USE OF PROBABILISTIC CASH FLOWS IN ANALYZING INVESTMENTS UNDER CONDITIONS OF RISK AND UNCERTAINTY*

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Managers of business firms, large or small, farm or nonfarm, must make investment decisions under conditions of risk and uncertainty. However, in evaluating investments, the assumption of perfect knowledge has often been used to simplify the analysis. For example, an estimate of average annual net returns is frequently discounted into perpetuity to evaluate a real estate investment alternative. Capital budgeting literature suggests a number of approaches to evaluating alternative investments. However, use of concepts such as the payback period, average rate of return, internal rate of return and net present value embodies the assumption of perfect knowledge.

Relatively little has been written regarding the evaluation of investments under conditions of uncertainty. Perhaps this is understandable since introduction of uncertainty transforms net returns from a single value to a probability distribution related to stochastic variables affecting the proposed investment. If those variables can be satisfactorily simulated, an analysis can be conducted in a probabilistic mode. The purpose of this manuscript is to suggest the use of probabilistic cash flows as an approach to analyzing investments under conditions of risk and uncertainty. A stochastic simulation model is constructed to illustrate the methodology. The approach may be used to evaluate farm and nonfarm investment decisions when sufficient information is available regarding sources of variability at the firm level. We illustrate the methodology for a nonfarm investment decision and suggest how it may be applied for agricultural investment decisions.

ALTERNATIVE APPROACHES TO INCORPORATING RISK AND UNCERTAINTY[1]

The concept of expected net returns represents an attempt to recognize the probabilistic nature of annual net returns for an investment under conditions of uncertainty [8]. Expected annual net return is calculated by weighting annual net returns by the probability of their occurrence and by summing over all levels of return. Since businessmen seldom know the probability distribution of net returns flowing from a proposed investment, use of subjective probabilities is required. While use of subjective probabilities represents an improvement over the assumption of perfect knowledge, objective probabilities would be preferred for most investment analyses.

The use of the standard deviation and the coefficient of variation as measures of variability has been suggested by Myers [17] and Rubinstein [23]. The standard deviation is computed as the dispersion about the expected annual net return. These methods again presume the investor knows the probability distribution for net returns with certainty. For purposes of comparing alternative investments, the assumption that net returns are normally distributed is frequently made [8, p. 155]. While expected

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[1] Space does not permit a comprehensive literature review regarding consideration of risk and uncertainty in economics analysis. The interested reader is referred to a recent review by Walker and Nelson [27]. The focus here is on analysis of individual investments. Readers interested in more detail regarding optimal investment strategies and the portfolio selection problem should see Baumol [3], Hirshleifer [11], Markowitz [16] and Tobin [26].

annual net return and standard deviation of return are intuitively annual numbers that change as the investment matures, capital budgeting literature does not describe how to evaluate changes in these values over time with respect to investment decisions.

Expected annual net returns can be adjusted for some *a priori* level of risk by using the risk-adjusted discount rate approach [1, 14, 23]. The subjective level of risk the investor expects can be added to the discount rate (if risk is perceived to be higher than normal) in computing the present value of expected annual net returns. An alternative method of discounting net returns is to use the certainty-equivalent approach [6, 22]. Net returns are adjusted for risk by an equivalence ratio and the product is discounted using a risk-free discount rate. The equivalence ratio is the coefficient relating an investor's indifference between a certain cash flow and a risky one, and theoretically comes from an investor's risk indifference curve. Robichek and Myers argue this method represents an improvement over the risk-adjusted discount approach, although it is not without limitations. The equivalence ratio is theoretically derived from an individual investor's indifference curve for risk and money income. Other studies have shown that individuals' risk preference functions change over time, are nonlinear, and differ for each investment [10, 19].

Monte Carlo simulation techniques offer an alternative method of analyzing investments under conditions of uncertainty. Swirles and Lusztig [25], Sprow [24], Cassidy, et al. [7] and Jones [12] describe the advantages of using those techniques in assessing risk associated with capital expenditures under conditions of uncertainty. Using their approach, objective and subjective probability distributions are specified for the stochastic variables influencing the performance of an investment. Random values for these variables are drawn and manipulated through appropriate accounting procedures to obtain simulated values for the net present value of the proposed investment. By repeating the process many times, a probability distribution for net present value is generated, thus allowing an investor to evaluate risk associated with a particular investment. Risk associated with the investment is reduced to a single value. For example, results may indicate there is a 90 percent chance that the investment will be successful, or that net present value will be positive. Despite the potential advantages, simulation has been used only to a limited extent by business and banking to evaluate investment opportunities [12, 13, 15, 18, 20, 21].

One of the reasons for limited acceptance is aggregation of simulation results into a single measure of risk. Probabilistic cash flow analysis represents a more meaningful approach to the analysis of investment decisions. The general development of a simulation model to generate probabilistic cash flows is discussed below. Then, a brief explanation of the model used in this study and the results obtained are presented.

GENERAL METHODOLOGY

The first step in developing a stochastic model for investment analysis is identification of critical variables that are expected to influence success or failure of the investment. Probability distributions for critical variables thought to be stochastic must be developed. Stochastic variables will differ for alternative situations, but might include rainfall, temperature, wind, level of production, demand for grain storage, price of inputs or finished products, performance of mechanical equipment, availability of transportation facilities, etc. Development of probability distributions is discussed by Jones [12], Poulinquen [20], Sprow [24] and Burt and Finley [5].

The next step is to formally link probability distributions for stochastic variables to known or fixed variables that influence the proposed investment. The mathematical relationships involved need not be complicated. For example, in evaluating possible expansion of a grain elevator, one would link acres of wheat in the area to a probabilistic estimate of yields. The product of acreage and the stochastic yield parameter would generate a distribution of production for the region. An econometric equation relating yield to rainfall might be used to obtain a stochastic value for yield.

The fourth step is to specify accounting relationships associated with the proposed investment. Accounting records are helpful at this point if the analysis is for an existing business considering an expansion. Cash receipts equations must be outlined, as well as cost accounting equations. Net returns, before or after taxes, should be the end result of this step.

The simulation model may then be programmed in a computer language for rapid analysis, or a formal flow sheet may be constructed for hand calculations. Stochastic values for critical variables may be drawn from the specified distribution by use of random number tables or random number generator programs, available at most computer installations. By repeating the process of drawing random values for critical variables and calculating the resulting net returns, the proposed investment can be analyzed under all possible futures. The type of project being analyzed dictates whether calculations are simulated on a daily, weekly, monthly or annual basis.

Probabilistic cash flow analysis comes from the probability distributions of annual net returns. A separate distribution is estimated for each year of the project's expected life, thus allowing one to determine the variance of annual net returns about the mean, by year. Management may discover that even though average annual net returns are about equal for all years, variability of net returns may grow larger (or smaller) throughout the project's life, thus, making the investment more (or less) risky for the prevailing money risk indifference surface of the firm.

The following sections of this paper demonstrate development of a simulation model for a proposed ice making plant and probabilistic cash flow results from the model. The methodology introduced here can be used in agricultural businesses and can be readily added to existing micro and macro simulation models.

MODEL DEVELOPMENT

Management identified several critical variables that influence the proposed ice plant. These variables include production, inventory level, mechanical failures, sales of ice, growth in demand, price of inputs and the price of the firm's output. The manager indicated that production guidelines are based on sales for the same period (month) during the previous year and the levels of actual and desired inventories. Desired inventory levels were specified by the manager as a function of sales for the same period in the previous year. Since the firm has little or no control over input prices, 1975 input costs are inflated by five percent per year and the firm's per-unit profit margins are held constant by adjusting the price of the output.

The critical variables considered to be stochastic are sales of ice, annual growth in sales and mechanical failures. Statistical analysis of the existing firm's sales records indicates seasonal variations in sales were responsive to air temperature. Probability distributions of average temperature for five different periods of the year are estimated from climatological data [9]. A regression equation relating ice sales to temperature is used to obtain a stochastic value for sales.[2] The firm's sales records indicate an annual growth in sales for the past four years of 3.3 percent per year. Growth of demand is assumed a function of population, income and competition by other firms. These variables are considered in developing a probability distribution for annual growth in demand.

With help of the manager, a triangular distribution (minimum .01, mode .03, maximum .05) was specified. A value for annual growth in demand is drawn at random for each iteration and is used to adjust the stochastic demand.[3] The random occurrence of mechanical failures for new ice-making equipment is specified by the manufacturer in his warranty statement. A breakdown time of five percent is considered a reasonable value, and a probability distribution, assuming 0.05 breakdown and 0.95 operating time, is used.

Stochastic variables are linked to fixed variables by way of mathematical equations. Production, if a breakdown does not occur, is computed on a daily basis as a function of expected demand, existing inventory and desired inventory level. Supply equals production plus inventory on hand. Daily sales is a random value coming from random daily temperatures and an equation relating sales and temperature, adjusted for a stochastic annual growth in sales. Inventory at the end of the day is supply minus sales.

The accounting equations in the ice plant model calculate variable production and delivery costs and sales receipts on a daily basis. Annual summaries of costs, receipts and net returns (before taxes) are calculated and recorded for future analysis. The simulation process is repeated for ten years (the expected economic life of the ice plant), at which time the stream of simulated net returns over the period is discounted at a rate of 10 percent to obtain the net present value. The results are replicated 100 times to generate the probabilistic cash flows used in the analysis.

RESULTS OF THE SIMULATION ANALYSIS

The simulation analysis provides data to estimate probability distribution of net present value for the

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[2]The random sample of one hundred daily high temperatures for five subperiods was read into a matrix (100 x 5) in the simulation model. A random temperature is drawn by the following process: an internal counter for month of the year indicates the column or subperiod of the year from which a value is to be selected. The appropriate row is indicated by a uniform random number from 1 to 100. This method results in a temperature drawn at random from an empirical approximation of the true probability distribution. Over a large number of iterations this procedure reproduces the probability distributions implicit in the random samples. This procedure would be useful when a probability distribution is needed to represent a population, such as rainfall or temperature, which has a large number of observations. This procedure reduced the cost of drawing random temperatures by 66 percent over using a Gaussian normal distribution.

[3]Poliques [20], Sprow [24] and Jones [12] describe the use of triangular distribution to represent stochastic phenomena when technical consultants or management is unclear as to the true distribution, but can identify the minimum, mode and maximum values for the random variable.

overall life of the proposed project and probability distributions for annual net returns for each year simulated. The distributions of net present values and cash flows are discussed below.

Distribution of Net Present Values

Figure 1 presents the cumulative probability density function of net present value and the rate of return for the proposed investment. This cumulative function indicates probability of the investment generating a negative net present value is about .23. Conversely, there is a 77 percent chance the investment will yield a positive net present value. Probability that the project will be very successful (a net present value of $40,000 or more) is about .10 or one chance in ten, while probability of the investment resulting in more than a $10,000 loss is about .10, or one chance in ten. From Figure 1, the probability of net present value falling within some range can also be determined. For example, probability of a net present value between $20,000 and $40,000 is about .46.

By dividing each net present value in Figure 1 by the initial investment, the rate of return is computed. The probabilities now refer to the rates of return. The probability of a rate of return being less than 8 percent is .46 and the probability of a rate of return exceeding 16 percent is about .08.

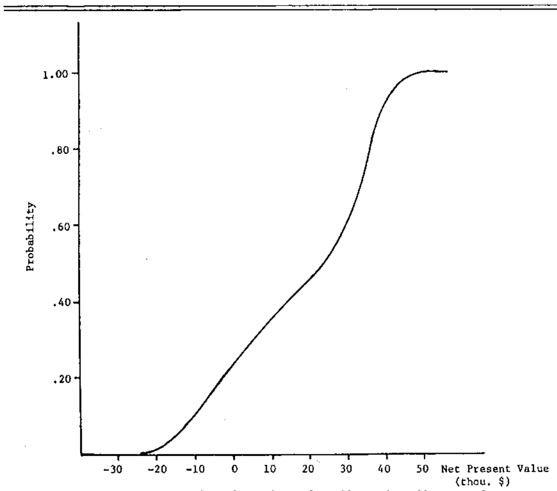

FIGURE 1. CUMULATIVE PROBABILITY DISTRIBUTION OF NET PRESENT VALUE AND RATE OF RETURN ON INVESTMENT

Probabilities associated with various levels of net present value can be considered as objective measures of the risk of the proposed investment. Management can incorporate the results from Figure 1 with its current utility function for money income and risk to determine whether or not the project presents too much risk with respect to some satisficing rate of return. If cumulative probability density functions for several investments are presented in a manner similar to Figure 1, management can compare risks associated with each for some specific rate of return or net present value.

Distribution of Cash Flows

Another factor that may affect the investment decision is distribution of annual net returns, or cash flows over the life of the project. Such information may be obtained by analyzing the simulated annual net returns of the proposed project for each year of the project's economic life. Probability distributions of annual cash flows were estimated from simulation results and are presented in Table 1 for a probabilistic analysis of annual net returns.

For year one, there is a one percent chance that cash flows will fall below $22,400 and only a one percent chance that they will exceed $25,200. In year one, the mean value of cash flows is $23,920. There is about a 20 percent chance that cash flows will fall within $200 of this mean. For the second year, there is only a one percent chance that cash flow will fall below $25,410 (Table 1). In year two, the probability that cash flow will exceed $26,710 is .90, an event that had almost a zero probability in year one.

An investor may have in mind a satisficing level of cash flow for a particular investment. Information in Table 1 can help determine probability of the investment being a success or a failure with respect to this criterion. The satisficing cash flow vector could be specified (for example $23,000 in year one and $5,000 for each additional year), and the probability of achieving this cash flow can be computed from Table 1.

The cash flow distribution changes with respect to range and degree of dispersion over the life of the project. The range between the minimum cash flow and the maximum increases steadily from year one to year ten (Table 1). Also, the degree of dispersion about the mean, the standard deviation, increases from year one to year ten.[4]

Results of a probability analysis of annual cash flow may be a major consideration in deciding

[4] The probability distribution in Table 1 may be thought of as one-half of a cone, with year one at the point and year ten at the large end. The cone does not represent a normal distribution (or even a symmetrical distribution) because the mean for each year lies to the left of the cash flow value associated with a 50 percent probability.

TABLE 1. PROBABILITIES ASSOCIATED WITH THE ANNUAL NET RETURNS OVER THE LIFE OF THE INVESTMENT

	Probability [a]											Statistics	
Years	.01	.10	.20	.30	.40	.50	.60	.70	.80	.90	.99	mean	std. dev.
						(dollars)							
1	22,400	23,000	23,240	23,440	23,700	23,950	24,130	24,310	24,580	24,790	25,200	2 ,920	661.82
2	25,410	26,710	27,000	27,540	28,200	28,540	29.070	29,450	29,700	30,020	30,500	28,460	1,280.67
3	29,700	30,650	31,210	31,850	32,640	33,390	34,340	34,720	36,290	35,290	35,800	33,300	1.968.67
4	32,470	33,470	34,590	35,160	36,550	37,390	38,350	39,080	39,510	40,390	42,100	37,170	2,574.13
5	35,760	37,540	38,520	39,910	41,330	42,760	44,020	44,610	45,380	46,250	48,620	42,300	3,417.93
6	40,000	42,150	43,340	45,580	46,880	48,670	50,290	51,250	52,400	53,190	55,660	48,190	4,332.28
7	42,830	45,420	47,620	49,000	51,630	54,130	56,340	57,300	58,230	59,920	62,090	53,280	5,344.87
8	47,450	50,110	52,760	54,730	57,180	60,530	63,200	64,430	65,660	67,730	71,590	59,660	6,644.56
9	52,630	54,960	57,830	60,220	63,300	67,000	70,110	71,660	73,220	75,480	79,070	66,200	7,721.44
10	56,700	59,350	63,260	66,960	69,990	74,050	76,850	79,290	81,860	82,980	88,700	72,930	8,871.29

[a]The table is read as the probability P(x) of income being less than some value y. For example, there is a 30 percent chance of income being below $23,440 in the first year.

whether or not to invest. Cash flow considerations are particularly important if the firm has a low equity position, or the decision-maker has no other sources of income, and is thus vulnerable to financial ruin. If the firm is quite stable, the probabilistic cash flow analysis may be useful in acquiring commercial financing for the proposed project. Another use of a probabilistic analysis of annual cash flows is development of a more reasonable repayment schedule for the investment being considered. Also, the type of results presented here can be used by farmers and agricultural businesses to analyze proposed changes in their production or marketing structures.

Two or more investments can be compared by using stochastic simulation techniques. For example, if a decision-maker wishes to consider two different investments, each could be modeled and separate probability distribution for net present value estimated. By comparing the two distributions graphically or statistically in terms of mean, standard deviation and coefficient of variation, the decision-maker could select one investment over the other. Probability analysis of each project's annual cash flow may be useful in comparing two proposed investments if they are otherwise equal with respect to levels of risk and rate of return.

If the decision to invest in a project is made as a result of a simulation analysis, the model could be used to analyze changes in input costs, input and output mix, and management decision rules. For example, changes in annual net returns for the proposed ice plant could be estimated for changes in bag weight, price of output, or changes in input costs, such as electricity, gasoline, labor or water.

SUMMARY AND CONCLUSION

This paper presents methodology for developing a probabilistic analysis of annual cash flows for a proposed investment under conditions of risk and uncertainty. This methodology can be applied to agricultural or nonagricultural investment decisions. Results of risk analysis of the overall project and a probabilistic analysis of annual cash flows provide information a decision-maker can use in deciding whether or not to invest in a proposed project. A comparison of two or more investments can be made by simulating each and comparing risks and probable cash flows. The final decision to invest or not remains with the decision-maker and his own utility function for money income and risk.

This paper suggests simulation of investments under conditions of uncertainty should not be limited to multi-million dollar corporations, federal agencies and the World Bank; but can be useful for analyzing decisions facing small businesses. Simulation models can be contracted to universities or consultants. The problem of simulation expertise and computer facilities need not be a limiting factor. If an investment is undertaken after a simulation model has been developed, the initial cost of development can be recouped by using the model to analyze year-to-year changes in the business—including change in input costs, output prices, input mix, or changes in production and inventory decision rules.

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