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UNIVERSITY PARK
LOS ANGELES, CALIFORNIA
90089-0152

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TURNING POINTS IN ECONOMIC TIME SERIES, LOSS STRUCTURES AND BAYESIAN FORECASTING

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ABSTRACT

Methods for forecasting turning points and future values of economic time series are developed which take account of a forecaster's loss structure. For example, it is found that the decision to forecast a downturn in an economic series is very sensitive to the form of the forecaster's loss structure as well as to the predictive probability of a downturn. Using an autoregressive-leading indicator model and data on real output growth rates for eighteen countries, turning point forecasts were made for each year, 1974-84. Overall, 66% of the 68 downturn and no-downturn forecasts were correct and 75% of the 82 upturn and no-upturn forecasts were correct.

Turning Points in Economic Time Series, Loss Structures and Bayesian Forecasting

by

Arnold Zellner, Chansik Hong and Gaurang Mitu Gulati*
University of Chicago

1. Introduction

In this paper we consider the problem of forecasting future values of economic time series and turning points given explicit loss structures. Kling (1987, pp.201-204) has provided a good summary of past work on forecasting turning points by Moore (1961, 1983), Zarnowitz (1967), Moore and Zarnowitz (1982), Wecker (1979), Neftci (1982), and others. In this work there is an emphasis on the importance and difficulty of forecasting turning points. Also, in our opinion, not enough attention has been given to the role of loss structures in forecasting just turning points and in forecasting turning points <u>and</u> the future values of economic variables.

The plan of our paper is as follows. In Section 2, we introduce loss structures and explain how optimal forecasts of the occurrence of turning points and future values of economic time series can be computed. Applications of our procedures are presented in Section 3 based on annual output growth rate data for eighteen countries and an autoregressive model with leading indicator variables employed in our previous work--see Garcia-Ferrer et al. (1987) and Zellner and Hong (1987). In Section 4, a summary of results and some concluding remarks are presented.

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2. Turning Points, Loss Structures and Forecasting

As Wecker (1979) and Kling (1987) recognize, given a model for past observations and a definition of a turning point, probabilities relating to the occurrence of future turning points can be evaluated. In our work, we, along with Kling (1987) evaluate these probabilities using predictive probability density functions for as yet unobserved, future values of variables which take account of uncertainty regarding the values of model parameters as well as the values of future error terms. We also indicate how to take account of model uncertainty in evaluating probabilities relating to future events such as the occurrence or non-occurrence of a turning point at a future time.

Let the given past measurements of a variable, say the growth rate of real GNP, be denoted by $\underset{\sim}{y}' = (y_1, y_2, \ldots, y_{n-1}, y_n)$ and $z \equiv y_{n+1}$ be the first future value of the series. Then, definitions of a downturn (DT) and of an upturn (UT) and their negations, based on y_{n-1}, y_n and z are given by,

$$y_{n-1} < y_n \text{ and } \begin{cases} z < y_n \equiv \text{Downturn } (DT_1) \\ z \geq y_n \equiv \text{No Downturn } (NDT_1) \end{cases} \tag{2.1a}$$

and

$$y_{n-1} > y_n \text{ and } \begin{cases} z > y_n \equiv \text{Upturn } (UT_1) \\ z \leq y_n \equiv \text{No Upturn } (NUT_1) \end{cases} \tag{2.1b}$$

where the subscript 1 on DT, UT, etc., denotes that one past observation, y_{n-1} and one future observation, $z \equiv y_{n+1}$ have been employed in defining downturns and upturns.

Given a model assumed to generate the observations, let $p(\underset{\sim}{y}|\underset{\sim}{\theta})$ be the likelihood function for the past data, where $\underset{\sim}{\theta}$ is a vector of parameters, $\pi(\underset{\sim}{\theta}|I)$ a prior distribution for $\underset{\sim}{\theta}$ and $p(\underset{\sim}{\theta}|D) \propto \pi(\theta|I)p(y|\underset{\sim}{\theta})$ the posterior distribution for $\underset{\sim}{\theta}$, where $D = (\underset{\sim}{y},I)$ represents the past sample and prior information. Then with $p(z|\underset{\sim}{\theta})$ representing the probability density function (pdf) for $z \equiv y_{n+1}$, given $\underset{\sim}{\theta}$, the predictive pdf for z is given by

$$p(z|D) = \int_{\Theta} p(z|\underset{\sim}{\theta})p(\underset{\sim}{\theta}|D)d\underset{\sim}{\theta} \qquad -\infty < z < \infty \tag{2.2}$$

where $\underset{\sim}{\theta} \subset \Theta$, the parameter space. As is well known, $p(z|D)$ can be viewed as an average of the conditional pdf, $p(z|\underset{\sim}{\theta})$, with the posterior pdf, $p(\underset{\sim}{\theta}|D)$ serving as the weighting function.

The predictive pdf in (2.2) can be employed to obtain probabilities associated with the events in (2.1). For example, if $y_{n-1} < y_n$, the probability of a downturn, P_{DT_1} is given by

$$P_{DT_1} = \int_{-\infty}^{y_n} p(z|D)dz \tag{2.3}$$

while the probability of no downturn is $P_{NDT_1} = 1 - P_{DT_1}$. Note that with $y_{n-1} < y_n$, the probability of an upturn, given the definitions in (2.1), is zero.

Given $y_{n-1} < y_n$ and the probability of a downturn, P_{DT_1} as given by (2.3), we now wish to make a decision as to whether to forecast a downturn or no downturn. To solve this problem, consider the loss structure shown in Table 1. The two possible outcomes are DT_1 and NDT_1. If the act forecast a DT_1 is chosen, loss is scaled to be 0 if the forecast is correct and to be $c_1 > 0$ if it is incorrect. If the act forecast a NDT_1 is chosen, loss is 0 if the outcome is NDT_1 and $c_2 > 0$ if it is DT_1. In many

Table 1

Forecasting Loss Structure Given $y_{n-1} < y_n$[a/]

Acts	Possible Outcomes	
	DT_1	NDT_1
Forecast DT_1	0	c_1
Forecast NDT_1	c_2	0

[a/] c_1 and c_2 are given positive quantities.

circumstances, $c_1 \neq c_2$. From (2.3), probabilities associated with the outcomes DT_1 and NDT_1 are available and can be used to compute expected losses associated with the two acts shown in Table 1 as follows:

$$EL|\text{Forecast } DT_1 = 0 \cdot P_{DT_1} + c_1(1-P_{DT_1}) = c_1(1-P_{DT_1}) \tag{2.4a}$$

and

$$EL|\text{Forecast } NDT_1 = c_2P_{DT_1} + 0(1-P_{DT_1}) = c_2P_{DT_1}. \tag{2.4b}$$

If (2.4a) is less than (2.4b), that is $c_1(1-P_{DT_1}) < c_2P_{DT_1}$ or, equivalently

$$1 < \frac{c_2}{c_1}\left(\frac{P_{DT_1}}{1-P_{DT_1}}\right) \tag{2.5}$$

then choosing the act forecast a DT_1 will lead to lower loss than choosing the act forecast NDT_1. Note that if $c_2 = c_1$, this rule leads to a forecast of a DT_1 if $P_{DT_1} > 1/2$. On the other hand, if c_2/c_1 is much larger than 1, say $c_2/c_1 = 2$, then the condition in (2.5) would be satisfied for $P_{DT_1}/(1-P_{DT_1})$

$> 1/2$ or $P_{DT_1} > 1/3$. This example indicates that the decision to forecast a downturn is very sensitive to the value of the ratio c_2/c_1 as well as to the value of P_{DT_1}. Thus, considering just the value of P_{DT_1} in forecasting turning points is not usually satisfactory except in the special case of symmetric loss, $c_1 = c_2$.[1/]

We now turn to consider forecasting the value of $z \equiv y_{n+1}$ given $y_{n-1} < y_n$ and the probability of a DT_1 from (2.3) employing squared error loss functions. Let

$$L_{DT_1} = k_1(\hat{z}-z)^2 \qquad k_1 > 0 \tag{2.6a}$$

be the loss incurred if a downturn occurs and $\hat{z}$ is used as a point forecast of z and

$$L_{NDT_1} = k_2(\hat{z}-z)^2 \qquad k_2 > 0 \tag{2.6b}$$

be the loss incurred if no downturn occurs and $\hat{z}$ is used as a point forecast of z. Expected loss is

$$EL = P_{DT_1}k_1E(\hat{z}-z)^2|DT_1 + (1-P_{DT_1})k_2E(\hat{z}-z)^2|NDT_1. \tag{2.7}$$

On minimizing (2.7) with respect to the choice of $\hat{z}$, the minimizing value for $\hat{z}$, denoted by $\hat{z}^*$, is[2/]

$$\hat{z}^* = \frac{P_{DT_1}k_1\bar{z}_{DT_1} + (1-P_{DT_1})k_2\bar{z}_{NDT_1}}{P_{DT_1}k_1 + (1-P_{DT_1})k_2}, \tag{2.8}$$

[1/] Since analysis of forecasting an upturn, UT_1, given that $y_{n-1} > y_n$ is similar to that for forecasting a downturn, it will not be presented.

[2/] To derive $\hat{z}^*$ in (2.8) express (2.7) as $EL = P_{DT_1}k_1[E(z-\bar{z}_{DT_1})^2|DT_1 + (\hat{z}-\bar{z}_{NDT_1})^2] + (1-P_{DT_1})k_2[E(z-\bar{z}_{NDT_1})^2|NDT_1 + (\hat{z}-\bar{z}_{NDT_1})^2]$ and minimize with respect to $\hat{z}$.

a weighted average of the conditional mean of z given $z < y_n$, $\bar{z}_{DT_1}$, and the conditional mean of z given $z \geq y_n$, $\bar{z}_{NDT_1}$ with weights $P_{DT_1}k_1$ and $(1-P_{DT_1})k_2$. Note that if $k_1 = k_2$, $\hat{z}^* = \bar{z}$, the mean of the predictive pdf $p(z|D)$ in (2.2). However, if $k_1 \neq k_2$, $\hat{z}^*$ in (2.8) will not be equal to $\bar{z}$. For example, if $P_{DT_1} = 1/2$, (2.8) reduces to $\hat{z}^* = (k_1\bar{z}_{DT_1} + k_2\bar{z}_{NDT_1})/(k_1+k_2)$ which differs from $\bar{z}$. Thus while $\bar{z}$ is optimal relative to an overall squared error loss function ($k_1 = k_2$), it is not optimal in the case that different loss functions ($k_1 \neq k_2$) are appropriate for downturn and no-downturn situations.

In (2.6a-b), we have allowed for the possibility that loss functions may be different for downturn and no-downturn cases. However, the use of symmetric, squared error loss functions may not be appropriate in all circumstances. If $L_{DT_1}(z,\hat{z})$ and $L_{NDT_1}(z,\hat{z})$ are general convex loss functions for DT_1 and NDT_1, respectively, then expected loss is given by

$$EL = P_{DT_1}EL_{DT_1}(z,\hat{z})|DT_1 + (1-P_{DT_1})EL_{NDT_1}(z,\hat{z})|NDT_1 \qquad (2.9)$$

where the expectations on the right-side of (2.9) are computed using the conditional predictive pdfs $p(z|z<y_n,D)$ and $p(z|z\geq y_n,D)$. Then EL in (2.9) can be minimized, analytically or by computer methods to obtain the minimizing value of $\hat{z}$.

To illustrate the approach described in the previous paragraph, it may be that given a DT, over-forecasting is much more serious than under-forecasting by an equal amount. A loss function, the LINEX loss function, employed in Varian (1975) and Zellner (1986), is a convenient loss function which captures such asymmetric effects. It is given by:

$$L_{DT_1} = b_1[e^{a_1(\hat{z}-z)} - a_1(\hat{z}-z) - 1] \qquad \begin{array}{l} b_1 > 0 \\ a_1 > 0. \end{array} \qquad (2.10)$$

When $\hat{z}=z$, loss is zero and when $\hat{z}-z > 0$, a case of over-forecasting, loss rises almost exponentially with $a_1 > 0$. When $\hat{z}-z < 0$, loss rises almost linearly. Choice of the value of a_1 governs the degree of asymmetry. For example, when a_1 has a small value, the loss function in (2.10) is close to a symmetric squared error loss function as can be seen by noting that $e^{a_1(\hat{z}-z)} \doteq 1 + a_1(\hat{z}-z) + a_1^2(\hat{z}-z)^2/2$.

With NDT_1, it may be that under-forecasting is a more serious error than over-forecasting by an equal amount. The following LINEX loss function provides such asymmetric properties,

$$L_{NDT_1} = b_2[e^{a_2(\hat{z}-z)} - a_2(\hat{z}-z) - 1] \qquad \begin{matrix} b_2 > 0 \\ a_2 < 0. \end{matrix} \tag{2.11}$$

The loss functions in (2.10) and (2.11) can be inserted in (2.9) and an optimal point forecast can be computed. Note that the necessary condition for a minimum of (2.9) is

$$P_{DT_1}\frac{dEL_{DT_1}|DT_1}{d\hat{z}} + (1-P_{DT_1})\frac{dEL_{NDT_1}|NDT_1}{d\hat{z}} = 0. \tag{2.12}$$

If the derivatives in this last expression are approximated by expanding them around $\hat{z}_1$ and $\hat{z}_2$, values which set them equal to zero, respectively, the approximate value of $\hat{z}$, $\hat{z}^*$, which satisfies (2.12) is given by

$$\hat{z}^* = w\hat{z}_1 + (1-w)\hat{z}_2 \tag{2.13}$$

where $w = P_{DT_1}b_1a_1^2/[P_{DT_1}b_1a_1^2 + (1-P_{DT_1})b_2a_2^2]$. Thus it is seen that when $[P_{DT_1}/(1-P_{DT_1})](b_1a_1^2/b_2a_2^2]$ is large, $\hat{z}^*$ is close to $\hat{z}_1$ and when it is small, $\hat{z}^*$ is close to $\hat{z}_2$, with $\hat{z}_1 = -\ell n(Ee^{-a_1z}|DT_1)/a_1$ and $\hat{z}_2 = -\ell n(Ee^{-a_2z}|NDT_1)/a_2$. Again, the optimal point forecast in (2.12) is sensitive not only to the values of P_{DT_1} and $1-P_{DT_1}$, but also to the values of the loss functions' parameters, a_1, a_2, b_1, and b_2.

In summary, it has been shown how the loss structure in Table 1 can be used to obtain forecasts of turning points and how the probability of a DT and various loss functions can be employed to obtain optimal point forecasts. The main conclusion which emerges is that forecasts of turning points and of a future value are quite sensitive to assumptions regarding loss structures.

Above we have concentrated attention on the first future observation, $z = y_{n+1}$ given $y_{n-1} < y_n$ or given $y_{n-1} > y_n$--see (2.1). Given a predictive pdf for the next q future observations, $\underset{\sim}{z}' = (z_1, z_2, \ldots, z_q) \equiv (y_{n+1}, y_{n+2}, \ldots, y_{n+q})$, namely $p(\underset{\sim}{z}|D)$, $-\infty < z_i < \infty$, $i = 1,2,\ldots,q$, it is possible to calculate the probability that $z_{i-1} < z_i > z_{i+1}$, that is z_i is a peak value, $i = 2,3,\ldots,q-1$, using the predictive pdf, $p(\underset{\sim}{z}|D)$ and the definition in (2.1a). Also, the probability that $z_{i-1} > z_i < z_{i+1}$, that is z_i is a trough value can be evaluated.

The definitions in (2.1) can be broadened to include more past and future observations, as Wecker (1979) and Kling (1987) indicate. As an example consider, with $z_1 \equiv y_{n+1}$ and $z_2 \equiv y_{n+2}$,

$$y_{n-2} < y_{n-1} < y_n \text{ and } \begin{cases} z_1 < y_n \text{ and } z_2 < z_1 \equiv DT_2 \\ \text{otherwise} \equiv NDT_2 \end{cases} \tag{2.14}$$

where $DT_2 \equiv$ downturn based on two previous observations and two future observations relative to the given value y_n and $NDT_2 \equiv$ no such downturn. Then given $y_{n-2} < y_{n-1} < y_n$, the probability of a DT_2, P_{DT_2}, is given by

$$P_{DT_2} = \int_{-\infty}^{y_n} \int_{-\infty}^{z_1} p(z_1,z_2|D)dz_2dz_1 \qquad (2.15)$$

where it has been assumed that $-\infty < z_i < \infty$, $i = 1,2$.[4/] This probability can be employed along with the loss structure in Table 1 to choose between the acts forecast DT_2 and forecast NDT_2 so as to minimize expected loss. Also, this analysis can be combined with the problem of obtaining optimal point predictions of z_1 and z_2. Further, the probability that the future values of the series satisfy $z_{i-2} < z_{i-1} < z_i > z_{i+1} > z_{i+2}$, that is that z_i is a peak value, can be computed. Finally, using m past values and m future values, a DT_m can be defined and its probability calculated and used in forecasting.

Above, we have considered just one model for the observations, $\underset{\sim}{y}' = (y_1,y_2,\ldots,y_{n-1},y_n)$. Often forecasters utilize several alternative models, $M_1,M_2,\ldots,M_r$, for example autoregressive models of differing orders, autoregressive models with various leading indicator variables, etc. Let $p_i(\underset{\sim}{y}|M_i,D_i)$ be the marginal pdf for $\underset{\sim}{y}$, based on model M_i and sample and prior information D_i.[5/] Then the posterior probability associated with the i'th model M_i, denoted by P_i is given by

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[4/] Note that $y_{n-2},y_{n-1} < y_n > z_1,z_2$ is an alternative definition of a DT_2 and its probability of occurring is $\int_{-\infty}^{y_n}\int_{-\infty}^{y_n} p(z_1,z_2|D)dz_1,dz_2$, somewhat different from (2.15).

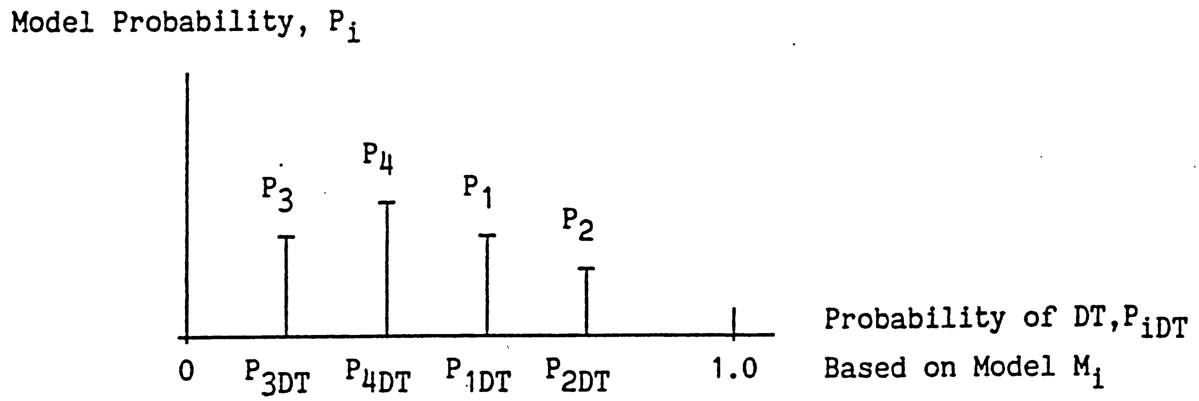
[5/] As is well known, $p_i(y|M_i,D_i) = \int_{\Theta_i} p_i(\underset{\sim}{y}|M_i,\underset{\sim}{\theta}_i)\pi(\underset{\sim}{\theta}_i|I_i)d\underset{\sim}{\theta}_i$ where $p_i(\underset{\sim}{y}|M_i,\underset{\sim}{\theta}_i)$ is the likelihood function given model i, $\underset{\sim}{\theta}_i$ is a vector of parameters with prior pdf $\pi(\underset{\sim}{\theta}_i|I_i)$, where I_i denotes the prior parameters, $D_i = (\underset{\sim}{y},I_i)$ and $\underset{\sim}{\theta}_i \subset \Theta_i$, the parameter space.

$$P_i = \Pi_i p_i(\underset{\sim}{y}|M_i,D_i) / \sum_{i=1}^{r} \Pi_i p_i(\underset{\sim}{y}|M_i,D_i) \qquad (2.16)$$

where Π_i, $i = 1,2,\ldots,r$ is the prior probability associated with M_i. Also, for each model the predictive pdf for $z = y_{n+1}$ can be derived and is denoted by $p_i(z|M_i,D_i)$. It can be used, for example, to compute the probability of a DT, P_{iDT}. The P_{iDT}'s and the P_i's in (2.16) allow us to form a probability mass function as shown in Fig. 1 for the case of r=4. Fig. 1

Fig. 1

Probability Mass Function for Probabilities of a DT



reveals the effects of model uncertainty on the probability of a DT. That is, instead of having a single probability of a DT, when there is model uncertainty, we have several probabilities of a DT, $P_{1DT}, P_{2DT}, \ldots, P_{rDT}$ and their respective probabilities, $P_1, P_2, \ldots, P_r$. The usual practice of selecting one model and viewing it as "absolutely true" in deriving a single probability of a DT obviously abstracts from model uncertainty and is inappropriate when model uncertainty is present.

Formally, we have the marginal predictive pdf, $p(z|D)$, given by

$$p(z|D) = \sum_{i=1}^{r} P_i p_i(z|M_i,D_i) \qquad (2.17)$$

where P_i is given in (2.15) and D is the union of the D_i. Then, for example, the probability of a DT_1, P_{DT_1} is given by

$$P_{DT_1} = \int_{-\infty}^{y_n} p(z|D)dz \qquad (2.18)$$
$$= \sum_{i=1}^{r} P_i P_{iDT_1}$$

where $P_{iDT_1} = \int_{-\infty}^{y_n} p_i(z|M_i,D_i)dz$ is the probability of DT_1 based on model M_i. It is seen from the second line of (2.18) that P_{DT_1} is an average of the P_{iDT_1} with the posterior model probabilities, the P_i's serving as weights. Further, various measures can be computed to characterize the dispersion and other features of the P_{iDT_1}'s. For example their variance is given by

$$Var(P_{iDT_1}) = \sum_{i=1}^{r} P_i(P_{iDT_1} - P_{DT_1})^2. \qquad (2.19)$$

Given a loss structure such as that in Table 1, P_{DT_1} in (2.18) can be employed to make an optimal choice between forecast DT_1 or forecast NDT_1. Similar analysis yields results for forecasting an upturn or no upturn. For broader definitions of turning points, predictive pdfs for several future observations would replace $p_i(z|M_i,D_i)$ in (2.17) and the integral in (2.18) would have to be modified along the lines shown in (2.15).

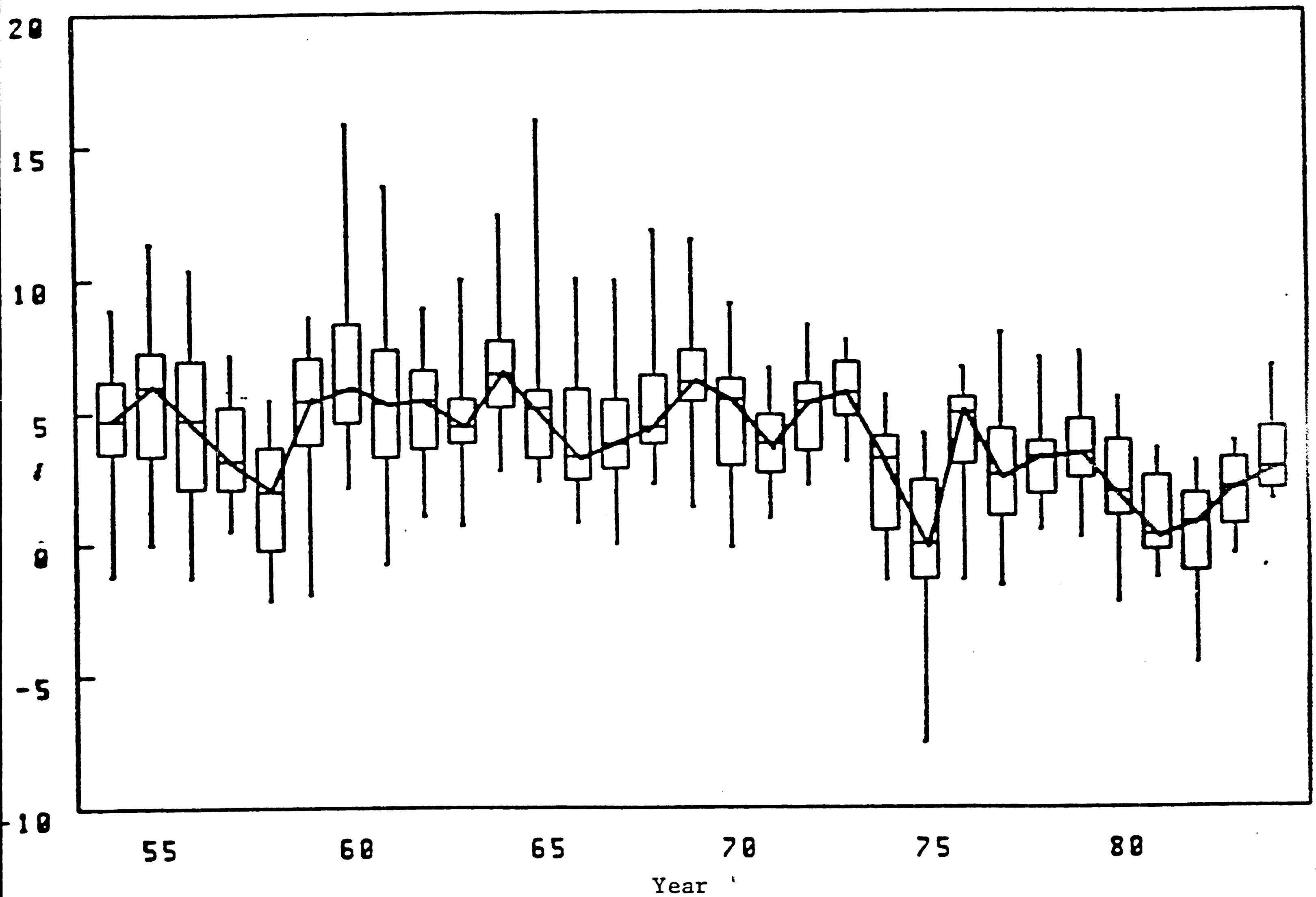
Finally, the problem of forecasting turning points in two or more time series is of interest. Given a definition of a turning point, and a model for two or more time series, probabilities of downturns or upturns can be computed from the joint predictive pdf of future values of the several time series. Then, given a loss structure, optimal turning point forecasts can be derived. In the case of two time series, for example the rates of growth of output and of inflation, the set of possible forecasts will involve forecasting downturns for both variables, a downturn for one variable and no downturn for the other, or no downturn for both variables. Given m such possible forecasts and m possible outcomes, an m×m loss structure can be defined, an expanded version of the 2×2 case shown in Table 1. Using probabilities associated with possible outcomes, the forecast that minimizes expected loss can be determined along the lines shown in (2.4) for the 2×2 case. Also, the multiple models, multiple time series case can be addressed using a generalization of the multiple models, one time series case analyzed above.

3. Data and Applications

In this Section some of the techniques described above will be applied in the analysis of data relating to annual output growth rates for 18 countries used in our previous work, Garcia-Ferrer et al. (1987) and Zellner and Hong (1987).

Shown in Fig. 2 is a boxplot of the annual rates of output growth for 18 countries over the period, 1954-84. It is seen that the annual median growth rates, given by the horizontal line in each box, appear to follow a cyclical path with peaks in 1955, 1960, 1964, 1969, 1973, 1976, and 1979 and troughs in 1958, 1963, 1966, 1971, 1975, and 1981. The average time between peaks is 4.2 years and between troughs is 4.4 years. To provide more detail, Fig. 3 provides the number of countries experiencing peaks and troughs in each year. It is seen that many countries experienced peaks in years close to or at 1955, 1960, 1964, 1969, 1973, 1976, 1979, and 1984. As regards troughs, they were encountered for many countries in years close to or at 1954, 1958, 1962, 1966-67, 1971, 1975, 1977,

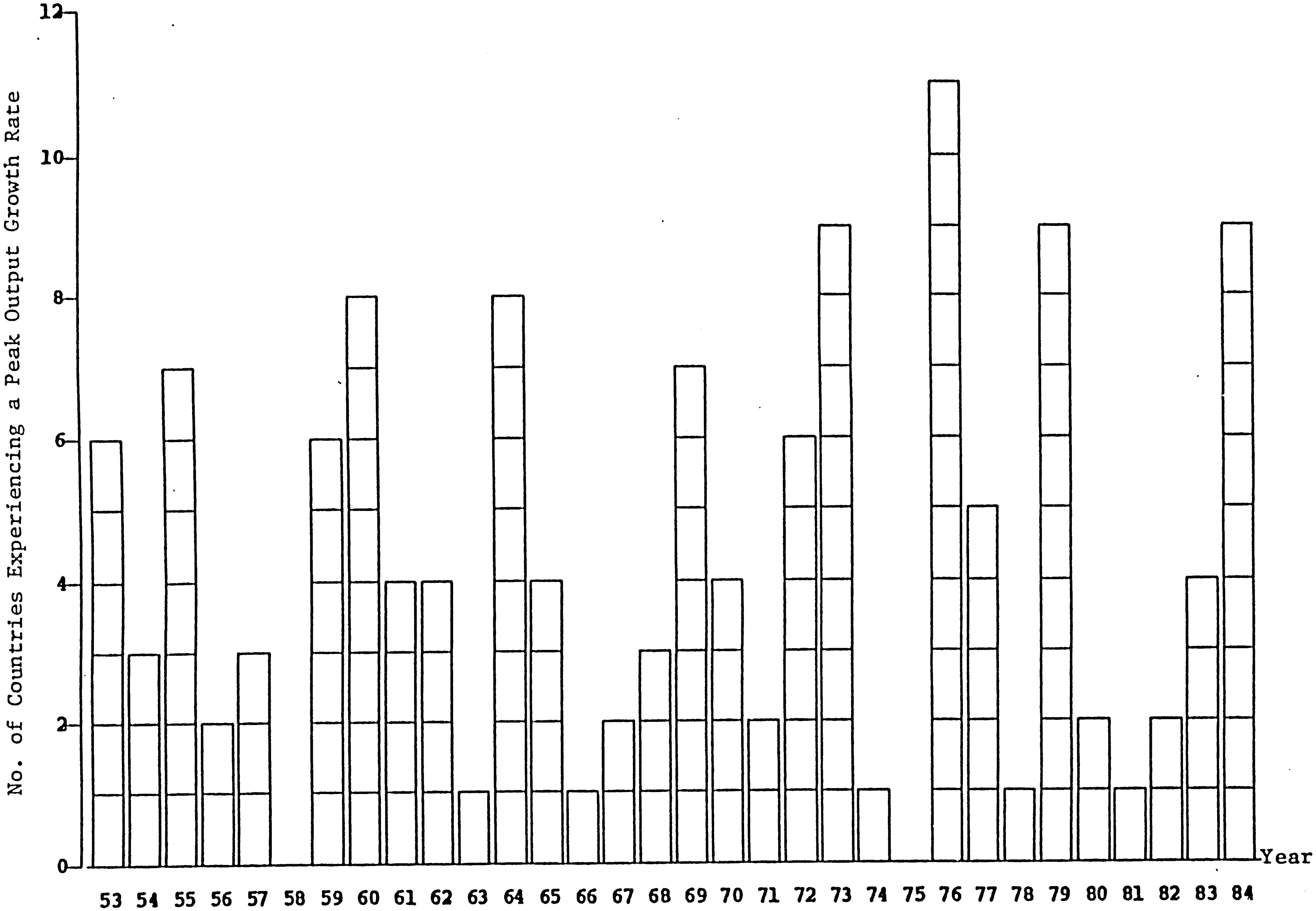
Fig. 2
Annual Growth Rates of Real Output for Eighteen Countries, 1954-1984[a]



[a] Annual rates of growth of real GNP or GDP for the following countries have been utilized: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, United Kingdom and United States. Data from the U. of Chicago, Graduate School of Business IMF data base.

Fig. 3a

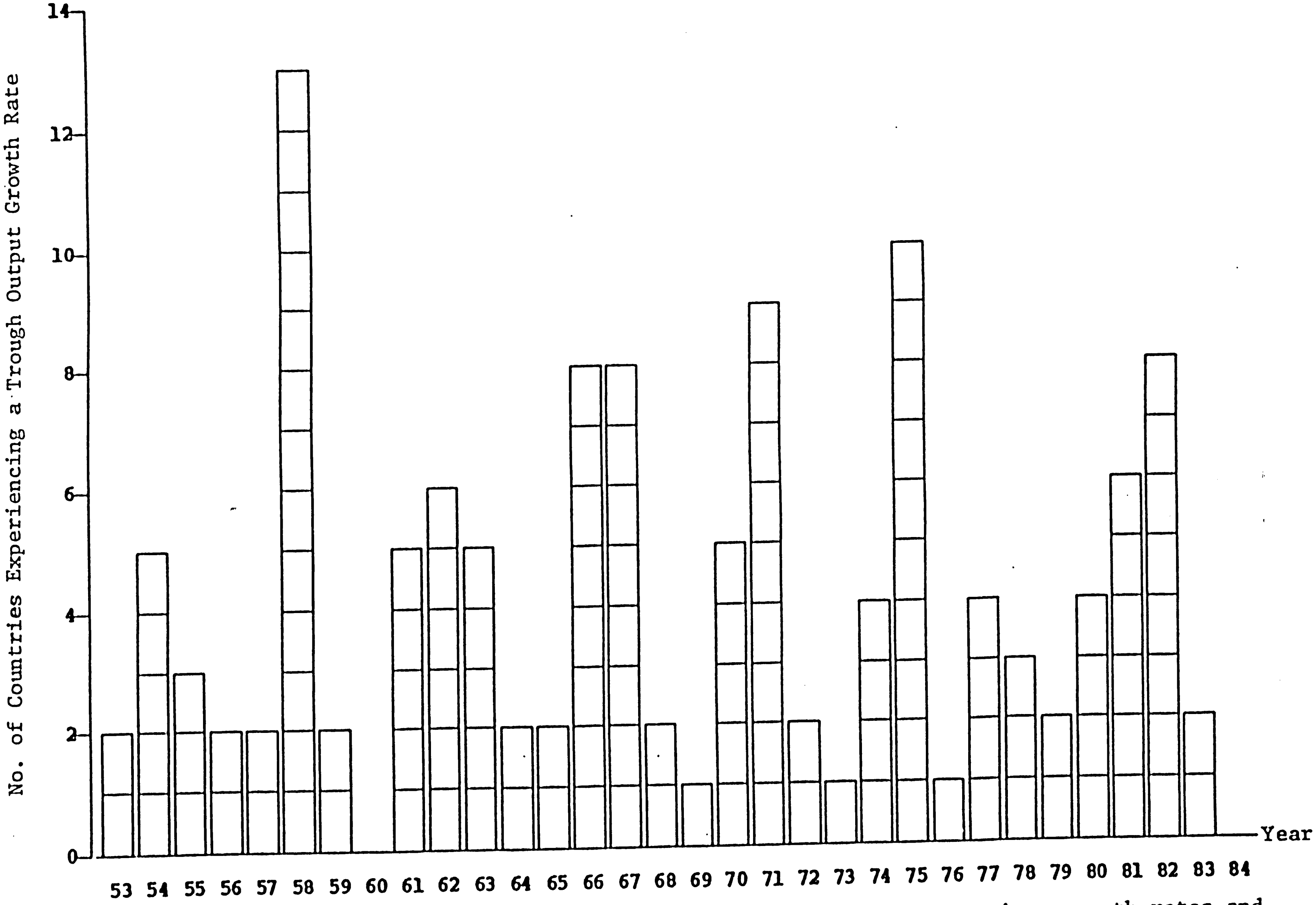
Number of Countries Experiencing a Peak Output Growth Rate by Years, 1953-84 for 18 Countries[a]



[a] A peak is defined to be an annual growth rate which is larger than the two previous growth rates and the following growth rate. That is y_n is a peak growth rate if $y_{n-2}, y_{n-1} < y_n > y_{n+1}$. The data on real GNP or GDP were obtained from the U. of Chicago, Graduate School of Business IMF data base and relate to the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Sweden, Spain, Switzerland, United Kingdom and United States.

Fig. 3b

Number of Countries Experiencing a Trough Output Growth Rate by Years, 1953-84 for 18 Countries[a]



a/ A trough is defined to be an annual growth rate which is smaller than the two previous growth rates and the following growth rate. That is y_n is a trough growth rate if $y_{n-1}, y_{n-2} > y_n < y_{n+1}$. See Fig. 3a for source of data and a listing of the 18 countries.

and 1982. These descriptive measures reflect the well known fact that economies' output growth rates, while not perfectly synchronized, tend to move up and down together, as noted by Burns and Mitchell (1946), Zarnowitz (1985) and others.

We now turn to the problem of forecasting turning points for the 18 countries' growth rates of output. Here a downturn is defined to be a sequence of observations satisfying $y_{n-1}, y_{n-2} < y_n > y_{n+1}$ and an upturn a sequence of observations satisfying $y_{n-2}, y_{n-1} > y_n < y_{n+1}$. Our forecasting model, an autoregression of order three with leading indicator variables, denoted by AR(3)LI model, used in our previous work, Garcia-Ferrer et al. (1987) and Zellner and Hong (1987) is

$$y_{it} = \alpha_{i0} + \alpha_{1i}y_{it-1} + \alpha_{2i}y_{it-2} + \alpha_{3i}y_{it-3} + \beta_{1i}SR_{it-1} \qquad i = 1,2,\ldots,18 \qquad (3.1)$$
$$+ \beta_{2i}SR_{it-2} + \beta_{3i}GM_{it-1} + \beta_{4i}WR_{t-1} + \varepsilon_{it} \qquad t = 1,2,\ldots,T$$

where the subscripts i and t denote the i'th country and t'th year, respectively and

y_{it} = growth rate of real output
SR_{it} = growth rate of real stock prices
GM_{it} = growth rate of real money
WR_t = "world return," the median of the SR_{it}'s.
ε_{it} = error term.

The ε_{it}'s are assumed independently drawn from a normal distribution with zero mean and variance σ_i^2 for all i and t. Using annual data, 1951-1973, a diffuse prior pdf for the parameters, the predictive pdfs for 1974 and subsequent years were computed and probabilities of downturns and upturns were computed. Then using the loss structure in Table 2 with $c_1=c_2$ forecasts of downturns (DTs) and upturns (UTs) were made. In this case, a DT is forecasted when $P_{DT} > 1/2$ and NDT is forecasted when $P_{DT} < 1/2$. Also, an UT is forecasted when $P_{UT} > 1/2$ and NUT when $P_{UT} < 1/2$. For all 18 countries over the period 1974-85, under the definition two preceding observations below (or above) a current observation and the following observation below (or above) the current observation, our forecasting procedure yielded the results shown in Table 2.

Table 2

Forecasts of Turning Points for 18 Countries' Output Growth Rates, 1974-85

A. Downturn (DT) and No Downturn (NDT)

Forecast	Correct	Incorrect	Total
DT	35	5	40
NDT	10	18	28
Total	45	23	68
Percent	(66%)	(34%)	(100%)

B. Upturn (UT) and No Upturn (NUT)

Forecast	Correct	Incorrect	Total
UT	43	20	63
NUT	17	2	19
Total	60	22	82
Percent	(75%)	(25%)	(100%)

We see from panel A of Table 2 that 45 out of 68 or 66% of the DT/NDT forecasts are correct. Of the 40 DT forecasts 35 of 40 or 88% are correct. However, only 10 of 28 or 36% of the NDT forecasts are correct. As regards forecasts of UT's or NUT's, 60 of 82 forecasts, 75% are correct. Of the 63 UT forecasts, 43 or 68% are correct while for the NUT forecasts, 17 of 19, or 90% are correct. Thus except for the NDT forecasts, the turning point forecasts are quite good.

Fig. 4 provides additional information regarding the forecasts of DTs and NDTs. It is seen that when the probability of a DT is between 0.40 and 0.60, approximately half of the forecasts are correct. When the probability of a downturn exceeds 0.60, most of the forecasts are correct. However, when the probability of a downturn is less than 0.40, a substantial number of errors are made, that is the NDT forecasts were not particularly good.

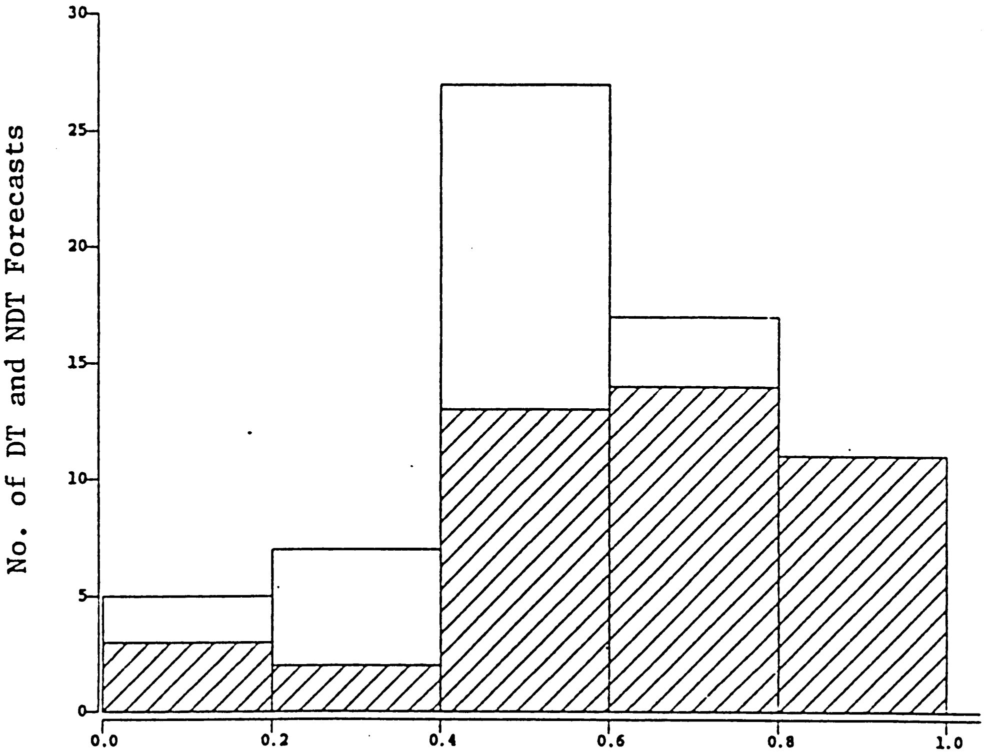
Fig. 5 provides information regarding the forecasts of UTs and NUTs. When the probability of an upturn is between 0.40 and 0.60, approximately half of the forecasts are correct. When the probability of an upturn is greater than 0.60, most of the forecasts are correct and when it is less than 0.40, all forecasts are correct.

Overall the forecasts, except the no-downturn forecasts, are quite good. With reference to Table 1, these forecasts were computed under the assumption that $c_1=c_2$, that is that the cost of incorrectly forecasting a downturn is the same as that of incorrectly forecasting no downturn. This leads to forecasting a downturn if the probability of a downturn is greater than 1/2--see (2.5). If the cost, c_2, of incorrectly forecasting no down-

Fig. 4

Frequency of Correct and Incorrect Forecasts of Output Growth Rate Downturns And No-Downturns by Calculated Probabilities of Downturns For 18 Countries, 1974-85[a]

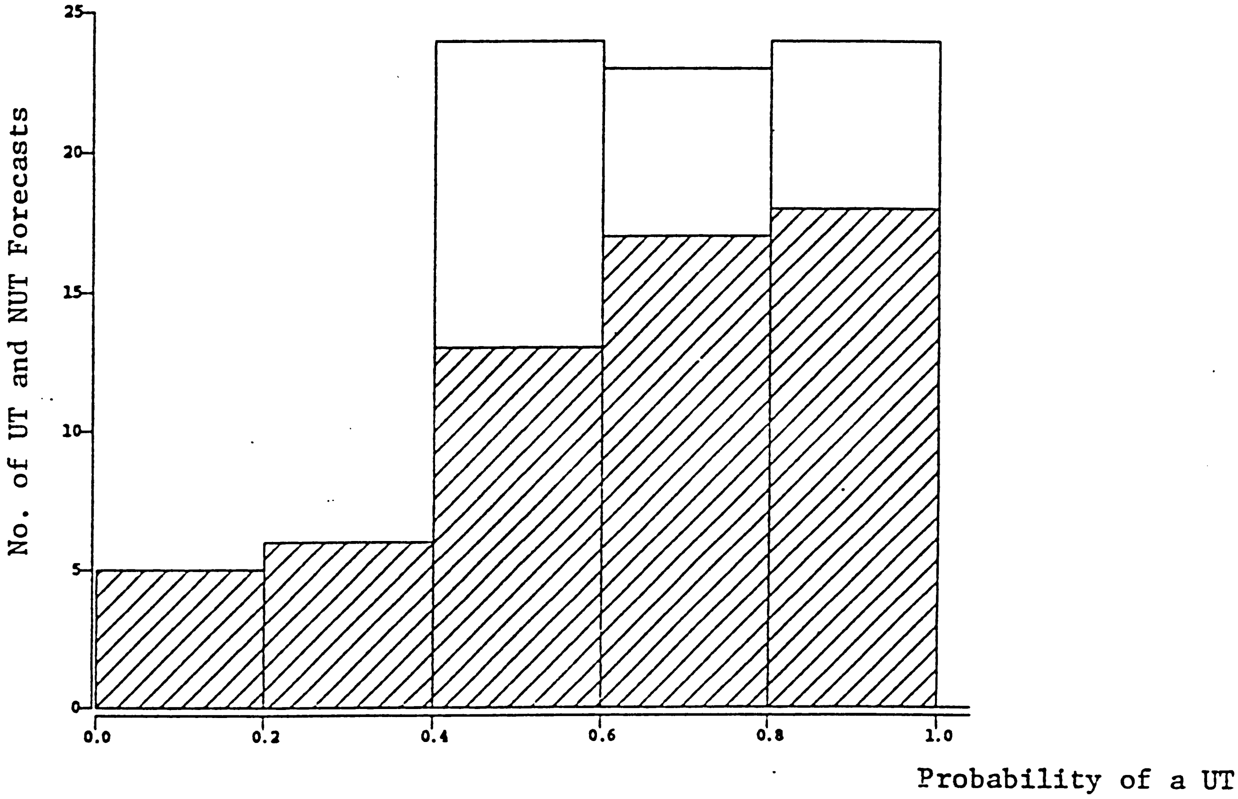
(Shaded areas represent correct forecasts)



[a] The probability of a downturn (DT) at time $t = n$ is given by $P_{DT} = Pr(y_{n+1} < y_n | y_{n-2}, y_{n-1} < y_n, D)$, where $D \equiv$ past data, which is calculated from the predictive pdf for y_{n+1}, based on the AR(3)LI model in (3.1) and a diffuse prior pdf. P_{DT} was calculated for each country and year for which $y_{n-2}, y_{n-1} < y_n$. When $P_{DT} > 1/2$, a DT is the forecast and when $P_{DT} < 1/2$, NDT is the forecast.

Fig. 5
Frequency of Correct and Incorrect Forecasts of Output Growth Rate Upturns (UTs) and No-Upturns (NUTs) by Calculated Probabilities Of Upturns for 18 Countries, 1974-85[a]

(Shaded areas represent correct forecasts)



[a] The probability of an upturn (UT) at time $t = n$ is given by $P_{UT} = Pr(y_{n+1} > y_n | y_{n-2}, y_{n-1} > y_n, D)$, where $D \equiv$ past data, which is calculated from the predictive pdf for y_{n+1} based on the AR(3)LI model in (3.1) and a diffuse prior pdf. P_{UT} was calculated for each country and year for which $y_{n-2}, y_{n-1} > y_n$. When $P_{UT} > 1/2$, UT is the forecast and when $P_{UT} < 1/2$, NUT is the forecast.

turn is greater than c_1, the cost of incorrectly forecasting a downturn, (2.5) leads to forecast of a downturn for $P_{DT} > c_1/(c_1+c_2) = 1/(1+c_2/c_1)$. If, for example $c_2/c_1 = 1.5$, a downturn would be forecasted when $P_{DT} > .4$ and no downturn when $P_{DT} < .4$. With this rule DTs would be more frequently forecasted and NDTs less frequently forecasted relative to the situation in which $c_1=c_2$. In future work, we shall evaluate turning point forecasts for various values of the ratio c_2/c_1.

4. Summary and Concluding Remarks

The problems of forecasting turning points and future values of economic time series were considered with the major result being that such forecasts are very sensitive to properties of loss structures. An operational procedure for forecasting turning points was formulated and applied to forecast turning points in output growth rates for 18 countries, 1974-85 utilizing an AR(3)LI model fitted with pre-1974 data and updated year-by-year in the forecast period. In general, with the exception of no-downturn forecasts, the results were encouraging, namely 66% of the DT and NDT forecasts correct and 75% of the UT and NUT forecasts correct. These results indicate that our AR(3)LI model and our forecasting techniques may be of practical value to applied economic forecasters not only in providing forecasts regarding turning points but also in computing probabilities associated with future, as yet unobserved values of economic variables.[6]

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[6] See Garcia-Ferrer et al. (1987) and Zellner and Hong (1987) for an evaluation of the quality of point forecasts derived from the AR(3)LI model in (3.1).

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