A BIO-ECONOMIC DYNAMIC PROGRAMMING ANALYSIS OF THE SEASONAL SUPPLY RESPONSE BY FLORIDA DAIRY PRODUCERS

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Abstract

Seasonal price premiums have been proposed as a means of dampening the highly seasonal patterns of milk production in Florida. A Markov decision bio-economic model of the breeding and replacement decisions was solved via stochastic dynamic programming and used to analyze the potential supply response to seasonal price premiums. The results of the analysis suggest that the seasonal milk supply in Florida is highly price inelastic.

Key words: dairy production, price analysis, bio-economic model, herd breeding and replacement

Dairy production follows a seasonal pattern. Milk production is typically higher in the spring and early summer months due to weather, breeding patterns, and the availability of forage.[1] Seasonality can also be found in the demand for dairy products. Fluid milk consumption traditionally has a lower demand in the summer months due to consumer preference for substitute drinks and a higher demand in the cooler months due to the school lunch program. In many regions, seasonal patterns in production and consumption do not coincide and result in excess capacity problems and additional costs to the dairy processing industry. These costs are then partially transferred to consumers in the form of higher prices, or borne by taxpayers through the dairy price support system. A production pattern in line with seasonal consumption patterns would benefit both consumers and producers.

Efforts at leveling seasonal milk production include the adoption of a seasonal price plan, which has been used by some federal or state milk marketing orders to provide dairy producers with an economic incentive to increase production during periods of low production and to discourage production in the high season. Seasonal pricing plans include the Louisville plan and the base excess plan. Under the Louisville plan, a specific amount is withheld from the blend price during the months of normally high production. In the months when milk production is at its lowest level, a premium is paid. Under the base excess plan, every year each producer establishes a base equal to his average daily delivery of milk during the season of high production for the market. In the base-paying months, a producer is paid a higher price for the portion of milk that does not exceeds the base, and a lower price for deliveries that exceed the base. By 1986, 18 out of 48 federal milk marketing orders had adopted some form of seasonal price plan (Kaiser et al.). There are serious debates about the effectiveness of these seasonal price plans in the states already having such systems, and about the feasibility of initiating a seasonal price plan in other states.

DAIRY MARKETING IN FLORIDA AND PROBLEM STATEMENT

The Florida Marketing Order is primarily a Class I market with 90 percent of total production in the major Florida milk cooperatives marketed as fluid milk (Kilmer and Blake). Florida cooperatives have been able to negotiate full supply contracts with Florida milk processors. Hence, the cooperatives act as the sole procurement agents for most fluid milk processors operating in the state.

In tropical weather zones, dairy cows follow a natural cyclical conception pattern, which indicates a higher conception rate in the mild winter months of December-April, calving in the early winter, and

[1] The pattern of production seasonality shifts with changes in latitude, e.g. in Canada there is a surfeit of milk production in the summer, while there is a paucity in the winter. See Prindle et al.

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peak milk production in February-April. During this period, Florida milk cooperatives are able to fully supply processors with locally produced milk. The hot, humid summers in Florida, however, depress milk production and inhibit reproduction, which reduces production in the next lactation. The result is a pronounced seasonal reduction of milk production in the late summer (July through October). In some years, production in the early spring months may exceed production in late summer by 50 percent. The difference between production and utilization is met by imports from other milk producing areas.

The milk deficit for Florida dairy cooperatives in most months can be met by imports from nearby states. In summer, however, these states face similar production problems and import sources are more distant. A problem faced by Florida milk cooperatives is that as they move away from Florida in search of supplemental milk supplies, the cost of imported milk increases. As the level of imports has increased in recent years (approximately 25 percent of annual consumption) the cost of importing milk has caused increasing concern (Kilmer and Blake). In some years, the cost of milk procured from more distant locations (primarily the Minnesota-Wisconsin area) plus transportation exceeds the negotiated price of the full supply contracts. In this case, prices paid to cooperative members are reduced.

The primary problem addressed in this paper is the effectiveness of introducing a system of price premiums for seasonal deliveries of milk in Florida as a means of ameliorating the seasonal swings of milk supplies. The analysis is conducted through a study of the farm-level response of seasonal milk production with seasonal price premiums. The current payment system used by Florida dairy cooperatives is essentially a constant price system adjusted for the cost of imports. In this study, several hypothetical premium-penalty seasonal price plans are tested for their projected effectiveness under the assumption that dairy producers are profit maximizers, who respond to price and cost changes subject to the economic and biological constraints of their operations. To test the strength of these constraints on seasonal production, the sensitivity of the model to alternative specification of feed costs, replacement heifer costs, carcass value, calving rate, and lactation curves is evaluated.

PREVIOUS WORK AND MODEL SPECIFICATION

The issue of seasonal milk production has been addressed by other authors. Prindle described various seasonal plans in place for certain marketing orders in 1977. Prindle and Livezey and Caine and Stonehouse used linear programming techniques to conduct a representative farm analysis of seasonal price patterns and supply response. Hall et al. used budgeting techniques to compute returns to dairy producers under different seasonal production patterns. Kaiser et al. surveyed dairy producers in the New York-New Jersey market order to detect the potential response to various plans intended to reduce seasonal fluctuations in milk production.

Other milk supply response studies have used annual data. Chavas et al. and Weersink and Howard estimated regional milk supply functions and studied supply elasticities across regions. Chavas et al. concluded that milk supply is inelastic in the short-run but elastic in the long-run. Luh and Stefanou used an econometric model to investigate dairy farmers' risk attitude in the presence of output price uncertainty and they found that output price variation has a significant impact on production decision making.

In this study, a different approach is taken. Rather than estimating supply functions by conventional econometric methods using lagged prices, quantities as explanatory variables, a dynamic programming model is used to simulate farmers' optimum behaviors, and their responses to different price scenarios. This approach analyzes biological factors as well as economic variables which affect supply response by dairy producers. The dominating factors in seasonal production variation are also identified. It is recognized that dairy producers can alter the seasonal pattern of production through two vehicles: the timing of breeding and of replacement. The breeding decision determines the optimum time to breed a cow, while the replacement decision determines when to replace the cow with a heifer, and hence maintain optimal herd life. Thus to examine the impact of seasonal price premiums on the pattern of production, it is essential to analyze the replacement and breeding decisions. Changes in the implied seasonal milk production distribution are calculated by assuming that farmers will follow the optimal replacement and breeding plan. Dynamic programming is used to obtain the optimal plan.

Numerous studies have been published related to replacement of dairy cows (Smith; Stewart et al. 1977, 1978; McArthur; Kristensen and Ostergaard). Recent work has characterized the dairy replacement and breeding problem as a Markov decision process solved via dynamic programming. In the Markov decision model, each dairy cow is assumed to pass through a number of states which differ according to the age of the cow, period of lactation, production level within a lactation cycle, pregnancy status, and other factors. The dynamic programming problem is to solve

$$(1)\ F_t(X_i) = \max_k \{S_{it}(k) + \delta \sum_{j=1}^{R} P_{ij}(k) F_{t+1}(X_j)\}$$

for $X_i = 0, 1,..., R$, and R is the number of possible states. $F_t(X_i)$ is the present discounted value of a cow in state X_i stage t, which follows the optimal policy; $S_{it}(k)$ is the pay-off associated with cow in state X_i in stage t under decision k; $P_{ij}(k)$ is the transition probability that a cow in state X_i in stage t will be in state X_j in state t+1 under decision k; δ is the discount factor.

The algorithm used to solve equation (1) begins in the terminal stage (stage T). It is typically assumed that all cows are replaced and hence slaughtered in stage T. The algorithm next moves to stage T-1 and identifies those states which represent cows that should be kept and those cows which should be replaced. Furthermore, if the cow is in the breeding period, the decision whether to breed or leave her open (not pregnant) is identified. The algorithm continues for stages T-2, T-3, etc. The algorithm terminates when the optimal policy converges, that is, the pattern of replace states, breeding states, and keep states is unaffected by the assumption that all cows in the terminal stage are slaughtered. This is a probabilistic dynamic programming model where the probability system follows a Markov process. This algorithm is called the method of successive approximations. For further discussion see Hillier and Liebermann (pp. 570-75) or Dreyfus and Law (pp. 179-85).

The size of the model is dictated by the number of possible states for dairy cows. Several factors called state variables are used to delineate states. State variables are used to represent the economic and biological status of the cow. For example, Smith expressed predicted milk production as a function of the two previous lactation production levels and the previous calving interval. So they are included among the state variables.

This approach towards delineation of state variables demonstrates one strategy to satisfy the Markov requirement. The Markov assumption requires that the process must be such that the optimal decision made at any stage is dependent only on the state of process at that stage and that an optimal policy is followed thereafter. In dairy cows, future production depends on a number of factors including the past history of the cow. To satisfy the Markov assumption, lagged productions are defined into the same state, which results in the number of states being greatly increased.

A recent study by Van Arendonk (1986,1987) represents the single most complete study of dairy cow replacement and breeding. In this study, Van Arendonk's model was adapted to dairy production in Florida. Five factors are used to delineate state variables: lactation number, period of lactation, milk production level during the present lactation, breeding period, and month of previous calving. Since the month of calving uses calendar month, the length of each stage is one month.

Production level is divided into 15 categories. Mean production is realized at the eighth category, and seven equal-sized intervals are determined above and below the mean. Rebreeding is assumed to occur between three and eight months after calving. This is treated as a stochastic event based upon estimated conception rates. The length of each lactation depends upon the success of rebreeding. Maximum calving interval is 16 months. The combination of month of previous calving and current pregnancy status gives rise to 70 possible states within a lactation. Maximum life of a dairy cow is assumed to be 12 parities (approximately 14 years old). There are 12 possible months of previous calving. Therefore, the total number of states is 15 x 12 x 12 x 70 = 151,200.

It is common practice in dairy production to have a limited breeding period for open cows. The relevant decision space for cows in the breeding interval is three-dimensional and includes the decisions: breed (INS), replace (REP) and leave open (OPEN). All other cows are outside the breeding interval. The relevant decisions for these cows are keep (KEEP) and replace.

Given the current state of an individual cow, the optimum decision and the corresponding maximum expected value of the cash flow from stage t is given by the solution of the dynamic programming problem (1). Each decision k results in different values of $S_{it}(k)$, $P_{ij}(k)$ and $F_{t+1}(X_j)$. For instance, if the decision of replacing the cow is made for a cow in state X_i in stage t (k = REP), the net present value of the cash flow from stage t ($Rep_t(X_i)$) is:

$$(2)\ Rep_t(X_i) = \{S_{it}(REP) + \delta \sum P_{ij}(REP) F_{t+1}(X_j)\}.$$

Using this format, the stochastic dynamic programming problem given by (1) can be adapted to the dairy cow replacement/breeding problem as:

-open cow during insemination period:

$$(3)\ F_t(X_i) = \text{Max}\ (OPEN_t(X_i), INS_t(X_i), REP_t(X_i);\quad t = 1,...,T-1$$

- pregnant or open cow outside the insemination period:

$$(4)\ F_t(x_i) = \text{Max}(\ KEEP_t(x_i)\ REP_t(x_i)\ ),\quad t = 1,\ldots,T-1.$$

The specification of the present value of the cash flow for the decision variables ($Open_t(X_i)$, $Ins_t(X_i)$, $Rep_t(X_i)$, $Keep_t(X_i)$) in a dynamic programming framework is presented in Appendix A. The objective function given in equation (3) applies to cows in their respective breeding periods. All other cows are not bred, and equation (4) is appropriate in this case.

In the adaptation of the model to Florida, economic and biological data of dairy production were used in the estimation of the parameters used in the dynamic programming model. Certain assumptions, consistent with dairy production in the state, were made. Cows are divided in two classes based on age and parity: heifers and cows in their second lactation or higher. Lactation production equations have been estimated by DeLorenzo and Maley for each class. In these equations, daily production depends upon month of freshening and days in milk. Seasonal conception rates for Florida dairy cows were estimated from Dairy Herd Improvement Association (DHIA) records. Since gestation in cattle is approximately nine months, the conception rate in January multiplied by the lactation production of a cow freshening in October gives expected milk production in November. Expected monthly milk production for the herd is the sum of the milk production from all cows milking in that month.

The biological and economic data used to specify to optimization model given in equations (3) and (4) are a composite of all dairy producers which participate in the DHIA. Two cautions should be noted. First, the results of the optimization model are extrapolated to the state. Differences across individual producers are not recognized. Hence, the supply response projected by the model is subject to aggregation error inherent in studies which use micro-level data. Second, given that DHIA participants are likely to be among the better managers, it is possible that the parameters estimated for the biological relationships are not truly representative. For this particular study, the bias introduced from this sampling problem was likely to have minimal effect on the conclusions.

Feed requirements are based on daily energy and protein requirements for maintenance and milk production. Total digestible nutrients (TDN) and crude protein (CP) requirements are taken from the National Academy of Sciences. To determine feed costs, actual rations used in Florida dairies were examined. It was determined that soybean meal, corn, and corn silage comprise a typical ration in Florida. Using average Florida prices from 1986-1987 as reported in *Feedstuffs*, the cost of the ration was determined. The value of TDN and CP in the typical ration were estimated to be $.06/lb for TDN and $.12/lb for CP. Seasonal TDN and CP prices were obtained by adjusting these average prices with a seasonal feed index. These values were multiplied by the respective nutrient requirements and added to give daily feed cost.

Milk prices in the base model are based on monthly blend prices received by Florida dairy producers in 1987 as reported by DeLorenzo. These blend prices are net of transport costs from the farm to milk processors and price adjustments imposed by the cooperative due to milk imports. The average annual price in 1987 was $13.80 per cwt which ranged from a high of $14.37/cwt in December to a low of $13.39/cwt in July.

Prices for culled cows are taken from average monthly prices for slaughter utility cows from 11 Florida livestock auctions over the period 1980-1987 (Department of Agriculture). Both heifer and bull calves were assumed to be sold at birth. The practice of not raising replacement heifers is predominant in Florida dairy herds. Prices for replacement heifers are not published. Average annual replacement heifer prices are approximately $1000 per head (DeLorenzo). Seasonal replacement heifer prices were calculated by multiplying the average replacement price by a seasonal price index for Florida feeder calves (Simpson and Alderman).

A four percent real rate was used to discount future revenues. Florida dairies have exhibited increased milk production per cow over the last 20 years. To account for this fact, it was assumed that for heifers entering the herd, milk production increases 100 kg per head per year (DeLorenzo). No other costs, including labor, facilities, and land were considered, because these costs do not affect replacement policy.

The optimal decision plan derived from the dynamic programming model was used to simulate farmers' responses to seasonal price changes. Operationally, this analysis involved specification of the seasonal price system into a system of models which include lactation equations, feed requirements and costs, milk revenues, etc. (as described in the previous section). These models were used to estimate the performance, revenues, and costs of dairy cows under different production and price situations, and thereafter calculate optimal decisions on rebreeding and replacements by using equations (3)-(4). Lastly, expected monthly milk production was computed based upon the optimal breeding and replacement policy. Because milk production associated with each state is known, computation of expected milk production from the optimized system involves determination of the probability that a cow will be in a particular state. Then expected milk production in month s is the sum over all states of the probability that a cow will be in state i in month s times milk production associated with state i in month s.

After the optimal solution (i.e. the optimal decision for each state) was determined, the probability that a cow will be in a particular state remained to be determined. These post-optimization probabilities are called the steady-state probabilities. The approximate steady-state probabilities are the probabilities of a cow being in state i after a large number of transitions, and these probabilities are independent of the initial states. The expected production of each state is the product of steady state probability and production level in that state. For details regarding estimation of the steady state probabilities, see DeLorenzo et al.

A program was written in FORTRAN to solve the dynamic programming model. The program consists of three modules. The first module was used to compute the expected costs and returns and expected milk production associated with each state of the system. The second module employed the method of successive approximations to determine the optimal replacement and rebreeding strategy. The third module estimated the steady-state probabilities and the expected monthly milk production associated with the optimal solution.

EMPIRICAL RESULTS

Expected monthly milk production per cow based on optimal replacement and rebreeding decisions is shown in Figure 1. The highest production level was obtained in February, while the lowest production was realized in July and August. Cows that freshened in the late summer months were more likely to be replaced than cows that freshened in the winter. Highest net present value was obtained for heifers calving in October through December and the lowest net present value was estimated for heifers calving in February through April. This result is consistent with the observation that those animals that calve in fall are lactating through the peak winter period, while those calving in spring suffer through the low production summer period in both their first and second lactations.

A seasonal coefficient of milk distribution was calculated to give an index of seasonality. The seasonal coefficient is defined as

$$(5)\quad sc = \left(\sum_{i=1}^{4} MP_i - \sum_{i=7}^{10} MP_i \right) / \sum_{i=7}^{10} MP_i$$

for MP_i represents milk production in calendar month i. January through April (i=1,...,4) and July through October (i=7,...,10) were used in the index because these periods correspond to the months of highest and lowest production, respectively. A positive coefficient implies a seasonal production pattern, a zero coefficient implies no seasonal pattern, and a negative coefficient implies a seasonal production pattern which is higher in the fall and lower in the spring. This seasonal coefficient was used to measure the supply responses of dairy farmers to various price plans and various seasonal factors. The base model seasonal coefficient is 0.33.

The current Florida blend price structure is that summer prices are slightly lower than winter prices. A run of the model was made in which constant prices over all months was specified. The results of this simulation are virtually identical to the base run. Another price set was examined in which milk deliveries in the July to November period were paid a 20 percent premium relative to deliveries in other months. This price set (equivalent to base-excess plan) had little effect on the seasonal production pattern. Production increased by three percent in July and August and by two percent in September. The seasonal coefficient was only reduced to 0.30 (Table 1).

The Louisville plan is a system of premiums and penalties imposed on seasonal deliveries of milk. Given the small response to premiums only, two Louisville-type plans were analyzed. The first price set was a 20 percent premium in the July-November period and a 10 percent penalty in all other months. The second price set imposed a combination of a 50 percent premium and a 30 percent penalty over the same delivery months.

The results of the two antiseasonal price sets and the base run on seasonal milk distribution are shown in Figure 1. The 20 percent premium - 10 percent penalty price set resulted in a larger response than did the premium-only price set. In August, production increased by 6.5 percent and in January, production decreased by 5.4 percent compared to the base run. The seasonal coefficient is 0.23. The change in production is small relative to seasonal imports. The 50 percent premium - 30 percent penalty price set had a much larger effect on the seasonal milk distribution. Total production over the July to October period increased by 14 percent; production over the December to February period decreased by a similar

Table 1. Seasonal Coefficients Under Different Economic and Biological Variation Scenarios

Scenarios	Seasonal Coefficients	Percentage Change w.r.t. BASE
BASE	0.332	
+20PT[a]	0.305	-8.00
+20PT10[b]	0.236	-28.80
+15PT10	0.259	-22.00
+50PT30	0.060	-81.82
MILK PRICE[c]	0.343	3.43
FEED COST	0.339	2.10
HEIFER PRICE	0.359	8.20
CARCASS VALUE	0.332	0.05
CALVING RATE	0.217	-34.60
LACTATION	0.104	-68.70

[a]The notation +20PT represents a 20 percent premium July-Nov. and no change in other months.
[b]The notation +20PT10 represents a 20 percent premium in July through November and a 10 percent penalty across all other months.
[c]The notation "MILK PRICE" represents the production distribution calculated by deleting seasonal variations of milk price.

percentage. While seasonal production was not flat in this price scenario, the increased production in the late summer months represented 50 percent of seasonal imports. The seasonal coefficient dropped to 0.06. A supply response of this magnitude would eliminate the need for imports from distant locations such as Minnesota and Wisconsin. It is unlikely, however, that a price plan with premiums and penalties of this magnitude could be successfully implemented by the cooperative.

Several experiments were conducted on the model to test its sensitivity to changes in other economic factors such as feed costs, replacement heifer costs, and carcass value. The results of these experiments indicate that the model is relatively insensitive to changes in these factors. The seasonal coefficients remained approximately the same in these cases. For further details of these analyses, see Gao.

The impact of biological factors such as seasonal calving rates and seasonal lactation was also examined. The model was run under the assumption that conception rates are constant over the year. Compared to the base run, milk production showed less seasonal fluctuation, and the trough in production occurred in September and October compared to July and August in the base run. The seasonal coefficient decreased to 0.21. Another run of the model was conducted in which lactation curves were invariant to the month of freshening. The results of this experiment showed a trough in milk production in June and July. The seasonal coefficient is 0.10. Expected monthly milk production for these two scenarios and the base run is shown in Figure 2. When both lactation and conception were constant across seasons, milk production was nearly flat over the 12 months. This means that seasonal production (lactation) and reproduction (conception) are the dominant factors influencing the seasonal milk production. Seasonal economic factors such as feed costs, heifer costs, carcass value and milk prices have much smaller effects on seasonal milk production.

CONCLUDING REMARKS

Florida dairy cooperatives face a problem of insufficient, locally available supplies of fluid milk in the late summer. Since they act as sole procurement agents, they must share the cost of imports from distant supply sources among their members. There is considerable interest in the impact of seasonal pricing schemes on supply response and its potential for reducing seasonal imports.

The results of this study suggest that the seasonal supply of fluid milk by Florida dairy producers is highly price inelastic. A system of large price premiums and penalties is required to stimulate a modest supply response. Given the magnitude of the premium-penalty (50 percent premium and 30 percent penalty) that is required to significantly alter the seasonal milk distribution, it is unlikely that such an approach could be implemented. Another conclusion is that a Louisville type premium-penalty combination plan would be effective compared with a premium-only base-excess plan.

The test on the sensitivity of seasonal variations of other economic and biological variables shows that seasonal milk production in Florida is severely constrained by biological rather than economic factors. Any economic effort to significantly change the seasonal distribution through economic incentives alone is likely to be unsuccessful.

The biological and economic relationships used in the replacement model are based on a sample of DHIA data, and hence represent a composite of the dairy industry in Florida. Although the usual limitations apply when the results of a micro-level analysis are extrapolated, the aggregation error for the conclusion of this paper, however, is small.[2] This study did not consider the potential impact of tech-

[2] If well-managed farms are constrained by technological conditions and are supply price inelastic, the same would apply to the poorly-managed farms.

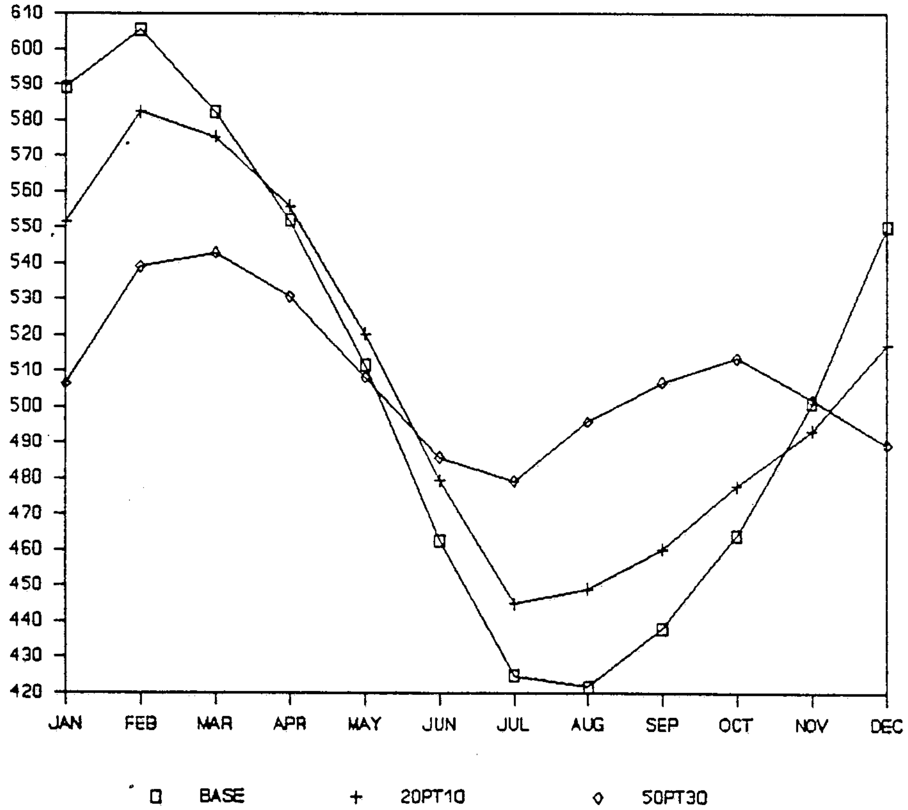

Figure 1. Predicted seasonal milk distrubution (Kg/cow) from the base model, 20 percent premium-10 percent penalty price set, and 50 percent premium-30 percent penalty price set

nical change on seasonal milk production. It is possible that seasonally adjusted prices would stimulate the adoption of technology which can mitigate the effect of heat and humidity on lactation and reproduction. In this sense, the conclusions drawn from the model do not reflect the long run.

Given the apparent inelastic nature of seasonal milk supply in Florida, it is reasonable to question the advisability of the full-supply contracts negotiated by Florida dairy cooperatives. The seasonal procurement problem is a direct consequence of the full-supply contracts. Cooperatives are faced with the tradeoff of the benefits associated with a guaranteed local market for fluid milk during the winter months against the cost of importing milk during the summer months. Investigation of this question is left to further research.

APPENDIX A

The net present value of cash flow from stage t in equation (3) and (4) is defined by solving the following stochastic dynamic programming problems

(A1) $Open_t(x_i) = R_t(x_i) + \delta(1 - PI(X_i))F_{t+1}(X_j) + PI(X_i)(S_t(X_i) - L_t(X_i) + FH_{t+1})$

(A2) $Ins_t(X_i) = PC(X_i)[R_t(X_i)\delta(1 - PI(X_i))F_{t+1}(X_j) + PI(X_i)(S_t(X_i) - L_t(X_i) + FH_{t+1})] + (1 - PC(X_i))Open_t(X_i)$

(A3) $Keep_t(X_i) = R_t(X_i) + \delta(1 - PI(X_i))\sum_{j=1}^{R} P_{ij}(F_{t+1}(X_j) + D(X_i)) + PI(X_i)[(S_t(X_i) - L_t(X_i) + FH_{t+1})]$

(A4) $Rep_t(X_i) = S(X_i) + FH_t$

(A5) $FH_t = -C + \sum_{m=1}^{M} PH(m)[R_t(X_i) + \delta(1 - PI(X_i))F_{t+1}(X_j) + PI(X_i)(S_t(X_i) - L_t(X_i) + FH_{t+1})]$

and

FH_t = the expected present value of cash flow from stage t until stage T for a replacement heifer under an optimum policy;

X_i = state variable;

X'_i = state when the cow conceived at the beginning of stage t;

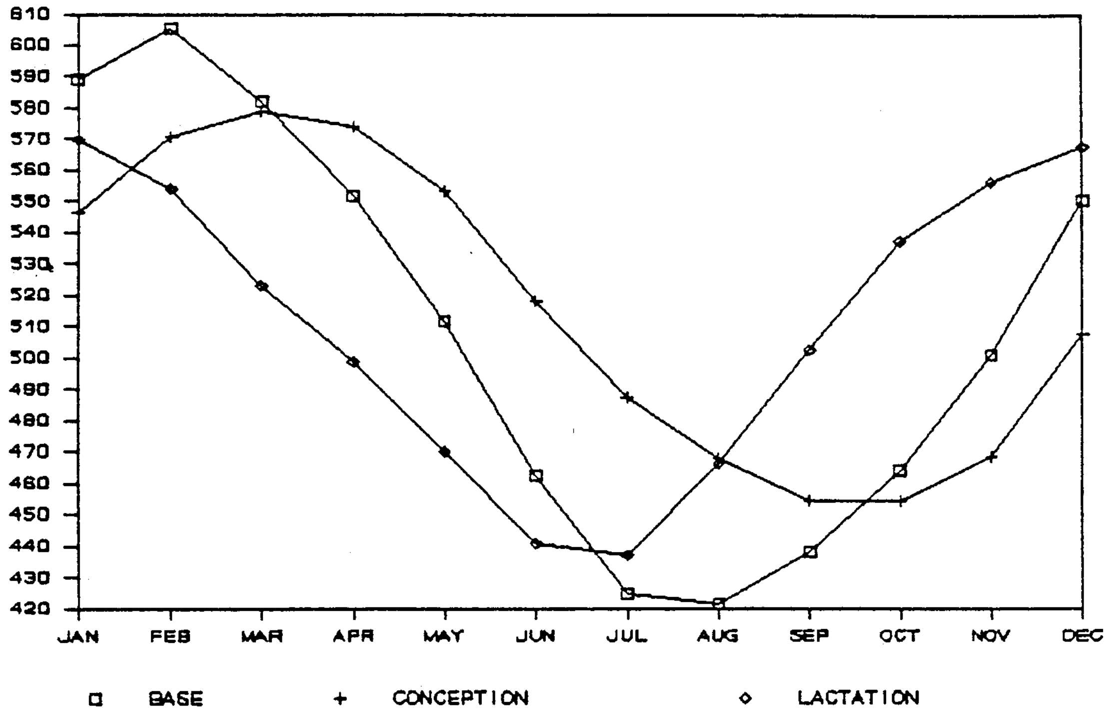

Figure 2. Predicted seasonal milk distribution (Kg/cow) from the base model, constant conception rate, and non-seasonal lactation

$R_t(X_i)$ = net revenues from milk production during stage t;

$S_t(X_i)$ = carcass value of a cow in state i in stage t;

$PI(X_i)$ = marginal probability of involuntary disposal associated with state i;

$PC(X_i)$ = marginal probability of conception associated with state i;

$L_t(X_i)$ = financial loss associated with involuntary disposal of a cow in state i during stage t;

P_{ij} = probability of transition to state j in the next stage given that the current state of the cow is X_i.

PH(m) = probability of a replacement heifer with production level m;

C = price of a replacement heifer;

δ = discount factor;

$D(X_i)$ = deviation in net revenues due to the length of the previous lactation.

Equations (A1), (A2), (A3), and (A4) are the expressions for the expected net present value associated with leaving a cow open (not rebreeding), insemination, keeping a cow that is not in the breeding period, and replacement, respectively. Equation (A5) gives the expected net present value of a replacement heifer which enters the herd in stage t.

To further clarify the dynamic programming model, consider equation (A1). This expression gives the expected net discounted revenue associated with leaving a cow open (not rebreeding) in period t. On the right-hand-side of equation (A1), the first term in brackets is the revenue from milk production in period t ($R_t(X_i)$). The second term is the probability that the cow will not die (or be removed from the herd for health reasons) times return from the optimal policy starting in stage t+1. The third term is the probability of involuntary disposal times the net revenue (or minus the cost) associated with losing an animal.

Expression (A2) is the expected net discounted revenue associated with the decision to rebreed an open (not pregnant) cow in period t. It is the discount rate times the marginal probability of conception times the expected revenue associated with successful rebreeding plus the marginal probability of unsuccessful rebreeding times the expected revenue from an open cow. The expected revenue associated with successful rebreeding reflects the fact that the state of the cow changes from X_i to X'_i because the cow has conceived. Otherwise this expression is analogous to the term inside the brackets in equation (A1) in that milk revenues from current and future periods are included for surviving animals, and the net revenues from dead animals are computed.

Expression (A3) is appropriate for cows outside the breeding interval. It represents the expected net discounted revenue associated with keeping a cow an additional period. The first term inside the brackets is the revenue from milk production in the current period. The second term is the probability that the cow is not involuntarily removed times future expected net revenue. Future expected revenue is the sum over all relevant states R of the probability of transition from state X_i to state X_j in stage t+1 times the value of optimal policy followed thereafter corrected for any deviation in milk revenues due to the length of the previous lactation. The third term is the probability of involuntary disposal times the net revenue (minus the cost) associated with losing an animal. The reason that production probability transition matrix P_{ij} and the revenue deviation ($D(X_i)$) enter into this equation, but not into equation (A1) and (A2), is that the cow left open during breeding period will be replaced automatically after sixteen months in lactation. The production transition matrix is zero in this case. After a cow is successfully inseminated, she will follow a more or less fixed lactation production; there is no variation in production.

Expression (A4) simply states that the expected return from replacement of a cow in state X_i in period t is the sum of the salvage value plus the expected return from a replacement heifer (FH_t). The expected return from a replacement heifer is given in expression (A5). The first term on the right-hand side of equation (A5) is the cost of replacement heifer. The second term is the discount rate times the sum over all possible production levels of the probability that the entering heifer is at production level m times a term which is analogous to equation (A1).

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