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REQUIREMENTS FOR CONTRIBUTIONS

Articles in the field of agricultural economics, suitable for publication in the journal, will be welcomed.

Articles should have a maximum length of 10 folio pages (including tables, graphs, etc.), typed in double spacing. Contributions, in the language preferred by the writer, should be submitted in triplicate to the Editor, c.o. Department of Agricultural Economics and Marketing, Pretoria, and should reach him at least one month prior to date of publication.

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The Determination of the Economic Optimum Level of Fertilizer Application for Silage Production in the Eastern Cape[1]

by

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INTRODUCTION

The rapid increase in the use of fertilizers is one of the causal factors in the tremendous increase in the yield of various crops during the past few years. It is estimated that 25 per cent[2] of the increase in agricultural production in the United States can be attributed to a greater use of fertilizers. The sale of fertilizers through the trade in South Africa from 1952 to 1964 has increased more than threefold. The value of fertilizers now being used by the agricultural industry amounts to approximately R55 million annually. It is, therefore, of the utmost importance to agriculture that this factor must be applied as economically as possible. In this article an endeavour is made to put forward the economic implications of fertilization.

The importance of fertilization is being widely recognised and much experimental work on technical grounds has been carried out. In spite of all the tests already undertaken, few of the layouts and experiments lent themselves to economic analysis. Since 1962 small-scale trials have been laid out which can be interpreted economically.

Although these types of trials in South Africa are still in the early stages, advanced work has already been done in the United States. Many publications have appeared with analyses and findings of economically interpretable fertilizer trials. In this sphere pioneer work has been done especially by E.O. Heady et al.[3] of the Iowa State University.

The aim of this type of research, known as production function analysis, is the determination of the most profitable level(s) of fertilizer application(s) of one or more elements. This optimum point(s) can be determined graphically in the case where one element is used, but in the more complicated cases these points must be calculated mathematically. These points are ascertained where the marginal revenue from the various fertilizer elements are equal and also equal to the marginal cost.

This type of analysis has, however, the following limitations:

1) The writers are indebted to Messrs. S.P. van Wyk and J.A. Groenewald of the Division of Agricultural Economic Research and the University of Natal respectively for helpful suggestions.

2) Robertson, L.S., Johnson, G.L. and Davis, J.F. - Economic and Technical Analysis of Fertilizer Innovations and Resource Use. Iowa State College Press, 1957.

3(a) Heady, E.O. and Dillon, J. - Agricultural Production Functions. Iowa State University Press, Ames, 1964.

(b) Carter, H.O., Dean, W.G. and McCorkle, C.D. Jr. - Economics of Fertilization for Selected California Crops. California Agricultural Experiment Station, Mimeographed Report No. 230, March, 1960.

(i) The subjectiveness of the choice of a function - although different types of functions can be made to fit, no function corresponds precisely to the yield data.

(ii) The uncontrollable nature and instability of the various factors of production which can influence the yield, for example, rainfall, temperature, soil types and other factors. The level of fertilization which under certain conditions is economical can nevertheless prove unprofitable in other cases in the same area. The usefulness of these results depends on the homogeneity of conditions, for example, soil, rainfall, cultivation methods, seed, etc. Nevertheless it is not a specific disadvantage to this type of trial because in all fertilizer trials we have to contend with the same difficulties.

THEORETICAL FRAMEWORK FOR THE DETERMINATION OF OPTIMAL FERTILIZER APPLICATIONS

For the determination of the economic optimum fertilization, the following basic information is necessary:

(a) The physical fertilizer/yield reaction function
(b) The price of the product
(c) The price of the fertilizer element(s).

Fertilizer applications, on an economic basis, can be done in the simpler cases with only a single factor and with more than one factor in the more complicated cases.

1. Single factor analysis

Here the yield is regarded as a function of a single variable factor while other factors, for instance soil, seed, cultivation, rainfall and other fertilization factors are held constant as is shown in equation I.

$$Y = f\,(X_1/X_2 X_3 \ldots X_n, P, Q, R, S) \ldots \quad \text{I}$$

where Y = yield
X_1 = first element
X_2 = second element
X_3 = third element
X_n = n'th element
P = soil conditions
Q = seed type
R = cultivation methods
S = uncontrollable factors (such as rainfall, temperature, plagues, etc.).

In equation I the factors to the left of the bar are variable and those to the right are held constant.

Trials can only be interpreted economically in the rational stage of this function. In the stage of increasing returns of the function it is profitable to produce more on the ground of physical considerations only. For the same reason it is unprofitable to produce in the stage of negative marginal returns. With restricted capital resources, it is possible to produce more in the first stage by holding the quantity of the variable factor (fertilizer) constant and reducing the fixed factor (for example surface area).

In the third stage a greater physical product is obtained by keeping the fixed factor (e.g. land) constant, whilst reducing the application, of the variable factor (fertilizer) - even if the variable factor can be had entirely free of charge.

The point of maximum profit for a single variable factor is given in equation II.

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4) If the point of profit maximization is determined algebraically with the help of a continuous function, the point is defined as follows, where X tends to zero $\frac{dy}{dx} = \frac{Px}{Py}$

$$\frac{\Delta Y}{\Delta X} = \frac{Px}{Py}$$ [4]

or $\Delta YPy = \Delta XPx$ II

where Py = Price of product
Px = Price of factor
ΔY = Physical marginal product
ΔX = Physical marginal application

Equation II shows the point of maximum profit where the value of the marginal yield ($Py \Delta Y$) is equal to the marginal cost ($Px \Delta X$).

2. Two and more factor analysis

In this case the yield is regarded as a function of two or more factors while the other factors are held constant.

Hypothetically a production surface can take on the form where the elements are perfect complements. By substitutes are meant the various combinations of elements which result in the same yield. The principle of substitution is of the greatest importance to the farmer to know which combination of elements is cheapest to attain a desired yield.

Perfect substitution takes place when two elements substitute for each other at a constant rate, for example, a substitution rate between elements A and B of 2:1 means that one unit of element B always replaces two units of element A while the yield remains unchanged. In practice only one element - the cheapest - will be applied.

By perfect complement is meant that the substitution ratio between two elements is zero. In this case a quantity of one element must be combined with a fixed quantity of another element to

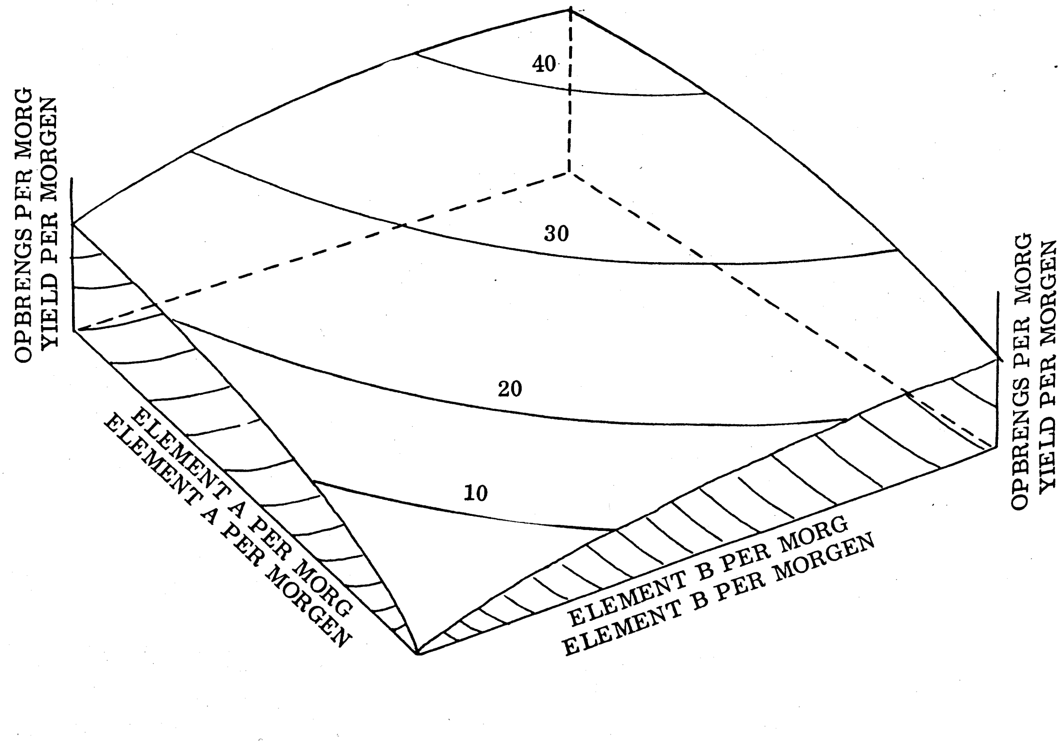

FIG. 1 - Production level with different substitution ratios for two elements.

ensure a given yield. No higher yield is thus attained by increasing only one element.

By far the most common case is where a diminishing yield for one or for more elements is obtained. This case is illustrated graphically in figure 1. The yield contours are all convex to the origin which indicated that one element substitutes for the other at a decreasing ratio to produce the same yield. This means that for every given yield an increasing amount of one element is necessary to replace the other element. In figure 1 it is shown that the lower yields of 10 and 20 can be attained by administering sufficient quantities of only one element or a combination of both elements. At higher yields such as 30 and 40, complementarity appears and minimum quantities of both elements are required. Further the surface of substitution diminishes until a maximum yield is given by a single point. This point can only be attained by a single combination of factors.

The economic optimum condition for the application of two elements to attain a given yield is shown in equation III.

$$\frac{\Delta X_1}{\Delta X_2} = \frac{Px_2}{Px_1}$$ [5]

or $\Delta X_1 \, Px_1 = \Delta X_2 \, Px_2$ III

The combination in equation III indicates the minimum cost point at a given yield. Px_1 and Px_2 are the prices for elements 1 and 2 respectively and the ratio $\frac{\Delta X_1}{\Delta X_2}$ is the marginal rate of substitution between the two elements. Graphically it is also the slope at any given point on the yield contour surface. The equation $\Delta X_1 \, Px_1 = \Delta X_2 \, Px_2$ means that the cost of the added element is equal to the cost of the replaced element. At this point the marginal costs of the two elements are the same. Graphically this point is determined where the yield contour $\frac{\Delta X_1}{\Delta X_2}$ and the price ratio line $\left(\frac{Px_2}{Px_1}\right)$ are tangent to each other. This price ratio line indicates all possible combinations of the two elements which are obtainable with a given cost.

Since it is difficult to carry out the graphical method the point of maximum profit is calculated algebraically.

ECONOMIC FERTILIZER RATES BASED ON DATA FROM VARIOUS FERTILIZER TRIALS [6]

1. Optimal use of a single factor

Japanese millet silage experiments carried out in the Elliot district of the Eastern Cape were used for this analysis. The results were collected during the 1962/63 production season.

The layout of the experiment made allowance for nine levels of application of nitrogen namely 0, 41, 82, 123, 164, 205, 246, 287 and 304 pounds of pure nitrogen per morgen in the form of ammonium sulphate. These applications

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5) Where a continuous yield curve forms the basis for recommendations $\frac{\Delta X_1}{\Delta X_2} = \frac{Px_2}{Px_1}$ where ΔX_1 and ΔX_2 are infinitely small, (ΔX_1 and ΔX_2 tend to zero). Algebraically the equation is thus $\frac{dX_1}{dX_2} = \frac{Px_2}{Px_1}$

6) These trials were laid out by officials of the Department of Agricultural Technical Services of the experimental farm Döhne.

were separately combined with 3 levels of applications of phosphate namely 0, 80 and 160 pounds P_2O_5 per morgen in the form of granulated superphosphate. For each level of phosphate a separate production function was calculated.

(a) Production function for the zero level of phosphate application (Po)

The functional relationship between the yields in the form of tons of silage per morgen and applications of nitrogen (pure nitrogen) per morgen is shown in equation IV.

$$M = 8.84 + 0.186N - 0.00036303N^2 \ldots \text{ IV}$$

M = Estimate of silage in tons per morgen

and N = Levels of nitrogen application in pounds per morgen.

The minus sign before N^2 shows diminishing marginal yields in respect of nitrogen applications.

The economic optimum point is determined by equating the marginal yields of nitrogen $\left(\frac{\Delta M}{\Delta N} \text{ or } \frac{dM}{dN}\right)$ to the inverse price ratio $\left(\frac{Pn}{Pm}\right)$, where Pn = price of pure nitrogen per pound and Pm = price of silage per ton.

Equation V shows the marginal product of nitrogen as calculated from equation IV by means of differentiation.

$$\frac{dM}{dN} = 0.1816 - 0.000726\,N \ldots\ldots\ldots \text{ V}$$

Equation VI shows the price ratio[7] of nitrogen to silage.

$$\frac{Pn}{PM} = 0.0240 \ldots\ldots\ldots\ldots\ldots\ldots \text{ VI}$$

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7) Price of pure nitrogen per pound = 8.452 cents.
Estimated value of silage per ton = 350 cents.

The economic optimum application of nitrogen is now calculated from equation VII.

$$0.1816 - 0.000726N = .0240 \ldots\ldots\ldots \text{ VII}$$

N = 217.1 lb N per morgen

or = 1086 lb ammonium sulphate per morgen.

By substituting this optimum application in equation IV the optimal yield of 31.2 tons silage per morgen is calculated as follows:

$$M = 8.84 + 0.186\,(217.1) - 0.00636303\,(217.1)^2$$

= 31.2 tons silage per morgen.

(b) Production function at the 80 lb P_2O_5 level of phosphate application (P_1)

In equation VIII the production function obtained by the 80 lb P_2O_5 per morgen application is shown.

$$M = 7.83 + 0.2470N - 0.0004748N^2 \ldots \text{ VIII}$$

The maximum profit position is calculated as in the previous example.

$$\frac{dM}{dN} = 0.2470 - 0.0009496N$$

$$0.2470 - 0.0009496N = 0.0240$$

$\therefore$ N = 234.8 lb N per morgen

or = 1173 lb ammonium sulphate per morgen

The optimum yield is calculated by substitution as follows:

$$M = 7.83 + 0.2470\,(234.8) - 0.0004748\,(234.8)^2$$

$\therefore$ M = 39.7 tons silage per morgen.

(c) Production function at the 160 lb P_2O_5 level of phosphate application (P_2).

In equation IX the production function obtained by the 160 lb P_2O_5 per morgen application is shown.

$$M = 10.71 + 0.2731N - 0.0005657N^2 \quad .. \text{IX}$$

The maximum profit position is calculated as in the previous example.

$$\frac{dM}{dN} = 0.2731 - 0.0011314N$$

$$0.2731 - 0.0011314N = 0.0240$$

$\therefore N = 220.1$ lb N per morgen

or $= 1100$ lb ammonium sulphate per morgen

$\therefore M = 10.71 + 0.2731 (220.1) - 0.0005657 (220.1)^2$

$= 43.4$ tons silage per morgen.

Graphical representation. - The production functions Po, P_1 and P_2 (0, 80, 160 lb P_2O_5 respectively) as shown in equations IV, VIII and IX are shown in figure 2.

From figure 2 it is obvious that with higher nitrogen applications the yield of silage increases but at a decreasing rate. After about sixth level ($\pm$246 lb N per morgen) the marginal yield becomes negative and the total yield decreases. The gap between the P_2 and P_1 yield curves is smaller than that between the Po and P_1 curves. This shows that there is a decreasing marginal yield with regard to phosphate.

The maximum profit position is determined where the marginal yield line $(\frac{dM}{dN})$ and the price ratio line $(\frac{Pn}{Pm})$ intersects. In figure 2 the intersections for the different levels of phosphate are shown by dotted lines.

Summary of experimental results. - A summary of the experimental results are presented in table 1.

TABLE 1 - Summary of three maximum profit points at different levels of phosphate

Pounds N per morgen	Pounds P_2O_5 per morgen	Yield per morgen, tons silage
217.1	0 (Po)	31.2
234.8	80 (P_1)	39.7
220.1	160 (P_2)	43.4

In table 2 the relative profitability of the three maximum profit positions is calculated.

According to table 2 the net income for the P_1 and P_2 levels are respectively R15.36 and R21.45 above that of the Po level. The P_2 level of phosphate application is thus the most profitable level of application. If the experimental design had made allowance for higher levels of P_2O_5 the most profitable level of phosphate application would probably have been higher.

2. Optimal use of two variable factors

In this analysis Japanese millet silage trials which were also laid out in the Elliot district of the Eastern Cape were employed. The results were collected during the 1963/64 production season. Yields obtained in the trials were relatively low because of unfavourable weather conditions.

The experimental design[8)] made allowance for nine levels of phosphate

8) Harold, O.C., Dean, G.W. and McCorkle, C.O. - Economics of fertilization for selected California crops, California Agricultural Experiment Station, Mimeographed report No. 230, March 1960, p.70.

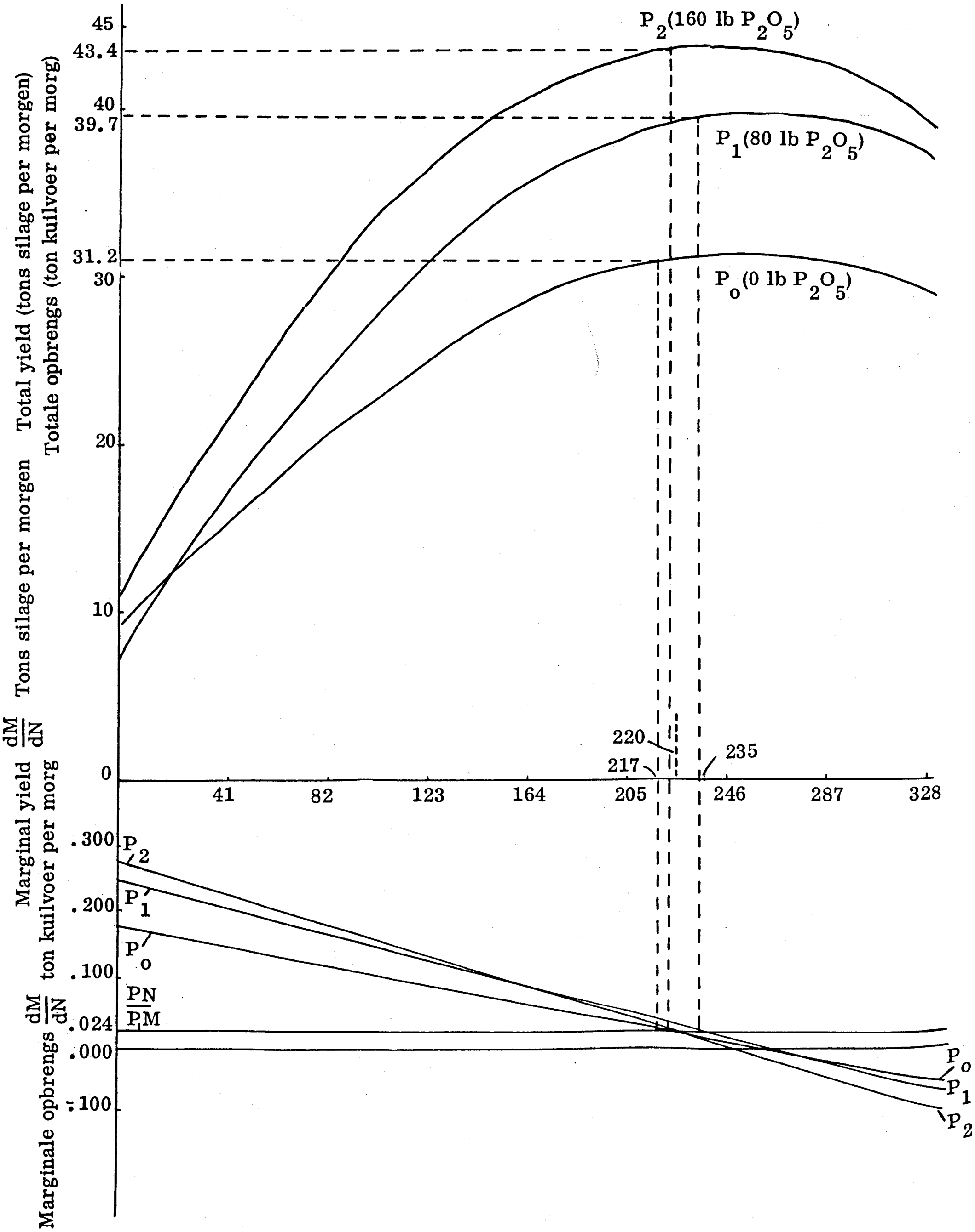

FIG. 2 - Total and marginal yield curves of millet, silage, Elliot, Eastern Cape 1962/63

TABLE 2 - The relative profitability of the three maximum profit positions

		Levels of phosphate application in P_2O_5 per morgen		
		O (Po)	80 (P_1)	160 (P_2)
1. Levels of nitrogen per morgen lb N		217.1	234.8	220.1
2. Cost of nitrogen per morgen	R	18.35	19.84	18.60
3. Cost of phosphate per morgen	R	0	4.40	8.80
4. Total fertilizer costs	R	18.35	24.24	27.40
5. Extra costs of fertilizer per morgen above Po level	R	0	5.89	9.05
6. Extra harvesting costs at R1.00 per ton	R	0	8.50	12.20
7. Total extra costs above Po level	R	0	14.39	21.25
8. Value of yield per morgen	R	109.20	138.95	151.90
9. Total extra income above Po level	R	0	29.75	42.70
10. Additional income minus additional costs above Po level	R	0	15.36	21.45

Explanations -

2 = Pounds nitrogen per morgen x 8.452 cents per pound nitrogen.
3 = Pounds P_2O_5 per morgen x 5.546 cents per pound P_2O_5.
4 = 2 + 3.
5 = P_1 -Po and P_2 -Po.
6 = 39.7 tons - 31.2 tons = 8.5 tons and 43.4 tons - 31.2 = 12.2 tons.
(Although not altogether realistic, the extra harvesting cost is taken as R1.00 per ton of silage).
7 = 5 + 6.
8 = Yield in tons at R3.50 per ton.
9 = R138.95 - R109.20 = R29.75 and R151.90 - R109.20 = R42.70.
10 = 9 - 7.

combined with nine levels of nitrogen and three levels of potash. A total of 342 observations were made. The nitrogen applications had no significant effect and are therefore left out of the following calculations. Phosphate was applied in the form of granulated superphosphate with levels 0, 15, 30, 45, 60, 75, 90, 105 and 120 pounds P per morgen. Potash was applied as Muriate of potash (50% K) with levels 0, 150 and 300 pounds K_2O per morgen (Muriate of potash will henceforth be indicated by the symbol "K").

On this experimental data a quadratic function was fitted such as shown in equation X.

$$M = 14.9 + .01055K - .00000964K^2 + .113P - .000420P^2 - .00000823PK \quad \ldots\ldots\ldots\ldots\ldots X$$

Where M = estimated value of millet silage in tons per morgen
K = pounds K per morgen, and
P = pounds p per morgen.

Equation X shows diminishing yields from potash and phosphate, separately and in fixed combinations.

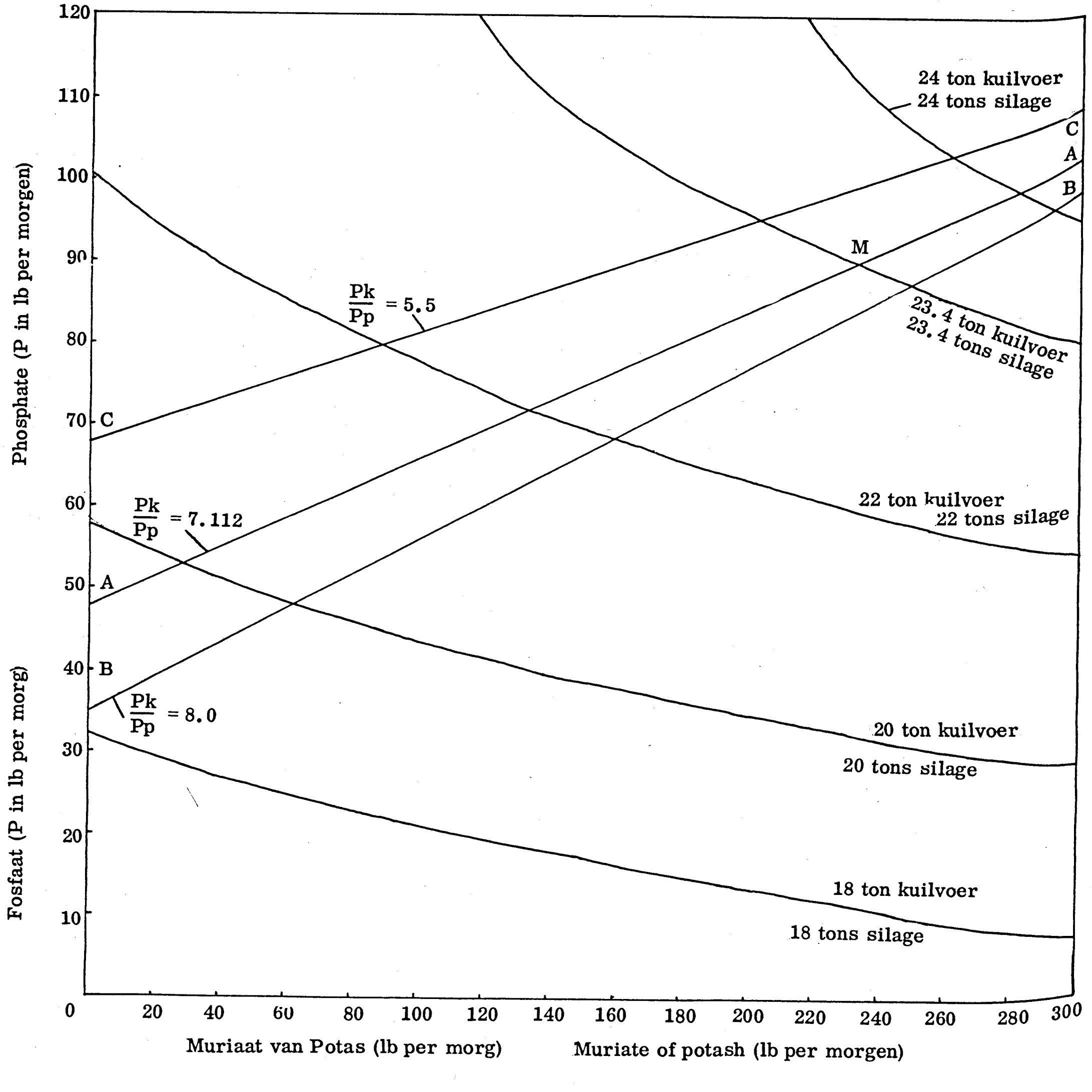

FIG. 3 - Isoquants and extension curves for millet silage, Elliot, Eastern Cape, 1963/64

Equal yield contours (isoquants) can be obtained from equation X by keeping the yield constant and varying the applications of P and K. These yield contours are presented in figure 3 for yields of 18, 20, 22, 23.4 and 24 tons silage per morgen.

The mathematical form of equation X is such that the lines for equal yields (isoquants) can be convex or concave to the origin, in other words $\frac{dK}{dP}$ is negative. The same yield can thus be obtained by changing the combination of P and K; P and K are, therefore, substitutes for each other. In this example a decreasing rate of substitution is found; if the quantity of element K is increased in the combination relative to the element P then even more of element K is required to substitute for one unit of element P to ensure the same yield.

The slope of iso-yield contours for any combination of P and K is given in equation XI.

$$\frac{dK}{dP} = \frac{.113 - .00084P - .00000823K}{.01055 - .00001928K - .00000823P} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{XI}$$

Equation XI is derived from equation X. For example one combination of P and K which yields 22 tons of silage is 71.2 lb P and 137.3 lb K. The applications of P and K are shown to one decimal place so that the marginal rate of substitution can be exactly equal to the inverse price ratio. Fertilization will, however, not take place in such exact quantities. By putting this quantity of P and K in equation XI the $\frac{dK}{dP}$ is equal to 7.112; this means that at this point on the contour surface 1 lb P substitutes for 7.112 lb K. Any other combination of K and P on the 22 ton silage per morgen contour would have given some other marginal rate of substitution. The lowest cost combination of P and K is at the point where the slope of the curve (the marginal rate of substitution between the two elements) is equal to the inverse price ratio of the two elements.

The least-cost point is thus calculated by equating equation XI to $\frac{Pp}{Pk}$ as in equation XII.

$$\frac{.113 - .00084P - .00000823K}{.01055 - .00001928K - .00000823P} = \frac{Pp}{Pk} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{XII}$$

With the price ratio $\frac{Pp}{Pk} = 7.112$, the point where the yield of 22 tons silage is produced by 71 lbs P and 137 lbs K is the least-cost combination. It is possible to deduce the equation from a line which shows the least-cost combination of P and K for every level of silage yield with a given price ratio. The line AA in figure 3 shows the minimum cost point with the price ratio $\frac{Pp}{Pk} = 7.112$. If the price ratio of P:K should change then the least-cost combination would also change in favour of the element which has now become relatively cheaper.

These isoclines are deduced by making $\frac{Pp}{Pk} = \beta$ and then solving for K in equation XIII.

$$K = \frac{.0105\beta - .00000823P\beta - .113 \quad .0084P}{.00001928\beta - .00000823} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{XIII}$$

The corresponding P value is then obtained from the K value (equation XIII) and any desired yield level substituted for K and M in equation X. Isoclines are obtained in this manner with P and K values calculated for a number of yield levels. Such isoclines are calculated, and also illustrated in figure 3, for three price ratios, i.e.

$$\frac{Pp}{Pk} = 5.5, 7.112 \text{ and } 8.0$$

If the original price ratio of 7.112 (AA in figure 3) changes to 8.0 (BB in figure 3) it indicates that element K has become relatively cheaper than element P and the isocline shifts to the right of AA to BB. If the price ratio changes from $\frac{Pp}{Pk}$ = 7.112 to $\frac{Pp}{Pk} = 5.5$ (CC in figure 3) this would again mean that element P becomes relatively cheaper than element K and the isocline thus shifts to the left of AA to CC.

According to figure 3, the isoclines converge (AA, BB and CC) with higher yields because the isoquants at the higher yields become more convex to the origin.

According to figure 3 at the higher yield levels, for example at yield 23.4 tons and 24 tons silage per morgen, a minimum of each element is required. This indicates a degree of complementarity in the production of silage. The law of diminishing returns shows clearly in figure 3 where the distance between the resulting yield contours of 18, 20, 22 and 24 tons silage widens as the yields increase.

Although fertiliser applications in practice cannot be applied in such exact quantities these quantities show the extent of change in fertilizer combinations to ensure minimum costs if yields change.

The position of the minimum cost combination for P and K is presented in table 3.

In table 3 the combinations for P and K for five yield levels are shown as calculated from equation X. The total costs of P and K for points on the chosen yield level (isoquants) are also calculated and are shown in table 3. From table 3 it is possible to obtain the approximate minimum cost combination - that is the point where $\frac{dK}{dP} = \frac{Pp}{Pk}$

The isocline $\frac{Pp}{Pk}$ (line AA in figure 3) never cuts the 18 ton yield contour $\frac{dK}{dP}$ - thus phosphate is the cheapest fertilizer at this yield contour.

According to table 3 the minimum cost combination of a 22 ton silage yield contour lies between P = 70, K = 146 and P = 80, K = 82. The exact combination is P = 74 lb and K = 134 lb. From table 3, then, an approximate minimum cost combination can be obtained for every yield contour where $\frac{dK}{dP} = \frac{Pp}{Pk} = 7.112$.

In the above discussion the minimum cost combination for the two elements at given yield levels was determined. The question now arises, what is the most profitable level of production?

In the case of a single element the maximum profit is attained when the value of the marginal yield is equal to the marginal cost. The same principle applies in the case of two variables. Maximum profit is reached when the value of the additional yield is equal to the additional costs of both elements as combined in the least-cost combination, in other words the combination indicated by the isocline. The maximum profit position is described more specifically for elements P and K for silage production in equation XIV [9].

$$\frac{\frac{\delta M}{\delta K} \cdot Pm}{Pk} = \frac{\frac{\delta M}{\delta P} \cdot Pm}{Pp} = 1 \ldots \text{XIV}$$

The partial differentiation $\frac{\delta M}{\delta P}$ means a change in silage yield which is associated with an infinitely small change in P where all other factors, including K, are held constant; similarly $\left(\frac{\delta M}{\delta K}\right)$

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9) Equation XIV can be extended for any number of variables as follows:

$$\frac{\frac{\delta M}{\delta K_1} \cdot Pm}{Pk_1} = \frac{\frac{\delta M^2}{\delta K_2} \cdot Pm}{Pk_2} = \ldots \frac{\frac{\delta M}{\delta K_n} \cdot Pk}{Pk_n} = 1$$

K_1 = first element

K_2 = second element

K_n = n'th element.

TABLE 3 – Costs and marginal rates of substitution for chosen yield levels with different P and K combinations

P (pounds P)	K (pounds K)	Cost of P at 13.3 c per lb	Cost of K at 1.38 c per lb	Total costs c per lb	$\frac{dK}{dP}$	$\frac{Pp}{Pk}$
			Silage – 18 tons			
10	252	133	348	481	18.270	7.112
15	171	199	236	435	13.885	7.112
20	108	266	149	415	11.479	7.112
25	55	332	76	408	9.861	7.112
30	8	399	11	410	8.643	7.112
			Silage – 20 tons			
35	200	465	276	741	12.793	7.112
40	141	532	194	726	10.429	7.112
45	93	598	128	726	8.874	7.112
50	52	665	72	737	7.724	7.112
55	16	731	22	753	6.811	7.112
			Silage – 22 tons			
60	235	798	324	1122	10.980	7.112
70	146	931	201	1132	7.402	7.112
80	82	1064	113	1177	5.430	7.112
90	36	1197	50	1247	4.267	7.112
100	3	1335	4	1339	2.996	7.112
			Silage – 23.4 tons			
85	264	1130	364	1494	8.641	7.112
98.1	232.2	1184	320	1504	7.112	7.112
105	155	1396	214	1610	3.511	7.112
110	140	1463	193	1656	2.800	7.112
120	117	1596	161	1757	1.538	7.112
			Silage – 24 tons			
100	273	1330	377	1707	5.990	7.112
105	254	1396	350	1746	4.742	7.112
110	239	1463	330	1793	3.699	7.112
115	228	1529	315	1844	2.788	7.112
120	216	1596	298	1894	1.930	7.112

means the change in silage yield which is associated with an infinitely small change in K where all factors, including P, are held constant. Equation X gives the solution of both the optimum combination of elements and the optimum level of production with given product and fertilizer prices.

The maximum profit position for this equation is reached as follows: According to equation XIV

$$\frac{\delta M}{\delta K} = \frac{Pk}{Pm} \text{ and } \frac{\delta M}{\delta P} = \frac{Pp}{Pm} \quad \ldots\ldots \text{ XV}$$

to ensure maximum profit. These unknowns are calculated empirically below.

The marginal yield of potash $\left(\frac{\delta M}{\delta K}\right)$ is deduced from equation X as follows:

$$\frac{\delta M}{\delta K} = .01055 - .00001928K - .00000823P$$

Similarly the marginal yield of P is deduced:

$$\frac{\delta M}{\delta P} = .113 - .000840P - .00000823K$$

The price rations[10)] of silage to potash (K) and phosphate (P) are respectively as follows:

$$\frac{Pk}{Pm} = .00534 \text{ and } \frac{Pp}{Pm} = .038$$

The maximum profit position is thus where:

$$\frac{\delta M}{\delta K} = \frac{Pk}{Pm}$$

$$\therefore .01055 - .00001928K - .00000823P = .00534 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{ XVI}$$

$$\text{and } \frac{\delta M}{\delta P} = \frac{Pp}{Pm}$$

$$\therefore .113 - .000840P - .00000823K = .038 \quad \ldots\ldots\ldots\ldots \text{ XVII}$$

Through the simultaneous solution of equations XVI and XVII the optimum application of P and K respectively is 89.1 lb P per morgen and 232.2 lb K per morgen.

By putting these values into equation X, 23.39 tons - silage per morgen is obtained. In figure 3 the isocline intersects the 23.39 ton isoquant at the point where P = 89 lb and K = 232 lb which agrees with the algebraic calculations.

SUMMARY AND CONCLUSIONS

The importance of fertilization is still increasing at a rapid rate and the value of fertilizers used at present by the agricultural industry amounts to approximately R55 million annually. It is, therefore, of the utmost importance that this factor be applied as economically as possible.

In the foregoing analyses different fertilizer trials have been analysed. These trials were laid out by officials of the Department of Agricultural Technical Services of the experimental farm Döhne, in collaboration with the Division of Agricultural Economic Research.

In this study two types of fertilizer trials have been analysed namely the single factor and double factor. A single factor (N) Japanese millet silage trial was laid out in the Elliot district of the Eastern Cape and the results of this trial were obtained during the 1962/63 production season. In the experimental design nine levels of nitrogen viz. 0, 41, 82, 123, 164, 205, 246, 287 and 304 pounds pure nitrogen in the form of ammonium sulphate were combined separately with three levels of phosphate - viz. 0, 80 and 160 pounds P_2O_5 per morgen in the form of granulated superphosphate. At each of the three levels of phosphate a production function was fitted which illustrated the effect of nitrogen on the yield. From these functions it was evident that with higher nitrogen applications the yield of si-

10) Price of K per pound = 1.38 cents.
Price of P per pound = 13.3 cents.
Price of silage per ton = 350 cents.

lage increased, but at a diminishing rate. After about the sixth level (± 246 pounds N per morgen) the total yield decreased and the marginal yield became negative with higher applications. The gap between the 80 and 160 pound P_2O_5 yield curves was smaller than that between the 0 and 80 pound P_2O_5 curves. (See figure 2). This indicates that with phosphate there was a decreasing marginal yield. The maximum profit point is determined where the marginal cost of nitrogen $Pn \Delta N$ is equal to the marginal yield of silage ($Pm \Delta M$) for each level of phosphate application. More exactly it is determined where $\left(\frac{dM}{dN} = \frac{Pn}{Pm}\right)$. The most profitable levels of production at the 0, 80 and 160 pounds of phosphate (P_2O_5) per morgen were found at 217, 235 and 220 pounds N per morgen respectively. The resulting yield of silage per morgen at these levels of nitrogen and phosphate applications were respectively 31.2, 39.7 and 43.4 tons per morgen. From these results the net income from the 80 and 160 pound phosphate levels were R15.36 and R21.45 more than that of the O level of phosphate. (See table 2). For the experimental design the 160 pound phosphate application combined with the 220 pound nitrogen application provided the most profitable level of production. If the experimental design had made allowance for higher levels of phosphate the most profitable level of phosphate application would probably have been higher.

Another experiment with Japanese millet silage with more than one variable factor (N, P and K) was carried out in the Elliot district of the Eastern Cape and the results were collected during the 1963/64 production season. In this trial yields were relatively low due to unfavourable weather conditions. The experimental design made allowance for nine levels of phosphate combined with nine levels of nitrogen and three levels of potash. There was a total of 342 observations. The nitrogen applications had no significant effect and are therefore left out of the further analysis. Phosphate was applied in the form of granulated superphosphate with levels 0, 15, 30, 45, 60, 75, 90, 105 and 120 pounds P per morgen. Potash applied as muriate of potash (50%K) with levels 0, 150 and 300 pounds K_2O per morgen. To this experimental data a production function was fitted for silage against pounds K_2O and P. Equal yield contours (isoquants) were obtained from this function by keeping the yield constant at given levels, and the levels of application of P and K were varied. Such isoquants were calculated for yields of 18, 20, 22, 23.4 and 24 tons silage per morgen (figure 3). The isoquants were convex to the origin and therefore a decreasing rate of substitution existed between the two elements at a given yield. The principle of diminishing returns was shown clearly by the fact that the distances between the isoquants widen as the yield increased (figure 3).

The lowest cost combination of P and K is at the point where the slope of the isoquant (marginal rate of substitution between the two elements) is equal to the inverse price ratio of the two elements. This combination of the two elements P and K were calculated at each of the isoquants with the prices of K = 1.38 cents per pound and P = 13.3 cents per pound. By linking these points the expansion path of production is found. Although each yield level has a minimum cost combination of P and K only one of the points yields maximum profits. Maximum profit is reached when the additional yield is equal to the additional costs of both elements as combined in the least-cost combination. This position was found at a yield of 23.4 tons silage per morgen and a combination of 89 pounds P and 232 pounds K per morgen.

Variations in the optimum production level will occur from year to year depending on weather conditions. This explains the difference in the optimum production of the two trials which were laid out in consecutive years.

Because of the great variations in soil types and climatic factors more experimental work is necessary to obtain more reliable standards of fertilizer application.

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