THE EFFECT OF SPATIAL VARIABILITY OF IRRIGATION APPLICATIONS ON RISK-EFFICIENT IRRIGATION STRATEGIES

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Abstract

The effect of irrigation system uniformity on the selection of risk-efficient irrigation strategies is evaluated using crop simulation and stochastic dominance procedures. Alternative strategies are evaluated under assumptions of both uniform and non-uniform application. Results indicate that the variability of net returns resulting from the employment of a specified schedule increases when irrigation uniformity is explicitly represented. Solutions derived using economic efficiency and stochastic dominance criteria indicate that the uniformity with which irrigations are applied contributes to the application of water-intensive irrigation schedules.

Key words: irrigation uniformity, crop simulation, grain sorghum, irrigation strategies, stochastic dominance.

Economic analysis of irrigation scheduling has frequently concluded that irrigators could reduce water and energy use significantly, and hence increase net returns, by timing irrigation applications in accordance with plant water needs. However, despite increasing water scarcity and escalating costs of irrigation, many producers continue to irrigate intensively. Some researchers have suggested that irrigator risk aversion may serve as a possible explanation of this seemingly irrational behavior; irrigators may apply excessive water quantities to reduce the incidence of low yields and/or high variability of net returns.

Three recent studies have evaluated the influence of irrigator risk preferences on irrigation scheduling. Prickett et al. used stochastic dominance procedures to investigate the risk efficiency of alternative soybean irrigation strategies and determined that irrigation water was a risk-reducing input. Irrigators' propensity to apply more water was shown to increase as they became more risk averse. Boggess et al. also concluded that risk preferences affect irrigation scheduling decisions, but determined that risk-averse decision makers irrigate less frequently and apply less water than that prescribed by the irrigation strategy that maximizes net returns. Finally, Harris and Mapp employed first- and second-degree stochastic dominance analysis in determining that several water-conserving strategies dominate conventional intensive irrigation schedules for grain sorghum. Application of stochastic dominance with respect to a function yielded the same stochastically efficient set over all preference intervals. The risk-efficient set of schedules was not affected by the risk preferences of irrigators. Thus, at least in some settings, risk aversion alone may not explain the water-intensive irrigation schedules often followed by irrigators.

An additional explanation of discrepancies between efficient irrigation strategies derived from economic analysis and actual irrigation practices may be the lack of uniformity with which irrigations are applied. Economic prescriptions for efficient water use have been based on the assumption that water is uniformly applied to the field. In actual practice, however, a portion of the field receives an irrigation depth below the quantity desired and a portion receives a depth in excess of the anticipated water quantity. Thus, many irrigators may over-apply water in some portions of the field to assure that the entire field receives some minimum water application. The objective of this paper is to evaluate the effect that spatial variability of irrigation applications has on economically efficient irrigation strategies. From this analysis, the degree to which non-uniform irrigation applications may explain current intensive irrigation practices may be ascertained.

A grain sorghum crop simulation model is

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used to estimate crop yields under the assumption of both uniform and non-uniform application depths. Net return distributions are derived for alternative irrigation strategies under both uniform and non-uniform irrigation conditions. Risk efficient sets of irrigation strategies are then identified under assumptions of uniform and non-uniform irrigations consistent with the risk preferences of specified groups of decision makers. Comparison of the risk efficient sets indicates the influence of spatial variability of irrigation applications on irrigation scheduling decisions.

METHODOLOGY

Chambers notes that randomness can enter the production process through the input hiring decision. Let the input vector (in this case, water application quantities in various periods of the irrigation season) be X*, while the effective amount of input applied is X. Crop production depends solely on the effective amount of water applied. Assume that the effective amount of water applied varies randomly around the amount actually applied according to:

$$(1)\quad X = X^* + \epsilon,$$

where ϵ is a vector of random variables. The problem may be conceptualized as a case where the irrigator may apply a mean level of water but is unable to control random quantity differentials. Thus, the irrigator's objective function may be expressed as

$$(2)\quad \max_{X^*} E\{U[P \cdot f(X^* + \epsilon) - r \cdot X^*]\},$$

where E is the expectations operator, U is utility, P is output price, r is the cost of all irrigation inputs, and f( ) is a response function relating crop yield to the effective amount of water applied.

If an irrigator can incorporate the uniformity characteristics of the irrigation system into the decision-making process, the degree of randomness is reduced and the problem takes on a different complexion. The effective quantity of water applied may be respecified as

$$(3)\quad X = X' + \epsilon',$$

where $\sigma^2_{\epsilon'} < \sigma^2_{\epsilon}$. The revised objective function becomes

$$(4)\quad \max_{X'} E\{U[P \cdot f(X' + \epsilon') - r \cdot X']\}.$$

Comparison of solutions derived from each specification indicates the degree that irrigator behavior is affected by the uniformity with which irrigations are applied.

Crop Simulation Model

To determine the influence of alternative irrigation strategies and uniformity conditions on crop yield, a grain sorghum crop growth model was employed. The SORGF model, originally developed by Arkin et al. and later modified by Maas and Arkin, has been successfully applied to irrigated conditions in western Oklahoma (Harris and Mapp; Hornbaker). Sorghum was selected as the study crop because of its importance to irrigated agriculture in the region and the large amount of previous research addressing scheduling sorghum irrigations. As the sorghum model has been discussed in detail elsewhere (Harris and Mapp; Arkin et al.; Maas and Arkin), only a sketch is given here. The discussion of model development will instead focus on the modifications of the original model to represent spatial variability of irrigation applications.

The grain sorghum plant growth model simulates the daily growth and development of a single sorghum plant based upon the prevailing climatic and soil moisture conditions. The growth data from the single sorghum plant are then extrapolated to field-level information using specified plant population data. The model represents sorghum plant development in five stages: stage one—emergence to differentiation; stage two—differentiation to end of leaf growth; stage three—end of leaf growth to anthesis; stage four—anthesis to physiological maturity; and stage five—physiological maturity and beyond.

The sorghum growth model begins each year of simulation on May 1 by accepting initial values for various agronomic, edaphic, and climatic variables.[1] Soil moisture is calculated on a daily basis until planting occurs

[1]Initial soil moisture (May 1) is estimated from an equation developed by Mapp et al. that relates soil moisture on May 1 to rainfall events in April. The previous year's crop, weather, and irrigation decisions are assumed to not affect initial soil moisture levels.

on June 15. A row spacing of 30 inches and a plant population of 100,000 per acre are assumed. Each day of the growing season is simulated sequentially, using the ending agronomic conditions of the previous day as the starting point for the next day's calculations. Emergence of the grain sorghum plant is calculated based upon accumulated heat units and available soil moisture. After emergence, daily leaf development is simulated based upon further accumulation of heat units. The calculation of potential and net photosynthesis is used to estimate daily dry matter development of the grain sorghum plant. Using daily climatic data and estimated agronomic conditions, an estimate of daily potential evapotranspiration (ET_t) is derived. This estimate is employed in the following soil-water balance equation to calculate the daily extractable soil water level (SW_t):

$$(5)\ SW_t = SW_{t-1} - ET_t + R_t + IRR_t,$$

where R_t and IRR_t are the quantities of effective rainfall (rainfall-runoff) and irrigation, respectively, occurring during day t. The estimate of the quantity of extractable soil water is employed in a relationship to estimate the reduction in net photosynthesis resulting from insufficient soil moisture. Net photosynthesis is converted to dry matter weight, which is then allocated to particular points of the plant according to the stage of plant development. Crop yield is estimated from the portion of dry weight allocated to the grain head during the third and fourth stages of plant development.

Modeling the Spatial Variability of Irrigation

To investigate the effect of non-uniform application depths on irrigation decision making, it is necessary to represent the spatial distribution of irrigation applications in a field setting. Several characteristics interact to determine the uniformity with which sprinkler irrigations are applied. Some of the important physical features of the irrigation system include nozzle type and size, sprinkler head spacing, and system operating pressure. System operating conditions such as the age and maintenance of the system may also influence uniformity. Finally, wind speed, wind direction, and soil characteristics are environmental factors that affect water distribution.

Numerous engineering studies have focused on the description and modeling of the areal distribution of water applications. The most widely accepted measure of the spatial variability of irrigation applications is the coefficient of uniformity (UC), which may be expressed as:

$$(6)\ UC = 1 - \frac{\sum_{i=1}^{n} |w_i - \overline{w}|}{n\overline{w}},$$

where w_i is the application depth of water at the i-th observation point in the field, $\overline{w}$ is the mean application depth, and n is the number of observations (Pair; Merriam et al.). The coefficient of uniformity provides a single numerical measure that may be used to compare the uniformity properties of various irrigation systems.

While useful in general irrigation system appraisal, UC conveys little information concerning the actual distribution of water applications. Several recent studies have focused on system uniformities using theoretical procedures and field measurements to determine the field distribution of irrigations applied with various irrigation systems (Heerman and Hein; Ring and Heerman; Merriam et al.; Pair). Research findings of the uniformity of a particular irrigation system are often reported by plotting water application levels at various locations of the irrigated field.

In this analysis, an empirically derived application pattern reported by Ring and Heerman was employed to represent the areal distribution of a representative center pivot irrigation system as shown in Figure 1. The

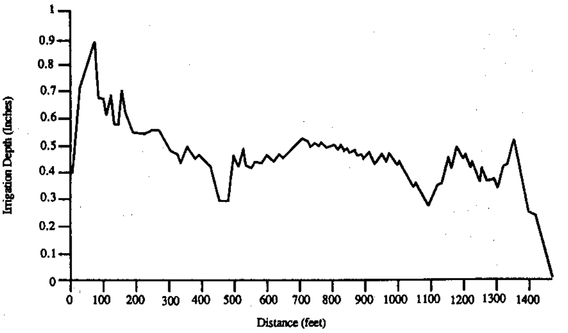

Figure 1. Empirically Derived Water Application Pattern for a Representative Center Pivot Irrigation System (Source: Ring and Heerman).

application pattern depicts the amount of water that is received at various distances from the pivot point on a 130-acre field irrigated with a center pivot system. Based upon the application pattern, a uniformity curve relating the dimensionless irrigation depth to the fraction of the field area receiving at least that depth was derived as shown in Figure 2.

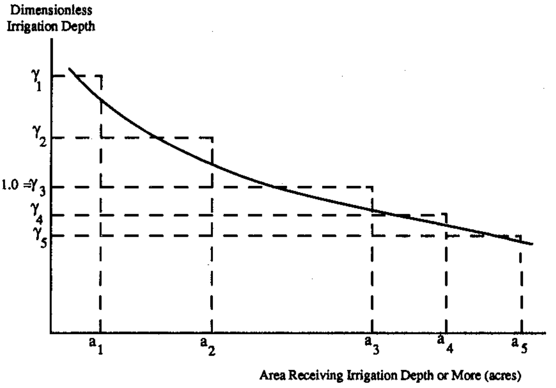

Figure 2. Uniformity Curve and Its Approximation Using Scalars Representing the Percentage of Water Received by Each Section of the Field.

The uniformity curve was approximated by parceling the field into five sections; for each section a scalar representing the percentage of the desired application depth actually received was estimated.[2] The quantity of water actually applied to the i-th section of the field is estimated as the product of the scalar (γ_i) and the desired application depth (w′). For example, if $\gamma_2 = 1.1$ and a two inch application was desired, field section two would receive an actual depth of 2.2 inches.

Pair observed that sprinkler uniformities are significantly affected by wind speed and direction and reported UC values measured under alternative wind conditions. Using Pair's results, uniformity curves were estimated to reflect the spatial distribution of irrigations applied under four alternative wind conditions. Wind conditions were defined based upon increments of five miles per hour average wind speed during the irrigation. Uniformity coefficients describing the percentage of the desired irrigation depth received in each field section under four alternative wind conditions are given in Table 1.[3] The uniformity curves were programmed into the growth model so that when an irrigation was applied the appropriate uniformity curve was selected (based upon the average wind speed over the duration of the irrigation) and applied to the desired irrigation depth. For example, if average wind speed during a two-inch irrigation was characterized as wind condition c(2), field section 1 would receive an irrigation depth of 3.64 inches (1.82×2), section 2 would receive 2.36 inches (1.18×2), and so on. The variability of irrigation depths over the field is shown to become more pronounced as average wind speed is increased.

Expected field-level yields under non-uniform application conditions [$E(Y)_n$] may be represented as:

$$(7)\quad E(Y)_n = \sum_{i=1}^{5} f_i(\gamma_{ij} \cdot w') \cdot A_i,$$

where γ_{ij} is the percentage of the desired application depth applied to the i-th section of the field under wind condition j and A_i is the number of acres in the i-th section of the field. Alternatively, yields may be estimated under uniform application conditions [$E(Y)_u$] as:

$$(8)\quad E(Y)_u = f(w') \cdot A,$$

where A is the total acreage of the field.

Alternative Irrigation Strategies

Grain sorghum growth is simulated on a 130-acre field located on a representative irrigated farm in the Oklahoma Panhandle. Average annual precipitation in the study area is 15

TABLE 1. UNIFORMITY COEFFICIENTS (γ_{IJ}) DESCRIBING THE PERCENTAGE OF THE DESIRED APPLICATION DEPTH RECEIVED BY EACH SECTION OF THE 130 ACRE FIELD

Wind	Field Section				
Condition[a]	1	2	3	4	5
c(1)	1.70	1.10	1.00	.85	.55
c(2)	1.82	1.18	1.00	.84	.51
c(3)	1.94	1.28	1.00	.83	.47
c(4)	2.00	1.38	1.00	.82	.46

[a]Average daily wind speed pertaining to the four wind conditions are c(1), 0-5 mph; c(2), 5-10 mph; c(3), 10-15 mph; and c(4), greater than 15 mph.

[2]The acreage of the i-th section of the field (A_i) is calculated as $a_{i-1} - a_i$ (see Figure 2). Acreages employed in the analysis are A_1 – 3 acres, A_2 – 12 acres, A_3 – 75 acres, A_4 – 31 acres, and A_5 – 9 acres.

[3]Each set of coefficients represents a discrete approximation of the uniformity curve applicable to a particular wind condition.

inches; however, only four inches of this amount occurs during the growing season for grain sorghum. A clay loam soil having an available water holding capacity of 7.65 inches is assumed. The field is irrigated from a 1,000 gallon per minute well having a static lift of 250 feet. The center pivot application system is characterized by an operating pressure of 60 pounds per square inch, a pumping efficiency of 70 percent, and an average application efficiency of 75 percent. The outcome of concern to the decision maker is assumed to be field-level returns to overhead, management, and fixed costs of irrigation (FLNR), estimated as:

$$(9)\ FLNR = [P \bullet f(w) - VC \bullet w - HC \bullet f(w) - PC] \bullet 130,$$

where P is the price of grain sorghum ($/bu), w is the average seasonal irrigation depth (acre inches), f(w) is sorghum yield (bu/A) expressed as a function of seasonal irrigation depth, VC is the variable cost of irrigation ($/acre inch), HC is harvest and hauling cost ($/bu), and PC represents all other production costs ($/acre). The price of grain sorghum is held constant at $3.90 per hundredweight throughout the analysis. Irrigation variable costs (fuel, lubrication, repairs, and labor) were estimated using the Oklahoma State University Irrigation Cost Generator (Kletke et al.) as $3.96 per acre inch. Non-irrigation production costs were estimated from published enterprise cost estimates for sorghum production in the Oklahoma Panhandle (Oklahoma State University Cooperative Extension Service).

Random weather variability for analyzing the various irrigation strategies is represented using 20 years of daily precipitation, temperature, solar radiation, and average wind speed data from the study area (National Weather Service). Each year is assumed to be an independent, equally likely outcome. Each irrigation strategy is replicated twenty times under uniform and non-uniform application conditions, and the resulting net return distributions provide the basis for the stochastic efficiency analysis.

In developing the irrigation strategies to be considered in the analysis, irrigators are assumed to be capable of monitoring soil moisture levels and scheduling irrigations based upon soil moisture readings and crop growth stage. Three variables were parameterized in generating the irrigation strategies for the analysis: (1) the soil moisture triggering the time of irrigation, (2) the depth of individual applications, and (3) the growth stage during which irrigations are initiated. Several previous studies focusing on the efficient scheduling of grain sorghum irrigations were used to develop relevant ranges for these scheduling parameters. Maas and Arkin indicated that sorghum yield was insensitive to irrigation schedules based upon soil moisture ratios exceeding 45 percent. Also, Hornbaker found that application depths between 2.8 and 2.1 acre inches yielded economically efficient irrigation strategies for the Oklahoma Panhandle. As a result, irrigation schedules based upon soil moisture extraction ratios ranging from 25 to 45 percent (in 5 percent increments) were included in the analysis. For each soil moisture criterion, application depths of 2.8 and 2.1 acre inches were considered, as well as strategies that eliminated irrigations in the initial stage of plant growth.

RESULTS

The irrigation scheduling criteria used to derive the irrigation strategies evaluated in the analysis are listed in Table 2. To expedite discussion, each strategy is assigned an acronym defining the scheduling criterion employed. The acronym consists of three

TABLE 2. SUMMARY OF THE DERIVED MEAN IRRIGATION DEPTHS AND NET RETURN DISTRIBUTIONS FOR GRAIN SORGHUM IRRIGATION STRATEGIES UNDER UNIFORM AND NON-UNIFORM CONDITIONS, OKLAHOMA PANHANDLE

		Uniform		Non-Uniform	
Schedule[a]	Mean Irr. Depth (acre inches/acre)	Mean	S.D.	Mean	S.D.
		$/acre			
2.8-45-N	14.8	90.57	22.65	87.39	22.90
2.8-45-D	12.1	90.36	24.27	83.26	28.16
2.1-45-N	14.5	87.66	23.21	77.52	26.18
2.1-45-D	11.4	88.29	24.51	85.10	26.80
2.8-40-N	13.7	91.56	22.77	84.12	23.34
2.8-40-D	11.8	91.17	28.87	78.75	30.43
2.1-40-N	13.4	90.36	24.27	81.74	24.65
2.1-40-D	11.2	92.26	28.10	86.88	28.51
2.8-35-N	12.6	92.58	21.72	84.67	22.24
2.8-35-D	10.9	89.50	28.91	79.70	29.02
2.1-35-N	12.2	84.26	25.40	76.81	28.11
2.1-35-D	10.5	83.20	30.15	76.25	32.44
2.8-30-N	12.7	93.02	20.13	80.90	24.40
2.8-30-D	10.6	95.24	29.43	88.29	30.56
2.1-30-N	11.8	95.19	22.21	85.47	24.63
2.1-30-D	9.7	96.84	29.72	84.86	28.57
2.8-25-N	11.1	90.36	25.27	84.00	29.10
2.8-25-D	10.0	88.05	30.21	80.29	34.61
2.1-25-N	10.2	85.80	28.10	78.11	32.09
2.1-25-D	9.1	83.42	34.13	74.25	39.69

[a]Schedule name consists of the depth of individual application, the extractable soil moisture ratio, and a character denoting when irrigation was initiated.

parts: (1) the depth of individual irrigations, (2) the extractable soil moisture ratio, and (3) a character denoting when irrigation was initiated. For example, 2.1-45-D represents a strategy that applies 2.1 inch irrigations each time the soil moisture ratio falls to 45 percent and delays irrigations until following the first stage of crop growth. The acronym 2.1-45-N denotes the same strategy with the exception that irrigations are initiated at the beginning of the first growth stage.

For each strategy, the mean and variance of return to land, management, and risk under assumptions of both uniform and non-uniform application are given. Mean net returns range from \$83.42 to \$96.84 per acre under the assumption of uniform applications and from \$74.25 to \$88.29 per acre when the spatial variability of irrigations is represented. Average annual water application is also reported and ranges from 9.1 to 14.8 acre inches.

Mean net returns derived under the assumption of uniform irrigation exceed non-uniform returns for each of the irrigation strategies. Although non-uniform irrigation results in a portion of the field receiving water quantities in excess of the mean application, the net effect of non-uniform applications is a decrease in yields below those estimated under uniform application. Yield reductions from under-application of a portion of the field exceed yield increases resulting from over-application. This occurs because a large portion of over-applied water is lost as deep percolation, while the root zone of the sections receiving inadequate water quantities are not filled, resulting in some degree of crop water stress prior to the next irrigation.

Despite lower mean returns, the variability of net returns under assumptions of non-uniform application generally exceeds that derived in the uniform application scenario. When the spatial variability of irrigations is represented, yield dispersion across years is augmented by the increased yield variability within a field. However, comparison of the moments derived under uniform and non-uniform conditions indicates that the effect of representing non-uniformity is not consistent across strategies. In particular, the decrease in mean net return and increase in return variability appears to be more pronounced in some of the low water-use schedules. Since the representation of non-uniform irrigations affects both the location and shape of the distribution of annual net returns, consideration must be given to how spatial variability will influence the irrigator's preference ordering of alternative irrigation strategies.

Under uniform irrigation conditions, the irrigation strategy selected by the profit maximizing producer is 2.1-30-D. This strategy applies 2.1 inch irrigations based upon an extractable soil moisture ratio of 30 percent and initiates irrigations following the first stage of plant growth. Mean net returns derived from the schedule are \$96.84. The profit maximizing schedule is a low water-use strategy, applying an average annual irrigation depth of 9.7 inches. Mean net returns are maximized because the schedule applies water in quantities sufficient to avoid large yield reductions but minimizes average annual irrigation costs by eliminating excessive irrigations. The strategy results in a large degree of return variability, as evidenced by the relatively large standard deviation of net returns.

The irrigation strategy that maximizes average annual net returns under non-uniform application assumptions is 2.8-30-D. This strategy is based upon the same scheduling criterion as the profit maximizing strategy in the uniform scenario but applies water in 2.8 inch applications rather than 2.1 inches. As a result, the mean annual irrigation depth is increased approximately 10 percent. Applying larger irrigations is necessary to avoid water stress in portions of the field receiving application depths below the mean application quantity. Therefore, irrigators basing scheduling decisions on a criterion of maximizing economic efficiency are affected by the uniformity with which irrigations are applied. Mean net returns are maximized by applying excessive water quantities in some portions of the field to avoid large yield reductions in under-irrigated portions of the field.

Stochastic Efficiency Analysis

The application of stochastic dominance to evaluate and rank alternative production strategies has been well established in the literature. The use of stochastic efficiency to order activities was first formalized by Quirk and Saposnik and has been extended by several researchers to place alternative restrictions on decision-maker preferences (Fishburn; Hadar and Russell; Hanoch and Levy; King and Robison). Specific stochastic efficiency criteria included in this analysis are first-degree stochastic dominance, second-degree stochastic dominance, and stochastic dominance with respect to a function. The stochastic efficiency analysis involves the simultaneous comparison of the cumulative distribution functions of net returns for each of the alternative irrigation stategies listed in Table 2. Stochastic dominance procedures are applied to net return distributions derived under assumptions of both uniform and non-uniform application.

First- and Second-Degree Stochastic Dominance

Risk efficient sets of irrigation strategies derived from the application of first- and second-degree stochastic dominance analysis are presented in Table 3. Under uniform application assumptions, the first-degree stochastically efficient set is comprised of six irrigation strategies. Three of the schedules apply 2.8 inch irrigations based upon extractable soil moisture ratios ranging from 25 to 35 percent.

TABLE 3. RISK EFFICIENT GRAIN SORGHUM IRRIGATION STRATEGIES UNDER FIRST- AND SECOND-DEGREE STOCHASTIC DOMINANCE FOR UNIFORM AND NON-UNIFORM IRRIGATION CONDITIONS, OKLAHOMA PANHANDLE

Risk Efficiency Criteria	Risk Efficient Strategies: Uniform Irrigation	Risk Efficient Strategies: Non-Uniform Irrigation
First-Degree Stochastic Dominance	2.1-40-D, 2.8-35-N 2.8-30-N, 2.8-25-N 2.1-30-D, 2.8-30-D	2.8-45-N, 2.8-35-N, 2.8-30-D
Second-Degree Stochastic Dominance	2.1-40-D, 2.8-35-N 2.1-30-D	2.8-45-N, 2.8-35-N, 2.8-30-D

The profit maximizing schedule (2.1-30-D) and strategies 2.1-40-D and 2.8-30-D represent the remaining strategies comprising the risk efficient set. Three irrigation strategies exhibit first-degree stochastic dominance when non-uniform applications are represented—2.8-45-N, 2.8-35-N, and 2.8-30-D. Under the more restrictive assumptions of second-degree stochastic dominance, three strategies are eliminated from the risk-efficient set derived under the uniform scenario. The resulting risk efficient set consists of strategies 2.1-40-D, 2.8-35-N, and 2.1-30-D. Imposing the additional assumption of risk aversion on producer preferences had no effect on the set of preferred strategies under the non-uniform irrigation scenario.

By comparing risk efficient strategies derived under uniform and non-uniform assumptions, inferences can be made concerning the influence of non-uniform irrigations on the relative ranking of irrigation strategies. Only one of the six first-degree stochastically efficient strategies derived under assumptions of uniform application is also risk-efficient under non-uniform conditions. Similarly, only one strategy exists in both sets derived using second-degree stochastic dominance procedures. Clearly, representation of the uniformity with which irrigations are applied has a significant effect on the selection of the irrigation strategies by risk-averse decision makers.

Efficient strategies derived under non-uniform conditions apply as much or more water as their counterparts in the uniform scenario. Extractable soil moisture ratios dictating the time of irrigation are higher, indicating more frequent irrigation. Also, under non-uniform conditions, only schedules that apply 2.8 inch irrigations are efficient. Increased variability in yield, and hence net returns, from applying smaller irrigation amounts eliminates strategies applying 2.1 inch applications from the stochastically efficient set. Irrigation strategies comprising the first-degree stochastically efficient set under non-uniform conditions apply an annual irrigation depth averaging 1.4 inches in excess of the efficient set derived under uniform conditions.

Stochastic Dominance with Respect to a Function

Stochastic dominance with respect to a function is an evaluative criterion which orders uncertain choices for classes of decision makers whose absolute risk-aversion functions are within specified upper and lower bounds (King and Robison). The absolute risk aversion function is defined by the expression:

$$(10)\ R = -u''(y)/u'(y),$$

where $u'(y)$ and $u''(y)$ are the first and second derivatives of a von Neumann-Morgenstern utility function, $u(y)$. The risk intervals and efficient sets derived with the stochastic dominance with respect to a function technique are given in Table 4. The four risk intervals employed are used to represent the preferences of risk-preferring, risk-neutral, slightly risk-averse, and strongly risk-averse irrigators and are based upon the empirical work of Cochran et al.[4] A computer algorithm developed by King and Robison was used to derive risk efficient irrigation strategies for irrigators characterized by the four risk intervals.

TABLE 4. RISK EFFICIENT GRAIN SORGHUM IRRIGATION STRATEGIES FOR DECISION MAKERS CHARACTERIZED BY ALTERNATIVE RISK PREFERENCES UNDER UNIFORM AND NON-UNIFORM IRRIGATION CONDITIONS, OKLAHOMA PANHANDLE

Classification of Decision Maker	Pratt/Arrow Risk Coefficient	Risk Efficient Strategies	
		Uniform Irrigation	Non-Uniform Irrigation
Risk Preferring	−.0008 to −.0001	2.1-30-D 2.8-25-N	2.8-30-D 2.8-35-N
Risk Neutral	−.0001 to .0001	2.1-30-D	2.8-35-N
Slightly Risk Averse	.0001 to .0004	2.1-30-D	2.8-35-N
Strongly Risk Averse	.0004 to .001	2.1-30-D	2.8-35-N

The use of stochastic dominance with respect to a function reduces the risk efficient set of irrigation strategies to a single schedule for three of the four risk intervals under both uniformity scenarios. When irrigations are applied uniformly, the same strategy is identified as risk efficient for risk intervals characterizing strongly risk-averse, slightly risk-averse and risk-neutral decision makers. The risk efficient strategy involves the application of 2.1 inch irrigations based upon a 30 percent soil moisture ratio (2.1-30-D). Strategy 2.8-25-N is added to complete the risk efficient set for risk preferring decision makers. Under the non-uniform irrigation scenario, the stochastically efficient strategy is 2.8-35-N for both strongly and slightly risk-averse decision makers, as well as risk-neutral irrigators. Risk-preferring decision makers would select either 2.8-35-N or 2.8-30-D. These results indicate that risk-averse irrigators place emphasis on the avoidance of low returns. Strategies identified as risk efficient under assumptions of both uniform and non-uniform applications are characterized by infrequent occurrences of low returns resulting from low yields or excessive water applications.

Risk-efficient irrigation strategies are shown to be somewhat insensitive to irrigator risk preferences. Under both scenarios, the strategy identified as risk efficient under preferences characterized as strong risk aversion, mild risk aversion, and risk neutrality are identical. Only in the case of risk-preferring decision makers is the efficient set affected by irrigator risk attitudes. In both cases, a strategy is added to the single strategy identified for risk-averse decision makers to form the risk efficient set.

The efficient set of strategies is affected by the prevailing assumptions concerning the uniformity with which irrigations are applied. As in the economic efficiency and first- and second-degree stochastic dominance solutions, efficient strategies under non-uniform conditions apply larger water quantities than those derived under uniform application assumptions. Strategy 2.8-35-N applies irrigation water more frequently and in larger amounts than 2.1-30-D, resulting in an increase in the average quantity of water applied annually of 2.9 inches. From these results, one can infer that the application of intensive irrigation schedules can at least in part be explained by the non-uniformity with which irrigations are applied.

CONCLUSIONS

Results from the analysis illustrate that although risk aversion may partially explain the intensive irrigation practices of study-area irrigators, additional factors must also influence irrigation scheduling decisions. One of these factors is the uniformity with which irrigations are applied. Results presented indicate that the non-uniformity of irrigation is a primary source of risk faced by study-area irrigators. Variability of net returns resulting from the employment of the irrigation schedules considered increases when the climatic and technological factors affecting irrigation uniformity are represented. Solutions derived using economic efficiency and stochastic dominance criteria indicate that efficient strategies derived under non-uniform conditions apply water quantities in excess of strategies derived under uniform conditions. Thus, one can infer that the non-uniformity with which irrigations are applied contributes to the application of high water-use schedules.

The effect of spatial variability of water applications on irrigation decision making was analyzed based upon a set of representative irrigation conditions. Thus, application of the results are limited to the specific agronomic situation, climatic conditions, and irrigation technology specified in the study. It is expected, however, that the effect of irrigation non-uniformity would be more pronounced if the analysis were applied to surface irrigation technologies or more water stress-sensitive crops. This analysis was also limited to the specific set of circumstances where considerations of crop mix, water scarcity, and water allocation among crops do not enter into the

[4]To prevent the scaling problem identified by Raskin and Cochran, per-acre returns were converted to per-field (130 acre) returns.

irrigation scheduling decision. Incorporation of these considerations would require that water use decisions be represented on a whole-farm basis.

This paper illustrates the need to incorporate the spatial variability of irrigation applications when conducting normative analysis of irrigation scheduling. Failure to do so may result in erroneous water-use prescriptions. The results also indicate the need for irrigators to incorporate uniformity considerations in their irrigation scheduling decisions. Proper use of irrigation uniformity information can reduce the risk inherent in irrigated production and, hence, decrease water-use by increasing the precision with which irrigations may be applied.

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