PLANT NUTRIENT DEMAND FUNCTIONS FOR TENNESSEE WITH PRICES OF JOINTLY APPLIED NUTRIENTS

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Abstract

Several studies have estimated plant nutrient demand functions for nitrogen, phosphate, and potash. All included own-price effects but excluded prices of jointly applied nutrients. In this study, nutrient demand functions, which include prices of all three nutrients, are estimated for Tennessee by seemingly unrelated regression. Results suggest that cross-price effects are important in determining plant nutrient demand, at least in the case of Tennessee, and that multicollinearity need not be a hindrance in all cases to including cross-price effects in plant nutrient demand models.

Key words: fertilizer, plant nutrients, inputs, factor demand, elasticities, multicollinearity.

Nitrogen (N), phosphate (P_2O_5), and potash (K_2O) are essential for plant growth and health and are commonly applied in mixture. Several studies have estimated separate demand functions for these major plant nutrients (e.g., Heady and Yeh, 1959; Carman; Roberts and Heady; Gyawu et al.). All included the nutrient's own price but excluded prices of the other two nutrients from the models. While two studies (Roberts and Heady; Gyawu et al.) recognized that prices of jointly applied nutrients are important, such prices were excluded because of multicollinearity.

The purpose of this paper is to present estimates of N, P_2O_5, and K_2O nutrient demand functions for Tennessee which include prices of all three nutrients. The theoretical model is first discussed. Then, the empirical model is specified and estimated. The possibility of multicollinearity and its effects on estimates are addressed. Elasticities are presented and briefly compared with those from other studies.

THEORETICAL MODEL

Demand for an input used in production is derived from demand for the final product. Farmers are assumed to be rational profit maximizers, with a general profit function expressed as:

$$(1)\ \pi_y = P_y f(X_1, X_2, \ldots, X_n) - \sum_{i}^{n} P_i X_i$$

where P_y and P_i are the prices of output and the i-th input, respectively, f is a production function, and the X_is are quantities of inputs. The first order conditions for profit maximization state that each input should be used to the level where the marginal physical product equals the input-output price ratio. Assuming satisfaction of the second-order conditions, the n first-order conditions can be solved simultaneously to obtain input demand functions, with the quantity of input demanded as a function of its own price, other input prices, and the output price as expressed by equation (2).

$$(2)\ X_i = f(P_1, P_2, \ldots, P_n, P_y)\ i = 1,\ldots,n.$$

These theoretical input demand functions are homogeneous of degree zero (Henderson and Quandt, p. 69), suggesting that one price can be used as the numeraire and only relative prices are important.

As the price of an input changes, the demands for all inputs change through the substitution effect and the expansion effect. The

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substitution effect relates to how input use changes along a given isoquant, while the expansion effect is concerned with how inputs adjust when output expands. These two effects work together to produce a negative relationship between the price of an input and its own quantity. If two inputs are complements, the substitution and expansion effects work together to yield a negative cross-price relationship, but they work in opposite directions when inputs are substitutes. For substitutes, the cross-price relationship is positive when the substitution effect dominates and it is negative when the expansion effect dominates (Gisser, pp. 248-51). Thus, two inputs can simultaneously be technical substitutes and economic complements, having a negative cross-price relationship (Doll and Orazem, p. 119). Also, because expansion of output normally requires additional quantities of inputs, a positive relationship between the output price and the demand for an input is expected.

EMPIRICAL MODEL

Annual time series data[1] for the period 1965-84 were used to estimate equations (3)-(5). These equations were specified based on the theory of derived demand and the literature cited previously. Other fertilizer demand studies by Griliches (1958 and 1959), Rausser and Moriak, and Gunjal et al. were considered when specifying equations:

$$(3)\ X_1 = a_1 + b_1P_1 + c_1P_4 + d_1Y + g_1Z + e_1,$$

$$(4)\ X_2 = a_2 + b_2P_2 + c_2P_5 + d_2Y + e_2, \text{ and}$$

$$(5)\ X_3 = a_3 + b_3P_3 + c_3P_6 + d_3Y + e_3;$$

where X_1, X_2, and X_3 are thousands of pounds of N, P_20_5, and K_2O used in Tennessee, respectively; P_1, P_2, and P_3 are current-period ammonium nitrate, concentrated superphosphate, and muriate of potash prices paid by farmers in Tennessee ($/ton); P_4 is a weighted average of P_2 and P_3 , with quantities of P_2O_5 and K_2O as weights; P_5 is a weighted average of P_1 and P_3, with quantities of N and K_2O as weights; P_6 is a weighted average of P_1 and P_2, with quantities of N and P_2O_5 as weights; Y is thousands of acres of cropland harvested in Tennessee; Z is the ratio of soybean harvested acreage to other harvested acreage in Tennessee; e_i is a random error; and a_i, b_i, c_i, d_i, and g_i are parameters to be estimated. All prices are divided by the index of prices received for crops in Tennessee (1977=1.0), lagged one period. For convenience, time subscripts are suppressed.

The above specification is similar to that of Heady and Yeh (1959) in that total use is estimated for each nutrient, rather than per acre use, with crop acreage (Y) appearing on the right-hand side. It differs from their specification because an index of prices received for crops, lagged one period, is used as the numeraire and replaces lagged cash receipts from farming, which Heady and Yeh used as a proxy for the expected output price. Prices of ammonium nitrate, concentrated superphosphate, and muriate of potash are used as proxies for nutrient prices because they represent the dominant forms of direct nutrient application in Tennessee (Tennessee Valley Authority). Weighted averages of other nutrient prices, P_4, P_5, and P_6, reduce the likelihood of multicollinearity and still provide estimates of cross-price effects. The variable Z captures the effects on the demand for N resulting from the substitution of soybeans acreage for acreage of other crops. It enters equation (3) because soybeans are legumes requiring little N relative to other major crops and because soybean acreage has increased from 24 percent of total Tennessee harvested acreage in 1965-67 to 38 percent in 1982-84 (U. S. Department of Agriculture, 1965-83; Tennessee Valley Authority). Changes in soybean acreage are not expected to affect the demand for P_20_5 and K_2O differently from changes in other crop acreage. Signs of the coefficients for the P_is and Z are expected to be negative, while those for Y are anticipated to be positive.[2]

[1] Data were obtained from *Fertilizer Summary Data* (Tennessee Valley Authority), *Agricultural Statistics* (U. S. Department of Agriculture, 1965-83), *Agricultural Prices, Annual Summary* (U. S. Department of Agriculture, 1964-84), and *Tennessee Agricultural Statistics* (Tennessee Department of Agriculture). Nutrient price data are for April 15 of each year until 1976, after which they are for May 15. Also, beginning with 1977, nutrient prices are averages over the East South Central Region which includes Tennessee, Kentucky, Alabama, and Mississippi.

[2] Although N, P_20_5 and K_2O have been shown to be technical substitutes under certain conditions (Pesek and Heady), they are expected to be economic complements (Doll and Orazem, pp. 118-20) because they are commonly applied in mixture, which is expected to result in a dominant expansion effect.

ESTIMATION AND RESULTS

Under the assumption of perfect competition, the individual farmer is a price taker and the quantity of a nutrient used by an individual farmer does not influence its price. This is not likely to be true when the decisions of all farmers are taken together. Griliches (1958) indicates simultaneity may exist at the national level. However, he suggests that fertilizer prices are "administered" and fairly unresponsive to changes in quantity in the short run. Hence, they can be regarded as predetermined and the simultaneous relationship between prices and quantities may be ignored. Rausser and Moriak also assume prices are predetermined, following the reasoning of Griliches. This reasoning is even more compelling at a more disaggregated level for an individual state such as Tennessee.

With prices predetermined, ordinary least squares would appear to be an appropriate estimation method. However, as indicated by Roberts and Heady (p. 269), error terms are likely to be correlated across nutrient demand equations and seemingly unrelated regression may provide more efficient parameter estimates. The ordinary least squares residuals from equations (3)-(5) were found to be significantly correlated at the 1 percent level (Johnston, pp. 41-42) indicating cross-equation correlation of error terms.

Results of estimating equations (3)-(5) by seemingly unrelated regression (White) are presented in Table 1. All coefficients have the expected signs and all, except the coefficients for the own price and the constant in equation (3), are significant at the 10 percent level or better. In no case is the hypothesis of nonautocorrelation rejected at the 5 percent level of significance. However, the Durbin-Watson statistics of equations (3) and (4) fall in the inconclusive region. Predicted values fit the observed data reasonably well, as suggested by R^2s greater than 0.7.

As indicated earlier, multicollinearity was the major reason for excluding cross-price effects from nutrient demand equations in previous studies. Multicollinearity is a problem if it results in imprecise and unstable estimates which lead to incorrect inferences about population parameters. The coefficients in Table 1 are estimated with enough precision to suggest significance of most coefficients. However, multicollinearity could still affect the standard errors of the coefficients, especially in equation (3).

A two-step procedure suggested by Belsley et al. is used to identify coefficients which are likely to be adversely affected by multicollinearity (Johnston, pp. 249-50). Multicollinearity diagnostics, including eigenvalues, condition indexes, and proportions of variances of estimated coefficients associated with each eigenvalue for equations (3)-(5) are presented in Table 2. The first step is to identify condition indexes which are large, say greater than 20. Large condition indexes indicate that the X'X matrix is close to being singular and that multicollinearity could be a problem. The next step is to identify the coefficients which might be adversely affected. This is done by observing coefficient variance proportions associated with each large condition index. If an eigenvalue with a large condition index has

TABLE 1. RESULTS OF ESTIMATING EQUATIONS (3)-(5) BY SEEMINGLY UNRELATED REGRESSION, TENNESSEE, 1965-84

	Dependent variable (equation number)		
Explanatory variable	X_1(3)	X_2(4)	X_3(5)
P_i[a]	−6.035	−16.697[d]	−15.198[e]
	(−0.611)[b]	(−1.594)	(−1.687)
P_k[c]	−22.425[d]	−37.414[e]	−41.620[f]
	(−1.463)	(−2.414)	(−4.266)
Y	0.030[f]	0.019[f]	0.026[f]
	(6.581)	(6.916)	(11.265)
Z	−40.378[f]		
	(−2.997)		
constant	36.711	85.198[f]	72.418[f]
	(1.620)	(3.911)	(3.934)
DW	2.319	2.426	2.164
R^2[g]	0.748	0.721	0.864

[a] $P_i = P_1$ for X_1, P_2 for X_2, and P_3 for X_3.
[b] Asymptotic t statistics are in parentheses. One-tailed t tests are used to determine significance, except for the constants, in which case two-tailed tests are used.
[c] $p_k = P_4$ for X_1, P_5 for X_2, and P_6 for X_3.
[d] Significant at the 10 percent level.
[e] Significant at the 5 percent level.
[f] Significant at the 1 percent level.
[g] R^2 is only used as a measure of goodness-of-fit. It is obtained by regressing predicted results on observed data.

associated with it two or more coefficients for which large proportions (say greater than 50 percent) of their variances are explained, then multicollinearity could affect those coefficients (Leong, p. 14). In equation (3), three condition indexes are large, but only eigenvalue 5 has two or more variables with large proportions of their variances explained. Linear dependency is indicated among P_4, Y, Z, and the constant. Still, only the constant term is not estimated precisely enough to suggest significance at the 10 percent level or better. Also, it appears that P_1 is not linearly related with the other variables and that its nonsignificant coefficient results from forces other than multicollinearity. Multicollinearity diagnostics for equation (4) suggest that P_2 and P_5 are linearly related. Again, the coefficients of these variables are estimated with enough precision to indicate significance at the 10 percent level or better. Finally, the collinearity between P_3 and P_6 in equation (5), implied by the contents of Table 2, is of little concern given that all coefficients in that equation are significant at the 5 percent level or better. In summary, it appears that multicollinearity has seriously affected only the constant term in equation (3). Standard errors of other coefficients also may have been affected; nevertheless, they are sufficiently small, relative to their estimated coefficients, to suggest significance at the 10 percent level or better.

Elasticities of nutrient demand, evaluated at the means of the data, are reported in Table 3. The quantity of N demanded is least responsive to price changes, possibly because: (1) N is subject to significant loss through leaching, volatilization, and denitrification, requiring application each year to maintain the desired level of N in the soil and (2) crop yields are more responsive to N than to P_2O_5 and K_2O. Therefore, it is relatively more costly in terms of lower yields to reduce or forego application of N. On the other hand, because P_2O_5 and K_2O are relatively immobile in the soil and not subject to excessive loss except through soil erosion and plant use, farmers can vary application rates substantially from year to year as fertilizer and crop prices fluctuate and still maintain crop yields. Hence, own-price elasticities for P_2O_5 and K_2O are larger in absolute value than for N. In the longer run, however, lost nutrients must be replaced to maintain fertility.

Quantities of P_2O_5 and K_2O are most responsive to prices of other nutrients possibly because they have typically been applied in mixture, with fewer alternatives available for direct application.[3] In all cases, cross-price elasticities are larger than own-price elasticities, emphasizing the high degree of dependence among nutrients.

Acreage elasticities of unity would indicate that nutrient demands change at the same rate as acreage, which is the expected result. Miller et al. present a technique for calculating confidence intervals for elasticities obtained from linear functions and evaluated at the means of the data. Ninety-five percent confidence intervals for acreage elasticities given in Table 3 all include 1.0, suggesting that the acreage elasticities are not significantly different from unity.

TABLE 2. MULTICOLLINEARITY DIAGNOSTICS FOR EQUATIONS (3), (4), AND (5), USING DATA SCALED TO UNIT LENGTH, TENNESSEE, 1965-84

Equation number	Eigenvalue	Condition index	Coefficient variance decomposition P_1	P_2	P_3	P_4	P_5	P_6	Y	Z	Constant
3	1 4.8980	1.00	0.00			0.00			0.00	0.00	0.00
	2 0.0875	7.48	0.01			0.01			0.00	0.20	0.00
	3 0.0080	24.79	0.70			0.09			0.13	0.00	0.16
	4 0.0048	32.11	0.19			0.10			0.45	0.24	0.29
	5 0.0022	46.82	0.10			0.81			0.54	0.56	0.55
4	1 3.9790	1.00		0.00			0.00		0.00		0.00
	2 0.0133	17.31		0.03			0.04		0.95		0.02
	3 0.0047	29.12		0.29			0.04		0.01		0.87
	4 0.0026	39.30		0.69			0.92		0.04		0.10
5	1 3.9760	1.00			0.00			0.00	0.00		0.00
	2 0.0145	16.53			0.16			0.01	0.80		0.01
	3 0.0056	26.56			0.65			0.60	0.19		0.03
	4 0.0040	31.68			0.19			0.39	0.01		0.95

[3] Percentages of nutrients applied in mixture over the 1965-84 period average 37.4, 87.6, and 59.1 percent for N, P_2O_5, and K_2O, respectively. The percentage of K_2O applied in mixture has decreased to a level closer to that of N in recent years (Tennessee Valley Authority).

TABLE 3. ESTIMATED ELASTICITIES OF NUTRIENT DEMAND WITH RESPECT TO THE EXPLANATORY VARIABLES, EVALUATED AT THE MEANS OF THE DATA, TENNESSEE, 1965-84

Explanatory variable	Dependent variable (equation number)		
	$X_1(3)$	$X_2(4)$	$X_3(5)$
P_i[a]	−0.08	−0.29	−0.17
P_k[b]	−0.29	−0.51	−0.60
P_{cr}[c]	0.37	0.80	0.77
Y[d]	1.25	0.92	1.10
Z	−0.21	—	—

[a] P_i = P_1 for X_1, P_2 for X_2, and P_3 for X_3.
[b] P_k = P_4 for X_1, P_5 for X_2, and P_6 for X_3.
[c] P_{cr} is the index of crop prices received by farmers in Tennessee, lagged one period (1977 = 1.0).
[d] End points for 95 percent confidence intervals for acreage elasticities are (0.845, 1.660), (0.639, 1.210), and (0.890, 1.307) for X_1, X_2, and X_3, respectively.

Table 4 compares own-price elasticities estimated from previous studies with those estimated from equations (3)-(5). Own-price elasticities fall in or below the lower range of those obtained from other plant nutrient demand studies. This might result from: (1) bias in other studies caused by exclusion of other nutrient prices, (2) the long history of fertilizer use in Tennessee (Heady and Yeh, 1960), (3) different sample periods, and (4) different model specifications and data.

CONCLUSIONS

The objective of this research was to estimate plant nutrient demand functions for Tennessee which included prices of the three major nutrients. This was done, adding to previous research in the area of plant nutrient demand estimation. The results presented in this paper are useful because they suggest that cross-price effects are important in determining nutrient demand, at least in the case of Tennessee, and that multicollinearity need not be a hindrance in all cases to including cross-price effects of other major nutrients in nutrient demand models.

The equations presented could be used for impact analysis, capturing effects that were not possible to obtain from previously estimated nutrient demand equations. For example, a sudden increase in the price of N would not only affect the quantity of N used, but also the quantities of P_2O_5 and K_2O. Such analysis might be appropriately conducted, using the equations estimated for Tennessee, if the assumption of inconsequential simultaneity at the national level made by Griliches (1958) and others were indeed correct. Then, a rise in the price of N would not affect prices of P_2O_5 and K_2O. Otherwise, nutrient price impact analysis using the equations presented in this study might more appropriately be conducted in conjunction with a national model which accounted for nutrient interrelationships, as well as simultaneity between current prices and quantities.

TABLE 4. COMPARISON OF ESTIMATED OWN-PRICE ELASTICITIES FOR N, P_2O_5, AND K_2O FROM PREVIOUS STUDIES WITH THOSE OF THE CURRENT STUDY

Study	Period	Area	Own-price elasticities		
			N	P_2O_5	K_2O
Heady and Yeh[a]	1926-56	United States	−0.45	−0.45	−0.40
Carman[b]	1955-76	11 Western states:			
		Minimum	−0.20	−0.29	−0.21
		Maximum	−1.84	−2.38	−3.27
Roberts and Heady[c]	1952-76	United States:			
		Corn	−1.15	−1.13	−1.30
		Wheat	−0.23	−0.74	−0.24
		Soybeans	−0.20	−0.82	−0.96
Gyawu et al[d]	1960-80	United States	−0.30	−0.09	−0.78
Current study	1965-84	Tennessee	−0.08	−0.29	−0.17

[a] Quantities are total amounts of N, P_2O_5, and K_2O applied to crops. The U. S. fertilizer price index is used as a proxy for the own-price of each plant nutrient.
[b] Quantities for each state are sales of N, P_2O_5, and K_2O per acre of cropland. Ammonia sulphate, superphosphate, and muriate of potash prices are used as proxies for N, P_2O_5, and K_2O prices, respectively. Elasticities presented for each plant nutrient represent the range of 11-state estimates.
[c] Quantities are N, phosphorus, and potassium applied per acre of corn, wheat, and soybeans. Price variables are averages of compound prices converted to elemental prices.
[d] Quantities are total amounts of N, P_2O_5, and K_2O applied directly to crops. Quantities applied in mixture are excluded. Prices of anhydrous ammonia, diammonium phosphate, and muriate of potash are used as proxies for own prices. Quarterly observations are used in contrast to annual data for other studies.

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