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U.S. MONEY DEMAND INSTABILITY:

A FLEXIBLE LEAST SQUARES APPROACH†

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ABSTRACT

This paper uses a *flexible least squares* (FLS) time-varying linear regression technique to investigate coefficient stability for the Goldfeld U.S. money demand model over the period 1959:Q2 to 1985:Q3. Time-paths traced out by the FLS coefficient estimates exhibit shifts in 1974 and 1983; but these shifts are small relative to a persistent downward trend in the estimated coefficient for the inflation rate, indicating potential model misspecification. The FLS estimates also indicate that the "unit root" nonstationarity problem reported by OLS money demand studies disappears if the coefficient estimates are allowed to exhibit even small amounts of time-variation.

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U.S. MONEY DEMAND INSTABILITY: A FLEXIBLE LEAST SQUARES APPROACH

I. INTRODUCTION

The extensive empirical literature on U.S. money demand has posited two key qualitative requirements for a desirable money demand model. First, the model should specify a simple (preferably linear or log-linear) relationship between money demand and a small number of theoretically plausible variables. Second, the parameters characterizing the relationship should exhibit stability over time.

Conventional statistical techniques such as OLS impose these requirements as inflexible restrictions. Time-constancy is imposed on the model parameters prior to estimation; and distributional restrictions are imposed on residual error terms, so that these terms are interpreted as random shocks which disturb an otherwise true relationship rather than as discrepancies resulting from model misspecification.

This paper uses a recently developed "flexible least squares" technique [Kalaba and Tesfatsion (1988b,1989)] to investigate the plausibility of log-linearity and parameter stability for U.S. money demand over 1959:Q2 to 1985:Q3. Log-linearity and parameter stability are imposed in a flexible way, so that the data, interacting with the theoretical specifications, indicate the degree to which the theoretical specifications can be simultaneously satisfied.

Two basic hypotheses are formulated concerning money demand: a measurement hypothesis that observations on real money demand have been generated in accordance with the well-known Goldfeld (1976) log-linear regression model; and a dynamic hypothesis that the coefficients characterizing the regression model have evolved only slowly over time, if at all. Two types of residual modelling error are thus associated with each possible sequence of coefficient vector estimates. Residual *measurement* errors are discrepancies between the log of observed real money demand and the estimated log-linear regression model at each point in time. Residual *dynamic* errors are discrepancies between estimated coefficient vectors at successive points in time.

The flexible least squares (FLS) solution is defined to be the collection of all coefficient sequence estimates which yield vector-minimal sums of squared residual measurement and dynamic errors for the given observations—i.e. which attain the "residual efficiency frontier." The frontier characterizes the efficient attainable trade-offs between residual measurement error and residual dynamic error. In particular, the frontier reveals the cost in terms of residual measurement error that must be paid in order to obtain the *zero* residual dynamic error (time-constant coefficients) required by OLS estimation. The time-paths traced out by the FLS estimates along the frontier indicate how the regression coefficients could have evolved over time in a manner *minimally incompatible* with the log-linearity and coefficient stability hypotheses.

The Goldfeld money demand model represents a good test case for the FLS technique. The model has been extensively investigated by previous researchers using OLS; see, for example, Roley (1985). These previous OLS studies raise three interesting questions which FLS is used to address in Section IV, below.

First, shifts in the OLS coefficient estimates in 1974, during the time of the first OPEC oil price shock, and to a lesser extent in 1983, have been repeatedly documented using variants of the Chow test (1960). For the Chow test, the sample period is divided into two parts, and OLS (time-constant) coefficient estimates for each subperiod are determined and compared. In contrast, the only restriction imposed on the time-variation of the FLS coefficient estimates for the Goldfeld model is a simple smoothness prior. *Do these FLS estimates exhibit shifts in 1974 and 1983 despite the fact that no prior information concerning shift times is incorporated into the estimation procedure?*

The answer is affirmative. At each point along the residual efficiency frontier the time-paths traced out by the FLS coefficient estimates *do* exhibit a clear-cut downward shift in 1974 and a partial upward shift beginning in 1983.

Second, the FLS technique generates an estimated time-path for each regression coefficient over the sample period; hence, unlike the Chow test, the FLS technique can be used to detect unanticipated qualitative movements in individual coefficients at dispersed points in time. *Do the FLS estimates for the Goldfeld model only exhibit simple simultaneous shift movements, the alternative hypothesis to coefficient constancy under the Chow test?*

The answer is negative. The FLS estimates exhibit systematic idiosyncratic time-variations in addition to the simultaneous shift movements in 1974 and 1983. Specifically, the 1974 and 1983 shifts are small in comparison with the pronounced and persistent downward trend exhibited by the FLS-estimated coefficients for the inflation rate over the entire sample period. Moreover, each of the FLS-estimated coefficients exhibits either a downward or an upward trend over the subperiod 1975:Q1-1983:Q4, strongly indicating that an assumption of coefficient constancy over this subperiod is not warranted. The apparent shifts in 1974 and 1983 could thus be artifacts of model misspecification rather than structural breaks in the money demand relationship itself.

Finally, the OLS solution for the Goldfeld model corresponds to the extreme point of the residual efficiency frontier where residual dynamic error is zero (regression coefficients are time-constant). *Are the properties displayed by the OLS solution essentially retained as one moves along the frontier, allowing progressively greater amounts of time-variation in the coefficient estimates?*

The answer is again negative. Important features of the OLS estimates for the Goldfeld model are *not* robust with respect to a relaxation of the constant-coefficient assumption. For example, it is well-documented [see, e.g., Roley (1985)] that the OLS solution for the Goldfeld model over both the full sample period and the pre-1974 and post-1974 subperiods implies that nominal money balances essentially follow a simple random walk $M_{t+1} \approx M_t$, indicating the presence of a severe "unit root" nonstationarity problem. The FLS estimates indicate, however, that this unit root problem disappears when the coefficient estimates are allowed to exhibit even small amounts of time variation.

The FLS approach to time-varying linear regression is a conceptually and computationally straightforward procedure for guarding against coefficient instability in unrestricted form. It is hoped that the results reported in this paper will encourage other researchers to undertake additional theoretical and empirical studies of the FLS procedure.

Section II describes the money demand model in more detail. To ensure the paper is self-contained, Section III reviews the FLS estimation technique. Section IV summarizes the principal empirical findings of the current study. Concluding comments are given in Section V.

II. AN EMPIRICAL EXEGESIS OF U.S. MONEY DEMAND

The elusive ideal of a simple, empirically stable U.S. money demand function has bewitched and bedeviled postwar macroeconomists in the way the Philosophers' Stone bewitched the ancient alchemists. The search for such a function has been an integral part of the U.S. macroeconomic policy agenda throughout the postwar period. As chronicled by Boorman (1982) and Judd and Scadding (1982), this search has ranged widely, and was resolved for only a short while in an influential paper by Goldfeld (1973).

Throughout the money demand debates of the 1960's and early 1970's, three criteria for a desirable money demand model were maintained:

- a simple relationship (preferably linear or log-linear) between real money balances and theoretically plausible variables;
- stability of the estimated relationship over time;
- a good in-sample fit combined with good out-of-sample forecasting capabilities.

In 1973 the Goldfeld model appeared to satisfy all three criteria. Beginning in 1974, however, forecasts based on the Goldfeld model began systematically to over-predict actual real balances, the error increasing to nearly ten percent by 1976:Q2. The period 1974 to 1978 has become known as "The Case of the Missing Money." More recently, forecasts based on the Goldfeld model have tended to under-predict real balances, this episode being termed "The Great Velocity Decline."

These difficulties with the Goldfeld model have led to renewed efforts to discover an empirically stable money demand function. The search has occupied a disproportionate amount of empirical macroeconomists' time since 1973, and has become something of a rite of passage into the discipline. A discussion of this literature is given in Tesfatsion and Veitch (1988, Section 2).

The objective of the current study is qualitatively different from previous money demand studies. In no way do we attempt to *construct* a stable money demand function. Rather, we use a recently developed "flexible least squares" technique to conduct an empirical investigation of the two central tenets underlying much of the money demand literature—log-linearity and coefficient stability—for a particular set of regressor variables stressed in previous studies.

In keeping with the first tenet, most post-1974 empirical studies of money demand have taken the Goldfeld log-linear regression model as a starting point. Consequently, we also adopt a version of this model as our benchmark [cf. Goldfeld (1976) and Roley (1985)]. The version we choose contains four regressor variables, in addition to a constant term: the 90-day T-bill rate r_t^{tb}, real GNP q_t, lagged real M1 money balances m_{t-1}, all in log-level form, and the inflation rate π_t. The inflation rate π_t is given by $log(P_t/P_{t-1}) \equiv [p_t - p_{t-1}]$, where P_t denotes the time-t GNP deflator. Thus, our benchmark model is[1]

$$m_t \approx b_{t1} + b_{t2}r_t^{tb} + b_{t3}q_t + b_{t4}m_{t-1} + b_{t5}\pi_t \ , \ t = 1, ..., T. \tag{2.1}$$

Letting $y_t \equiv m_t$ denote the time-t observed dependent variable, x_t denote the 5×1 vector of time-t regressor variables, and b_t denote the 5×1 vector of time-t regressor coefficients, the *prior measurement specification* (2.1) can equivalently be expressed as

$$y_t \approx x_t' b_t \ , \ t = 1, ..., T. \tag{2.2}$$

[1]Model (2.1) has been criticized on the basis that it is the reduced form of an underlying supply and demand system in the money market rather than a true money demand relation. See, for example, Cooley and LeRoy (1981) for an informative discussion of the potentially serious identification and simultaneity problems associated with the specification of money demand functions. As will be clarified below, our results have implications for either interpretation of model (2.1).

The second tenet of the money demand literature is that the estimated coefficient vectors should not change very much from one time period to the next. Rather than impose strict time-constancy on the coefficients, as in OLS estimation, we capture this requirement through a *prior dynamic specification* (smoothness prior) for successive coefficient vectors:

$$b_{t+1} \approx b_t \ , \ t = 1, ..., T-1. \tag{2.3}$$

A basic problem is to determine whether the theory is compatible with the observations. That is, does there exist *any* coefficient sequence estimate $(b_1, ..., b_T)$ which satisfies the prior theoretical specifications (2.2) and (2.3) in an acceptable approximate sense for the realized sequence of observations $(y_1, ..., y_T)$? How might such a coefficient sequence estimate be found?

III. FLEXIBLE LEAST SQUARES

A. The Basic Flexible Least Squares Approach

One approach is as follows. Associated with each possible coefficient sequence estimate $b = (b_1, ..., b_T)$ are two basic types of model specification error. First, b could fail to satisfy the prior measurement specification (2.2). Second, b could fail to satisfy the prior dynamic specification (2.3).

Suppose the cost assigned to b for the first type of error is measured by the sum of squared residual measurement errors

$$r_M^2(b;T) = \sum_{t=1}^{T} [y_t - x_t' b_t]^2 \ , \tag{3.1}$$

and the cost assigned to b for the second type of error is measured by the sum of squared residual dynamic errors

$$r_D^2(b;T) = \sum_{t=1}^{T-1} [b_{t+1} - b_t]' D [b_{t+1} - b_t] \ , \tag{3.2}$$

where D is a suitably chosen positive definite scaling matrix (see below). Define the *(time T) residual possibility set* to be the collection

$$R(T) = \{r_D^2(b;T), r_M^2(b;T) \mid b \epsilon E^{T6}\} \tag{3.3}$$

of all possible configurations of squared residual dynamic error and measurement error sums attainable at time T, conditional on the given observations $y_1, ..., y_T$. The residual possibility set (3.3) is qualitatively depicted in Figure 1(a).[2]

—Insert Figure 1 About Here—

If the prior theoretical specifications (2.2) and (2.3) are correct, then, by construction, the squared residual errors associated with the actual coefficient sequence will be approximately zero. In general, however, the lower envelope for the residual possibility set $R(T)$ will be bounded away from the origin in E^2.

The lower envelope gives the locus of vector-minimal pairs (r_D^2, r_M^2) of dynamic and measurement error sums attainable at time T, conditional on the given observations. The left uppermost endpoint of the frontier gives the minimum possible value for r_M^2 subject to $r_D^2 = 0$, hence this endpoint reveals the cost in terms of residual measurement error that must be paid in order to attain the *zero* residual dynamic error (time-constant coefficients) required by OLS estimation. The right lower endpoint of the frontier gives the minimum possible value for r_D^2 subject to $r_M^2 = 0$, i.e., the minimum amount of time-variation in the coefficients which must be allowed in order to have zero residual measurement error (an exact fit for the log-linear regression).

Hereafter this lower envelope, denoted by $R^F(T)$, will be referred to as the *(time T) residual efficiency frontier.* The coefficient sequence estimates whose costs attain this frontier will be referred to as *FLS estimates.*

[2] When least-squares formulations such as (3.1) and (3.2) are used to measure costs of estimation, a common reaction is that the analysis is implicitly relying on normality assumptions for the residual measurement and dynamic errors whose "likelihoods" are measured by (3.1) and (3.2). To the contrary, (3.1) and (3.2) are simply cost (inverse utility) functions which assign penalties for theory-data discrepancies on the basis of residual error magnitudes without regard for signs. Nothing is necessarily implied about the intrinsic properties of the residual errors themselves, e.g., that they are symmetrically distributed about zero. Alternative cost specifications could certainly be considered.

The dynamic scaling matrix D appearing in the sum of squared residual dynamic errors (3.2) can be specified so that the residual efficiency frontier is essentially invariant to the choice of units for the regressor variables.[3] The frontier then becomes an intrinsic characterization for the time-varying linear regression problem, depending only on the prior theoretical specifications (2.2) and (2.3), the cost assessments (3.1) and (3.2) for model specification errors, and the observations $y_1, ..., y_T$ obtained to date.

B. Parametric Representation for the Residual Efficiency Frontier

How might the residual efficiency frontier be found? In analogy to the usual procedure for tracing out Pareto-efficiency frontiers, a parameterized family of minimization problems is considered.

Thus, let $\mu \geq 0$ be given; and let each possible coefficient sequence estimate $b = (b_1, ..., b_T)$ be assigned an *incompatibility cost*

$$C(b; \mu, T) = \mu r_D^2(b; T) + r_M^2(b; T) \tag{3.4}$$

consisting of the μ-weighted average of the associated measurement error and dynamic error sums (3.1) and (3.2). Expressing these sums in terms of their components, the incompatibility cost $C(b; \mu, T)$ takes the form

$$C(b; \mu, T) = \mu \left[\sum_{t=1}^{T-1} [b_{t+1} - b_t]' D [b_{t+1} - b_t] \right] + \sum_{t=1}^{T} [y_t - x_t' b_t]^2. \tag{3.5}$$

[3] To accomplish this invariance; it suffices to specify D to be the diagonal matrix whose ith diagonal term d_{ii} consists of the averaged sum of squared values for the ith regressor variable over times $t = 1, ..., T$; i.e. $d_{ii} = [(x_{1i})^2 + ... + (x_{Ti})^2]/T, i = 1, ..., 5$. A change of unit for any regressor variable x_i other than lagged real money, i.e. a multiplication of x_{ti} at each time t by some common factor δ_i, then only results in the multiplication of the FLS estimate for the ith regressor coefficient b_{ti} at each time t by the common factor $1/\delta_i$. Moreover, multiplication of the real money variable m_t by some common factor θ at each time t only results in the multiplication of the FLS coefficient estimates for all regressors other than lagged real money by the common factor θ. The residual efficiency frontier is invariant to the change of unit in the first case, and expands or contracts by the factor θ^2 in the second case; and the qualitative time-variations displayed by the FLS coefficient trajectories which attain the frontier are not affected in either case. The present study uses this specification for D.

As (3.5) indicates, the incompatibility cost function $C(b;\mu,T)$ generalizes the goodness-of-fit criterion function for OLS estimation by permitting the coefficient vectors b_t to vary over time.

Suppose the regressor matrix $[x_1,...,x_T]$ has full rank. Then, if the smoothness weight μ is strictly positive, the incompatibility cost function (3.4) is a strictly convex function of the coefficient sequence estimate b, and there exists a unique estimate b which attains the minimum cost. Let this unique estimate be denoted by

$$b^{FLS}(\mu,T) = (b_1^{FLS}(\mu,T),...,b_T^{FLS}(\mu,T)). \tag{3.6}$$

If μ is zero, let (3.6) denote the estimate b which minimizes the sum of squared residual dynamic errors $r_D^2(b;T)$ subject to $r_M^2(b;T) = 0$. Hereafter, (3.6) will be referred to as the *FLS estimate conditional on* μ *and* T.

Finally, let the sums of squared residual measurement errors and dynamic errors corresponding to the FLS solution (3.6) be denoted by

$$r_M^2(\mu,T) = r_M^2(b^{FLS}(\mu,T);\ T); \tag{3.7a}$$

$$r_D^2(\mu,T) = r_D^2(b^{FLS}(\mu,T);\ T). \tag{3.7b}$$

By construction, a point (r_D^2, r_M^2) in E^2 lies on the residual efficiency frontier $R^F(T)$ if and only if there exists some $\mu \geq 0$ such that $(r_D^2, r_M^2) = (r_D^2(\mu,T), r_M^2(\mu,T))$. The residual efficiency frontier $R^F(T)$ thus takes the parameterized form

$$R^F(T) = \{r_D^2(\mu,T), r_M^2(\mu,T) \mid 0 \leq \mu < \infty\}. \tag{3.8}$$

The parametrized residual efficiency frontier (3.8) is qualitatively depicted in Figure 1(b). As μ approaches zero, the incompatibility cost function (3.4) ultimately places no weight on the prior dynamic specification (2.3); i.e. r_M^2 is minimized with no regard for r_D^2. Thus, r_M^2 can generally be brought down close to zero and the corresponding value for r_D^2 will be relatively large. As μ becomes arbitrarily large, the incompatibility cost function (3.4) places absolute priority on the prior dynamic specification (2.3); i.e. r_M^2 is minimized subject to $r_D^2 = 0$. The latter case coincides with OLS estimation in which a single time-invariant coefficient vector is used to minimize the sum of squared residual measurement errors r_M^2.

In Kalaba and Tesfatsion (1989) a procedure is developed for sequentially generating the FLS solution (3.6). The algorithm gives directly the estimate $b_t^{FLS}(\mu, t)$ for the time-t coefficient vector b_t, conditional on the observations $y_1, ..., y_t$, as each successive observation y_t is obtained. The algorithm also yields smoothed (back-updated) estimates for all intermediate coefficient vectors for times 1 through $t-1$, conditional on the observations $y_1, ..., y_t$, as well as an explicit smoothed estimate for the actual dynamic relationship connecting each successive coefficient vector pair.[4]

C. Regime Shift Simulation Experiments

A FORTRAN program "FLS" has been developed for implementation of the FLS sequential solution procedure [Kalaba and Tesfatsion (1989, Appendix B)]. Simulation experiments demonstrating the ability of the FLS estimates to track linear, quadratic, and sinusoidal time variations in the true regression coefficients, despite noisy observations, are summarized and graphically depicted in Kalaba et al. (1989).

The present study uses this program to generate the FLS estimates reported in Section IV for the money demand model (2.1). For this empirical application the ability of the FLS estimates to detect regime shifts in the coefficients is particularly important.

In Kalaba and Tesfatsion (1989, Section 7) it is established, analytically, that the time-paths traced out by the FLS estimates at any finite μ-point along the residual efficiency frontier accurately reflect a single shift in the true coefficient vector occurring at an

[4] The relationship between FLS, with its focus on obtaining efficient estimates in terms of two-dimensional cost vectors (r_D^2, r_M^2), and Kalman-Bucy filtering with its focus on obtaining maximum *a posteriori* probability estimates, is clarified in Kalaba and Tesfatsion (1988a), where each approach is derived as a distinct special case of a general multicriteria approach to dynamic estimation.

arbitrary unanticipated time S, for all process lengths $T > S$. In addition, numerous simulation experiments have been conducted by the present authors which demonstrate the ability of the FLS estimates to detect more complex shifts in the true coefficient vectors, despite noisy observations.

–Insert Figure 2 About Here–

For example, an experiment was conducted with two cyclical regressors, thirty observations, and a balanced smoothness weight $\mu = 1.0$, for which the first component of the true coefficient vector, b_{t1}, was simulated to shift from 2.0 to 4.0 at time t = 7, and the second component of the true coefficient vector, b_{t2}, was simulated to shift from 3.0 to 5.0 at time t = 18. The observations y_t were subject to normally distributed random shocks representing about a 5% measurement error. As depicted in Figure 2, the FLS estimates accurately mimicked the separate shifts in the true coefficient vector components at times 7 and 18.

IV. EMPIRICAL RESULTS

A. General Considerations

FLS coefficient estimates for the benchmark money demand model (2.1) were derived using quarterly U.S. data for the period 1959:Q2 to 1985:Q3 for a range of smoothness weights μ along the residual efficiency frontier. For illustration, Figure 3(a-e) depicts time paths for the FLS coefficient estimates corresponding to the balanced smoothness weight $\mu = 1.0$, where there is a one-for-one trade-off between measurement and dynamic error.

— Insert Figure 3 about here —

As established in Kalaba and Tesfatsion (1989, Section 6), the FLS estimate for each coefficient b_{ti} at each time t contracts toward its corresponding OLS value b_i^{OLS} as the smoothness weight μ becomes arbitrarily large, where the OLS solution $(b_1^{OLS}, ..., b_5^{OLS})$ minimizes the sum of squared residual measurement errors for the constant-coefficient version of model (2.1). As will be clarified below, the time-paths traced out by the FLS estimates retain their qualitative shapes as μ becomes arbitrarily large. Time-patterns are flattened out as the OLS extreme point ($\mu = +\infty$) is approached, but they are not lost.

Three different levels of analysis are used in subsequent sections to compare the FLS estimates along the residual efficiency frontier (hereafter REF) to the estimates obtained at the OLS extreme point.

At the most general level, the full-sample REF is plotted to see how good a description the OLS solution provides for the observations. If the true observation-generating process is given by model (2.1) with time-constant coefficients, then the entire frontier should lie close to the origin. If the true observation-generating process is model (2.1) with coefficients which exhibit systematic time-variations, then, starting at the OLS extreme point, large percentage decreases in measurement cost r_M^2 should be attainable with only small percentage increases in dynamic cost r_D^2; consequently, the REF should be fairly steeply sloped in a neighborhood of the OLS extreme point. Finally, failure of the log-linear measurement postulate should result in one or both types of cost being large at each point along the frontier.

Summary descriptive statistics are next constructed for the time-paths traced out by the FLS estimates along the REF for the full sample period 1959:Q2-1985:Q3, as well as for subperiods of interest. For each μ in a specified grid, and for each coefficient $i = 1, \ldots, 5$, the average value attained by the FLS estimates for $\{b_{1i}, \ldots, b_{Ti}\}$ is compared to the OLS estimate b_i^{OLS}. The empirical standard deviation of these FLS estimates about their average value provides a summary measure of the extent to which the FLS estimates deviate from constancy for the given μ.

Finally, the time-paths traced out by the FLS estimates along the REF are directly examined for evidence of systematic time-variation. To facilitate this analysis, the smoothness weight $\mu \in [0, +\infty)$ is transformed into the variable $\delta \equiv \mu/[1 + \mu]$, so that $\delta \in [0, 1)$. Parameterizing the REF in terms of δ permits a direct characterization of the percentage of the REF over which the estimates exhibit any particular property of interest—e.g. the percentage of the REF over which the FLS estimates for the coefficient on lagged real money lie uniformly within some specified distance of the corresponding OLS estimate.

B. OLS Coefficient Estimates for the Benchmark Money Demand Model

The constant-coefficient version of the benchmark money demand model (2.1) was first estimated using OLS to obtain reference estimates for comparison against FLS. Table 1 summarizes the results for the OLS estimation of model (2.1) over the period 1959:Q2 to 1985:Q3 using quarterly U.S. data. Results for the OLS estimation of the model over the pre-1974 (1959:Q2 - 1973:Q4) and post-1974 (1974:Q1 - 1985:Q3) subperiods are also presented in this table. These results are consistent with those of Roley (1985) and other previous OLS studies.

— Insert Table 1 About Here —

The full sample OLS solution yields a fitted equation for which all the regressor variables are highly significant and have time-constant coefficient estimates of the correct sign. The DW and Q-statistics suggest that serial correlation is a problem in the residuals. This is confirmed by a residual plot which indicates over-prediction for 1974-78 (the Missing Money episode) and under-prediction for 1983-1985 (the Great Velocity Decline episode).

The full-sample fitted equation indicates the presence of a unit root nonstationarity problem. The coefficient b_4 on lagged real money is nearly unity while the coefficient b_5 on the inflation rate is close to negative one, indicating that the data favor what is essentially a random walk in nominal money balances with a drift factor influenced by interest rate and scale (real GNP) considerations. If the presence of lagged real money as a regressor in model (2.1) is rationalized in terms of a partial adjustment story,[5] then these coefficient values imply an implausibly low adjustment parameter value $\theta \approx 0$. Conventional time-

[5] Lagged real money balances might be included as a regressor variable because transactions costs prevent agents from fully adjusting their actual nominal money balances to their desired nominal money balances at each time t. For example, suppose the log of desired real money balances m_t^* is a linear function of r_t^{tb} and q_t; i.e. $m_t^* = a_1 + a_2 r_t^{tb} + a_3 q_t$. Suppose also that, due to transactions costs, agents only partially adjust their time-t nominal money balances M_t to the desired level $M_t^* \equiv m_t^* + p_t$. Specifically, suppose $M_t - M_{t-1} = \theta(M_t^* - M_{t-1})$, where the adjustment parameter θ lies in the interval [0,1]. Combining these two equations yields a constant coefficients version of model (2.1) with $b_i = \theta a_i$, i=1,2,3, and $b_4 = -b_5 = [1-\theta]$.

series techniques would suggest first differencing the variables to arrive at a stationary model.

The OLS solution obtained for the pre-1974 subperiod is roughly similar to the OLS solution obtained for the full sample period. The pre-1974 estimated model suffers from a higher degree of serial correlation in its residuals. The nonstationarity problem is more severe for this subperiod, with the coefficient b_4 on lagged money exceeding unity.

The OLS solution obtained for the post-1974 subperiod is also similar to the full-sample OLS solution, except that the OLS estimate for the constant term coefficient b_1 is not significantly different from zero. Serial correlation is less of a problem, but the unit root aspect of the estimated relationship is still troubling.

The null hypothesis of time-constant coefficients was tested using an approach recently proposed by Ploberger et al. (1989) which does not require an explicit specification of an alternative hypothesis. The null hypothesis was rejected at the 99 percent confidence level. A Chow test was then used to test for a structural break in the model between the two subperiods. Results of this more specific test supported the hypothesis of a break. Nevertheless, there does not appear to be a pronounced downward movement in the coefficients as the Missing Money episode might suggest. Note that the OLS estimated coefficients for the first three regressors actually increase in value from one subperiod to the next.

C. An FLS Robustness Study of the OLS Coefficient Estimates

Although the OLS solutions for the pre-1974 and post-1974 subperiods indicate a break in the coefficients in 1974, it is important to remember that the coefficients are held constant within each subperiod. Recall from Section III that the OLS solution corresponds to an extreme point of the REF, where the penalty μ imposed for time-variation in the coefficients is infinite. The primary question addressed in this subsection concerns the robustness of the OLS full-period and subperiod coefficient estimates in comparison with the FLS estimates obtained for a range of δ-points along the REF, where $\delta \equiv \mu/[1+\mu]$.

The REF generated for the money demand model (2.1) over the full sample period is presented in Figure 4. The most striking aspect of the REF is its extreme attenuation in a neighborhood of the OLS point $\delta = 1.0$. Note the difference in scale used for the measurement and dynamic error axes. Thus, permitting even a very small degree of time-variation in the coefficients for model (2.1) results in large decreases in measurement error.

— Insert Figure 4 About Here —

The attenuation of the REF is broadly consistent with the notion that the money demand relationship may have been subject to one or more breaks over the full sample period. Other types of model misspecification could also account for this finding. Whatever its cause, however, the attenuation makes clear that the OLS estimates are unlikely to be representative of the FLS estimates over much of the REF.

Table 2 presents the average value over time of each FLS coefficient estimate, together with its associated empirical standard deviation, for a range of points along the REF. Four aspects of the table are particularly striking:

- The average value of the constant term coefficient b_{t1}, which is negative for OLS, is positive for 94% of the estimates along the REF.
- The average value of the coefficient b_{t3} for the scale variable (real GNP) increases rapidly away from the OLS extreme point, eventually attaining a three-fold increase.
- There is a notable drop in the average value for the coefficient b_{t4} on lagged real money away from the OLS extreme point. At the OLS point this value is nearly unity; along 80% of the REF the average value of this coefficient lies in the range (.59,.81), with a very small empirical standard deviation (less than 0.001).
- The empirical standard deviation for the coefficient b_{t5} on the inflation rate is extremely large over most of the REF due to systematic time-variation in the coefficient rather than to large random fluctuations about the average value; see Section IV.D, below. Whether this systematic time-variation represents non-stationarity in the (possibly) stochastic coefficient b_{t5} or a more fundamental

misspecification of model (2.1) as a whole remains an open question.

— Insert Table 2 About Here —

The Table 2 results confirm the initial impression that the full sample OLS solution for the money demand model (2.1) is not robust with respect to a relaxation in the assumption of time-constant coefficients. This is not surprising given the Chow test results reported in Section IV.B, which indicate a break in the money demand relationship in 1974. What is surprising is the firm rejection of the OLS unit root finding by a large percentage (80%) of the FLS estimates along the REF. Unit root problems essentially disappear when the coefficients are allowed to exhibit even small amounts of time-variation.

The hypothesis of a break in the money demand relationship in 1974 can be investigated by comparing OLS against FLS for the pre-1974 and post-1974 subperiods using the average values and empirical standard deviations calculated for the FLS estimates over each subperiod. Table 3 presents these subperiod summary statistics for the FLS estimates corresponding to three $\delta-$points along the REF.

— Insert Table 3 About Here —

Comparing the subperiod OLS solutions presented in Table 1 with the corresponding FLS subperiod summary statistics presented in Table 3 indicates that the important differences between the OLS and FLS estimates found for the full sample period carry over to these subperiods. For example, the negative OLS estimate for the constant term coefficient b_1 contrasts sharply with the positive average value for b_{t1} displayed by the FLS estimates along more than 90% of the REF in both subperiods. Most strikingly, for each subperiod the near-unity subperiod OLS estimate for the coefficient b_4 on lagged real money is much higher than either the absolute or the average subperiod FLS estimates attained for this coefficient along most of the REF.

There are, however, some notable differences between the subperiods. The post-1974 FLS estimates obtained for the coefficient b_{t5} on the inflation rate have an average value which is markedly more negative, with a much higher empirical standard deviation, than obtained for the pre-1974 subperiod. Moreover, unlike the earlier subperiod, the absolute average values obtained for the coefficients b_{t4} and b_{t5} on lagged real money and inflation are strikingly different, indicating that a partial adjustment story is *not* a proper characterization for money demand behavior in this later subperiod. Note that the OLS estimates in Table 1 suggest this characterization is appropriate for both subperiods.

The subperiod OLS coefficient estimates presented in Table 1 do not display a uniform downward shift in 1974. In contrast, Table 3 indicates that the average FLS estimates are roughly unchanged or lower in the post-1974 subperiod for all five coefficients. This is consistent with a "Case of the Missing Money" story of a downward shift in the money demand relationship in 1974. The downward shift captured by the FLS estimates is not, however, consistent with the conjecture of a downward shift in a partial adjustment parameter on nominal money balances.[6]

In summary, the FLS results presented in Tables 2 and 3 strongly imply that important features of the full-period and subperiod OLS solutions are not robust with respect to a relaxation of the assumption of time-constant coefficients.

D. Time-Paths Traced Out by the FLS Estimates: Stability Implications

FLS generates an estimated time-path for each regression coefficient over any specified sample period, conditional on $\delta \equiv \mu/[1+\mu]$. The smoothness of the time-paths is jointly regulated by δ; otherwise no cross-coefficient restrictions are imposed. These time-paths enable an investigator to discern qualitative movements in individual coefficients —e.g., unanticipated shifts at dispersed points in time—which may be difficult to detect from summary characterizations of the estimated coefficient time-paths such as average values and empirical standard deviations.

[6] Recall from footnote 5 that a partial adjustment on nominal balances implies all coefficients except those on lagged real money and the inflation rate are pre-multiplied by the partial adjustment parameter θ. The coefficient on lagged real money is $[1-\theta]$ and the coefficient on the inflation rate is $-[1-\theta]$. Thus, a downward shift in θ would *decrease* the coefficient on the inflation rate and the (positive) coefficients on the constant term and real GNP, and *increase* the coefficient on lagged real money and the (negative) coefficient on the interest rate. The results in Table 3 clearly do not support such an interpretation.

Time-paths for the FLS coefficient estimates over the full sample period are presented in Figure 3(a-e) for the balanced-weight point $\delta = .5$; residual measurement errors associated with these FLS estimates are plotted in Figure 3(f). *The general qualitative patterns exhibited by these time-paths occur at all δ-points along the REF other than the OLS extreme point $\delta = 1.0$. Only the scale of the patterns changes as δ ranges over the REF.* Specifically, as δ approaches 1.0, the FLS estimated time-path for the ith coefficient b_{ti} shrinks in a uniform manner toward the constant-level time-path traced out by the OLS estimate for b_i, $i = 1, \ldots, 5$. The convergence in the average values for the FLS coefficient estimates is seen in Table 2; the same type of convergence occurs for their time-paths.

The most striking common feature exhibited by the estimated coefficient time-paths for the first four regressors is a downward shift beginning in 1974. There is also a common upward movement in these time-paths beginning roughly in 1983.

These common coefficient movements in 1974 and 1983 appear to represent "breaks" in the money demand relationship. They occur over a short time span and are much larger than coefficient movements occurring over the rest of the sample period. Moreover, the finding of breaks in 1974 and 1983 dovetails with the conclusions reached by other money demand studies using OLS techniques [e.g. Roley (1985)].

It is difficult to claim, however, that these common coefficient movements represent *significant* breaks in the money demand relationship. The movements of the estimated coefficients in 1974 and 1983, while large compared to within-sample variability, are not large relative to the absolute values of the estimated coefficient levels. For example, the estimated time-path for the coefficient b_{t3} on real GNP in Figure 3(c) undergoes a substantial movement after 1974 relative to any other time during the sample period; but this movement occurs in the fourth decimal place of an estimate whose average value is 0.13.

In contrast, the estimated time-path for the coefficient b_{t5} on the inflation rate trends sharply downward over the entire sample period at each point along the REF. For example, at the balanced weight point $\delta = .5$ the FLS estimate for b_{t5} begins with the value -0.3 in

1959:Q2 and ends with the value -1.7 in 1985:Q3 [see Figure 3(e)]. It is this trend movement which is primarily responsible for the large empirical standard deviations reported in Tables 2 and 3 for b_{t5}. This pronounced systematic movement in the FLS estimate for b_{t5} suggests that model (2.1) is misspecified.[7] The apparent shifts in 1974 and 1983 could thus be artifacts of model misspecification rather than structural breaks in the money demand relationship itself.

The Federal Reserve switched in 1979 from targeting a broad range of economic indicators to targeting only a narrow definition of base money. In 1982 it switched back to broader targets. Recent OLS studies of money demand [Roley (1985) and Peek and Wilcox (1987)] have surmised that these policy switches introduced additional variability into the residual term of the money demand relationship, but did not result in any significant structural break.

The FLS-estimated time-paths for all five coefficients are more volatile in the post-1974 subperiod than in the pre-1974 subperiod, and the volatility is particularly pronounced from 1979 to the end of the sample period. A consideration of the FLS residual measurement errors graphed in Figure 3(f) supports the inference that changes in the Federal Reserve System's operating procedure primarily induced volatility in these errors without fundamentally shifting the money demand relationship. A striking feature of the graph is the uniformity in measurement error variance before 1979 and after 1983, contrasted against the large increase in this variance over the period 1979 to 1983.

V. CONCLUDING REMARKS

Empirical work in economics commonly relies on data which have not been generated within the framework of controlled experiments. The inability to replicate the same experiment a large number of times is particularly troublesome when attempting to specify the

[7] Along these lines, Baba, Hendry, and Starr (1988) estimate a model incorporating a measure of interest rate volatility for which the coefficients are stable over the full sample period. Our results in Section IV.D suggest that more stable coefficients might be achieved for model (2.1) by incorporating a measure of *inflation rate* volatility.

statistical properties of the model to be estimated. An investigator typically has a well-defined theoretical model, but little information about specific probabilistic properties for residual error terms.

The FLS approach, with its minimal information requirements, is thus a useful technique for empirical economists. This is not to say that FLS should supplant more conventional statistical approaches. Rather, the two approaches are complementary. FLS provides an agnostic way to explore the compatibility of theory and data without requiring distributional assumptions for residual error terms. In particular, FLS can be used to check the stability of a theoretical model's parameters prior to embarking upon a more structured statistical approach. FLS also provides a good post-estimation check of parameter stability for an empirical model which appears to satisfy the normal battery of statistical checks.

The Goldfeld log-linear regression model for U.S. money demand is a natural test case for the FLS technique because one has, from previous studies, a broad agreement among researchers concerning the behavior of OLS coefficient estimates for the model across subperiods. As shown in Section IV, the FLS coefficient estimates clearly exhibit the shifts surmised by earlier OLS studies, even though the only restriction placed on the time-variation of the FLS estimates is a simple smoothness prior. The FLS estimates also provide important new evidence on the fragility of certain inferences drawn from previous OLS studies, e.g., the OLS indication of a unit root nonstationarity problem.

Among the many methodological issues raised in the course of this FLS time-varying linear regression study, two seem particularly important.

The FLS procedure yields a range of estimates for the sequence of regression coefficients: namely, all estimates whose cost vectors (r_D^2, r_M^2) attain the REF. This range of estimates reflects the intrinsic uncertainty inherent in the very formulation of the model. More precisely, the REF estimates constitute a "population" of possible coefficient sequence estimates characterized by a basic efficiency property: For the given observations, these are the coefficient sequence estimates which are *minimally incompatible* with the prior theoretical specifications. *Without additional prior information, restricting attention to any proper subset of these estimates is a purely arbitrary decision.*

More systematic procedures need to be developed for interpreting and summarizing the properties displayed by the FLS estimates. The REF is parametrized by a single parameter δ varying over the unit interval, permitting a direct characterization of the percentage of FLS estimates along the REF which exhibit a property of interest to any stated degree; i.e., one can construct an empirical distribution for the property. Such constructs could provide useful summary characterizations for the FLS estimates. Alternatively, in some cases a more direct statistical analysis of the FLS estimates might be possible. See, for example, the statistical interpretation of the FLS estimates provided in Dorfman and Foster (1989) under the assumption that the true measurement errors are independently and identically distributed random variables whereas the true dynamic errors $[b_{t+1} - b_t]$ evolve in an unknown deterministic manner.

A second related issue concerns the use of FLS for adaptive model respecification. Starting from the prior theoretical beliefs of a linear regression model with slowly evolving coefficients, the solution procedure for the FLS estimates developed in Kalaba and Tesfatsion (1989) recursively generates estimated time-paths for each coefficient, estimated time-paths for residual measurement and dynamic modelling errors, and explicit posterior dynamic relations connecting successive coefficient vectors. This posterior information might fail to support the prior theoretical beliefs. For example, in the present study the FLS estimate for the coefficient b_{t5} on the inflation rate exhibits a sharp downward trend over the full sample period at each point along the REF, in contradiction to the prior belief of slow coefficient evolution. How might this posterior information be used to respecify the model?

Future studies will explore these issues.

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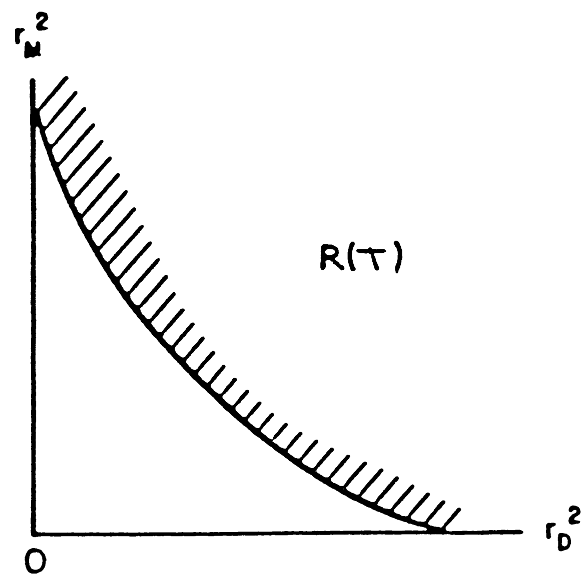
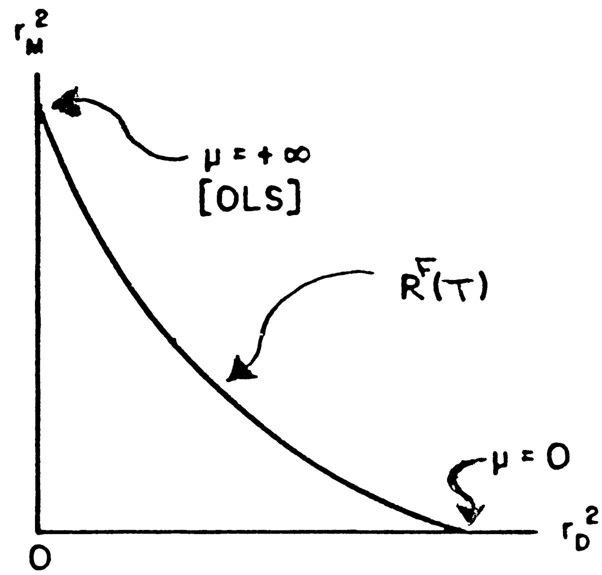


(a) Residual Possibility Set R(T)



(b) Residual Efficiency Frontier $R^F(T)$

Fig. 1: Schematic Illustration of the Residual Possibility Set and the Residual Efficiency Frontier

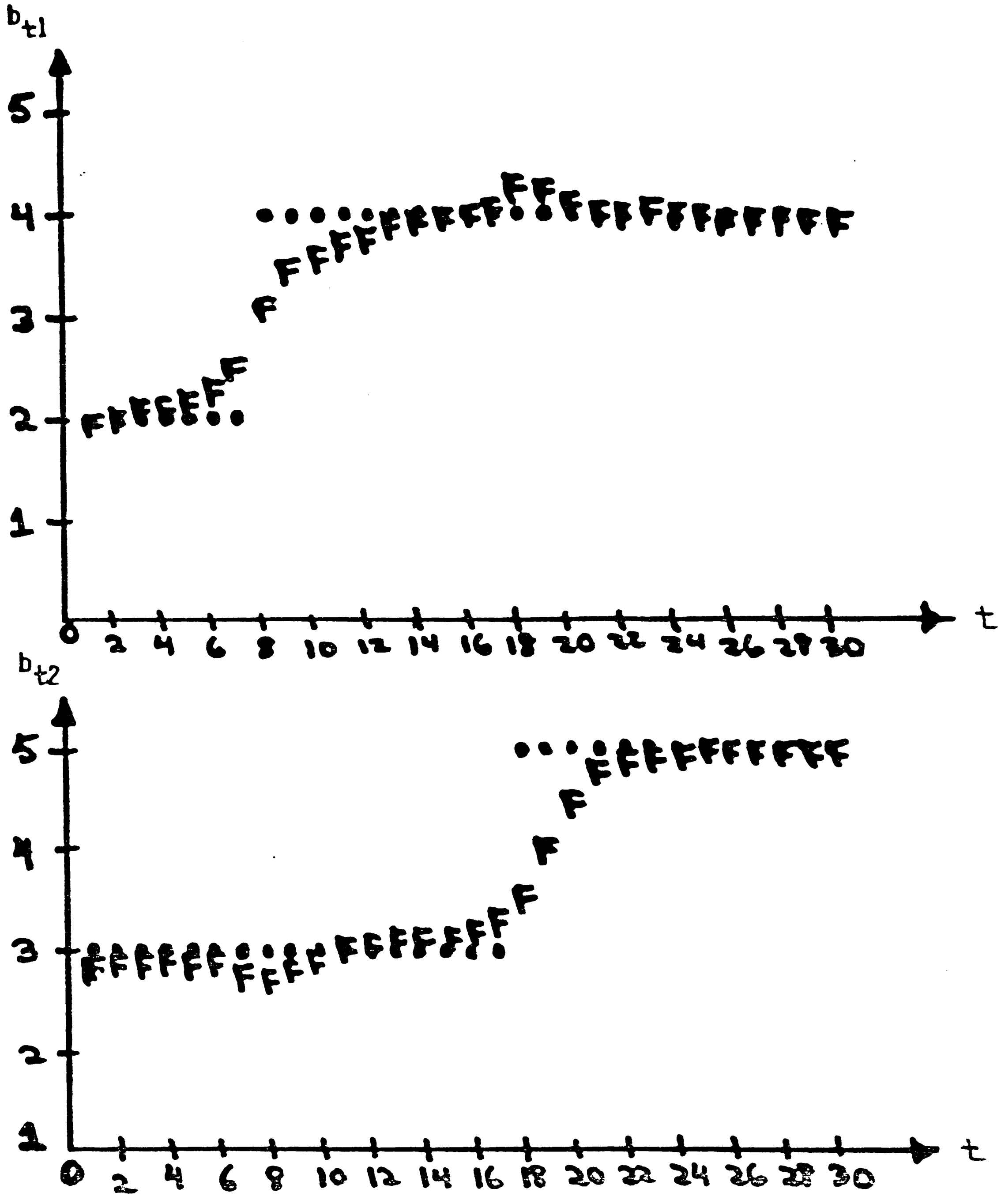
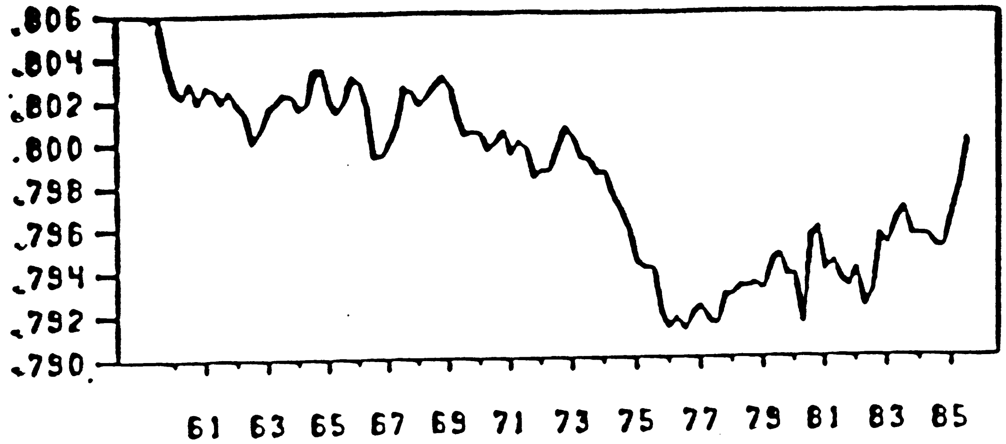
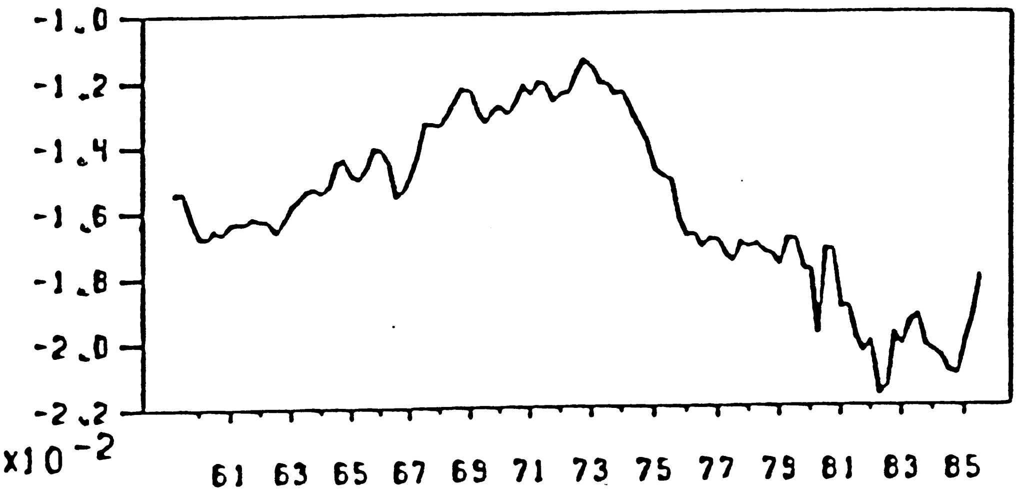


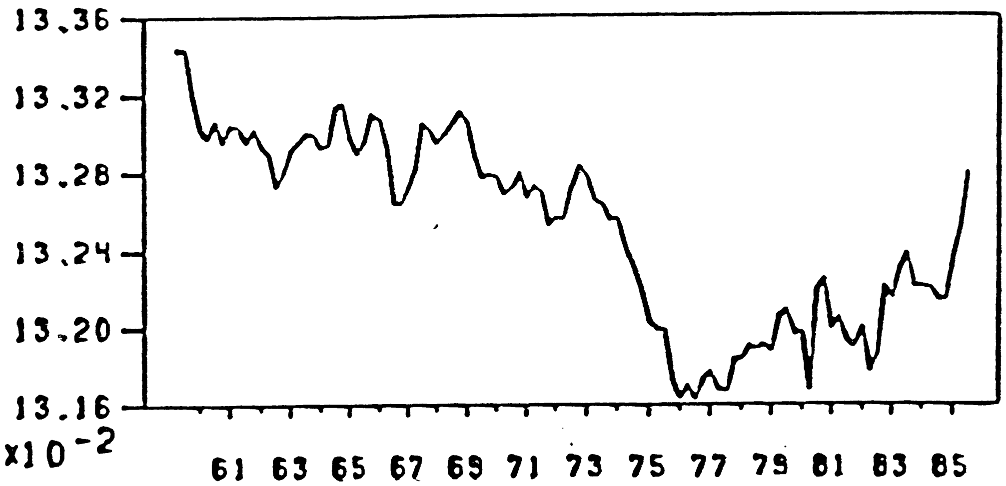
Fig. 2: FLS Coefficient Estimates for a Regime Shift Experiment With Balanced Smoothness Weight $\delta \equiv \mu/[1+\mu] = .50$



(a) Coefficient for Constant

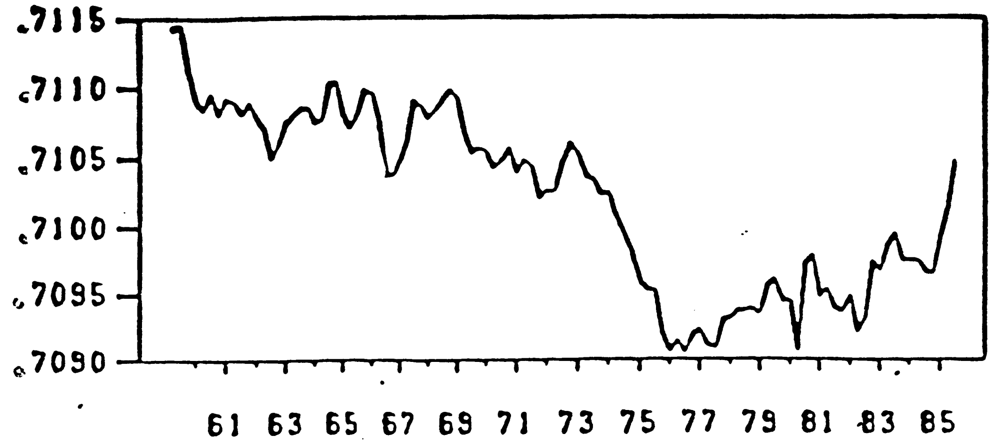


(b) Coefficient for Interest Rate

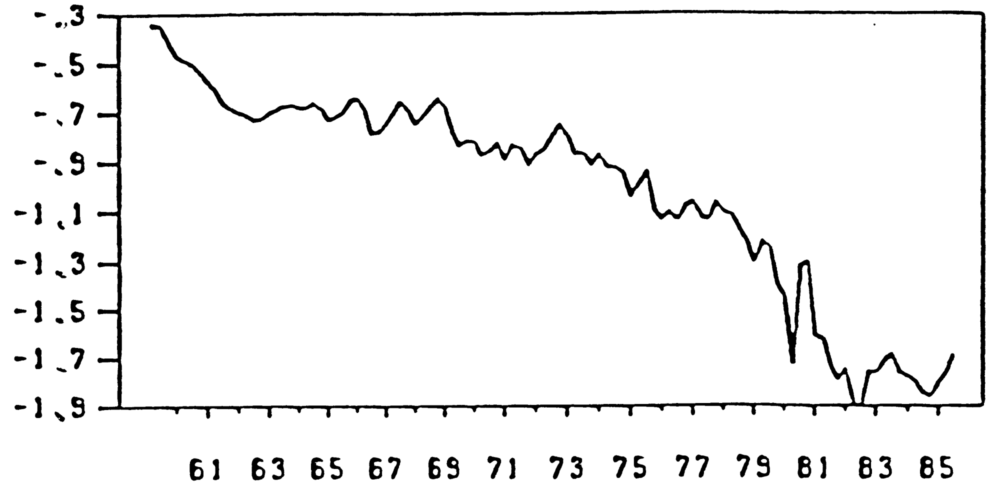


(c) Coefficient for Real GNP

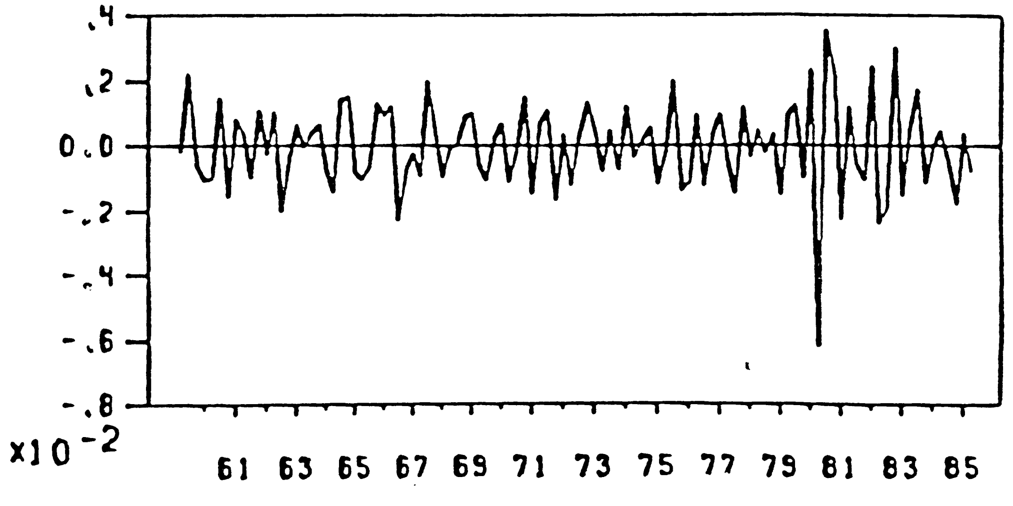
Fig. 3(a-c): FLS Estimates for Money Demand Model (2.1) Over 1959:Q2-1985:Q3
With Balanced Smoothness Weight $\delta \equiv \mu/[1+\mu] = .50$



(d) Coefficient for Lagged Real Money



(e) Coefficient for Inflation Rate



(f) Residual Measurement Error

Fig. 3(d-f): FLS Estimates for Money Demand Model (2.1) Over 1959:Q2-1985:Q3 With Balanced Smoothness Weight $\delta \equiv \mu/[1+\mu] = .50$

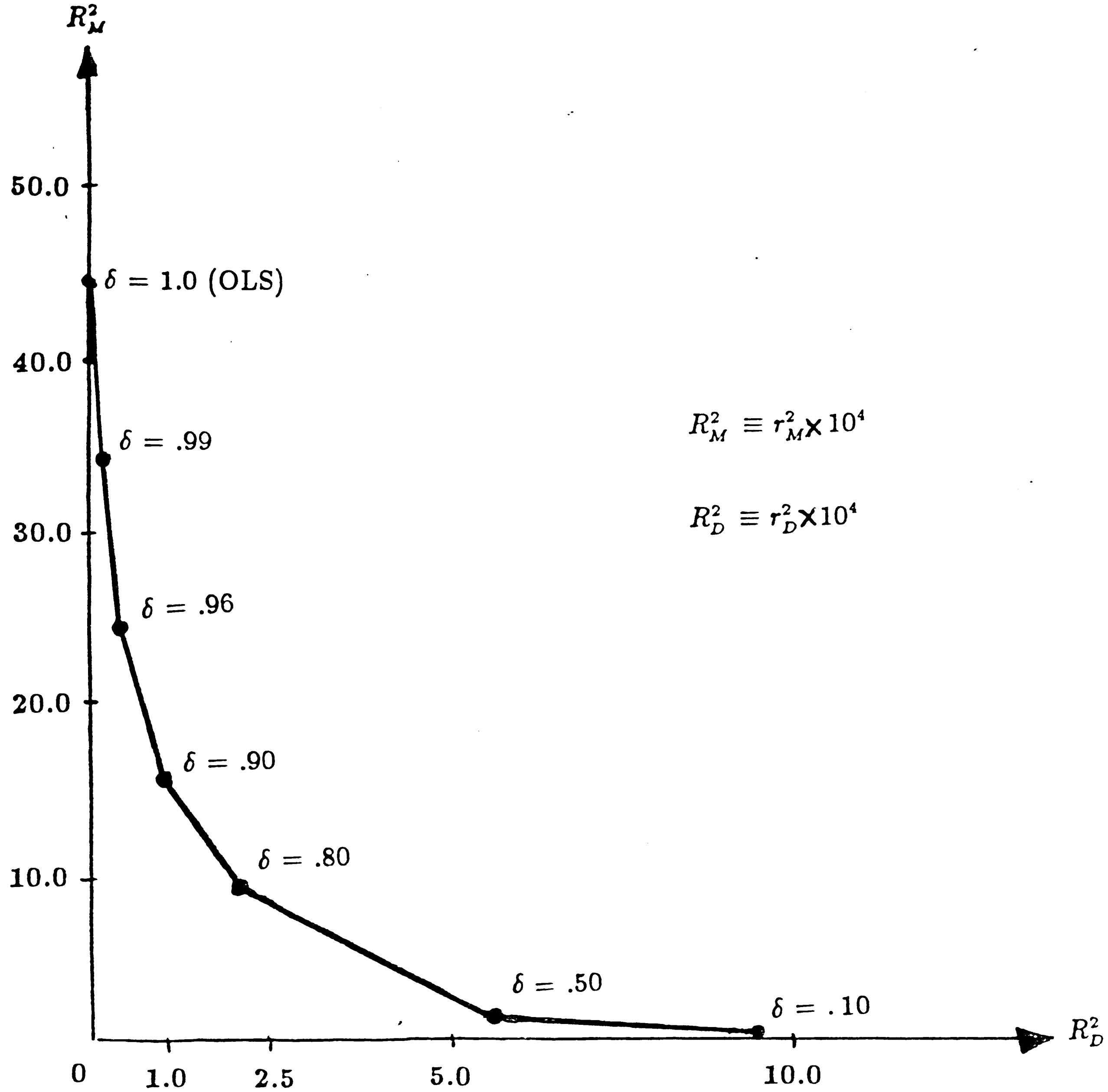


Fig. 4: Residual Efficiency Frontier for Money Demand Model (2.1) Over the Full Sample Period 1959:Q2-1985:Q3

Table 1

OLS Results

$m_t = b_1 +$	$b_2 r_t^{tb} +$	$b_3 q_t +$	$b_4 m_{t-1}$	$+ b_5(p_t - p_{t-1})$	R^2	SEE	DW	Q(30)
1959:Q2-1985:Q3								
-.273	-.011	.043	.995	-1.078	.978	.0066	1.74	35.12
(-2.78)	(-3.63)	(7.21)	(60.51)	(-8.84)				
1959:Q2-1973:Q4								
-.504	-.014	.039	1.038	-.957	.989	.0048	1.38	47.08
(-1.38)	(-3.87)	(2.05)	(12.83)	(-5.23)				
1974:Q1-1985:Q3								
-.053	-.012	.055	.944	-1.080	.959	.0083	1.96	21.67
(-.210)	(-2.05)	(2.23)	(26.44)	(-4.08)				

Notes:
Numbers in parentheses are t-statistics for the coefficient estimate.

Q is the Ljung-Box statistic defined as:

$$Q = T(T+2)\{\sum_{j=1}^{M} \frac{1}{T-j} r_j^2\}$$

$$with \;\; M = min(T/2, 3\sqrt{T}) = 30$$

m_t = natural log of M1, quarterly average from FRB of St. Louis divided by implicit GNP price deflator.

r_t^{tb} = natural log of 90-day Treasury bill yield, quarterly average from Federal Reserve Bulletin, Table 1.35.

q_t = natural log of real GNP, \$1982 billions, from Survey of Current Business, Table 1.1.

p_t = natural log of implicit GNP price deflator, 1982=100, from Survey of Current Business, Table 7.1.

Table 2

Coefficient Estimates Along
the Residual Efficiency Frontier
1959:Q2 - 1985:Q3

Average Values and Standard Deviations

δ	μ	$m_t = b_{t1}$ +	$b_{t2} r_t^{tb}$ +	$b_{t3} q_t$ +	$b_{t4} m_{t-1}$ +	$b_{t5}(p_t - p_{t-1})$
.1	.11	1.125 [.005]	-.014 [.003]	.145 [.0006]	.641 [.0008]	-.992 [.456]
.3	.43	.983 [.005]	-.015 [.003]	.140 [.0006]	.671 [.0008]	-1.003 [.443]
.5	1.00	.798 [.004]	-.016 [.003]	.133 [.0005]	.710 [.0006]	-1.013 [.423]
.7	2.33	.543 [.003]	-.017 [.002]	.121 [.0004]	.766 [.0005]	-1.018 [.385]
.9	9.00	.149 [.002]	-.019 [.002]	.096 [.0002]	.862 [.0003]	-.994 [.287]
.94	15.67	.038 [.001]	-.019 [.001]	.085 [.0002]	.893 [.0002]	-.976 [.241]
.98	48.98	-.100 [.0006]	-.017 [.0007]	.068 [.84E-04]	.937 [.0001]	-.951 [.152]
.99	98.90	-.146 [.0005]	-.015 [.0004]	.061 [.63E-04]	.953 [.43E-04]	-.954 [.109]
.998	496.5	-.216 -	-.013 -	.050 -	.977 -	-1.01 [.021]
1.0	$+\infty$	-.273	-.011	.043	.995	-1.078

Notes:
Numbers in square brackets are standard deviations about the average value.
The parameter $\delta \equiv \mu/[1+\mu]$ is the normalized smoothness weight.

Table 3

FLS Subperiod Average Values and Standard Deviations

$$m_t = b_{t1} + b_{t2} r_t^{tb} + b_{t3} q_t + b_{t4} m_{t-1} + b_{t5} (p_t - p_{t-1})$$

	b_{t1}	b_{t2}	b_{t3}	b_{t4}	b_{t5}
Results for: $\delta = .1$ $r_M^2 = .00000$, $r_D^2 = .00095$					
1959:Q2 - 1985:Q3					
coef. av.	1.125	-.014	.145	.641	-.992
s.d.	[.005]	[.003]	[.0006]	[.0008]	[.456]
1959:Q2 - 1973:Q4					
coef. av.	1.129	-.012	.145	.642	-.669
s.d.	[.002]	[.002]	[.0002]	[.0003]	[.137]
1974:Q1 - 1985:Q3					
coef. av.	1.120	-.016	.144	.640	-1.397
s.d.	[.002]	[.003]	[.0003]	[.0004]	[.387]
Results for: $\delta = .5$ $r_M^2 = .00019$, $r_D^2 = .00056$					
1959:Q2 - 1985:Q3					
coef. av.	.798	-.016	.133	.710	-1.013
s.d.	[.004]	[.003]	[.0005]	[.0006]	[.423]
1959:Q2-1973:Q4					
coef. av.	.801	-.014	.133	.711	-.707
s.d.	[.002]	[.002]	[.0002]	[.0003]	[.129]
1974:Q1-1985:Q3					
coef. av.	.794	-.018	.132	.710	-1.397
s.d.	[.002]	[.002]	[.0003]	[.0003]	[.341]
Results for: $\delta = .9$ $r_M^2 = .00161$, $r_D^2 = .00009$					
1959:Q2 - 1985:Q3					
coef. av.	.149	-.019	.096	.862	-.994
s.d.	[.002]	[.002]	[.0002]	[.0003]	[.287]
1959:Q2 - 1973:Q4					
coef. av.	.150	-.018	.096	.862	-.773
s.d.	[.002]	[.002]	[.0001]	[.0001]	[.102]
1974:Q1 - 1985:Q3					
coef. av.	.148	-.020	.096	.862	-1.271
s.d.	[.001]	[.001]	[.0002]	[.0002]	[.185]

Notes:
Numbers in square brackets are standard deviations about the average value.
The parameter $\delta \equiv \mu/[1+\mu]$ is the normalized smoothness weight.