STATISTICAL SIGNIFICANCE AND STABILITY OF THE HOG CYCLE

J. Scott Shonkwiler and Thomas H. Spreen

Abstract

Cyclical fluctuations in prices and production have long characterized the United States hog industry. Recent evidence suggests that the length of the hog cycle has changed. In order to determine whether the change in cycle length is statistically significant, the bootstrap technique is employed to derive confidence intervals for point estimates of the hog cycle. Application of the bootstrap technique to time series models is discussed and empirical results are presented. It is concluded that the hog cycle is undergoing rather complicated changes based on cycle lengths that are calculated to be statistically different from zero.

Key words: hog market, time series, bootstrapping, cycle, autoregressive model, structural change.

Cyclical patterns of output or price have been proposed to characterize many agricultural commodity markets. The hypothesis that some systematic periodic component can explain economic phenomena is an intriguing and popular notion, but proper assessment of such patterns is less understood. In particular, the hog cycle is an example of a phenomenon that has been widely discussed and measured. While it is agreed that hog production has evidenced a cyclical pattern, the relative importance of economic and biological factors causing the cyclical properties is subject to debate. Certainly, the biology of reproduction and growth provides a lower limit to the time taken for the expansion phase of the hog cycle. However, the expansion or contraction decisions of producers are made on the basis of expectations of future profitability.

The processes by which expectations are formed are unobserved, but the output adjustments they trigger are not. Gordon and Hines have maintained that rational behavior over time will destroy any systematic pattern observed in market data. Brock has suggested that this hypothesis may be too simplistic. He has written (p.360), *"At first blush one would argue that a stable hog cycle could not exist—because people could predict the ups and downs and behave in such a manner to destroy the regularity. But on the other hand, consider a farmer's problem. He may feel that the dip is coming again next year. Experience has told him so. But if he stays in the hog business and others exit, then he will make a profit. Although it seems very unlikely that a stable hog cycle could persist, it is not so clear that rational behavior over time automatically destroys a stable cycle."* In fact, Grandmont has recently shown, using an abstract model of a competitive economy, that cyclical output can occur even if economic agents possess perfect foresight. However, he also showed that when the structure that generates a stable cycle is changed, it may generate multiple unstable cycles in response. The transition from a stable cycle regime to the occurrence of multiple cycles provides *prima facie* evidence that the underlying structure has undergone some changes.

This study follows the hypothesis of Dixon and Martin that recent structural shifts in the United States hog industry have occurred in the post-World War II period. However, attention is focused on the question of whether these structural changes are in some sense statistically significant. That is, have they engendered meaningful changes in the hog

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cycle? In recent years, a number of authors have suggested that the duration of the hog cycle may be changing (Shonkwiler and Spreen; Spreen and Shonkwiler; Hayenga et al.). These studies, however, fail to provide statistical tests that, in fact, the length of the hog cycle has changed. In this paper, a method known as the bootstrap is outlined and employed to test for statistically significant changes in the length of the hog cycle. The approach taken is based on time series analysis of commercial hog production rather than on an econometric system. The number and duration of the cycles found and their significance lead to the conclusion that hog production in the United States is undergoing substantial changes that signal a meaningful change in market structure.

HOG CYCLE REVISITED

In the post-war period, hog production appeared to follow a fairly regular pattern. Writing in the early 1960's, Dean and Heady, Breimyer, and Harlow discussed systematic movements in annual hog slaughter. Breimyer noted that hog production in the 1950's was dominated by grain-belt farmers who used hogs as a means for marketing corn production and as an alternative enterprise for labor in the winter months. Harlow presented a detailed, diagrammatical argument to substantiate the existence of a 4-year cycle. Jelavich, Talpaz, and Wallis, using various statistical approaches, confirmed the existence of a 4-year cycle of production and prices in the hog industry.

Shonkwiler and Spreen analyzed more recent data and by using time series analysis calculated a 3.4-year cycle in the hog-corn price ratio. Spreen and Shonkwiler analyzed two periods from 1946 to 1962 and from 1964 to 1980. Using spectral analysis, they confirmed a 4-year cycle in the earlier period; however, a 3.2-year cycle was revealed in the latter period. Plain and Williams employed harmonic analysis on weekly data from 1970 to 1979 and found a 2.75-year cycle.

The studies which have analyzed more recent data tend to support the hypothesis that structural changes have occurred in the United States hog industry. Dixon and Martin, in an analysis of quarterly pork production using data from 1964 to 1978, found that a random coefficient model substantially reduced the mean square error compared to a fixed coefficient model. Hayenga et al. reported that seasonality in sow farrowings has diminished along with a shift to fewer and larger hog production units.

Stillman has suggested that the trend toward more concentrated production will continue. He has written that current production technology is capital intensive; therefore, production response is constrained more by the time required to build new facilities than by the reproductive cycle. Because the larger producer operates near capacity, Stillman argued that short-term production adjustments occasioned by moderate changes in costs or prices will take longer to complete. Of course, such reasoning appears contradictory with respect to other contentions that the cycle is shortening.

It is possible that structural changes in the United States hog industry will result in the gradual disappearance of the so-called hog cycle. Further, the process of structural change will result in irregular cycles that may exhibit longer and shorter durations than the historical 4-year cycle. This belief is based on the theory of the dynamic behavior of irregular cycles (May; Day; Grandmont). In order to develop this argument, time series models are estimated and their dynamic properties analyzed.

AUTOREGRESSIVE MODELS

A number of statistical techniques are available to study cyclical phenomena such as the hog market. These techniques include harmonic analysis, spectral analysis, and time series analysis. In this paper, time series methods are used to model the hog market.

Linear stochastic difference equations can be used to describe a time series process. The methods pioneered by Box and Jenkins can be used to identify and estimate the parameters of such equations. Time series methods hypothesize that a stochastic process can be described by an autoregressive-integrated-moving average (ARIMA) model of the form:

$$(1)\quad \theta(B)\ (1-B)^d\ y_t = \eta(B)\ u_t$$

where:

$$\theta(B) = 1-\theta_1 B - \theta_2 B^2 - \dots - \theta_p B^p,$$

$$\eta(B) = 1-\eta_1 B - \eta_2 B^2 - \dots - \eta_q B^q,$$

and $(1-B)^d$ is the differencing factor. Here, B denotes the backshift or lag operator such that $By_t = y_{t-1}$. This process is stationary if all roots of the polynomial equation,

$$1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_p B^p = 0,$$

lie outside the unit circle (Granger and Newbold, p. 25). If all roots of the polynomial equation associated with the moving average component, $\eta(B) = 0$, also lie outside the unit circle, the ARIMA model is invertible; that is, the model may be written as an infinite order moving average process or as an infinite order autoregressive process.

The identification of time series models is an inexact science (Box and Jenkins, p. 173). The procedure outlined by Box and Jenkins is to first assure that the series is stationary. If the raw data are not stationary, stationarity can usually be achieved by differencing the data, that is, applying the operator $1-B$ one or more times. Next, the autocorrelation and partial autocorrelation functions are used to identify the order of the autoregressive and moving average components. Then, the tentatively identified model is estimated. The last step is to use diagnostics to ensure that the fitted u_t have desirable statistical properties. More detail on time series techniques can be found in Box and Jenkins, Granger and Newbold, Nelson, or other books dealing with time series methods.

The invertibility property of stationary time series means that a particular time series can be represented in more than one form. In empirical applications, identification of the order of the autoregressive and moving average components is somewhat subjective. In fact, Wold's Theorem states that any stationary process may be approximated arbitrarily close by both a finite order autoregressive model and a finite order moving average model (Fuller). In this paper, it is instructive to use the autoregressive form:

$$(2)\quad (1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_r B^r)(1-B)^d y_t = u_t.$$

Using data on semi-annual hog production measured in millions of pounds over the period 1946-I to 1985-I, two time periods were selected with the requirement that each period would have 40 observations after accounting for all lags necessary for estimation. This appears to be about the minimum number of observations necessary to adequately fit autoregressive models (Wallis). The series were first differenced to induce stationarity and their partial autocorrelation functions were used to determine the appropriate lagged relationships (Box and Jenkins, p. 197). The parameters corresponding to the form of equation (1) were estimated using least squares, which can be shown to be a maximum likelihood estimator if certain assumptions concerning the initial conditions are made (Greenberg and Webster, p. 123).

The estimated parameters for the two models are presented in Table 1. The models appear to fit the first-differenced data well and are quite successful at generating uncorrelated residuals as evidenced by the small Box-Pierce Q statistics (Greenberg and Webster, p. 126). These statistics account for the number of autoregressive parameters estimated and are distributed asymptotically as chi square. Sufficient lags of the residual series were used such that the test statistic had 14 degrees of freedom. Thus, rejection of the hypothesis that the fitted residuals are white noise with 95 percent confidence is indicated if the calculated value of the statistic exceeds 23.7.

Note that several coefficients in each model have large standard errors relative to their estimated values. Although strategies exist for specifying subset models using some norm,

TABLE 1. ESTIMATED AUTOREGRESSION MODELS OF UNITED STATES PORK PRODUCTION

Item	AR6 Model 1949 II to 1969 I		AR8 Model 1965 II to 1985 I	
	Coefficient	Std. error	Coefficient	Std. error
Constant	102.56	40.8	67.67	57.6
Lag 1	−.0266	.163	.4466	.155
Lag 2	−.0260	.159	−.3617	.156
Lag 3	−.5572	.151	−.2538	.170
Lag 4	−.1844	.156	.2211	.177
Lag 5	−.2107	.160	−.1960	.175
Lag 6	−.3309	.150	−.2295	.172
Lag 7			.4704	.167
Lag 8			−.5057	.167
R^2	.417		.470	
RMSE	224.9		335.4	
Q(14)	9.76		7.26	

this was not attempted so that pretesting could be kept to a minimum. This is an important concern because the analysis of the sampling errors of the hog cycle depends upon the sampling errors of the estimated coefficients. It is well known that these standard errors are misrepresented if pre-testing is performed (Judge et al., Chapter 3).

In order to satisfy the stationarity condition, the complex roots associated with the polynomial in the lag operator must have moduli greater than one (using the form in equation (2); Greenberg and Webster, p. 78). The moduli of the roots of the estimated autoregressive models are presented in the first column of Table 2. These values show that the stationarity condition is satisfied.

The dynamic properties of an autoregressive time series model are related to the roots of the polynomial equation,

$$(3)\quad 1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_r B^r = 0.$$

Specifically, let $a \pm bi$ represent a pair of complex conjugate roots to equation (3). The modulus is defined $\lambda = (a^2 + b^2)^{1/2}$. The implied angle is $\theta = \cos^{-1}(a/\lambda)$. The cycle length associated with these roots is $\tau = 2\pi/\theta$ when θ is measured in radians (Chow, Chapter 2).

The cycles associated with the model presented in Table 1 have sampling errors which depend upon the variances of the parameters of the autoregressive models in a highly nonlinear way. Nevertheless, Theil and Boot have shown that asymptotic standard errors of the complex roots of a dynamic econometric system may be determined up to a second order approximation. Their approach could be adopted to a single autoregressive equation by expressing it as a multi-equation first-order system (Chow). Aside from the fact that approximate standard errors are based on large-sample assumptions, their method requires manipulations of complex matrices. Further, additional nonlinear transformations must be applied in order to relate the standard errors of the roots to the standard errors of the corresponding cycles (Neudecker and van de Panne).

This study employs the bootstrap method to derive standard errors of the cycles associated with the parameters of the autoregressive models (Effron; Effron and Gong; Freedman). The parameter of interest, β, can be generated by a set of observed data. Let F be the empirical probability distribution of the data. By putting a probability mass of 1/n on each observation (where n is the number of observations), "pseudo-data" can be generated by randomly sampling the observed data with replacement. The standard error of $\hat{\beta}$ can be bootstrapped by generating a large number of pseudo-data sets, calculating a value of the parameter associated with each set, $\hat{\beta}^{\bullet}$ and computing the standard error of the artificial values of $\hat{\beta}$ generated.

The obvious attraction of the bootstrap procedure is that it may be applied to any statistic. Thus, even though the standard error of a statistic may be impossible to express in closed form, the bootstrap estimate can be readily approximated using a Monte Carlo type approach. In the present case, interest is focused on determining the standard error of a function of the parameters of an autoregressive model. The approach is briefly outlined in the following discussion.

For example, consider a fitted linear model $Y = Xb + e$. Suppose attention is given to determining the sample variation of G(b),

TABLE 2. BOOTSTRAP CALCULATION OF THE STANDARD ERRORS OF THE ESTIMATED FREQUENCIES DERIVED FROM THE AUTOREGRESSIVE MODELS

Modulus	Angle	Cycle length (years)	95 percent confidence interval
AR6 Model (1949 II to 1969 I):			
1.25592	2.79537	1.1239	.995 to 1.253
	(.1636)[a]	(.066)	
1.29636	1.51265	2.0769	1.634 to 2.518
	(.1231)	(.225)	
1.06776	.74119	4.2386	3.313 to 5.164
	(.0826)	(.472)	
AR8 Model (1965 II to 1985 I):			
1.06155	2.73401	1.1491	1.110 to 1.188
	(.0471)	(.020)	
1.04636	1.71060	1.8365	1.713 to 1.960
	(.0584)	(.063)	
1.07874	1.06176	2.9589	2.520 to 3.398
	(.0805)	(.224)	
1.17347	.44377	7.0793	4.114 to 10.04
	(.0949)	(1.513)	

[a] Standard errors appear in parentheses.

where $G(\bullet)$ is some complicated function of the regression parameter vector. The bootstrap approach requires construction of a psuedo-data vector Y^* using the relation $Y^* = Xb + e^*$, where e^* is obtained by sampling e with replacement n times. These pseudo-data are used to estimate,

$$(4)\quad b^* = (X'X)^{-1} X'Y^*.$$

By repeated construction of Y^* and computation of equation (4) some large number of times, R, the variance of G(b) is given by:

$$(5)\ \mathrm{Var}[G(b)] = (R-1)^{-1} \sum_{r=1}^{R} [G(b)_r^*) - \sum_{r=1}^{R} G(b_r^*)/R]^2.$$

There is a requirement that X be nonrandom which cannot be satisfied if X contains lags of the dependent variable. Since the key idea in bootstrapping is to resample the residuals (as opposed to the data matrix) so that standard errors are conditioned by the model's own stochastic structure, stochastic regressors must be treated differently than nonrandom regressors (Freedman and Peters). Specifically, consider the first order autoregressive model,

$$(6)\quad y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t.$$

Residuals may be calculated by:

$$(7)\quad e_t = y_t - \hat{\alpha}_0 - \hat{\alpha}_1 y_{t-1}.$$

To construct a pseudo-data vector Y^* that preserves the stochastic assumptions of the original model, Freedman and Peters suggest generating Y^* recursively. That is, assume y_0 is fixed and use the relation,

$$(8)\quad y_t^* = \hat{\alpha}_0 + \hat{\alpha}_1 y_{t-1}^* + e_t^*,$$

where again e_t^* is drawn from e_t and $y_0^* = y_0$. This approach can be extended to more complicated dynamic models in a straightforward manner.

In operationalizing this approach, two additional items need to be addressed. First, some deflating in the observed residuals occurs from the fitting process. In order to compensate for this reduced variability, the rule of thumb of scaling e by the factor $(n/n-p)^{1/2}$, where p equals the number of autoregressive parameters estimated, is adopted (Freedman and Peters). Secondly, when the autoregressive parameters are estimated using the pseudo-data vector, Y^*, there are no parameter constraints imposed that require their implied roots to possess corresponding angles that are nonzero. In other words, there is a positive probability that a simulated angle equals zero (the case of a real root) and that its associated cycle length would be infinity.

The approach used is as follows. First, 200 Y^* vectors are constructed according to a generalization of equation (8) for each of the autoregressive models, equation (2). In each case the estimated residual vectors are scaled appropriately. Next, 200 autoregressions are estimated for each model and the roots associated with the parameters are calculated using the IMSL program ZRPOLY. The angles, $\hat{\theta}^*$, corresponding to the roots are calculated and empirical standard errors are calculated using the 200 simulated values. The standard error of the cycle length is calculated as:

$$(9)\quad se(\tau) = 2\pi\hat{\theta}^{-2}\, se(\hat{\theta}^*)$$

(Neudecker and van de Panne). In this way, angles of zero do not generate infinite standard errors for the cycle length.

The bootstrap results are presented in Table 2. Particular interest centers on cycles in the neighborhood of 4 years. Note that for the AR6 model from the earlier time period, a cycle of just over 4 years is identified. More importantly, its standard error is rather small; that is, the true cycle length lies between 3 1/3 years and approximately 5 years with 95 percent confidence. This finding is consistent with a number of earlier studies that proposed a 4-year hog cycle.

A completely different picture emerges when attention is focused on the more recent observation period. Here, a highly significant 3-year cycle is found in conjunction with a 7-year cycle. The longer cycle, however, has a large standard error. Nevertheless, the hypothesis that this cycle is greater than 4 years cannot be rejected at the .025 level of significance. Therefore, the results suggest both a lengthening and shortening cycle in more recent years. However, this finding reconciles the econometric studies that found a shorter hog cycle with the arguments by Stillman and Hayenga et al. that the hog cycle is becoming longer.

The empirical findings appear to be contradictory to the extent that no single change has occurred. In terms of the behavior of nonlinear mathematical difference equations,

TABLE 3. ESTIMATED BIAS OF THE AUTOREGRESSIVE MODELS ESTIMATED FROM ARTIFICIAL DATA

Angle	Std. deviation	Bias	$\sqrt{MSE}$
AR6 Model:			
2.79537	.1636	.0190	.1647
1.51265	.1231	.0084	.1234
.74119	.0826	.0070	.0829
AR8 Model:			
2.7304	.0471	.0050	.0474
1.71060	.0584	.0005	.0584
1.06176	.0805	.0046	.0806
.44377	.0949	.0116	.0956

these results are consistent with an unstable system. That is, when even a simple nonlinear difference equation is perturbed in a certain way, the cycles that it has been generating may change radically. The stable cycle is said to bifurcate (May) and may generate cycles that are harmonics of the original cycle. Although the results in Table 2 do not indicate harmonics of integer order, this may be verified in future studies. Note, however, that an 8-year cycle lies well within the 95 percent confidence interval for the estimated 7-year cycle.

Finally, as a means to assess the validity of the calculated standard errors, there must be some evidence that the artificial data created do represent the same structure as the original data. Toward this end, bias may be estimated by comparing the mean values of the angles generated from the simulated models to those calculated from the observed data, Table 3. These results indicate that bias is a negligible source of statistical error in comparison to the variabilities measured. Bias expressed as a percentage of root mean square error never exceeded 12.1 percent.

IMPLICATIONS AND CONCLUSIONS

Several recent studies have reported that the length of the hog cycle has changed. In this paper using bootstrap methods, it has been demonstrated that the dominant 4-year cycle in production which characterized the hog market over the early post World War II period has changed. A shorter cycle of approximately 3 years and a longer 7-year cycle are detected over the 1965 to 1985 period. Detection of two cycles over more recent data appears to reconcile the empirical findings of Shonkwiler and Spreen and Plain and Williams with the observations of Stillman and Hayenga et al. These results also provide empirical evidence that the United States hog industry has undergone a significant structural change.

A drawback of time series analysis is that it is a data-based tool. It is useful to explain the current state of a market, but since it is not based on a theoretical economic model, it is less helpful to explain why the market evolved to that state. In the context of this study, the results lead to the conclusion that the nature of the hog cycle has been altered significantly, but they do not explain why this change has occurred.

A plausible explanation for the emergence of both a 3-year cycle and a 7-year cycle is that in the early post World War II period, United States hog production was dominated by the corn belt farmers. Hog production on these farms was seasonal, with farrowing and weaning occurring in the winter months when surplus labor was available. Expanding production required 2 years which generated a 4-year production cycle. In the more recent observation period, seasonal hog operations have become less important relative to integrated farrow-to-finish operations. With advances in production techniques, the lag from farrowing to rebreeding retained sows is eight to nine months. Thus, approximately 1.5 years is required to expand production which generates a 3-year cycle and is more in line with the biological cycle. The 7-year cycle may correspond to the hypothesis of Stillman and represents expansion and contraction in facilities. A more comprehensive analysis is required to provide a more definitive explanation.

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