \quad Xj$$

In the above comparison X indicates an enterprise and σ ij the co-variance between Xi and Xj. This function can now be minimised for a specific income, subject to the usual linear restrictions of a linear programming problem. Income can be included with the linear limitations and then be treated parametrically, so that the enterprise combinations which have a minimum variance for a specific income, can be obtained.

In formulating a problem with, for instance two enterprises X_1 and X_2, with incomes W_1 and W_2 and variances σ_1^2 and σ_2^2 respectively, and with a co-variance of σ_{12} between W_1 and W_2 we then have the following:

Minimise $V(I) = \sigma_1^2 \; X_1^2 + \sigma_2^2 \; X_2^2 + \sigma_{12} \; X_1 \; X_2$

subject to $W_1 \; X_1 + W_2 \; W_2 = bl$ (income)

and $X_1 + X_2 \leqq b2$ (any other limitation)

4. THE MONTE CARLO METHOD

Linear programming has two weaknesses which are difficult to overcome, namely:

(a) Only one optimum answer is obtained; and
(b) enterprise levels can be given in fractions.

By using the Monte Carlo method - which in fact can be considered as a simulation of linear programming - these two weaknesses can be overcome. Besides this, the technique lends itself more readily to the incorporation of restraints which are of a more subjective nature. Risk factors can be handled with both the formulation of the problem and the final selection between alternative production plans.

In its most simple form the monte Carlo method includes the random selection of activities as well as the random selection of the levels at which these activities must be exercised. The answers are given in whole numbers. No matrix algebra is used and the general functioning of this technique is easily understood.

A simple example is described in the following pages in order to explain the operation of this technique. The Monte Carlo programme can be used in respect of data as compiled for a linear programme, but in order to make full use of the flexibility of the method, it is advisable to collect additional data for incorporation in the model.

Activities are divided into independent and dependent activities. The latter include inter alia purchase and selling activities, breeding stock and intermediate products, like in the case of linear programming. The level at which a dependent activity is operative, is a function of the level of an independent activity. A dependent activity can therefore not be selected before the independent activity from which it orginates. In Table 1, rows two and three represent the following:

Ci = gross margins of independent activities (i = 1 - - - 5).
Si = selling prices of independent activities (i = 6 - - - 8).
Bi = purchase prices of dependent activities (i = 6 - - - 8).

The restrictions are indicated in rows 4 to 6 and they are set out in precisely the same manner as in the case of linear programming. The dependent activities do not utilise resources directly and for this reason they have no input-output coefficients.

The A_{ij} coefficients in rows 7, 8 and 9 indicate the contributions made by the respective independent activities towards the dependent activities or vice versa. If, for example, activity X5 increases the quantity of activity X8, the coefficient A8,5 indicates the quantity "received" by A8 from X5. The same principle applies to the other coefficients in these rows.

TABLE 1 - Example of composition of data for solution by means of the Monte Carlo method

Row No.		Independent activities					Dependent activities		
1	Activity	X1	X2	X3	X4	X5	X6	X7	X8
2	Gross margin (selling price)	C1	C2	C3	C4	C5	S6	S7	S8
3	Purchase price	-	-	-	-	-	B6	B7	B8
	Restrictions:								
4	Land: Hectares	$\pm$A11	$\pm$A12	$\pm$A13	$\pm$A14	$\pm$A15			
5	Labour: Man hours	$\pm$A21	$\pm$A22	$\pm$A23	$\pm$A24	$\pm$A25			
6	Capital: Rand	$\pm$A31	$\pm$A32	$\pm$A33	$\pm$A34	$\pm$A35			
	Dependent activities:								
7	X6	$\pm$A6,1	$\pm$A6,2	$\pm$A6,3	$\pm$A6,4	$\pm$A6,5	-1		
8	X7	$\pm$A7,1	$\pm$A7,2	$\pm$A7,3	$\pm$A7,4	$\pm$A6,5		-1	
9	A8	$\pm$A8,1	$\pm$A8,2	$\pm$A8,3	$\pm$A8,4	$\pm$A8,5			-1
10	Minima	d1	d2	d3	d4	d5			
11	Maxima	f1	f2	f3	f4	f5			
12	Cumulative frequences	20	40	60	80	100			
13	Number of activities	K $\leqq$ 5							
14	Number of iterations	L							

In row 10 the di (i = 1 - - - 5) indicate the minimum level at which an activity may be incorporated in the production plan. Likewise the fi (i = 1 - - - 5) in row 11 indicate the maximum level at which an activity may be included in the production plan.

A weight is allocated to each activity for selection purposes and these weights are cumulated as shown in row 12 - in total the weights must be equal to 100.

In row 13 it is specified how many activities may be included in the production plan. In this example, K may thus be any round figure less than 6.

In row 14 it is specified how many production plans are to be prepared. L may be any large figure and will usually amount to several thousands. The figure is, inter alia, limited by the capacity of the computer used.

The selection of activities is very simple. Firstly, a figure between 0 and 100 is generated by means of the computer on a random basis. If the figure is 80, for example, X4 is selected. Thereupon a figure is again generated on a random basis, the figure to be between d4 and f4. This figure specifies at what level X4 is to be operated. The resource supplies needed by X4 are calculated and deducted from the total available. The process is now repeated for the remaining activities and resources and is maintained until the maximum number of activities (in this case not more than five) is reached. If there are still resources left, the activities are taken in the order selected and extended until every one is limited by at least one limitation. Hereafter, the levels of the dependent activities are calculated in relation to the independent activities.

In this way an acceptable production plan (not necessarily optimum) has now been prepared, consisting at most of five independent enterprises and a number of dependent activities.

The last step is to calculate the total gross margin, simply by multiplying the levels of the activities with their respective gross margins.

The entire procedure is then repeated from the beginning until the predetermined number of production plans (L in Table 1) is reached. Thereafter the 20 (or any desired number) rendering the highest gross profit are selected and analysed critically.

In the process of selection the cumulative frequencies play a very important role. By altering the weight allocated to an activity, the possibility of that particular activity being selected, is altered correspondingly.

It is clear, therefore, that not only one optimum result is now obtained as is the case with linear programming but a number of sub-optimum plans, and indeed as many as desired.

4.1 Practical application of the Monte Carlo method

The Monte Carlo method as described above, was applied to a farming unit in the Western Cape. Linear programming was used as control technique.

The basic data as is shown in Table 1, was set out for the Monte Carlo method. The ultimate table consisted of 24 independent and 14 dependent activities. Weights were allocated according to the extent of the risks involved and 2 000 iterations were carried out, from which the ten with the highest gross margin, were selected. It was found that the production plan which rendered the highest profit, had a gross margin of 88 per cent of that obtained with linear programming. However, all ten plans rendered a gross margin of 85 per cent or more of that obtained with linear programming.

5. CONCLUDING REMARKS

Any technique employed will yield results in accordance with the quality of the data used. Although quadratic programming is an outstanding technique for risk programming, it is doubtful whether this technique will soon establish itself, for the sole reason that the data required for the application of the technique are difficult to obtain.

Many of the disadvantages of linear programming can, however, be overcome by the Monte Carlo method. The latter does not only provide one optimum plan, but several which are sub-optimum. All results are given in whole figures. Minimum levels can be placed on activities, thus avoiding that activities appear in the production plan at unpractical low levels.

The greatest disadvantage of the Monte Carlo method is probably the fact that it can only be applied with the assistance of large computers. On the other hand, large linear programming problems can be processed with relative small computers.

In South Africa, relatively little use has been made of operations research techniques in agriculture. Linear programming is practically the only technique which is fairly well known and which is now fairly generally utilised by persons concerned with economic planning. The other techniques can, however, also be exploited to advantage, but only if South African agricultural entrepreneurs concentrate on keeping records, which are an essential requirement therefor.