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# A MODIFICATION OF THE MODIFIED STOLLSTEIMER LOCATION MOLED*

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The Stollsteimer plant location model is a normative tool appropriate for determining least-cost number, size and location of a subindustry's marketing facilities [5]. Several modification and extensions of the basic model have increased its value to the applied economist. Ladd and Halvorson developed a procedure to determine sensitivity of the optimal solution to variation in model parameters, i.e., the researcher may resolve how magnitude cost parameters are altered before the solution becomes non-optimal [3]. The basic model's solution procedure prevented application where large numbers of potential plant sites were involved. A recent modification by Warrack and Fletcher effects a reduction in required computer time by approximating optimization, thus increasing size of plant location problems investigated [6]. Polopolus extended the basic model to encompass multiple product plants [4] and, in collaboration with Chern, modified the basic Stollsteimer model to permit substitution of a discontinuous, long-run plant cost function for the strategically assumed continuous linear form [1]. Prior to the latter modification, the basic model accommodated only a long-run total plant cost function which was linear with a positive intercept.

The author applied the Chern-Polopolus modification, adapted for inclusion of the discontinuous plant cost function, and observed that the solution procedure did not resolve a minimum cost solution in the applied situation. This paper examines that case and formulates the computational scheme which enabled the researcher to lower total system costs below those obtained with the modified Stollsteimer model. The developed procedure would appear to be appropriate for plant location problems characterized by a discontinuous plant cost function, a small number of potential plant locations, and high processing costs relative to transportation cost. The following discussion is based on the assumption that plant economies for size exist.

## THE STOLLSTEIMER MODEL AND ITS MODIFICATION

In contrast to less realistic plant location models, which treat space as continuous for purposes of defining efficient market organization, the Stollsteimer model specifies a finites set of raw material sources and preselected potential plant locations[1] [5]. Assembly cost between each pair of supply sources and potential plant locations is constructed in a matrix format. The first step of the Stollsteimer models' solution procedure involves determination of raw product allocations which minimizes total assembly cost for each possible number of potential plant locations. This is obtained by way of a full enumeration of all possible combinations for each number of plant locations. The second step includes a determination of plant costs associated with those raw product allocations which minimize total assembly cost. Total plant costs associated with the slope component of the linear long-run plant cost function is invariant for a fixed volume of study area production, while the intercept value varies directly with plant numbers or locations. Therefore, total plant costs are simply a function of plant numbers. Because of the assumed plant cost form, the outcome of step one does not affect plant cost determination in step two. Finally, minimized assembly cost is aggregated with plant cost and plant numbers, sizes and locations minimizing combined

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[1] The model is equally capable of determining optimum industry organization with respect to either assembly or distribution systems, however, it is not applicable when both must be considered.

costs resolved.[2]

The minimized assembly cost relationship, developed during the first step of the outlined solution procedure, exhibits a negative slope. Total plant costs (developed in second step) exhibit a positive slope with respect to increasing plant numbers. If a plant were added to the optimal number, total system costs would increase because plant cost associated with this addition (intercept value of plant cost function) is greater than reduced transportation cost. Likewise, if a plant were removed, additional transportation cost would be greater than reduction in plant cost (intercept value of plant cost function). The Stollsteimer model's solution procedure guarantees a global minimum, given the assumptions regarding plant cost form.

A model's appropriateness for any location problem is in part dependent on the empirical plant cost function and its similarity with the assumed form. Long-run plant cost function is constructed from segments of a family of short-run functions. In a theoretical extreme, the long-run function is continuous, although virtually unlikely for most plant types [2]. The modified Stollsteimer model was developed for those plant location problems where the long-run cost function is discontinuous, i.e., where there are indivisibilities in plant durable equipment and number of available plant sizes are limited. Because the number of available technologies and equipment for constructing alternative plant sizes is limited, the long-run cost function has intermittent breaks. That is, the long-run function is discontinuous. The number of segments comprising the discontinuous function is identical with the number of available plant sizes. Because of technological limitations or institutional constraints, there is typically a maximum plant size beyond which expansion is not feasible.

Individual short-run plant costs or segments comprising the long-run function are generally estimated via the economic engineering technique. Typically, empirical total plant cost functions exhibit a positive intercept and constant marginal costs. The theoretical validity of this functional form has been established by French, Sammet and Bressler [2].

The modified model features a two-step solution procedure [1]. The first step involves operations on a transportation cost matrix analogous to that performed by the Stollsteimer model. By full enumeration, the least-cost locational pattern, least-cost allocation of raw product, and corresponding minimized total assembly cost are determined for each potential plant location. Secondly, a minimization of total assembly and plant cost with respect to number of locations, numbers of plants and plant size pattern is resolved. This is accomplished by first determining plant numbers and corresponding plant sizes which minimize total plant cost for each location within each optimum locational pattern. Because of the discontinuous cost function, a location may include more than one plant. After least-cost plant number and size patterns for each location within each optimal locational pattern have been determined, plant costs are aggregated. This procedure is repeated for the various number of considered potential plant locations. The overall optimum solution is determined by aggregating minimized assembly and plant cost and identifying the minimum.

Accomplishment of the initial step, with the modified solution procedure, assures that the allocation of raw product is least-cost. In addition, the solution procedure assures that the plant size pattern is least-cost, given the allocation of raw product determined in the initial step. But, because of the discontinuous plant cost function, the outcome of step one affects plant cost determination in step two. That is, if the plant size pattern were not determined by the least-cost raw material allocation, an alternative plant size and number pattern may yield lower cost. Allocation of raw product to each location within each optimal locational pattern is accomplished without consideration for plant cost. As a consequence, several factors affect plant cost unfavorably. They are: 1) selection of a plant size pattern which exhibits an excessive loss in industry economies of size[3] as plant locations increase; 2) a tendency for the creation of excess plant capacity and 3) within each optimal plant size pattern an inflexibility to allocate raw material between plants at alternative locations to lower industry plant costs.

In essence, the above two-step solution scheme answers two questions in sequence: 1) what allocation of raw material minimizes total cost of as-

[2] The outlined solution procedure is appropriate when there are economies of size in plant operation and plant costs are independent of location. The model may be modified to include those cases where there are no economies of size in plant operation and plant costs are dependent on location.

[3] Industry economies of size is a dissimilar concept to economies of size. The industry economies of size concept refers to the lowering of area processing costs as the number of plants in an industry decreases. See Chern and Polopolus [1] for discussion of total industry economies of size.

sembly for each potential plant location and 2) what plant size pattern minimizes industry aggregated plant cost for the least-cost raw product allocation for each potential plant location. An alternative procedure might answer the following questions sequentially: 1) what plant size pattern for each location minimizes industry processing cost and 2) which locational pattern for each optimum plant size pattern minimizes total cost of assembly. The alternative procedure reverses the sequence in which similar questions are answered. By reversing the steps in which assembly and plant costs are optimized, cost priorities are reversed. That cost initially optimized is given priority, since the second stage optimization is forced to accommodate the first stage decision. Thus, solutions rendered with the modified Stollsteimer place a priority on assembly cost, whereas, solutions obtained with the alternative procedure emphasize plant cost. The appropriate procedure depends on whether plant or assembly cost is the dominating cost factor. With the alternative procedure, raw material is allocated to accommodate a least-cost plant size pattern resolved in the first step. Clearly, this raw product allocation would increase assembly costs above those calculated with the modified Stollsteimer model. However, if the decrease in total plant costs were greater than the increase in total assembly costs, aggregated cost would be lower than those calculated with the modified Stollsteimer model.

## MATHEMATICAL REPRESENTATION OF PROBLEM AND ALTERNATIVE MODEL

Consider M production origins, N potential plant locations and k plant sizes, $S_1, S_2, \ldots, S_k$. We seek that number [T] of locations, that set of locations ($l[1], l[2], \ldots, l[T]$), and that set of sizes ($S[1], S[2], \ldots, S[T]$) at the chosen locations which will minimize total system transportation and processing costs. Namely, minimize

$$\sum_{r=1}^{T} [F_{S(r)} + \sum_{i=1}^{M} X_{ir}(T_{il(r)} + C_{S(r)})]$$

over variations in T; $l[1], \ldots, l[T]$; $S[1], \ldots, S[T]$: $X_{i1}, \ldots, X_{mt}$.

Where:

$F_{S(r)}$ = fixed cost in a plant of size $S_{(r)}$

$C_{S(r)}$ = unit cost in a plant of size $S_{(r)}$

$X_{ir}$ = expected volume of raw material from origin i processed at plant r

$T_{il(r)}$ = unit assembly cost for transporting the raw material from the *i*th origin to location $l_{(r)}$.

An admissible solution must be capable of processing the expected production, namely,

$$\sum_{i=1}^{M} \sum_{r=1}^{T} X_{ir} \leqslant \sum_{r=1}^{T} S(r)$$

Available and computable technology did not permit us to obtain a global minimum through simultaneous variation in all dimensions of the problem. An approximation to the global minimum was obtained in three distinct steps. (1) Temporarily fix T, the number of plants, and determine the plant sizes which will minimize total processing cost, for T = 1, 2, . . . , N; (2) for each T, locate the least-cost plant size pattern in the production region to minimize total assembly cost; (3) aggregate minimized plant and processing cost and determine the number of plants to find the overall least-cost solution.

The above approach may be represented mathematically as follows:

**Step I.**

For each T, T = 1, 2, . . . , N, find S[1], S[2], . . . S[T] and $X_{i1}, \ldots, X_{MT}$ to minimize total plant cost [TPC]. The chosen plant sizes (S[1], S[2], . . . , S[T]) must be available, namely, be selected from k plant sizes, $S_1, S_2, \ldots S_k$.

$$TPC = \sum_{r=1}^{T} [F_{S(r)} + C_{S(r)} \sum_{i=1}^{M} X_{ri}]$$

Denote the resulting minima, one for each value of T, by $TPC_T$.

**Step II.**

Using the least-cost plant pattern for each T, determined in Step I, find the locations $l[1], \ldots, l[T]$ which minimize total assembly cost [TTC]:

$$TTC = \sum_{r=1}^{T} \sum_{i=1}^{M} X_{ir} T_{il(r)}$$

Denote the respective minima by $TTC_T$.

**Step III.**

Using the results for each value of T found in Steps I and II, find that value of T which minimizes total cost [TC]:

$$TC_T = TPC_T + TTC_T$$

$$= \sum_{r=1}^{T} [F_{S(r)} + \sum_{i=1}^{M} X_{ir} (T_{il(r)} + C_{S(r)})]$$

Minimization of the total plant cost (TPC, Step I) function was executed by developing an algorithm which enumerated costs for all plant combinations capable of processing study area production for each of the T locations. Volume was allocated between plants in each plant size combination, so that processing costs were minimized. Given the least-cost plant size pattern and associated volumes, a fast transportation code calculated total assembly cost (TTC, Step II) associated with allocating these specified volumes among potential locations.[4] That locational configuration, minimizing total assembly cost for each of the T locations, was calculated via this procedure. The optimal or least-cost T was resolved by aggregating the minimized processing (Step I) and assembly (Step II) costs for each of the T locations.

## EMPIRICAL COMPARISON OF MODELS

Each model was applied to a southwestern irrigated valley to resolve least-cost number, size and location of central cotton gin plants. The valley is approximately 90 miles long and averages 2.5 miles wide. The study area was divided into 124 production sources and six potential plant locations selected on the basis of accessibility, zoning laws, and concentration of production.

The least-cost solution obtained with the modified Stollsteimer model involved three 40 bale per hour plants operating at a single location (Table 1). The forty bale per hour gin was the maximum available plant size and the most efficient when operated at near capacity. The least-cost organization rendered with the alternative solution procedure included the three 40 bale per hour plants but at separate locations. Each procedure gave identical total plant costs ($1,587,344). However, the solution involving plants at separate locations reduced transportation cost by $28,722. Therefore, the alternative procedure effected a cost reduction of $28,722, relative to the modified Stollsteimer model.

Characteristics of solutions obtained by each procedure are of interest. When one location is considered, the least-cost plant size and number are identical (Table 1). Therefore, if area production can be processed by a single plant, each solution procedure renders identical least-cost solutions. Differences occur when more than one plant is necessary to process area production. As the number of potential plant locations increases, the modified Stollsteimer model requires that the least-cost plant size pattern accommodate volumes at alternative locations which were allocated by minimization of total transportation costs. Generally, this results in a loss of industry economies of size and a corresponding increased total plant cost. For example, the two location solution obtained with the modified Stollsteimer model included 40 and 32 bale per hour plants and 40 and 16 bale per hour processing 73,355 and 56,645 units, respectively. Because of the loss in industry economies of size, total plant costs were $125,061 greater than the single location solution. Total cost of the two-location solution is larger than the single location solution because plant cost increase is more than assembly cost decrease. As a result, the multiplant, single location solution becomes optimal. The least-cost solution resolved with the modified Stollsteimer may involve more than one plant per location, whereas, the optimum obtained with the alternative procedure will include only one plant per location.

The first step of the alternative solution procedure involves selection of least-cost plant size patterns for each considered location. This is executed via a computer program enumerating plant cost associated with all plant size combinations capable of processing area production. In the above problem, seven plant cost functions were considered —199 of the plant size combinations were capable of processing area production. These calculations were executed on an IBM 360/65 in approximately three seconds. Next, least-cost plant size patterns were located among potential locations to minimize transportation cost. This was executed through using a fast transportation code programmed to calculate transportation cost associated with locating the least-cost plant size patterns among all combinations of potential locations. Locating least-cost plant size pattern among all potential locations by a full enumeration of all least-cost plant sizes would be represented by a permutation and would require excessive computer time. Therefore, attention was directed away from ordering or permutations and toward combinations or only a partial enumeration. For example, to locate six different plants among 12 potential locations would necessitate ${}_{12}P_6 = 665280$ separate transportation code solutions or approximately 924 hours of computer

[4] The transportation code was developed by Dr. V. Srinivasin, Graduate School of Management, University of Rochester, Rochester, New York.

**Table 1. COMPARISON OF OPTIMUM NUMBER, SIZE AND LOCATION OF COTTON GINNING PLANTS RESOLVED WITH THE ALTERNATIVE SOLUTION PROCEDURE AND THE MODIFIED STOLLSTEIMER MODEL**

| | Alternative Solution Procedure | | | | | | Modified Stollsteimer Model | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number of Locations (T) | Plant(s) Location Pattern | Plant Size Pattern | Volume Processed per Location | Plant Cost ($) | Transportation Cost Associated with Distance ($) | Total Systems Cost ($) | Plant(s) Location Pattern | Plant Size Pattern | Volume Processed per Location | Plant Cost ($) | Transportation Cost Associated with Distance ($) | Total Systems Cost ($) |
| 1 | A | 3 - 40 BPH | 130000 | 1587344 | 61240 | 1648584 | A | 3 - 40 BPH | 130000 | 1587344 | 61240 | 1648584 |
| 2 | B | 1 - 40 BPH | 44000 | | | | B | 1 - 40 BPH<br>1 - 16 BPH | 56645 | | | |
| | C | 2 - 40 BPH | 86000 | 1587344 | 40515 | 1627859 | C | 1 - 40 BPH<br>1 - 32 BPH | 73355 | 1712405 | 39327 | 1751732 |
| 3 | A | 1 - 40 BPH | 44000 | | | | D | 1 - 40 BPH | 39625 | | | |
| | B | 1 - 40 BPH | 44000 | | | | F | 1 - 40 BPH<br>1 - 24 BPH | 69558 | | | |
| | D | 1 - 40 BPH | 42000 | 1587344 | 32518 | 1619862 | G | 1 - 24 BPH | 20827 | 1726263 | 24942 | 1751205 |
| 4 | C | 1 - 32 BPH | 35200 | | | | C | 1 - 40 BPH | 37713 | | | |
| | E | 1 - 32 BPH | 35200 | | | | F | 1 - 40 BPH<br>1 - 8 BPH | 44766 | | | |
| | F | 1 - 32 BPH | 35200 | | | | G | 1 - 24 BPH | 18902 | | | |
| | G | 1 - 24 BPH | 24400 | 1680100 | 25762 | 1705862 | I | 1 - 32 BPH | 28626 | 1847380 | 19144 | 1866524 |
| 5 | A | 1 - 32 BPH | 35200 | | | | A | 1 - 32 BPH | 30966 | | | |
| | B | 1 - 32 BPH | 35200 | | | | B | 1 - 32 BPH | 26585 | | | |
| | E | 1 - 32 BPH | 35200 | | | | C | 1 - 24 BPH | 25181 | | | |
| | G | 1 - 16 BPH | 17500 | | | | G | 1 - 24 BPH | 18642 | | | |
| | H | 1 - 8 BPH | 6900 | 1810082 | 21508 | 1831590 | I | 1 - 32 BPH | 28626 | 1853142 | 15837 | 1868979 |
| 6 | A | 1 - 32 BPH | 35200 | | | | A | 1 - 24 BPH | 19522 | | | |
| | B | 1 - 24 BPH | 25000 | | | | B | 1 - 24 BPH | 19695 | | | |
| | C | 1 - 32 BPH | 35200 | | | | C | 1 - 24 BPH | 25181 | | | |
| | G | 1 - 16 BPH | 17193 | | | | F | 1 - 24 BPH | 18304 | | | |
| | H | 1 - 8 BPH | 214 | | | | G | 1 - 24 BPH | 18642 | | | |
| | I | 1 - 16 BPH | 17193 | 1928836 | 17782 | 1946618 | I | 1 - 32 BPH | 28626 | 1993941 | 13599 | 2017540 |

execution time on an IBM 360/65. If combinations were considered, ${}_{12}C_6 = 924$ separate solutions would be necessary. Since each solution requires 3-6 seconds of execution time, the combination ${}_{12}C_6$ would consume approximately 77 minutes.[5] Clearly, the alternative solution procedure is only appropriate where small numbers of potential plant locations are considered.

In case of the cotton gin location problem, the overall least-cost solution consistently included the overall optimum plant size pattern. Attention, then, was forced on optimally locating just this plant size pattern. This substantially reduced computation requirements.

The appropriateness of the developed procedure is dependent on the specific characteristics of the plant location problem to be resolved. Generally, if the modified models solution exhibits a substantial loss in industry economies of size, as considered locations increase and per unit mile costs are low, then the developed procedure will tend to be appropriate. For example, in the above problem, the optimum resolved with the alternative procedure included three plants at three separate

[5] Srinivasin modified the employed transportation code so that it is about 2.1 times as fast. See Srinivasin, V. and G. L. Thompson, "Benefit-Cost Analysis of Coding Techniques for the Primal Transportation Algorithms," *Journal of the Association for Computer Machinery*, 20:194-213, (1973).

locations incurring a plant cost of $1,587,344 and transportation cost of $32,518. In contrast, the three location solution, resolved with the modified Stollsteimer, involved four plants operating at an annual cost of $1,726,263—a cost disadvantage of $128,919 relative to the overall optimum solution. Because of the low unit-mile cost ($.025), the modified models transportation cost savings relative to that calculated with the developed procedure was only $7,576. Clearly, the transportation cost saving associated with the modified Stollsteimer is inconsequential relative to loss in industry economies of size.[6]

## SUMMARY

Recent modifications and extensions of the Stollsteimer model have improved the level of realism which can be incorporated into plant location problems. One of the most promising modifications involved substitution of a discontinuous long-run plant cost function for the strategically assumed linear continuous form. In a specific plant location problem, the modified Stollsteimer model did not render a minimum cost solution. An alternative solution procedure was developed which lowered total system costs below those calculated with the modified Stollsteimer. However, because of extensive computation requirements, the alternative procedure is limited to small plant location problems. Neither the modified Stollsteimer model nor the developed solution procedure attain a global minimum through simultaneous variation in all dimensions of the problem. The appropriate model depends on specific characteristics of the plant location problem. Generally, if unit transportation cost is low relative to unit plant cost, the developed formulation is appropriate; if the opposite cost situation exists, the modified Stollsteimer model is best adapted.

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[6] The reported solution procedure was developed and applied while the author was employed by the Department of Agricultural Economics at New Mexico State University.