DETERMINING OPTIMAL REPLACEMENT AGE OF BEEF COWS IN THE PRESENCE OF STOCHASTIC ELEMENTS*

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INTRODUCTION

Feeder calf producers face a problem common to all owners of productive assets: how long should the asset (brood cow) be used before it is replaced? Expected productivity of a brood cow can be measured by two parameters: (1) the likelihood that she will wean a calf, and (2) its anticipated weight when weaned. Both are independent functions of the cow's age which decline annually after reaching peaks in the early years of her productive life. The calf producer who expects to maximize profits in the long run must choose an optimal rate of replacement for his brood cows.

Several authors [3, 5, 7] suggest the optimal time to replace an asset is when net revenue in the current period falls short of the amortized present value of the next replacement. Rogers [7] used expected net revenues in each year of the cow's life to determine optimal replacement age with the above criterion. Two shortcomings of this method are that it compares current income with the amortized revenue of a single replacement, rather than a stream of replacements, and it fails to take account of the stochastic nature of the replacement process, i.e., early replacement due to asset failure.

The purpose of this paper is to adapt a more suitable model from the literature to a specific cow-calf enterprise for use in determining the optimal (long-run profit maximizing) replacement age of a brood cow. Sensitivity of the replacement decision to changes in certain variables will also be examined.

THE REPLACEMENT MODEL

Perrin [6, p. 64] showed that the present value of an asset that is to be exchanged every T years with a replacement having an identical stream of returns is

$$RPV_T = \frac{PV_T}{1\text{-}B^{-T}} \qquad (1)$$

PV_T is the present value of the original asset without replacement, i.e., for an asset which is used T years and then sold for salvage. The denominator converts the present value of the single asset into the present value of an infinite stream of replacements, (RPV_T). $B=(1+i)$ where i is the applicable discount rate.

Burt [7] demonstrated the changes needed in equation (1) when productivity of the asset is uncertain due to stochastic elements (in this case loss of fertility or death of the cow). Letting P_t be the joint probability that the cow will wean a calf and not die in period t, expected present value of the cow sold and not replaced after weaning her T^{th} calf is

$$\begin{aligned} EPV_T = {} & B^{-1}\ R_1 + B^{-2} P_1\ R_2 + B^{-3} \\ & P_1\ P_2\ R_3 + \ldots + B^{-T} \\ & P_1\ P_2 \ldots P_{T-1} R_T + B^{-T} P_1 \\ & P_2 \ldots P_{T-1}\ M_T - M_o \end{aligned} \qquad (2)$$

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where

R_t = return to the cow in period t

M_T = market value of value of the cow at the end of the T^{th} period, and

M_o = market value (acquisition cost) of a bred heifer when she enters the herd.

The denominator of equation (1) must also be written in expectation terms as

$$(1-EB^{-T}) = 1-[(1-P_1)B^{-1}+P_1(1-P_2)B^{-2} +...+ P_1 P_2...P_{T-2} (1-P_{T-1})B^{-(T-1)} +P_1 P_2...P_{T-1} B^{-T}] \quad (3)$$

where $(1-P_t)$ is the probability that actual replacement will occur in period t due to fertility problem of death of the cow. The probability of replacement in period T, $(1-P_T)$, is unity since that is the period of planned replacement. Thus equation (1) is written in expectation terms as

$$ERPV_T = \frac{EPV_T}{1-EB^{-T}} \quad (4)$$

DATA

The production cycle of the cow-calf enterprise is as follows: cows are bred in March and April to calve the following December and January. Calves are weaned on September 1 at an average age of eight months. Cull cows and calves are sold at that time and replacement heifers are retained (or acquired). Replacements are bred the following March and April to calve on their second birthday, at which time they enter the herd.

Brood cows may be removed from the herd prior to planned replacement date because of death or failure to produce a marketable calf. PKEEP, the calving percentage, represents the probability of producing a marketable calf, i.e., the number of calves weaned per cow bred, assuming no deaths. One published [7, p. 922] and two unpublished [2, 14] sets of observations on calving percentages were obtained and fitted separately as quadratic functions of the cow's age (Table 1). Due to the large disparity among the three data sets, published probabilities are used in our initial analysis. They are based on the largest number of management systems and widest geographic dispersion. Unpublished data are used subsequently to test the sensitivity of the replacement decision to alternate distributions on calving percentage.

TABLE 1. EFFECT OF COW'S AGE ON CALVING PERCENTAGE, COW DEATH LOSS, AND WEANED WEIGHT OF CALF

Cow Age	Calf Number	Calf Weaned Weight	Calving Percentage Initial[a]	Alternative 1[b]	Alternative 2[c]	Cow Death Loss
			(percent)			
2	1	425.0	85.5	78.2	69.1	2.25
3	2	444.0	89.0	83.6	75.1	2.25
4	3	465.0	92.7	86.5	79.8	2.30
5	4	488.0	94.5	87.0	83.3	2.35
6	5	488.0	94.3	84.9	85.5	2.45
7	6	488.0	93.0	80.4	86.4	2.80
8	7	488.0	90.8	73.4	86.0	3.25
9	8	488.0	87.0	64.0	84.4	3.70
10	9	488.0	82.0	52.0	81.6	4.35
11	10	465.0	76.6	37.6	74.4	5.80
12	11	465.0	70.0	20.7	72.0	6.30
13	12	465.0	63.6	1.3	65.4	6.50
14	13	465.0	56.2	.0	57.4	6.60
15	14	465.0	45.0	.0	48.2	6.60

[a]Source [8].
[b]Source [14].
[c]Source [2].

Fewer data are available on the probability of a cow dying at each year of age. Most studies have assumed a constant death loss at all ages. However, Rogers [7, p. 922] suggested that the probability increases with age and reported a mortality probability series for the first 15 years of the cow's life; these are used in our analyses and labelled PDIE (Table 1).

Probability, P_t, that the cow will be carried into the next period, i.e., probability that the cow produces a calf and does not die, is PKEEP (1—PDIE). (1—PKEEP)(1—PDIE) is the probability of unplanned replacement of a live cow in period t. These estimates are used in calculating EPV_T and $1-EB^{-T}$.

Net returns to the cow in each period, R_t, were calculated by the following equation

$$R_t = [PKEEP_t(1-PDIE_t) NR_t+(1-PKEEP_t) (1-PDIE_t) [UR_t-PDIE_t \cdot UD_t] \quad (5)$$

where

NR_t = net revenue from sale of a calf in period t, i.e., the difference between gross revenue from sale of calf and sum of nutritional and other variable costs for the cow-calf unit

UR_t = return from unplanned culling of a 1,000 pound cow that fails to wean her t^{th} calf and is calculated as the difference between cull value of cow and cost of replacement heifer. Heifers that fail to produce a calf on their second birthday are fattened and sold as 950-pound slaughter heifers (good-choice grade); all other culls receive utility grade price (Table 2)

UD_t = cost of acquiring a bred heifer to replace cows that die in period t, less nutritional and other variable costs saved because of the cow's death.

Acquisition cost of a replacement heifer, M_o, was budgeted since no satisfactory price series for bred replacement heifers is available. Budgeted cost of the replacement heifer is the sum of the value of a 476-pound weaned heifer, cost of adequate nutrition to allow the heifer to reach 850 pounds by the time she has her first calf (at two years of age) and all other expenses (including land and capital charges) of raising the heifer for 16 months. Fixed expenses are included in the heifer budget, since it is a proxy for market value.

Nutritional requirements for the cow-calf unit and replacement heifers were calculated using digestible energy standards [4]. It was assumed that the average weaned weight of steers and heifers from a mature cow is 488 pounds. This weight was adjusted by USDA adjustment factors [13] to account for the effect of a cow's age on weaned weight. An adequate diet without supplementation was assumed to be provided by Coastal bermudagrass pasture (available March-December) and ryegrass-oat pasture (available November-April). Monthly yields of these crops were obtained from the Angleton Research Station in East Texas. Cost per megacalorie for each type of pasture was obtained by dividing the estimated total cost per acre [9] by the annual production of digestible energy. The average 1975 cost per megacalorie of digestible energy was $.0092. Since this cost seemed conservative and will vary according to location and forage system, sensitivity of the replacement decision was also examined at higher forage costs.

Variable costs, exclusive of feed, for maintaining the cow-calf unit over the year were adapted from Extension Service beef budgets for East Texas [10] and amount to $33 per year. Variable and fixed costs, exclusive of feed, for raising the replacement heifer were estimated to be $66.

Prices used in calculating gross revenues were based on the 1955-74 price series from the San Antonio market [12]. When prices of various classes of livestock were deflated by the USDA index of prices paid for factors of production [11] and regressed linearly on time, no significant trends emerged. Means of the deflated series were inflated by the 1975 index of prices paid for factors of production to serve as estimates of "normal" 1975 cattle prices, and are reported in Table 2. Feeder prices used in the analysis are average steer and heifer prices.

TABLE 2. PRICES USED FOR VARIOUS CLASSES OF CATTLE

Cattle Class	Price Level[a]		
	Mean	High	Low
Steer and heifer feeders, 200-500 lbs.	50.26	57.88	43.67
Heifer feeders, 200-500 lbs.	48.12	55.16	42.52
Slaughter heifers, 800-1100 lbs., good and choice	48.71	52.00	44.38
Utility cow	32.85	38.24	29.09
Cutter cow	29.46	35.06	24.59

[a]Source [12]. Mean is average 1955-75 deflated price inflated by the 1975 index of prices paid for factors of production [11]. High prices are midway between mean and high deflated observation for the period, and low prices are midway between mean and low deflated observation, each inflated by the 1975 index of prices paid [11].

RESULTS OF ANALYSIS

Initial Solution

The algorithm used to solve $ERPV_T$ computes the present value of planned replacement after each of 14 calving periods. Optimal replacement period is selected by observing the highest value of $ERPV_T$. Results of the initial and selected sensitivity analyses are presented in Table 3.

Due to increasing calving percentages and weaned weights, annual net income, R_t, from the cow-calf enterprise increases from $87 for a two-year old cow to $116 for a five-year old cow. R_t declines thereafter as first the calving percent and then calf weights become smaller, causing a net loss of $6 for the 15-year old cow. The cost of obtaining a replacement heifer, M_o, is $394. Salvage value, M_t, is $342 for a 950-pound two-year old cull cow and $355 for all other culls having a mature weight of 1,000 pounds. R_t and M_T must be discounted and expectations taken as shown in equation 2 before $ERPV_T$ can be calculated.

TABLE 3. EXPECTED PRESENT VALUE, SELECTED RESULTS[a]

		With Initial Calving Percentage and Cattle Price Level:				With Mean Cattle Price Level and:			
Cow Age	Calf Number	Mean	Low (i=3.5%)	High	High Feed Costs i=4.0%	Calving Percentage Alternative 1, i=5.5%	Calving Percentage Alternative 2, i=5.5%	Declining Cull Price Alternative 1[b]	Declining Cull Price Alternative 2[c]
2	1	-127	-622	540	-700	408	376	609	-127
3	2	-95	-528	431	-577	-17	-537	247[d]	-168
4	3	-43	-398	442	-418	-45	-693	161	-59
5	4	35	-211	516	-199	14	-663	175	19
6	5	91	-76	572	-41	44[d]	-606	195	75
7	6	122	0	601	48	30	-555	203[e]	106
8	7	133	26	608	80	-23	-519	199	118
9	8	128	13	598	67	-100	-499	184	114
10	9	112	-36	573	14	-184	-495[d]	160	97
11	10	84	-117	536	-76	-254	-511	128	71
12	11	58	-199	501	-166	-296	-535	99	45
13	12	37	-269	473	-244	-313	-554	76	24
14	13	21	-323	452	-303	-317	-572	59	9
15	14	11	-360	440	-343	-317	-585	49	0

[a]Discount rate is 10 percent unless otherwise noted. Recommended replacement policies are underscored.

[b]Beginning with average of good-choice slaughter heifer and utility cow price for two-year old cull cow.

[c]Beginning with utility price for two-year old cull cow.

[d]Second-best policy over culling after first calf.

[e]Third-best policy over culling after first or second calf.

The initial analysis suggests an earlier optimal replacement age than indicated by Roger's model. Expected present value of a cow that will be replaced repeatedly every seven years is \$133 versus \$112 for the nine-year replacement policy suggested by Roger's model. $ERPV_T$ is negative for the first three calving periods, because the sum of net incomes and salvage value discounted and adjusted by expected calving percentages and death losses are less than the cost of a replacement (which is never discounted or adjusted—see equation 2). $ERPV_T$ becomes positive by the fourth calf, increases to a maximum value for the seventh calf and declines thereafter through the 14th calving period.

Solution Sensitivity

Cattle prices. The sensitivity of the replacement decision was tested by first looking at two alternative sets of cattle prices. Prices midway between the 20-year mean and the high and low observations for the period were also computed for each of the cattle classes discussed above (Table 2). Optimal replacement strategy under each set of alternative prices remains unchanged at seven years. With high cattle prices $ERPV_T$ is \$608. With lower cattle prices, expected present value of replacement is not positive unless the discount rate is no higher than 3½ percent, i.e., it would be impossible to achieve a rate of return greater than 3½ percent if low cattle prices were expected to prevail over the long run.

Feed costs. The effect of higher nutrition costs on the replacement policy was examined by increasing the cost per megacalorie of digestible energy by 50 percent. The policy remained unchanged. However, $ERPV_T$ did not become positive until the discount rate was reduced to four percent when mean cattle prices were examined. A 10 percent rate of return was achieved for high cattle prices. It was not possible to achieve a positive $ERPV_T$ at any discount rate for low cattle prices. The analyses show that the expected present value of losses are minimized by a seven-year replacement policy.

Calving percent. Two alternative calving percent distributions were analyzed. The first alternative distribution [14] peaks with the fourth calf but at a lower value than the initial distribution considered (87 vs. 95 percent). The lower calving percentage means there is a greater probability that the cow will be culled earlier than planned. Annual net income to the operation is reduced as expected revenue from calf sale declines and unplanned replacement cost increases (equation 5). The effect of the lower calving percentage is to recommend planned replacement of the cow after she has weaned her first calf. Obviously this is feasible only if the producer is able to acquire bred replacement heifers from outside sources. A second-best policy is to replace the cow after she has weaned her fifth calf. Both these policies apply to all three sets of cattle prices analyzed. With low prices, no discount rate yielded a positive $ERPV_T$; thus, the policies minimize losses of a continuing operation. In the case of mean cattle prices, a positive $ERPV_T$ is indicated only when the discount rate is less than six percent (Table 3). High cattle prices permitted the operation to achieve a positive $ERPV_T$ with discount rate of 10 percent.

The second alternative distribution [2] peaks with the sixth calf. It is lower than either of the other distributions for the first four calves and higher for the last four. Again, the optimal policy for all cattle prices is to replace the cow after weaning her first calf. The second-best policy calls for replacement after the ninth calf is weaned. Low cattle prices never have a positive $ERPV_T$. Mean cattle prices have a positive $ERPV_T$ when the discount rate is less than six percent, although the nine-year policy is never positive. High cattle prices result in a positive $ERPV_T$ for both replacement policies.

Cull cow prices. In the initial analysis it was assumed that all cull cows receive the utility grade price regardless of their age. An alternative assumption permits cull price to decline linearly over the life of the cow in an attempt to account for deterioration in carcass quality with age. If cull price declines from the average of good-choice slaughter heifers and utility cow prices after the first calf to cutter price after the 14^{th} calf, (i.e., declining cull price alternative 1, Table 3), the relatively high income of the two-year old cull cow competes with that from future calf production. The optimal policy calls for the cow to be replaced after she weans her first calf. The second best policy calls for replacement after the second calf is weaned, but, like the optimal policy, this is not feasible unless replacements can be obtained from outside the herd. The third-best policy was also determined which calls for replacement after the sixth calf is weaned.

If the price of cull cows declines linearly with age between utility and cutter prices (i.e., declining cull price alternative 2, Table 3), the optimal policy is to replace after the seventh calf. This is the same as the results of the earlier analyses.

CONCLUSIONS

The optimal replacement age for breeding livestock, although earlier than Rogers' conclusion, appears quite stable with respect to feed and livestock prices. Considering an infinite planning horizon and the stochastic nature of brood cow replacement, results of this paper indicate that under a high level of management (i.e., low feed costs and high calving percentages), intended replacement should take place after the seventh calf is raised. The replacement decision is quite sensitive to calving percent distribution and to the cull cow price distribution (generally suggesting even earlier replacement) but insensitive to the range of cattle price levels and feed costs considered in this study.

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