ESTIMATING THE INVESTMENT BEHAVIOR OF FARM FIRMS USING THE CONCEPT OF RATIONAL DISTRIBUTED LAG FUNCTIONS

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INTRODUCTION

This study grew out of the need for a more realistic notion concerning the investment behavior of the individual farm firm. Once a stable investment behavior function is identified, it can be incorporated into dynamic growth models to describe and predict farm firm growth. Considerable effort has been devoted to estimating the appropriate investment behavior function for industrial corporations; however, a search of the literature revealed no estimates of such a function for individual farm firms.

The purpose of this study was to estimate the investment behavior function for individual farm firms using the concept of rational distributed lag functions developed by Jorgenson [8] to approximate the time structure of the investment process. To do this, a class of rational distributed lag functions was imposed on a multiple regression equation to obtain the lag distribution that best describes the time path of the investment response to changes in desired capital.

Theories to explain the investment activities of industrial corporations utilize some form of the fundamental flexible accelerator model developed by Chenery [3] and Koyck [11] as the basic investment behavior model. The model assumes the firm has a desired level of capital stock, K_t^*, and an actual level of capital stock, K_t. Assuming that actual capital is determined by a weighted average of all past levels of desired capital, the adjustment model is defined by equation (1.1).

(1.1)

$$I_t = \sum_{j=0}^{\infty} \beta_j (K_{t-j}^* - K_{t-j-1}^*)$$

where

$$I_t = (K_t - K_{t-1}).$$

If $\beta_1, \beta_2, \ldots, \beta_j, \ldots$ are not all equal to zero, only a part of the adjustment is completed during the current period; consequently, the adjustment process will be distributed over several time periods. Koyck [11], Chenery [3], Grunfeld [5], Nerlove [12, 13], et.al., assumed the form of the distributed lag function to be that of a declining geometric progression (series). However, it has clearly been demonstrated that a monotonically declining lag distribution may not best describe the observed data [7, 9, 10]. In fact, in comparing alternative theories of corporate investment, Jorgenson and Seibert [10] concluded that other forms of the lag distribution better described their data based on minimum residual variance around the regression. By imposing a class of rational distributed lag functions on the data, biases due to misspecification of the lag distribution can be avoided because the lag distribution is not constrained to a particular configuration.

Although at least four definitions of desired capital have been used to estimate the investment behavior function of industrial corporations [10], it was possible to test only two in this study. First, corresponding to the accelerator theory, desired capital was defined as proportional to gross farm income. Secondly, corresponding to the expected profits theory, desired capital was defined as proportional to net farm income.

DISTRIBUTED LAG THEORY

If the parameters of equation (2.1), which defines the time structure of the effect of a change in

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an independent variable X on a dependent variable Y, are unknown, equation (2.1) can be estimated only with a very large number of lagged values of X. Hence, some type of reduction technique is necessary. Jorgenson has developed a technique useful in the estimation of the parameters of any lag distribution.

(2.1)

$$Y_t = \beta\, (P_0 X_t + P_1 X_{t-1} + P_2 X_{t-2} + ... + P_j X_{t-j} + ...)$$

where

$$P_j \geqslant 0 \text{ and } \sum_{j=0}^{\infty} P_j = 1$$

Rational Distributed Lag Functions

Using the general notation of the distributed lag operator, L, where:

$$LX_t \;=\; X_{t-1},$$

$$L^2 X_t \;=\; X_{t-2},$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$L^j X_t \;=\; X_{t-j},$$

equation (2.1) can be expressed as equation (2.2). The part inside the parentheses represents an infinite polynomial in L written in open form.

(2.2)

$$Y_t = \beta\, (P_0 + P_1 L + P_2 L^2 + ... + P_j L^j + ...) X_t.$$

This polynomial, denoted P(L), can be expressed in the closed form as a ratio of two finite polynomials U(L)/V(L), which allows for the estimation of an infinite lag distribution with a finite set of data. Once the parameters of the polynomial in L expressed in the closed form are estimated, the transformation to the open form is possible by the division implied by P(L) = U(L)/V(L).

For example, consider the simple geometric distribution of Koych expressed in equation (2.3), which is a special case of the general class of distributed lag functions proposed by Jorgenson.

(2.3)

$$Y_t = \beta\, \Big[\, \lambda\, X_t + \lambda\,(1\text{-}\lambda)\, X_{t-1} + \lambda(1\text{-}\lambda)^2\, X_{t-2} + ... +$$

$$\lambda(1\text{-}\lambda)^j\, X_{t-j} + ...\,\Big]$$

where

$$0 \leqslant \lambda \leqslant 1.$$

The estimation of the effect of X on Y using the open form of the geometric distribution requires an infinite number of lagged values for X; however, with the use of the lag operator, equation (2.3) can be expressed as equation (2.4).

(2.4)

$$Y_t = \beta\, \Big[\, \lambda + \lambda\,(1\text{-}\lambda)\, L + \lambda\,(1\text{-}\lambda)^2\, L^2 + ... + \lambda(1\text{-}\lambda)^j L^j + ...\, \Big] X_t.$$

The part inside the brackets represents an infinite polynomial in L, which can be expressed in the closed form as U(L)/V(L) = $\lambda / 1 - (1 - \lambda)$ L. Applying the closed form of the geometric distribution to equation (2.4) and dividing through by V(L), the estimating equation becomes equation (2.5) which allows the effect of X on Y to be estimated with only one lagged value of the dependent variable.

(2.5)

$$Y_t = (1 - \lambda)\, Y_{t-1} + \beta\, \lambda X_t.$$

Once the parameter λ is estimated, the division implied by $\lambda/1 - (1 - \lambda)$ L enables the open form, equation (2.3), to be generated.

For the general class of distributed lag functions, the desired weights (P_0, P_1, P_2, ..., P_j, ...) are obtained by a two-step process [2, p. 611]. First, by carrying out the implicit division in P(L) = U(L)/V(L), the following polynomial in the lag operator L is generated:

$$A_0 + A_1 L + A_2 L^2 + ... + A_j L^2 + ...$$

By restriction of equation (2.1), the weights of the desired polynomial must sum to one. Consequently, the second step is to divide each element in the series by $\sum_{j=0}^{\infty} A_j$ to force the fulfillment of this restriction.

Therefore, the desired weights, or coefficients of the polynomial P(L) are:

$$\frac{A_0}{\sum A_j} + \frac{A_1}{\sum A_j} + \frac{A_2}{\sum A_j} + ... = P_0 + P_1 + P_2 + ... = 1.$$

The general class of distributed lag functions can be used to approximate any lag function to any desired degree of accuracy, but there probably will be

little interest in polynomials of orders higher than two or three. The eight functional forms listed in Table 1 will, therefore, be sufficient for most situations and were used in this study. Function 1 is the most general case. By allowing the parameters U_1, U_2, V_1, and V_2 of the most general case to assume zero values, the other seven functions can be estimated.

Constraints on the Parameters of the Lag Functions

For the sequence defined by $P(L)$ to be an acceptable distributed lag function (Nonnegative and convergent), it is sufficient for both sequences defined by $U(L)$ and $V(L)$ to be convergent and nonnegative [6]. This places rather strict constraints on the admissible range of values for the parameters of the lag distribution to be estimated. To aid the discussion, consider the rather simple rational function defined by equation (2.6):[1]

(2.6)

$$P(L) = U(L)/V(L) = 1/1 - V_1 L - V_2 L^2.$$

Table 1.   CLASS OF LAG FUNCTIONS

Function Number	U(L)	V(L)
1	$1 + U_1 L + U_2 L^2$	$1 - V_1 L - V_2 L^2$
2	$1 + U_1 L$	$1 - V_1 L - V_2 L^2$
3	1	$1 - V_1 L - V_2 L^2$
4	$1 + U_1 L + U_2 L^2$	$1 - V_1 L$
5	$1 + U_1 L$	$1 - V_1 L$
6	1	$1 - V_1 L$
7	$1 + U_1 L + U_2 L^2$	1
8	$1 + U_1 L$	1

For $P(L)$ to be an acceptable lag distribution, the roots of the auxiliary difference equation defined by $V(L)$ must be real and positive but less than one. If these roots are denoted by λ_1 and λ_2, it can be shown that V_1 equals $\lambda_1 + \lambda_2$ and V_2 equals $- \lambda_1 \lambda_2$.[2] Because $0 < \lambda_1 < 1$ and $0 < \lambda_2 < 1$, the following conditions must be satisfied for an acceptable lag distribution to be obtained:[3]

1. $0 < V_1 < 2$ 2. $(1 - V_1 - V_2) > 0$

3. $-1 < V_2 \, 1$ 4. $V_1^2 \geqslant - 4V_2$

When the more general lag function used in this study was considered, additional constraints had to be satisfied to obtain an acceptable set of weights for the estimated lag distribution. The division implied by $P(L) = U(L) / V(L)$ in the first step for obtaining the weights of the lag distribution produced the following when the second-ordered polynomial in both the numerator and denominator were used:

$$A_0 = 1 - V_1 - V_2 / 1 + U_1 + U_2,$$

$$A_1 = (U_1 + V_1) A_0,$$

$$A_2 = (U_2 + V_2) A_0 + V_1 A_1,$$

$$A_3 = V_2 A_1 + V_1 A_2,$$

$$\vdots$$

$$P_j = V_2 A_{j-2} + V_1 A_{j-1}.$$

Since the values of the parameters of $U(L)$ influence the weights of the estimated lag distribution only in the initial, first-lagged, and second-lagged periods, three additional constraints must be met for these weights to be positive. They are:

1. $(1 + U_1 + U_2) > 0$,

2. $-U_1 < V_1$, and

3. $-U_2 < V_2 + V_1 (U_1 + V_1)$.

If $U(L)$ is a first-ordered polynomial in L, the value of the parameters of $U(L)$ influence only the weights in the initial and first-lagged periods; hence, constraint 3 can be ignored and constraint 1 must be modified.

An acceptable lag distribution was evaluated in terms of these constraints for the eight functions in Table 1 and for the accelerator and expected profits theories of investment, respectively.

SOURCE OF DATA USED

This study utilized annual data from 180 Tennessee test demonstration farms for the four-year period 1965 through 1968. All of these farmers were required to keep accurate records on the farm businesses with the assistance of special extension

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[1] This discussion draws heavily on the work of Bauer [2] and Griliches [6].

[2] By setting $1 - V_1 L - V_2 L^2$ equal to zero and solving for the roots of this second-ordered polynomial, λ_1 and λ_2 can be obtained; hence $1 - (\lambda_1 + \lambda_2)L + \lambda_1 \lambda_2 L^2$.

[3] To comprehend how stringent these conditions are on the admissible range of values for the estimated parameters of the lag distribution, see Griliches [6].

agents. Records are consistent for all farms, checked for accuracy by component personnel, and systematically analyzed at the end of each calendar year.

Four classifications of farms were included in the study. Farms were classified on the basis of income source. If 50 percent or more of gross farm income was derived from Grade A dairy, manufacturing milk, swine, or beef, the farm was classified into one of these four groups.

THE STRUCTURAL MODEL

The structural model used in this study utilized the generalized flexible accelerator model as the basic model for estimating the time path of the investment response to changes in desired capital. But, in addition, several other variables were believed to have a significant effect on the farmers' willingness to invest. These were nonfarm income, size of the farm firm, age of the farmer, number of dependents of the farmer, and equity of the farmer in the farm business.

A dummy variable was added to the estimating equation for each of the four classifications of farms included in the study to account for the effect of farm classification on the investment expenditures of the cross section of farms.

The structural model is given by equation (2.7):

(2.7)

$$I_{it} = P(L) \beta_0(K_{it}^* - K_{it-1}^*) + \beta_1 Y_{it} + \beta_2 S_{it} + \beta_3 A_{it}$$

$$+ \beta_4 D_{it} + \beta_5 E_{it} + \beta_6 M_{it} + \beta_7 C_{it} + \beta_8 H_{it} + \beta_9 B_{it}$$

where:

K_{it} = actual capital;

K_{it}^* = desired capital defined as:

$K_{it}^* = I_{G_{it}}$ for the accelerator model and

$K_{it}^* = I_{N_{it}}$ for the expected profits model;

I_{it} = $K_{it} - K_{it-1}$;

$I_{G_{it}}$ = gross farm income;

$I_{N_{it}}$ = net farm income;

Y_{it} = nonfarm income of the family farm;

S_{it} = size of the farm business measured in total dollar value of all

inventories including land, buildings, equipment, livestock, feeds, and supplies;

A_{it} = age of the farmer;

D_{it} = number of dependents;

E_{it} = equity of the farmer in the farm business measured in dollars;

M_{it} = 1 if Grade A dairy, 0 otherwise;

C_{it} = 1 if dairy farm producing manufacturing milk, 0 otherwise;

H_{it} = 1 if hog farm, 0 otherwise;

B_{it} = 1 if beef farm, 0 otherwise;

i = 1 ... 180 for 180 Tennessee test demonstration farms;

t = time, for the years 1965 through 1968, and

$P(L)$ = a polynomial in the rational form $U(L)/V(L)$.

To impose the class of lag functions of Table 1 on the adjustment mechanism for converting changes in desired capital into changes in actual capital, all variables in equation (2.7) were multiplied by the appropriate $U(L)$ for each of the functional forms and the variables were lagged according to the operators. The resulting estimating equations were nonlinear in the parameters. For the most general functional form, 15 parameters were estimated using 32 independent variables. Because unique estimates of the structural and lag distribution parameters could not be obtained by ordinary linear estimating techniques, a nonlinear estimating procedure exposited by Clark Edwards [4] was used to get individual estimates of each of the parameters.

THE ESTIMATED DISTRIBUTED LAG FUNCTION

The estimations were completed for the eight functional forms of the lag distribution (Table 1) using the accelerator and expected profits theories of investment, respectively. When evaluated in terms of the constraints on the admissible range of values for the parameters of the lag distribution as previously described, only the following estimated function was acceptable, and it was acceptable only for the accelerator theory of investment:

$$P(L) = \frac{1.0 + 0.2052L}{1.0}.$$

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This function produced a lag distribution with only two weights. Changes in gross sales between the current and preceding period accounted for approximately 83 percent of the total effect of all changes in gross sales on net investment, while changes in gross sales lagged one period accounted for the remaining 17 percent. Although beyond two periods the weights of function 2 and function 5 alternated in sign at very small absolute values, these two functions tend to substantiate a two-period lag distribution. When "longer-than-optimum" distributions are estimated, the weights in the latter periods often go negative at small absolute values as pointed out by Almon [1].

ESTIMATES OF THE STRUCTURAL PARAMETERS

The estimated investment behavior function for the accelerator model is presented as follows with the standard errors of the structural parameters in parentheses:

$$I_t = 1.53P(L) \ (I_{G_t} - I_{G_{t-1}}) - 13.09Y + 0.115S-$$
$$\quad (.62) \qquad\qquad\qquad (6.64) \quad (0.141)$$

$$111.63A - 796.04D + 0.08E - 2,974M + 2,963C +$$
$$(202.74) \ (848.5) \quad (0.12) \quad (12311) \quad (12301)$$

$$13,913B + 2,394H$$
$$(13563) \quad (11830)$$

where:

$$P(L) = \frac{1.0 + 0.2052L}{1.0}, R^2 = .86.$$

The results can be interpreted to mean that as changes in gross sales increase by $1.00, net investment increases by $1.53, given time for this adjustment to occur. Nonfarm income had a negative effect on net investment possibly because nonfarm work was competitive with farm work. The coefficient relating size of the farm business to net investment indicates larger farmers tend to be more growth oriented. Age of the farmers included in this study had a negative effect on net investment; however, better results might be expected with a nonlinear relationship between age and net investment. The number of dependents of the farmer also had a negative effect on investment. The estimates of the four dummy variable coefficients indicate net investment by beef farmers was larger than for other types of farmers, other variables held

constant, followed by dairy farmers producing manufacturing milk, swine farmers, and Grade A dairy farmers.

CONCLUSIONS

Because of a lack of a priori information concerning the nature of the time structure of the investment response to changes in desired capital, a general class of distributed lag functions was imposed on the data to avoid biases due to misspecification of the lag distribution. Any arbitrary distributed lag function can be approximated to any desired degree of accuracy by a member of this class with relatively few lagged variables. However, the results of this study indicate a two-period lag distribution was sufficient for approximating the time required for the investment response to changes in gross farm income for the farmers studied.

If, as the study suggests, a short lag distribution best describes the investment behavior of farmers, the Almon method [1], which uses varying numbers of periods for only the lagged variable in the estimating equation, could be used with relatively few years' data. The use of this method would greatly simplify the estimation procedure because the need for a nonlinear estimating technique would be alleviated along with problems of multicollinearity and serial correlation in the residual terms due to the reduction technique of the general class of distributed lag functions.

When the rational distributed lag method is used to investigate farm firm investment behavior, the restrictions outlined previously on the lag function parameters to be estimated may have to be built into the estimation procedure to obtain acceptable lag distributions. This involves utilizing quadratic programming in which the sum of squares would be minimized subject to a set of linear constraints [13, p. 14].

Caution should be used when estimating the distributed lag function for farm firm investment by the rational distributed lag technique. Although this method allows the data to determine the form of the lag distribution, the researcher should not expect a clear-cut answer about its exact form. If the general shape of the lag distribution is known a priori to the study and sufficiently long periods of data are available for its estimation, the best results might be obtained by imposing a particular lag distribution on the data.

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