PERFORMANCE OF RISK-INCOME MODELS OUTSIDE THE ORIGINAL DATA SET

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Abstract

Selected risk programming solutions (i.e., profit maximization, Target-MOTAD, and MOTAD) are tested in an economic environment outside the data set from which they were developed. Specifically, solutions are derived from either a longer 10-year (1965-74) or shorter 6-year estimation period (1969-74), and then, they are tested for consistent risk-income characteristics over a later 10-year period (1975-84). Risk solutions estimated from earlier periods perform well in the later test period in spite of different economic conditions between time periods. However, favorable performance may be related to the specific example used in this analysis. Further testing for other farm situations is needed before general conclusions can be reached.

Key words: model testing, MOTAD, Target-MOTAD, risk programming.

Considerable attention has been focused on the relationship of expected utility theory and risk income frontiers derived from a given data set by linear or nonlinear risk programming. For example, Target-MOTAD solutions with known distributions of outcomes have been shown to be more theoretically appealing than MOTAD (Tauer; Watts et al.). In this context, LP (i.e., profit-maximizing) solutions can form all or part of the Target-MOTAD frontier. Should economic conditions, and therefore the probability distribution of returns, change between the initial estimation and subsequent application periods, the distribution of future states is not that used for modeling efforts. In such a situation, the relative performance of LP, Target-MOTAD, and MOTAD solutions can not be known *a priori*. It is conceivable that an efficient solution from a sample based analysis may not be part of an *ex post* optimal set when tested in an economic environment of a later test period.

Little, if any, emphasis has been directed toward how well estimated risk-income models actually perform when applied outside the original data set. While some research has examined how closely actual firm plans compare to risk programmed plans (Brink and McCarl; Lin et al.), more research is needed on the performance of risk models when used as normative tools.

The purpose of this article is to examine the performance of selected risk programming solutions that are estimated using data from one time period and tested over a subsequent time period. Specifically, *ex post* risk-income outcomes are examined for three models: (1) LP or profit maximizing, (2) minimum risk Target-MOTAD, and (3) MOTAD solutions. In addition, the effect of different length estimation periods with respect to the performances of each of the above models is considered.

A variety of approaches have been developed to include risk in management decisions. Some approaches use parameters of the probability distributions (e.g., E-V analysis) and others are based upon direct use of samples (e.g., MOTAD and Target-MOTAD). Usually, the probability distribution (either the distribution parameters or the sample to describe the underlying distribution) is developed from historical data. Implicitly, it is assumed that the probability distribution is static from the historical data period to the time of application. Furthermore, the historical data are assumed to describe the un-

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Alabama Agricultural Experiment Station Journal No. 1-861111.

derlying probability sufficiently to be useful in developing management strategies. In the case of parameter based risk analysis, the problem is having sufficient observations to estimate parameters with an acceptable degree of accuracy. Sample based risk analysis has a similar problem in that the sample must be sufficiently large to describe the underlying distribution. If the sample size is small, the optimization process may simply seek "holes" in the distribution. The larger the sample, the closer the approximation to a continuous distribution and the better the performance in representing the underlying distribution.

The question of what constitutes an adequate sample size of income observations to sufficiently describe the underlying distribution in risk programming analysis is not totally resolved. However, there is evidence that samples of 50 or more observations may be necessary (Jones). In practice, attaining samples of 20 or more observations from historical times series data is often difficult, as was the case in this analysis.

MODEL SETTING

A representative 480-acre irrigated farm in Wyoming's Big Horn Basin is used as the example for analysis. Enterprise alternatives include sugar beets, dry beans, malt barley, corn, and silage. An upper bound of 300 acres is imposed on each crop for rotation purposes. Labor is provided by the owner-operator and two full-time employees. Seasonal labor requirements by crop are developed from coefficients in Agee.

Annual per acre gross margins (i.e., gross returns over variable costs) are developed for the named crops over a 20-year period (1965-84) in the following manner.[1] First, nominal gross returns are developed from Big Horn County crop yields and seasonal prices (Wyoming Agricultural Statistics).[2] Second, nominal gross returns are converted to a real 1984 dollar basis using the implicit price deflator for GNP. Third, gross margins by crop are developed on a real 1984 dollar basis by subtracting 1984 based variable costs (Agee) from real 1984 dollar gross returns. The decomposition of gross margin relation-

TABLE 1. MEAN NET RETURNS, ASSOCIATED VARIABILITY (STANDARD DEVIATION AND COEFFICIENT OF VARIATION), AND CORRELATION COEFFICIENTS FOR ALTERNATIVE CROPS, SELECTED TIME PERIODS (1975-84, 1965-74, AND 1969-74), WYOMING BIG HORN BASIN

Time period and measure	Crop: Sugar beets	Dry beans	Malt barley	Corn	Silage
Ten-year test (1975-84):					
Mean net returns ($/acre)	448	174	178	128	135
Standard dev. ($/acre)	177	163	24	43	46
Coef. of var.	.395	.937	.135	.336	.340
	Correlation coefficients				
Sugar beets	1.0000	.5829	.6044	.6684	.3660
Dry beans	—	1.0000	.2197	.5734	.2340
Malt barley	—	—	1.0000	.6232	.3905
Corn	—	—	—	1.0000	.6857
Silage	—	—	—	—	1.0000
Ten-year estimation (1965-74):					
Mean net returns ($/acre)	398	166	88	92	67
Standard dev. ($/acre)	429	239	64	91	74
Coef. of var.	1.078	1.439	.727	.989	1.104
	Correlation coefficients				
Sugar beets	1.0000	.9639	.9709	.9080	.9264
Dry beans	—	1.0000	.9921	.8977	.8872
Malt barley	—	—	1.0000	.8888	.9031
Corn	—	—	—	1.0000	.9534
Silage	—	—	—	—	1.0000
Six-year estimation (1969-74):					
Mean net returns ($/acre)	550	240	108	130	97
Standard dev. ($/acre)	506	291	77	102	80
Coef. of var.	.920	1.213	.713	.785	.825
	Correlation coefficients				
Sugar beets	1.0000	.9921	.9987	.8864	.9476
Dry beans	—	1.0000	.9925	.9204	.9446
Malt barley	—	—	1.0000	.8986	.9488
Corn	—	—	—	1.0000	.9500
Silage	—	—	—	—	1.0000

[1] For purposes of simplicity and since not all producers in the Big Horn Basin choose to participate in government programs, government supports are not included in calculating corn income, thus resulting in a potential understatement of corn returns to the extent that some producers participate.

[2] It is recognized that county average yields can potentially understate yield variability incurred at the firm level. However, firm levels yields were not available for the study area.

ships into underlying trends of prices, yields, and costs is irrelevant to the optimum solution mix. Therefore, gross margins were deflated without concern for the source of systematic change, whether that source was yields, product price, or costs.

In Table 1, correlation coefficients, mean gross margins, standard deviations, and coefficients of variation are shown for Big Horn Basin crops over three different time periods. These include a 10-year (1975-84) test period and two separate estimation periods. The two estimation periods are a 10-year period (1965-74) and a 6-year period (1969-74). Gross margins between crops are all positively correlated, but to a lesser degree in the recent 10-year test period (1975-84) than in the earlier 10-year and 6-year estimation periods. Similar correlation coefficients and ranking of crops with respect to returns and variability are shown between the 10-year and 6-year estimation periods. During both periods, sugar beets and dry beans show higher returns and variability than other crops. Compared to the two estimation periods, the later 10-year test period exhibits some difference in ranking of crop returns and variability. Most notable is malt barley having higher mean returns than dry beans. In addition, relative variation (CV) is less for all crops during the 1975-84 test period versus earlier periods.

Annual per acre gross margins for each of the described crops are incorporated into a Target-MOTAD model, Table 2. Gross margins for the 1965-74 estimation period (rows 7-16) are used to develop a Target-MOTAD solution. However, gross margins for the 1975-84 test period (rows 18-27) are included only for the purpose of calculating *ex post* annual income from the estimated solution. The general form of the Target-MOTAD model featured in Table 2 is:

Minimize vy^- (sum of negative deviations from target income) such that:

$$
\begin{aligned}
&(1) & Ax &\leq b \\
&(2) & Rx + Iy^- &\geq T \\
&(3) & \bar{r}x &= E \\
&(4) & x, y^- &\geq 0
\end{aligned}
$$

TABLE 2. TARGET-MOTAD MODEL: TEN-YEAR (1965-74) ESTIMATION PERIOD AND TEN-YEAR (1975-84) TEST PERIOD, WYOMING BIG HORN BASIN

		Crop activities (acres)					Negative deviations from target income ($)				
Rows	Units	Sugar beets	Dry beans	Malt barley	Corn	Silage	D74	D73···	D65	Constraint type	RHS
		(1)	(2)	(3)	(4)	(5)	(6)	(7)···	(19)		
Negative deviation	($)						1	1 ···	1		Min
1) Land	(ac)	1	1	1	1	1				LE	480
2) Labor 1	(hr)	2.71	1.61	3.19	2.11	.46				LE	1,183
3) Labor 2	(hr)	1.68	2.28	.41	1.73	2.26				LE	1.183
4) Labor 3	(hr)	1.67	2.63	2.50	1.91	1.97				LE	1,569
5) Labor 4	(hr)	2.44	2.94	.12	.06	1.44				LE	1.183
6) Labor 5	(hr)	6.10			1.43					LE	1.183
7) R-74[a]	($)	1,365	671	230	218	195	1			GE	T[b]
8) R-73	($)	997	545	180	296	198		1		GE	T
9) R-72	($)	321	74	69	77	87				GE	T
10) R-71	($)	268	118	64	64	24				GE	T
11) R-70	($)	139	4	43	53	34				GE	T
12) R-69	($)	208	29	64	70	42				GE	T
13) R-68	($)	194	4	36	25	−5				GE	T
14) R-67	($)	230	84	70	41	18				GE	T
15) R-66	($)	217	7	50	65	74				GE	T
16) R-65	($)	37	119	71	12	6			1	GE	T
17) Mean inc.[c]	($)	$\bar{r}$	$\bar{r}$	$\bar{r}$	$\bar{r}$	$\bar{r}$				EQ	E[d]
18) Y-84[e]	($)	253	26	156	86	118				GE	0
19) Y-83	($)	268	70	158	116	118				GE	0
20) Y-82	($)	461	−65	181	105	88				GE	0
21) Y-81	($)	587	136	198	144	145				GE	0
22) Y-80	($)	858	389	207	191	188				GE	0
23) Y-79	($)	455	347	157	129	153				GE	0
24) Y-78	($)	326	93	179	52	96				GE	0
25) Y-77	($)	464	381	155	122	69				GE	0
26) Y-76	($)	355	80	166	147	213				GE	0
27) Y-75	($)	453	286	223	190	164				GE	0

[a] R-74, R-73, . . ., R-65 indicate enterprise gross margins vectors.
[b] T represents the target income parameter.
[c] $\bar{r}$ represents the mean enterprise gross margins for the period 1975-1984.
[d] E represents mean income parameter.
[e] Y-84, Y-83, . . ., Y-75 indicate observed enterprise gross margins used as accounting rows in the post-optimality analysis.

where:

- v = $1 \times s$ vector in which each element is 1 and where s is the number of years of gross margins;
- y^- = $s \times 1$ vector of annual income deviations below the fixed level of target income;
- A = $m \times n$ matrix of technical coefficients, where m is the number of constraints and n is the number of crop activities;
- x = $n \times 1$ vector of crop activities;
- b = $m \times 1$ vector of resource amounts;
- R = $s \times n$ matrix of annual gross margins for crop activities;
- I = $s \times s$ identity matrix;
- T = $s \times 1$ vector of target incomes;
- $\bar{r}$ = $1 \times n$ vector of mean gross margins for crops; and
- E = mean income for the total farm plan.

Target income in the model is set at $100,000 to derive three separate risk-income solutions in the analyis.[3] First, a LP (profit-maximizing) solution results from parameterizing mean income (Table 2) from zero to the point of infeasibility. Second, a "minimum risk" Target-MOTAD solution is established by parametrically reducing mean income from the LP maximum level to the point where negative deviations (from T=$100,000) are minimum. Third, a corresponding MOTAD solution is derived by setting mean income equal to the established target income amount ($100,000).

ESTIMATION PROCESS

Estimating LP, Target-MOTAD, and MOTAD solutions for purposes of testing their performance outside the original data set can be accomplished by using either an "updated" or a "non-updated" approach. With a non-updated approach, the same activity mix developed from either the 10-year (1965-74) or 6-year (1969-74) estimation period is used as the basis for generating realized annual income over the designated 10-year (1975-84) test period. Alternatively, an updated approach allows the activity mix to be revised annually by adding a later year and deleting the earliest year of the estimation period. To the extent that income variability is influenced by trends over time, it could be hypothesized that updating (or annually revising) activity mixes might provide better results because it takes advantage of new knowledge. Estimation of updated solutions is considered first. Presentation of non-updated solutions is deferred to a later section.

Employing the described Target-MOTAD model, updated LP, Target-MOTAD, and MOTAD solutions are developed over a series of separate 10-year estimation periods, Table 3. The first of the 10 updated solutions sets (LP, Target-MOTAD, and MOTAD) is developed from the 1965-74 estimation period. Resulting mean incomes from the 1965-74 period are $121,042 (LP); $114,953 (Target-MOTAD); and $100,000 (MOTAD). Corresponding aggregate negative deviations from the $100,000 target are $360,103 (LP); $357,918 (Target-MOTAD); and $397,955 (MOTAD) which indicates the MOTAD solution, having lowest income and highest deviations, is dominated in general terms by both the LP and Target-MOTAD solution. Crop mixes associated with LP, Target-MOTAD, and MOTAD solutions, Table 3, are then used as the basis for computing "realized" income ($166,734, $161,817, and $145,699, respectively) from gross margins in the year following the 1965-74 estimation period, 1975.[4] The updating process is repeated such that solutions based on ensuing 10-year estimation periods (1966-75, 1967-76, ..., 1974-83), determine realized income in each succeeding year (1976, 1977, ..., 1984).

As shown in Table 3, optimum LP activity mixes did not change across the 10 estimation periods, suggesting only minor shifts in relative returns over time. Compared to LP, Target-MOTAD solutions exhibit the same acreage of sugar beets (194) but a substitution of more malt barley acreage for fewer acres of dry beans. Although sugar beet returns are quite variable, Table 1, annual returns are sufficiently high so that even low income years generally compare favorably with average returns of other crops. Since Target-MOTAD does not consider high returns

[3] Selecting a specific target (e.g., $100,000) is somewhat arbitrary since it is essentially unique for an individual farm's financial situation. In the context of this analysis, it is considered to represent the minimum amount of income required to meet annual fixed cash obligations including fixed cash costs, family living requirements, and debt servicing.

[4] Realized incomes resulting from gross margins observed in years following the designated 10-year estimation period are not featured in Table 3, but instead are presented in the following section (Table 5) dealing with *ex post* performance of solutions.

Table 3. Optimum Ten-Year LP; Target-MOTAD; and MOTAD Activity Mixes with Associated Ten-Year Mean Income and Aggregate Negative Deviations from Target Income of $100,000, Over Ten Updated Estimation Periods, Wyoming Big Horn Basin

10-year estimation periods	10-year mean income	Σ Neg. dev. from T=$100,000	Crop mix Sugar beets	Dry beans	Malt barley	Corn	Silage
	($)	($)	acres				
1) 1965-74:							
LP	121,042	360,103	194	240	46	0	0
Targ-MOTAD	114,953	357,918	194	161	125	0	0
MOTAD	100,000	397,955	164	83	105	128	0
2) 1966-75:							
LP	133,524	299,086	194	240	46	0	0
Targ-MOTAD	127,357	293,154	194	161	125	0	0
MOTAD	100,000	343,450	155	0	24	166	135
3) 1967-76:							
LP	138,714	249,440	194	240	46	0	0
Targ-MOTAD	132,859	242,598	194	161	125	0	0
MOTAD	100,000	306,872	130	0	85	265	0
4) 1968-77:							
LP	150,493	217,422	194	240	46	0	0
Targ-MOTAD	140,836	209,488	194	161	125	0	0
MOTAD	100,000	270,528	115	0	89	263	13
5) 1969-78:							
LP	156,016	163,851	194	240	46	0	0
Targ-MOTAD	148,911	152,243	194	161	125	0	0
MOTAD	100,000	248,854	108	0	67	219	86
6) 1970-79:							
LP	168,800	114,113	194	240	46	0	0
Targ-MOTAD	160,692	105,112	194	168	118	0	0
MOTAD	100,000	214,721	92	13	59	162	154
7) 1971-80:							
LP	192,610	44,030	194	240	46	0	0
Targ-MOTAD	182,923	37,827	194	168	118	0	0
MOTAD	100,000	180,745	50	61	52	17	300
8) 1972-81:							
LP	199,900	27,247	194	240	46	0	0
Targ-MOTAD	189,254	17,010	194	156	118	0	12
MOTAD	100,000	141,835	44	37	44	55	300
9) 1973-82:							
LP	199,820	28,176	194	240	46	0	0
Targ-MOTAD	192,752	0	194	167	119	0	0
MOTAD	100,000	133,726	51	0	46	145	238
10) 1974-83:							
LP	174,266	52,084	194	240	46	0	0
Targ-MOTAD	170,050	17,035	194	161	125	0	0
MOTAD	100,000	131,401	64	0	122	294	0

as a source of risk, it is not surprising to find no reduction in Target-MOTAD sugar beet acreage.

Compared to LP and Target-MOTAD, MOTAD activity mixes tend to be more diversified with increased acreage of corn and/or silage largely replacing sugar beets and dry beans. Risk in a MOTAD setting is cast in the form of minimizing total deviations (both positive and negative) from mean income, as opposed to minimizing negative deviations from a fixed target (that does not necessarily correspond to mean income). As a result, low-return and less variable MOTAD solutions tend to be more diversified with lower-return and less-variable activities to achieve minimum deviations from mean income.

To test the effect of using a shorter estimation period, a parallel set of updated solutions (LP, Target-MOTAD, and MOTAD) is developed from 6-year estimation periods, Table 4. Activity mixes associated with the initial 6-year estimation period (1969-74) are used to compute realized income in 1975. Similar to using the 10-year estimation periods previously, the updating process is repeated such that solutions based on ensuing 6-year estimation periods (1970-75, 1971-76, ..., 1978-83), determine realized income in each succeeding year (1976, 1977, ..., 1984). The LP activity mixes are identical to the previously described 10-year scenario, with the exception of the tenth estimation period (1978-83). General shifts in the crop mix moving from LP to lower income Target-MOTAD solutions are similar (although not identical) in the 10-year (Table 3) and 6-year (Table 4) cases. Alternatively, updated MOTAD activity mixes are found to vary considerably between the 10-year and 6-year cases.

Performance of Updated Solutions

To evaluate *ex post* risk performance of updated LP, Target-MOTAD, and MOTAD so-

TABLE 4. OPTIMUM SIX-YEAR LP; TARGET-MOTAD; AND MOTAD ACTIVITY MIXES WITH ASSOCIATED SIX-YEAR MEAN INCOME AND AGGREGATE NEGATIVE DEVIATIONS FROM TARGET INCOME OF $100,000, OVER TEN UPDATED ESTIMATION PERIODS, WYOMING BIG HORN BASIN

6-year estimation periods	6-year mean income	Σ Neg. dev. from T=$100,000	Crop mix Sugar beets	Dry beans	Malt barley	Corn	Silage
	($)	($)	acres				
1) 1969-74:							
LP	169,176	153,414	194	240	46	0	0
Targ-MOTAD	158,872	152,243	194	161	125	0	0
MOTAD	100,000	211,946	95	0	105	280	0
2) 1970-75:							
LP	188,684	103,677	194	240	46	0	0
Targ-MOTAD	188,684	103,677	194	240	46	0	0
MOTAD	100,000	172,955	68	0	120	292	0
3) 1971-76:							
LP	199,710	37,859	194	240	46	0	0
Targ-MOTAD	195,565	36,907	194	213	32	0	41
MOTAD	100,000	115,349	56	0	35	129	260
4) 1972-77:							
LP	217,200	21,076	194	240	46	0	0
Targ-MOTAD	183,453	13,669	194	62	0	0	224
MOTAD	100,000	92,740	45	4	29	102	300
5) 1973-78:							
LP	218,950	10,436	194	240	46	0	0
Targ-MOTAD	207,899	0	194	168	118	0	0
MOTAD	100,000	83,892	0	112	124	0	224
6) 1974-79:							
LP	193,405	10,436	194	240	46	0	0
Targ-MOTAD	184,436	0	194	168	118	0	0
MOTAD	100,000	72,482	18	107	111	0	244
7) 1975-80:							
LP	165,477	10,436	194	240	46	0	0
Targ-MOTAD	159,593	0	194	168	118	0	0
MOTAD	100,000	55,451	62	61	46	11	300
8) 1976-81:							
LP	163,763	10,436	194	240	46	0	0
Targ-MOTAD	159,386	0	194	168	118	0	0
MOTAD	100,000	51,621	54	87	140	121	78
9) 1977-82:							
LP	161,451	23,910	194	240	46	0	0
Targ-MOTAD	158,999	0	194	167	119	0	0
MOTAD	100,000	58,620	70	49	142	219	0
10) 1978-83:							
LP	144,195	17,035	194	161	125	0	0
Targ-MOTAD	144,195	17,035	194	161	125	0	0
MOTAD	100,000	73,630	95	0	105	280	0

lutions, actual incomes realized over the 1975-84 test period (from solutions presented in tables 3 and 4) are summarized in Table 5. For the 10-year estimation period, the MOTAD solution ($96,571 mean income) is less variable in terms of standard deviation and CV ($34,180; .360)than either the LP ($65,749; .480) or Target-MOTAD solutions ($55,867; .407). If risk is considered in the context of income variability, the MOTAD solution would be least risky. However, if risk is considered in a "chance or amount of loss" context, the MOTAD solution is shown to be the most risky and is inferior to the LP and Target-MOTAD solutions. Specifically, the MOTAD solution yields much lower mean income than either the LP or Target-MOTAD solution ($96,571 vs. $136,916 and $137,247), in conjunction with greater aggregate negative deviations from the $100,000 target ($155,024 vs. $89,527 and $44,858). In addition, the low-income MOTAD solution missed the $100,000 target as frequently as the LP solution (5 of 10 years), and more frequently than the Target-MOTAD solution (5 of 10 versus 2 of 10 years).

Inferior performance of low-income MOTAD solutions within the designated test period is consistent with the phenomenon of LP and Target-MOTAD solutions dominating MOTAD solutions within designated estimation periods, as noted earlier in Table 3. Compared to LP and Target-MOTAD, low-income MOTAD solutions inherently include a larger share of low-income, less-variable activities to achieve lower variability from mean income.

It is interesting to note that the Target-MOTAD solution in this case shows not only the least amount of risk over the 1975-84 test period, but also results in greater mean income than the LP solution. Target-MOTAD

TABLE 5. ANNUAL NET INCOME (REALIZED DURING THE 1975-84 TEST PERIOD), WITH ASSOCIATED RISK MEASURES (STANDARD DEVIATION; COEFFICIENT OF VARIATION: RATIO OF YEARS THAT REALIZED NET INCOME FELL SHORT OF TARGET INCOME = $100,000; AND AGGREGATE NEGATIVE DEVIATIONS FROM $100,000): "UPDATED" LP, TARGET-MOTAD, AND MOTAD SOLUTIONS, CONSIDERING A LONGER (TEN-YEAR) AND SHORTER (SIX-YEAR) ESTIMATION PERIOD, WYOMING BIG HORN BASIN

	Updated 10-year estimation				Updated 6-year estimation		
	Solution method				Solution method		
Item	(1) LP	(2) Target MOTAD	(3) MOTAD	Item	(4) LP	(5) Target MOTAD	(6) MOTAD
	dollars				dollars		
Estimation period/test year:				Estimation period/test year:			
1) 1965-74/1975	166,734	161,817	145,699	1969-74/1975	166,734	161,817	119,647
2) 1966-75/1976	95,735	102,447	112,204	1970-75/1976	95,735	95,735	86,896
3) 1967-76/1977	188,456	170,816	105,900	1971-76/1977	188,456	178,796	65,070
4) 1968-77/1978	93,829	100,542	68,275	1972-77/1978	93,829	90,497	54,310
5) 1969-78/1979	178,660	163,830	101,096	1973-78/1979	178,660	165,029	95,752
6) 1970-79/1980	269,203	256,145	155,830	1974-79/1980	269,203	256,145	125,802
7) 1971-80/1981	155,630	160,078	93,783	1975-80/1981	155,630	160,078	98,792
8) 1972-81/1982	82,260	101,654	58,151	1976-81/1982	82,260	99,909	64,190
9) 1973-82/1983	76,095	82,441	64,192	1977-82/1983	76,095	82,441	70,005
10) 1974-83/1984	62,554	72,701	60,575	1978-83/1984	72,701	72,701	64,419
Mean income, 1975-84	136,916	137,247	96,571		137,930	136,315	84,488
Standard dev.	65,749	55,867	34,810		64,541	57,950	24,928
Coef. of var. (Pct.)	.480	.407	.360		.468	.425	.295
Years in 10 (1975-84) that Income < $100,000	5/10	2/10	5/10		5/10	5/10	8/10
Σ Neg. dev. from $100,000 ($)	89,527	44,858	155,024		79,380	58,717	200,566

cannot yield higher mean income than LP when estimated from a population of income observations. However, this phenomenon is possible when solutions are tested in a different environment than that used to estimate the solution. It should also be noted that the Target-MOTAD solution does not always dominate the LP solution, since in 4 of 10 years, the LP solution has higher returns.

Observed risk-income relationships among LP, Target-MOTAD, and MOTAD solutions derived from the 10-year estimation periods (Table 5, col. 1-3) are generally consistent with those shown for the 6-year estimation periods (Table 5, col. 4-6). However, comparing performance of related solutions estimated from 10 versus 6 years of data (e.g., 10-year LP versus 6-year LP, etc.) indicates mixed results. The 6-year LP solution performs marginally better than the 10-year LP solution, yielding higher mean income ($137,930 versus $136,916) and fewer aggregate deviations from $100,000 ($79,380 versus $89,527). However, 10-year Target-MOTAD and MOTAD solutions are superior to 6-year Target-MOTAD and MOTAD solutions in terms of: (1) higher mean income ($137,247 versus $136,315; $96,571 versus $84,488); (2) lower aggregate deviations from $100,000 ($44,858 versus $58,717; $155,024 versus $200,566); and (3) years in 10 below $100,000 (2/10 versus 5/10; 5/10 versus 8/10).

Performance of Non-Updated Solutions

The alternative to updating LP, Target-MOTAD, and MOTAD solutions in sequential fashion with revised activity mixes is employing a fixed activity mix based on observations from the initial 1965-74 10-year period, Table 3, and 1969-74 6-year period, Table 4. Net income realized over the 1975-84 test period as a result of non-updated LP, Target-MOTAD, and MOTAD solutions is featured in Table 6 for both the 10-year (1965-74) and 6-year (1969-74) estimation periods.

Similar to updated solutions, Table 5, non-updated Target-MOTAD solutions dominate non-updated MOTAD solutions (for both the 10-year and 6-year estimation periods), having higher mean income and lower aggregate negative deviations. Employing a 6-year (1969-74) versus a 10-year (1965-74) estimation period does not change either the LP mix or the Target-MOTAD activity mix. In both cases, the LP solution yields less mean income than Target-MOTAD ($136,916 versus $137,204) and higher aggregate deviations ($89,527 versus $44,335). However, consistent with the updated scenario above, the MOTAD solution estimated from 10 years of data outperforms the 6-year MOTAD solution, exhibiting higher mean income ($122,970 versus $97,140); fewer years in 10 below the $100,000 target (3/10 versus 7/10); and fewer aggregate negative deviations ($61,239 versus $121,844).

Comparison of updated solutions, Table 5, and corresponding non-updated solutions, Table 6, reveals little, if any, advantage for updating activity mixes in this example. Ten-year mean income and aggregate deviations for updated versus non-updated LP and Target-MOTAD solutions are not markedly different. Indeed, non-updated MOTAD solutions outperform updated MOTAD solutions in terms of higher 10-year mean income and lower aggregate deviations for both the 10-year and 6-year estimation period. It is not entirely clear why updated solutions did not perform better in the context of this example. In part, it may have been due to insufficient serial correlation of relative gross margins. In addition, risk-income relationships among gross margins may not have changed as dramatically in this particular example as might be true in other situations.

A final issue entails comparing "actual" risk-income performance of non-updated activity mixes relative to their "expected" risk-income performance. Table 7 shows non-updated LP, Target-MOTAD, and MOTAD solutions derived from the 1965-74 estimation period (from Table 3), and corresponding "expected" mean incomes and deviations. These mean incomes ($121,042, $114,953, and $100,000) and deviations ($360,103, $357,918, and $397,955) represents levels that could be expected to recur if conditions from the 1965-74 period were repeated over the following 1975-84 test period. In contrast, realized mean incomes ($136,916, $137,204, and $122,970) and deviations ($89,527, $44,335, and $57,564) are those actually observed over the 1975-84 test period as a result of employing non-updated LP, Target-MOTAD, and MOTAD crop mixes.

In this example, mean income actually realized over the 1975-84 test period exceeded that which was expected for all three models. In addition, realized risk was less than that expected. Although actual risk-income outcomes over the 1975-84 test period proved more favorable than expected from the ear-

Table 6. Annual Net Income (Realized during the 1975-84 Test Period), with Associated Risk Measures (Standard Deviation; Coefficient of Variation: Ratio of Years that Realized Net Income Fell Short of Target Income = $100,000; and Aggregate Negative Deviations from $100,000): "Non-Updated" LP, Target-MOTAD, and MOTAD Solutions, Considering a Longer (Ten-Year) and Shorter (Six-Year) Estimation Period, Wyoming Big Horn Basin

	Non-Updated 10-year estimation (1965-74)			Non-Updated 6-year estimation (1969-74)		
	Solution method			Solution method		
Item	(1) LP	(2) Target MOTAD	(3) MOTAD	(4) LP	(5) Target MOTAD	(6) MOTAD
	dollars			dollars		
Test year,						
1975	166,734	161,817	145,699	166,734	161,817	119,647
1976	95,735	102,447	101,080	95,735	102,447	92,314
1977	188,456	170,816	139,501	188,456	170,816	94,513
1978	93,829	100,542	86,569	93,829	100,542	63,304
1979	178,660	163,830	136,320	178,660	163,830	95,829
1980	269,203	254,997	219,029	269,203	254,997	156,731
1981	155,630	160,469	146,698	155,630	160,469	116,870
1982	82,260	101,654	102,616	82,260	101,461	92,190
1983	76,095	82,964	81,178	76,095	82,964	74,523
1984	62,554	72,701	70,014	62,554	72,701	64,483
Mean income, 1975-84	136,916	137,204	122,970	136,916	137,204	97,140
Standard dev.	65,749	55,571	43,999	65,749	55,571	28,205
Coef. of var. (Pct.)	.480	.405	.357	.480	.405	.290
Years in 10 (1975-84) that Income < $100,000	5/10	2/10	3/10	5/10	2/10	7/10
Σ Neg. dev. from $100,000 ($)	89,527	44,335	61,239	89,527	44,335	121,844

lier 1965-74 period, it should be recognized that the reverse could easily occur, given alternative situations. It is also conceivable that alternative activity mixes could exist that are more risk-efficient in the context of the 1975-84 test period, than those derived from the earlier 1965-74 estimation period.

CONCLUSIONS

Risk-income solutions are tested in an economic environment outside the original data set from which they are derived. Linear programming, Target-MOTAD, and MOTAD solutions estimated outside the test period perform well (in the context of mean income and negative deviations within the designated test period) relative to expected levels from earlier estimation periods. Results in this analysis are surprising, given the limited number of sample observations for estimating solutions. Indeed, results from this particular example could be misleading. For example, the inclusion of a dominant activity (sugar beets) may have had an overriding effect on maintaining more consistent solutions over time that might be typical for many other farm situations. It is not unreasonable to expect worse *ex post* performance of risk-programming solutions in other specific settings, and further testing is certainly warranted before general conclusions can be reached.

It is clear that low-income MOTAD solutions are inferior to Target-MOTAD solutions. Although MOTAD solutions have less income variability over the designated test period, they are actually more risky in a "chance or amount of loss" context by missing target income with a greater frequency and by a greater aggregate amount than Target-MOTAD solutions. In fact, compared to low-income MOTAD solutions, LP solutions can not always be categorized as risky. The income generating potential of LP solutions can be advantageous in reducing the risk of annual income falling below a designated target. It should be emphasized that the above conclusions about model dominance hold in general terms (mean net returns and aggregate deviations). Exceptions can exist for individual years in the test period.

A longer estimation period appears to be beneficial in terms of providing solutions having both higher mean income and fewer aggregate negative deviations over the designated test period. Yet, the benefit of using a longer estimation period is not as pronounced as might be expected. In some instances, solutions estimated from a shorter period of time perform no worse (and in some cases better) than solutions derived from a longer time period.

It should be re-emphasized that these results and conclusions are specific for a particular study area and period of time. Results could differ if tested over other regions and/or time periods. In alternative settings, risk-income relationships among crop activities may change between time periods to a greater extent than observed in this particular example. This could lead to worse performance of solutions in later time periods than featured here. In addition, differences in the performance of solutions derived from shorter versus longer estimation periods could be more pronounced than indicated by these results. Finally, it is possible that updated solutions could perform better relative to non-updated solutions if risk-income relationships changed more dramatically over the time horizon.

TABLE 7. COMPARISON OF EXPECTED RISK-INCOME OUTCOMES (1965-74 ESTIMATION PERIOD) VERSUS REALIZED RISK-INCOME OUTCOMES (1975-84 TEST PERIOD) BY SELECTED MODELS, WYOMING BIG HORN BASIN

Model	Mean income	Σ Neg. dev. from T=$100,000	Non-updated crop mix (1965-74) Sugar beets	Dry beans	Malt barley	Corn	Silage
	($)	($)	acres				
Linear Programming (LP):							
Expected (1965-74)[a]	121,042	360,103	194	240	46	0	0
Realized (1975-84)[b]	136,916	89,537	194	240	46	0	0
Target-MOTAD:							
Expected (1965-74)[a]	114,953	357,918	194	161	125	0	0
Realized (1975-84)[b]	137,204	44,335	194	161	125	0	0
MOTAD:							
Expected (1965-74)[a]	100,000	397,995	164	83	105	128	0
Realized (1975-84)[b]	122,970	57,564	164	83	105	128	0

[a] Expected mean income and negative deviations based on activity mixes derived from the 1965-74 estimation period (from Table 3).
[b] Realized mean income and negative deviations based on employing the same solution over the 1975-84 test period (from Table 6).

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