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# MODELLING RESEARCH GROUP



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## FORECASTING INTERNATIONAL GROWTH RATES USING BAYESIAN SHRINKAGE AND OTHER PROCEDURES

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January, 1988

## ABSTRACT

Bayesian and other procedures are developed and applied to forecast annual output growth rates for eighteen countries year by year for the period 1974-84. This work extends earlier work relating to nine countries' data, 1974-81. The new calculations indicate that previous methods work well in forecasting using the new, extended data base. Comparisons of the performance of various Bayesian shrinkage procedures, other procedures and OECD forecasts are provided. It is found that the autoregressive-leading indicator models used in past work continued to perform well in the present study based on an enlarged data base.

# Forecasting International Growth Rates Using Bayesian Shrinkage and Other Procedures

by

Arnold Zellner and Chansik Hong*

## 1. Introduction

In our past work, Garcia-Ferrer, Highfield, Palm, and Zellner (1987), we employed several methods to forecast annual growth rates of real output (GNP or GDP) for eight European Economic Community countries and the U.S. year by year for the period 1974-81. It was found that diffuse prior or least squares forecasts based on an autoregressive model of order three including leading indicator variables, denoted by AR(3)LI were reasonably good in terms of forecast root mean squared error (RMSE) relative to those of three naive models and of AR(3) models without leading indicator variables. Also, it was found that certain shrinkage forecasting techniques produced improved forecasting results for many countries and that our simple, mechanical forecasts compared favorably with OECD annual forecasts which were constructed using elaborate models and judgmental adjustments.

In the present paper our main objectives are to extend our earlier work by: (1) providing further analysis of shrinkage forecasting techniques, (2) providing forecasting results for an extended time period, 1974-84 for our past sample of nine countries, (3) applying our forecasting techniques to data relating to nine additional countries, and (4) reporting results of forecasting experiments using a simple modification of our AR(3)LI model.

The importance of checking the forecasting performance of our techniques using new data is reflected in objectives (2) and (3) above. The modification of our AR(3)LI model, mentioned in (4) was motivated by macroeconomic considerations embedded in structural models that are cur-

*Research financed by the National Science Foundation and by income from the H.G.B. Alexander Endowment Fund, Graduate School of Business, U. of Chicago. Luis Mañas-Antón, Carla Inclan and Michael Zellner provided valuable research assistance.

rently being formulated which yield reduced form equations similar in form to our AR(3)LI forecasting model and reduced form equations for other variables, e.g. the rate of inflation, employment growth, etc. that will be subjected to forecasting tests in the future.

The plan of the paper is as follows. In Section 2 our AR(3)LI model is explained and analysis yielding several shrinkage forecasts is presented. Also, an extended version of our AR(3)LI model is specified and it is shown how it can be employed to yield forecasts. Section 3 is devoted to a presentation of our data and in Section 4 previous forecasting results are compared with those relating to our broader data set and with those yielded by our extended AR(3)LI model. Finally, we provide a summary of results and some concluding remarks in Section 5.

## 2. Model Description and Forecasting Procedures

In this section, we shall describe the autoregressive-leading indicator (ARLI) model employed in our past work as well as some possible extensions of it. Then we shall consider various forecasting procedures for our ARLI models.

### 2.1 Model Description

In Garcia-Ferrer et al. (1987), the following AR(3)LI model was employed to generate one year ahead forecasts of the rate of growth of real output, $y_{it}$, for the eight years, 1974-81 for nine countries:

$$y_{it} = \beta_{0i} + \beta_{1i}y_{it-1} + \beta_{2i}y_{it-2} + \beta_{3i}y_{it-3} + \beta_{4i}SR_{it-1} + \beta_{5i}SR_{it-2} + \beta_{6i}GM_{it-1} + \beta_{7i}WR_{t-1} + u_{it} \qquad i = 1,2,\ldots,9 \quad t = 1,2,\ldots,T \tag{2.1a}$$

or

$$\underset{\sim}{y}_i = X_i\underset{\sim}{\beta}_i + \underset{\sim}{u}_i \qquad i = 1,2,\ldots,9 \tag{2.1b}$$

where, with L = lag operator and the subscript i,t denoting the value of a variable for the i'th country in the t'th time period,

$y_{it}$ = rate of growth of output = $(1-L)\log O_{it}$ with $O_{it}$ real output,

$SR_{it}$ = real stock return = $(1-L)\log(SP_{it}/P_{it})$ with $SP_{it}$ a stock price index and $P_{it}$ a general price index,

$GM_{it}$ = growth rate of real money supply = $(1-L)\log(M_{it}/P_{it})$ with
$M_{it}$ = nominal money supply,
$WR_t$ = world return = median of countries' real stock returns,
$SR_{it}$ in period t,
$\beta_{ji}$'s = parameters for i'th country j = 0,1,...,7,
$u_{it}$ = disturbance term.

In (2.1b) the model for each country is expressed in matrix notation with $y_{it}$ a typical element of $\underset{\sim}{y}_i$, $(1, y_{it-1}, y_{it-2}, y_{it-3}, SR_{it-1}, SR_{it-2}, GM_{it-1}, WR_{t-1})$ a typical row of $X_i$ and $u_{it}$ a typical element of $\underset{\sim}{u}_i$. Some comments regarding the model in (2.1) follow:

(1) An autoregression of order 3 was chosen to permit the possibility of having two complex roots associated with a cycle and a real root associated with a trend. Past calculations indicated that estimated roots had these properties for eight of nine countries. Also, use of just an AR(3) process without leading indicator variables did not perform well in actual forecasting. Use of leading indicator variables led to improved forecasts in most cases as measured by RMSEs of forecast.

(2) The disturbance terms in (2.1) were found to be practically serially uncorrelated for most countries and not highly correlated across countries, results based on least squares analyses of (2.1) using initial annual data 1951-73 for estimation. The introduction of the "common effect" variable, $WR_t$, reduced contemporaneous disturbance terms' correlations considerably.

(3) The leading indicator stock return variables and money growth rate variable apparently caught the effects of oil price shocks, policy changes, etc. in the period of fit 1951-73 and in our previous forecast period 1974-81. Here we are employing market variables to take rough account of expectational and other effects influencing countries' output growth rates.

(4) Macroeconomic considerations suggest that a measure of world output growth and changes in countries' real exchange rates affect countries' exports and these should be included in our ARLI model. Since these variables are close to being white noise, they may be buried in the

disturbance terms of (2.1). Below, we shall report some results using an ARLI model including a measure of world output growth.

(5) In our past work, forecasts from (2.1) using least squares and some shrinkage forecast procedures were reported. Also, some forecasts yielded by a time-varying-parameter version of (2.1) were reported. Some of these results will be presented below and compared with more recently obtained results.

In our forecasting experiments, we employ annual data, usually 1954-73, 20 observations with data for 1951-53 used for initial lagged values to fit our models.[1] Then the fitted models are employed to forecast outcomes for 1974 and subsequent years with the models reestimated year by year. Multi-year ahead forecasts have not as yet been calculated. For the forecast period 1974-81, eight years, least squares forecasts using (2.1) have yielded forecast RMSEs ranging from 1.47 to 2.92 with a median of 2.23 percentage points for eight EEC countries and the U.S.--see Table 2, line F, of Garcia-Ferrer et al. (1987). Our "η-shrinkage" forecasts described below, yielded forecast RMSEs ranging from 1.25 to 2.52 percentage points with a median of 1.78 percentage points--see Table 4, line G3 of Garcia-Ferrer et al. (1987). Similar results for an extended time period and for nine additional countries are presented below.

## 2.2 Derivation and Description of Shrinkage Forecasts

In this sub-section, we provide derivations of several shrinkage forecasts, including the "η-forecast" and the "γ-forecast." The performance of these forecasts will be compared with those of naive models and diffuse prior forecasts or least squares forecasts derived from the ARLI model, or variants of it, shown in (2.1).

The η-forecast involves averaging a forecast from (2.1), say a diffuse prior or least squares forecast for a particular country, $\hat{y}_{if}$, with the mean of all the N countries' forecasts, $\bar{\hat{y}}_f = \sum_{i=1}^{N} \hat{y}_{if}/N$, as follows:

[1]The 1954-73 period was used for all countries except Australia, 1960-73, Canada, 1959-73, Japan, 1956-73, and Spain, 1958-73.

$$\tilde{y}^*_{if} = \eta\bar{\hat{y}}_f + (1-\eta)\hat{y}_{if}$$
$$= \bar{\hat{y}}_f + (1-\eta)(\hat{y}_{if}-\bar{\hat{y}}_f). \qquad (2.2)$$

From the second line of (2.2), it is seen that for $0 \leq \eta < 1$, a country's forecast, $\hat{y}_{if}$, is shrunk toward the average forecast $\bar{\hat{y}}_f$ for all countries.

One way to obtain an optimal forecast in the form of (2.2) is to employ the following predictive loss function,

$$L = (y_{if} - \tilde{y}_{if})^2 + c\Big(\sum_{i=1}^{N} y_{if}/N - \tilde{y}_{if}\Big)^2 \qquad (2.3)$$

where $c > 0$ is a given constant, the $y_{if}$'s are the future unknown values, $i = 1,2,\ldots,N$, and $\tilde{y}_{if}$ is some forecast. Note that the loss function in (2.3) incorporates an element of loss associated with being away from the mean outcome in its second term. Under the assumption that the $y_{if}$'s are independent (common influences have been represented by input variables in (2.1)) and have predictive probability density functions (pdfs) with mean $m_i$ and variance $v_i$, $i = 1,2,\ldots,N$. The predictive expectation of the loss function in (2.3) is:

$$EL = v_i + (m_i - \tilde{y}_{if})^2 + c[E(\bar{y}_f - E\bar{y}_f)^2 + (E\bar{y}_f - \tilde{y}_{if})^2] \qquad (2.4)$$

where $\bar{y}_f = \sum_{i=1}^{N} y_{if}/N$. On minimizing (2.4) with respect to $\tilde{y}_{if}$, the result is:

$$\tilde{y}^*_{if} = \eta\sum_{i=1}^{N} m_i/N + (1-\eta)m_i \qquad (2.5)$$

where $\eta = c/(1+c)$. If diffuse priors for the $\underset{\sim}{\beta}_i$'s in (2.1) are employed, the means of the predictive pdfs are $m_i = \underset{\sim}{x}'_{if}\hat{\underset{\sim}{\beta}}_i$, $i = 1,2,\ldots,N$, where $\underset{\sim}{x}'_{if}$ is a vector of observed inputs for the first future period and $\hat{\underset{\sim}{\beta}}_i = (X'_iX_i)^{-1}X'_i\underset{\sim}{y}_i$, the least squares estimate for country i. Under these conditions (2.5) takes the form of (2.2) with $\hat{y}_{if} = \underset{\sim}{x}'_{if}\hat{\underset{\sim}{\beta}}_i$. This is the "diffuse prior $\eta$-forecast."

Another approach for obtaining relatively simple shrinkage forecasts is a slightly modified form of the Lindley-Smith (1972) procedure in which the coefficient vectors are assumed generated by:

$$\underset{\sim}{\beta}_i = \underset{\sim}{\theta} + \underset{\sim}{\delta}_i \qquad i = 1,2,\ldots,N \qquad (2.6)$$

with the $\underset{\sim}{\delta}_i$'s assumed independently distributed, each having a $N(\underset{\sim}{0},\phi^{-1}\sigma_u^2 I_k)$

distribution where $0 < \phi < \infty$, $\sigma_u^2$ is a common variance of $u_{it}$ for all i and t and $\underset{\sim}{\theta}$ is a $k \times 1$ mean vector. If the $u_{it}$'s are assumed normally and independently distributed, each with zero mean and common variance $\sigma_u^2$, then a conditional point estimate for $\underset{\sim}{\beta}' = (\underset{\sim}{\beta}_1', \underset{\sim}{\beta}_2', \ldots, \underset{\sim}{\beta}_N')$, denoted by $\tilde{\underset{\sim}{\beta}}_a$, an $Nk \times 1$ vector, is given by

$$\tilde{\underset{\sim}{\beta}}_a = (Z'Z + \phi I_{Nk})^{-1}(Z'Z\hat{\underset{\sim}{\beta}} + \phi J\tilde{\underset{\sim}{\beta}}) \tag{2.7}$$

where Z is a block diagonal matrix with $X_1, X_2, \ldots, X_N$ on the main diagonal, $\hat{\underset{\sim}{\beta}} = (Z'Z)^{-1}Z'\underset{\sim}{y}$, where $\underset{\sim}{y}' = (\underset{\sim}{y}_1', \underset{\sim}{y}_2', \ldots, \underset{\sim}{y}_N')$, $\hat{\underset{\sim}{\beta}}' = (\hat{\underset{\sim}{\beta}}_1', \hat{\underset{\sim}{\beta}}_2', \ldots, \hat{\underset{\sim}{\beta}}_N')$ with $\hat{\underset{\sim}{\beta}}_i = (X_i'X_i)^{-1}X_i'\underset{\sim}{y}_i$, $J' = (I_k, I_k, \ldots, I_k)$ and

$$\tilde{\underset{\sim}{\beta}} = (\sum_{i=1}^{N} X_i'X_i)^{-1} \sum_{i=1}^{N} X_i'X_i\hat{\underset{\sim}{\beta}}_i \tag{2.8}$$

a matrix-weighted average of the least squares estimates, the $\hat{\underset{\sim}{\beta}}_i$'s which replaces $\underset{\sim}{\theta}$ in (2.7). Also, $\tilde{\underset{\sim}{\beta}}$ in (2.8) can be obtained by regressing $\underset{\sim}{y}' = (\underset{\sim}{y}_1', \underset{\sim}{y}_2', \ldots, \underset{\sim}{y}_N')$ on X, where $X' = (X_1', X_2', \ldots, X_N')$, that is from one big regression in which it is assumed that the $\underset{\sim}{\beta}_i$'s are equal. Point forecasts can be obtained using the coefficient estimate $\tilde{\underset{\sim}{\beta}}_a$ for various selected values of $\phi$. When $\phi$ is very large, (2.7) reduces approximately to (2.8). Forecasts based on the estimate in (2.8) have been reported earlier in Garcia-Ferrer et al. (1987). It should be recognized that while the $\underset{\sim}{\beta}_i$'s are probably not all the same, the bias introduced by assuming them to be may be more than offset in a MSE error sense by a reduction in variance.

As an alternative to the assumptions used in connection with (2.6), following the "g-prior" approach of Zellner (1983, 1986a), we assume that the $\underset{\sim}{\delta}_i$'s in (2.6) are independently distributed with normal distributions $N[\underset{\sim}{0}, (X_i'X_i)^{-1}\sigma_\delta^2]$. With this assumption and the earlier assumption made about the $u_{it}$'s, the joint pdf for $\underset{\sim}{y}' = (\underset{\sim}{y}_1', \underset{\sim}{y}_2', \ldots, \underset{\sim}{y}_N')$ and $\underset{\sim}{\beta}' = (\underset{\sim}{\beta}_1', \underset{\sim}{\beta}_2', \ldots, \underset{\sim}{\beta}_N')$ is proportional to

$$\exp\{-[(\underset{\sim}{y}-Z\underset{\sim}{\beta})'(\underset{\sim}{y}-Z\underset{\sim}{\beta}) + \gamma(\underset{\sim}{\beta}-J\underset{\sim}{\theta})'Z'Z(\underset{\sim}{\beta}-J\underset{\sim}{\theta})]/2\sigma_u^2\} \tag{2.9}$$

where $\gamma = \sigma_u^2/\sigma_\delta^2$ and other quantities have been defined in connection with (2.6)-(2.7). On completing the square on $\underset{\sim}{\beta}$ in the exponential terms of (2.9), the mean of $\underset{\sim}{\beta}$ given $\underset{\sim}{y}$, $\gamma$ and $\underset{\sim}{\theta}$ is:

$$\bar{\beta} = [(Z'Z)^{-1}Z'y + \gamma J\theta]/(1+\gamma) \tag{2.10a}$$

with the i'th sub-vector of $\bar{\beta}$ given by:

$$\bar{\beta}_i = (\hat{\beta}_i + \gamma\theta)/(1+\gamma) \tag{2.10b}$$

where $\hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i$, the least squares estimate for the i'th country's data. Thus (2.10b) is a simple average of $\hat{\beta}_i$ and $\theta$ with $\gamma = \sigma_u^2/\sigma_\delta^2$ involved in the weights. When a diffuse prior pdf for $\theta$, $p(\theta) \propto$ const., is employed, the posterior pdf for $\theta$ can be derived from (2.9) and employed to average the expression in (2.10) to obtain the marginal mean of $\beta_i$ given $\gamma$ and the data, namely,

$$\bar{\beta}_i^m = (\hat{\beta}_i + \gamma\tilde{\beta})/(1+\gamma) \tag{2.11}$$

with $\tilde{\beta}$ given in (2.8), the estimate resulting from a big regression in which the $\beta_i$'s are assumed equal. As $\gamma = \sigma_u^2/\sigma_\delta^2$ grows in value, $\bar{\beta}^m \to \tilde{\beta}$ while as $\gamma = \sigma_u^2/\sigma_\delta^2 \to 0$, $\bar{\beta}^m \to \hat{\beta}_i$, the i'th country's least squares estimate.

If instead of assuming that $\delta_i$ has a $N(0,(X_i'X_i)^{-1}\sigma_\delta^2)$ distribution, we assume that the $\delta_i$'s are independently distributed, with a $N[0,(X_i'X_i)^{-1}\sigma_i^2]$ distribution, i = 1,2,...,N, then analysis similar to that presented in connection with (2.9) yields as the conditional mean of $\beta_i$,

$$\bar{\beta}_i^c = (\hat{\beta}_i + \gamma_i\theta)/(1+\gamma_i) \qquad i = 1,2,\ldots,N \tag{2.12}$$

where $\gamma_i = \sigma_u^2/\sigma_i^2$. If we further condition on $\theta = \tilde{\beta}$, with $\tilde{\beta}$ given in (2.8), then

$$\bar{\beta}_i^c = (\hat{\beta}_i + \gamma_i\tilde{\beta})/(1+\gamma_i) \qquad i = 1,2,\ldots,N \tag{2.13}$$

which is similar to (2.11) except that the $\gamma_i$'s are not all equal as is the case in (2.11).

Upon introducing prior pdfs for $\sigma_u$ and $\sigma_\delta$, or $\sigma_u$ and the $\sigma_i$'s it is possible to compute the marginal distributions of the $\beta_i$'s, a possibility to be explored in future work--see Miller and Fortney (1984) for interesting computations on a closely related problem. At present, we shall evaluate (2.13) for various values of $\gamma_i$ and determine the quality of resulting forecasts. That is, the $\gamma_i$-forecast for country i is:

$$\tilde{y}_{if} = x_{if}'\bar{\beta}_i^c \tag{2.14}$$

with $\bar{\beta}_i^c$ given in (2.13) and $x_{if}'$ a given input vector.

In summary, we shall use the $\eta$-forecast in (2.2), the $\gamma$-forecast based on (2.11) and the $\gamma_i$-forecast in (2.14) in our forecasting experiments. Also, note that the $\eta$-forecasting approach can be applied to the $\gamma$-forecasts.

## 2.3 Elaboration of the AR(3)LI Model

As mentioned previously, we think that it is advisable to add a variable reflecting world real income growth, denoted by $w_t$ to our AR(3)LI model in (2.1). Then our equation becomes

$$y_{it} = w_t\alpha_i + \underset{\sim}{x}'_{it}\underset{\sim}{\beta}_i + u_{it} \qquad \begin{array}{l} i = 1,2,\ldots,N \\ t = 1,2,\ldots,T \end{array} \qquad (2.15)$$

where $\underset{\sim}{x}'_{it}\underset{\sim}{\beta}_i$ represents the constant and other lagged variables in (2.1) and $\alpha_i$ is the i'th country's coefficient of the world income growth rate variable, $w_t$. To forecast one period ahead using (2.15), it is clear that $w_t$ must be forecasted. To do this we introduce the following equation for $w_t$ which will be estimated and used to forecast $w_t$ one period in the future:

$$w_t = \pi_0 + \pi_1 w_{t-1} + \pi_2 w_{t-2} + \pi_3 w_{t-3} + \pi_4 MSR_{t-1} + \pi_5 MGM_{t-1} + v_t \qquad t = 1,2,\ldots,T \qquad (2.16)$$

where $MSR_t$ = the median of all countries' real stock returns, $MGM_t$ = the median of all countries' real money growth rates, $v_t$ = disturbance term, and the $\pi_i$'s are parameters. Thus (2.16) indicates that we are employing an AR(3)LI model for $w_t$, the rate of growth of world real income. As a proxy for $w_t$, we employ the median of all countries' real output growth rates.

Viewing (2.15)-(2.16), it is seen that we have a "triangular" system. For the future period f = T + 1, we have

$$Ey_{if} = Ew_f E\alpha_i + \underset{\sim}{x}'_{if} E\underset{\sim}{\beta}_i \qquad i = 1,2,\ldots,N \qquad (2.15a)$$

$$Ew_f = \underset{\sim}{z}'_f E\underset{\sim}{\pi} \qquad (2.15b)$$

where $Ey_{if}$ and $Ew_f$ are means of the predictive pdfs for $y_{if}$ and $w_f$ respectively, $E\alpha_i$, $E\underset{\sim}{\beta}_i$ and $E\underset{\sim}{\pi}$ are posterior means of the parameters $\alpha_i$, $\underset{\sim}{\beta}_i$ and $\underset{\sim}{\pi}' = (\pi_0, \pi_1, \ldots, \pi_6)$, respectively. If the system in (2.15)-(2.16) is <u>fully recursive</u> and diffuse prior pdfs for all parameters are employed,

$E\underset{\sim}{\pi} = \hat{\underset{\sim}{\pi}}$, $E\alpha_i = \hat{\alpha}_i$ and $E\underset{\sim}{\beta}_i = \hat{\underset{\sim}{\beta}}_i$, where $\hat{\underset{\sim}{\pi}}$, $\hat{\alpha}_i$ and $\hat{\underset{\sim}{\beta}}_i$ are least squares estimates --see Zellner (1971, Ch. 8) and Bowman and Laporte (1975). Also $Ew_f = \underset{\sim}{z}_f'\hat{\underset{\sim}{\pi}}$ and $Ey_{if} = \underset{\sim}{z}_f'\hat{\underset{\sim}{\pi}}\hat{\alpha}_i + \underset{\sim}{x}_{if}'\hat{\underset{\sim}{\beta}}_i$, $i = 1,2,\ldots,N$. Thus forecasts under these assumptions are easily computed. If the system in (2.15)-(2.16) is not fully recursive, that is the $u_{it}$'s and $v_t$ are correlated then the expectations in (2.15) have to reflect the non-recursive nature of the system. In the present work, we shall use a "conditional" forecasting procedure which is equivalent to a 2SLS point forecast. That is, the parameters $\alpha_i$ and $\underset{\sim}{\beta}_i$ in (2.15) are estimated by 2SLS, a conditional Bayesian estimate--see Zellner (1971, p. 266) and these estimates along with a forecast of $w_t$ from (2.16) are employed to obtain a forecast of $y_{it}$ from (2.15). Such forecasts will be compared with those that assume that $w_t$'s value in a forecast period is perfectly known, a "perfect foresight" assumption. In current work, an unconditional Bayesian approach for analyzing (2.15)-(2.16) when the $u_{it}$'s and $v_t$ are correlated is being developed.

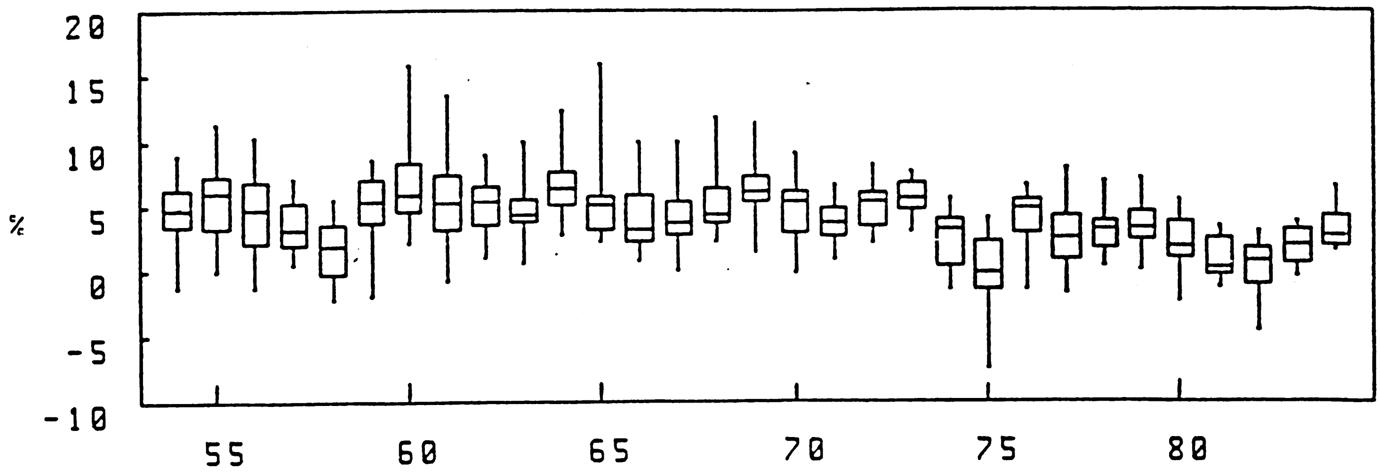
## 3. Data

Annual data for 18 countries employed in our work have been assembled in the main from the International Monetary Fund's International Financial Statistics data base and are available on a diskette from the authors for a nominal fee to cover costs. The output data include annual rates of growth of real output (GNP or GDP), of real stock prices and of real money for each country. In computing rates of growth of real stock prices, an index of nominal stock prices was deflated by an index of the price level for each country. Nominal money, $M_1$, was deflated by a general price index for each country to obtain a measure of real money.

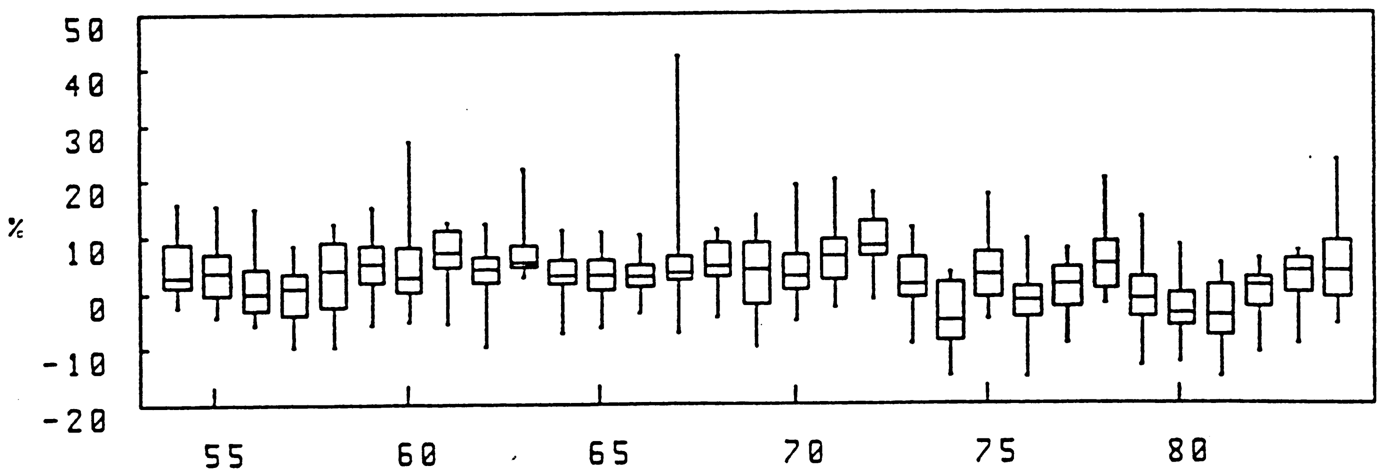
Boxplots of output growth rates, real stock price growth rates and real money growth rates are shown in Fig. 3.1. It is seen that the median growth rates exhibit a cyclical pattern with that for real stock prices having a considerably greater amplitude than those for output and real money growth rates. Also, as might be expected, the interquartile ranges of the real stock price growth rates are much larger than those of output and real money growth rates. Further, the interquartile ranges for growth rates of real stock prices appear to be slightly smaller in

Fig. 3.1 Boxplots of Data for 18 Countries, 1954-84

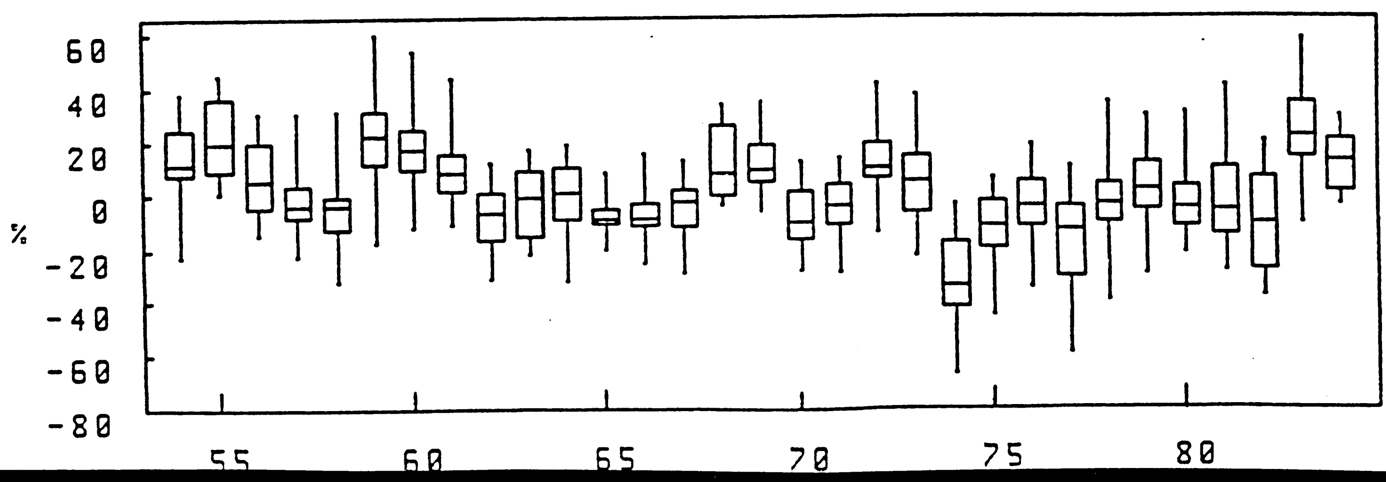
Annual Growth Rates of Real Output, 1954-1984




Annual Growth Rates of Real Money, 1954-1984




Annual Growth Rates of Real Stock Prices, 1954-1984



the vicinity of troughs than of peaks for the first half of the sample and the ranges for all three variables tend to be slightly larger in the vicinity of peaks than of troughs in many cases. Last, the plots of the real money and real stock price growth rates give some evidence of a slight lead relative to those of the output growth rates.

It will be noted from the plots in Fig. 3.1 that there are some apparently outlying points in the data with a number of them present in the data for the nine additional countries' data. These outlying data points are being subjected to close scrutiny and procedures for accommodating outlying data points are being considered for use in future work. In the present study, all data were employed, including outlying data without any special treatment given to them.

## 4. Forecasting Results

In this section, we first compare RMSEs of one-year-ahead forecasts, 1974-81, eight years, with those for the period 1974-84 for our original nine countries using various models and methods. Then for nine additional countries, forecasting results for the period 1974-84 are presented and compared with earlier results. Finally, the effects of introducing a world growth rate variable in our AR(3)LI model in (2.1a) on forecasting performance will be described.

### 4.1 Forecasting Results for an Expanded Data Set

Shown in Table 4.1 are the RMSEs of forecast for nine countries for the periods 1974-81 and for 1974-84. Here and elsewhere, all models were reestimated using data up to the forecast year. In the top panel of Table 4.1, results for the eight one-year-ahead forecasts, 1974-81 are shown. It is seen that the median RMSE for the AR(3)LI model, 2.23 percentage points is quite a bit lower than the median RMSEs for the naive models and for the AR(3) model. In addition, shrinkage or pooling techniques applied to the AR(3)LI model led to median RMSEs of 2.22 and 1.78, a very slight reduction in the former case and a somewhat larger reduction in the latter case, from 2.23 to 1.78 percentage points. Use of the $\eta$-shrinkage procedure, with $\eta=.5$, led to reduction of RMSEs for seven of the nine countries.

Table 4.1
Nine Countries' RMSEs of One-Year-Ahead Forecasts, 1974-81 and 1974-84

| Model | Bel. | Den. | Fr. | Ger. | Ire. | It. | Neth. | U.K. | U.S. | Median |
|---|---|---|---|---|---|---|---|---|---|---|
| | (Percentage points) | | | | | | | | | |
| | A. 1974-81 | | | | | | | | | |
| A. NMI ($\hat{y}_t=0$) | 3.09 | 2.83 | 2.96 | 2.95 | 4.38 | 3.72 | 3.77 | 2.21 | 3.48 | 3.09 |
| B. NMII ($\hat{y}_t=y_{t-1}$) | 4.25 | 3.73 | 2.43 | 3.26 | 2.06 | 4.88 | 4.04 | 3.91 | 3.60 | 3.73 |
| C. NMIII ($\hat{y}_t$= past average) | 3.23 | 3.48 | 3.05 | 3.87 | 1.88 | 3.90 | 3.74 | 2.95 | 2.81 | 3.23 |
| D. AR(3)[a] | 3.66 | 3.46 | 2.89 | 3.39 | 1.69 | 4.75 | 3.52 | 3.50 | 2.48 | 3.46 |
| E. AR(3)LI[b] | 1.56 | 2.92 | 2.43 | 1.47 | 1.83 | 2.57 | 2.63 | 2.23 | 1.82 | 2.23 |
| 1. Shrinkage(1)[c] | 1.69 | 2.37 | 1.35 | 2.03 | 1.77 | 2.22 | 2.87 | 2.26 | 2.75 | 2.22 |
| 2. Shrinkage(2)[d] | 1.68 | 2.21 | 1.61 | 1.25 | 1.52 | 2.01 | 2.52 | 2.46 | 1.78 | 1.78 |
| | B. 1974-84 | | | | | | | | | |
| A. NMI ($\hat{y}_t=0$) | 2.59 | 2.78 | 2.65 | 2.69 | 4.02 | 3.27 | 3.32 | 2.30 | 3.79 | 2.78 |
| B. NMII ($\hat{y}_t=y_{t-1}$) | 3.53 | 3.56 | 2.20 | 2.91 | 2.85 | 4.26 | 3.57 | 3.69 | 3.89 | 3.56 |
| C. NMIII ($\hat{y}_t$= past average) | 3.01 | 3.05 | 3.08 | 3.85 | 2.35 | 3.98 | 3.87 | 2.62 | 3.09 | 3.08 |
| D. AR(3)[a] | 2.98 | 2.96 | 2.47 | 3.10 | 2.29 | 4.34 | 3.35 | 3.21 | 3.01 | 3.01 |
| E. AR(3)LI[b] | 1.73 | 2.73 | 2.52 | 2.28 | 2.80 | 3.40 | 2.41 | 2.32 | 2.14 | 2.41 |
| 1. Shrinkage(1)[c] | 1.96 | 2.26 | 1.66 | 2.00 | 2.14 | 2.45 | 2.53 | 2.39 | 2.79 | 2.26 |
| 2. Shrinkage(2)[d] | 1.81 | 2.37 | 2.07 | 1.94 | 2.31 | 2.73 | 2.50 | 2.63 | 2.03 | 2.31 |

a. Least squares forecasts from an AR(3) model for each country.

b. Least squares forecasts from AR(3) model with leading indicator variables shown in equation (2.1a).

c. Least squares forecasts with use of coefficient estimate in (2.8).

d. Use of shrinkage equation in (2.2) with $\eta$=.5.

As regards the results for 1974-84, in the lower panel of Table 4.1, the AR(3)LI model's median RMSE is 2.41 percentage points, a good deal lower than those associated with the naive models and the AR(3) model. Also, the two shrinkage or pooling procedures produced modest decreases in median RMSEs, from 2.41 to 2.26 and 2.31 and in six of nine cases for the $\eta$-shrinkage procedure. In these respects, the forecasting results parallel those obtained for the shorter period, 1974-81. However, note that there is an increase in the median RMSEs for the AR(3)LI model in going from the period 1974-81 to the longer period 1974-84 which also shows up in seven of the nine countries' RMSEs.

In Table 4.2 RMSEs of forecast are shown for nine additional countries. Here the median RMSE for the AR(3)LI model, 3.33 is somewhat larger than that for the naive models and for the AR(3) model. However the shrinkage or pooled forecasts based on the AR(3)LI model show substantial reductions in median RMSEs, from 3.33 to 2.01 and 2.32 which are similar in magnitude to those reported for the original nine countries for the periods 1974-81 and 1974-84 in Table 4.1. In Table 4.2, on comparing the AR(3)LI country RMSEs with the corresponding shrinkage RMSEs, it is seen that there is a reduction of RMSEs in all but one case. Thus for the nine additional countries, shrinkage results in a somewhat greater improvement in forecasting results than in the case of the original nine countries.

In Table 4.3 summary forecasting results for different models applied to all 18 countries' data to forecast year-by-year for the period 1974-84 are reported. It is seen that the AR(3)LI model's median RMSE is about 13 percent or more below those of the AR(3) and naive models' median RMSEs, 2.62 versus 3.00 or greater. Also, the shrinkage or pooled forecasts median RMSEs, 2.14 and 2.32 are 18 and 11 percent lower, respectively than the AR(3) model's median RMSE. Further, the ranges of the AR(3)LI and shrinkage forecast RMSEs for the 18 countries are much smaller than those for the AR(3) and naive models.

In summary, the shrinkage or pooling techniques used earlier in Garcia-Ferrer et al. (1987) for nine countries for the period 1974-81 are effective in reducing overall median RMSEs for an extended forecast period, 1974-84, and for nine additional countries.

The $\eta$-shrinkage forecast RMSEs, reported above, are based on the same value of $\eta=.5$ for all countries. It is of interest to see how sensitive forecasting results for individual countries are to variation in the value of $\eta$. In Table 4.4 one-year-ahead forecast RMSEs are reported for selected values of $\eta$ in (2.2) for each of the 18 countries in our

Table 4.2
Nine Additional Countries' RMSEs of One-Year-Ahead Forecast, 1974-84

| Model | Austral. | Austria | Can. | Fin. | Jpn. | Nor. | Spain | Swed.[e] | Switz. | Median |
|---|---|---|---|---|---|---|---|---|---|---|
| | (Percentage points) | | | | | | | | | |
| A. NMI ($\hat{y}_t=0$) | 3.10 | 2.91 | 3.51 | 3.39 | 4.10 | 4.15 | 2.51 | 2.29 | 3.03 | 3.10 |
| B. NMII ($\hat{y}_t=y_{t-1}$) | 2.90 | 2.82 | 4.13 | 2.34 | 3.12 | 1.98 | 1.86 | 1.87 | 3.91 | 2.82 |
| C. NMIII ($\hat{y}_t$= past average) | 2.75 | 3.11 | 3.31 | 2.88 | 4.98 | 1.76 | 3.56 | 2.43 | 4.48 | 3.11 |
| D. AR(3)[a] | 2.87 | 3.16 | 3.55 | 2.58 | 3.26 | 1.75 | 2.50 | 2.22 | 4.24 | 2.87 |
| E. AR(3)LI[b] | 3.34 | 2.71 | 3.68 | 3.37 | 3.33 | 1.62 | 2.06 | 2.32 | 3.45 | 3.33 |
| 1. Shrinkage(1)[c] | 2.01 | 1.77 | 2.39 | 2.01 | 2.40 | 1.68 | 1.65 | 2.01 | 2.71 | 2.01 |
| 2. Shrinkage(2)[d] | 2.32 | 2.12 | 2.92 | 2.42 | 2.51 | 1.52 | 1.81 | 2.29 | 3.25 | 2.32 |

a. Least squares forecasts from an AR(3) model for each country.
b. Least squares forecasts from AR(3) model with leading indicator variables shown in equation (2.1a).
c. Least squares forecasts with use of coefficient estimate in (2.8).
d. Use of shrinkage equation in (2.2) with $\eta$=.5.
e. Based on 10 forecasts, 1974-83.

Table 4.3
Summary Statistics on Forecasting Results for 18 Countries, 1974-84

| Model | Median RMSE | Smallest RMSE | Largest RMSE |
|---|---|---|---|
| | (Percentage points) | | |
| A. NMI ($\hat{y}_t=0$) | 3.07 | 2.29 | 4.15 |
| B. NMII ($\hat{y}_t=y_{t-1}$) | 3.02 | 1.87 | 4.26 |
| C. NMIII ($\hat{y}_t$= past average) | 3.09 | 1.76 | 4.98 |
| D. AR(3)[a] | 3.00 | 1.75 | 4.34 |
| E. AR(3)LI[b] | 2.62 | 1.62 | 3.68 |
| 1. Shrinkage(1)[c] | 2.14 | 1.65 | 2.79 |
| 2. Shrinkage(2)[d] | 2.32 | 1.52 | 3.25 |

a. Least squares forecasts from an AR(3) model for each country.
b. Least squares forecasts from AR(3) model with leading indicator variables shown in equation (2.1a).
c. Least squares forecasts with use of coefficient estimate in (2.8).
d. Use of shrinkage equation in (2.2) with $\eta$=.5.

Table 4.4
Sensitivity of η-Forecast RMSEs to Value of η for AR(3)LI Model in (2.1a), 1974-84[a]

| Country | Values of η: η = 0 | η = .25 | η = .50 | η = .75 | η = 1.0 |
|---|---|---|---|---|---|
| | (Percentage points) | | | | |
| Belgium | 1.73 | 1.70 | 1.81 | 2.04 | 2.35 |
| Denmark | 2.73 | 2.49 | 2.37[b] | 2.41 | 2.59 |
| France | 2.52 | 2.29 | 2.07 | 1.90 | 1.77[b] |
| Germany | 2.28 | 2.04 | 1.94[b] | 2.01 | 2.24 |
| Ireland | 2.80 | 2.51 | 2.31 | 2.24[b] | 2.29 |
| Italy | 3.40 | 3.04 | 2.73 | 2.46 | 2.27[b] |
| Netherlands | 2.41[b] | 2.42 | 2.50 | 2.65 | 2.87 |
| U.K. | 2.32[b] | 2.44 | 2.63 | 2.89 | 3.20 |
| U.S. | 2.14 | 1.97[b] | 2.03 | 2.31 | 2.75 |
| Australia | 3.34 | 2.80 | 2.32 | 1.92 | 1.67[b] |
| Austria | 2.71 | 2.40 | 2.12 | 1.87 | 1.67[b] |
| Canada | 3.68 | 3.27 | 2.92 | 2.64 | 2.46[b] |
| Finland | 3.37 | 2.86 | 2.42 | 2.10 | 1.95[b] |
| Japan | 3.33 | 2.90 | 2.51 | 2.18 | 1.96[b] |
| Norway | 1.62 | 1.53 | 1.52[b] | 1.57 | 1.70 |
| Spain | 2.06 | 1.86 | 1.81[b] | 1.91 | 2.14 |
| Sweden[c] | 2.32 | 2.28 | 2.29[b] | 2.34 | 2.44 |
| Switzerland | 3.45 | 3.26 | 3.25[b] | 3.42 | 3.75 |
| Median RMSE | 2.62 | 2.43 | 2.32 | 2.21 | 2.28 |
| Range | 1.62-3.68 | 1.53-3.27 | 1.52-3.25 | 1.57-3.42 | 1.67-3.75 |

a. The η-forecast is $\tilde{y}_{if} = \eta\hat{y}_f + (1-\eta)\hat{y}_{if}$, where $\hat{y}_f$ is the mean of the 18 countries' least squares forecasts and $\hat{y}_{if}$ is the i'th country's least squares forecast.

b. Minimum of entries in each row. The median of these RMSEs is 1.96.

c. Based on 10 forecasts, 1974-83.

sample. When $\eta=0$, the forecasts are least squares forecasts for each country based on our AR(3)LI model in (2.1a). When $\eta=1$, the forecast for each country is the mean of the 18 countries' least squares forecasts from the AR(3)LI model in (2.1a). On viewing the median RMSEs at the bottom of Table 4.4, it is seen that the median RMSEs are 2.43 for $\eta=.25$, 2.32 for $\eta=.5$ and 2.21 for $\eta=.75$, all below the median RMSE for $\eta=0$, 2.62, that for individual countries' unshrunk least squares forecasts. For individual countries, the RMSEs change as the value of $\eta$ changes. For example in the case of Belgium, the RMSEs vary from 1.73 for $\eta=0$ to 2.35 for $\eta=1.0$ with a minimum of 1.70 for $\eta=.25$. Two countries' minimal RMSEs occur for $\eta=0$ and $\eta=.25$, six for $\eta=.50$, one for $\eta=.75$ and seven for $\eta=1.0$. The median RMSE of these minimal values is 1.96, somewhat below that associated with the use of $\eta=.5$ for all countries, namely 2.32 or of $\eta=.75$, 2.21.

Just as it is of interest to determine the sensitivity of countries' $\eta$-forecasts to the values of $\eta$ employed, it is of interest to determine how sensitive $\gamma$-forecasts for countries are to the value of $\gamma$ employed. The coefficient estimate in (2.13) permits $\gamma$ to be different for different countries. In Table 4.5, RMSEs of forecast are reported for each country for selected values of $\gamma$. A zero value for $\gamma$ yields AR(3)LI least squares forecasts while a very large value for $\gamma$ results in an AR(3)LI forecast based on the pooled coefficient estimate in (2.8). From the median RMSEs reported at the bottom of Table 4.5, it is seen that a common value of $\gamma=5.0$ yields a median RMSE equal to 2.13, not far different from that associated with $\gamma=10^6$, namely 2.14. However for individual countries, RMSEs show more substantial variation as $\gamma$ assumes different values. For example in the case of Germany, the RMSE is 1.80 when $\gamma=2.0$, quite a bit lower than the RMSE of 2.00 associated with $\gamma=10^6$. 10 of the 18 countries show minimal RMSEs for $\gamma=10^6$ while the remaining eight countries have minima in the vicinity of $\gamma=.5$ to $\gamma=1.0$ in five cases and of $\gamma=2$ to $\gamma=5$ for the remaining three. The median RMSE for these minimal values is 2.01, slightly lower than the median RMSE of 2.14 when $\gamma=10^6$ is used for all countries. In future work it may be worthwhile to estimate $\gamma$ for each country which will probably produce lower

**Table 4.5**
**Sensitivity of γ-Forecast RMSEs to Values of γ for AR(3)LI Model in (2.1a), 1974-84[a]**

| Country | $\gamma = 0$ | $\gamma = .50$ | $\gamma = 1.0$ | $\gamma = 2.0$ | $\gamma = 3.0$ | $\gamma = 5.0$ | $\gamma = 10^6$ |
|---|---|---|---|---|---|---|---|
| | (Percentage points) | | | | | | |
| Belgium | 1.73 | 1.60[b] | 1.62 | 1.69 | 1.74 | 1.81 | 1.96 |
| Denmark | 2.73 | 2.34 | 2.22 | 2.16[b] | 2.16[b] | 2.18 | 2.26 |
| France | 2.52 | 2.17 | 2.01 | 1.87 | 1.81 | 1.75 | 1.66[b] |
| Germany | 2.28 | 1.91 | 1.81 | 1.80[b] | 1.82 | 1.87 | 2.00 |
| Ireland | 2.80 | 2.47 | 2.34 | 2.24 | 2.20 | 2.17 | 2.14[b] |
| Italy | 3.40 | 3.00 | 2.82 | 2.67 | 2.61 | 2.55 | 2.45[b] |
| Netherlands | 2.41 | 2.34[b] | 2.35 | 2.38 | 2.41 | 2.44 | 2.53 |
| U.K. | 2.32 | 2.19 | 2.18[b] | 2.21 | 2.24 | 2.28 | 2.39 |
| U.S. | 2.14 | 1.98[b] | 2.06 | 2.24 | 2.35 | 2.48 | 2.79 |
| Australia | 3.34 | 2.72 | 2.46 | 2.25 | 2.16 | 2.09 | 2.01[b] |
| Austria | 2.71 | 2.35 | 2.19 | 2.03 | 1.96 | 1.89 | 1.77[b] |
| Canada | 3.68 | 3.08 | 2.83 | 2.63 | 2.54 | 2.47 | 2.39[b] |
| Finland | 3.37 | 2.70 | 2.42 | 2.20 | 2.12 | 2.06 | 2.01[b] |
| Japan | 3.33 | 2.89 | 2.71 | 2.57 | 2.51 | 2.46 | 2.40[b] |
| Norway | 1.62 | 1.54[b] | 1.54[b] | 1.56 | 1.58 | 1.61 | 1.68 |
| Spain | 2.06 | 1.85 | 1.76 | 1.70 | 1.68 | 1.66 | 1.65[b] |
| Sweden[c] | 2.32 | 2.18 | 2.12 | 2.07 | 2.05 | 2.03 | 2.01[b] |
| Switzerland | 3.45 | 2.98 | 2.81 | 2.71 | 2.68[b] | 2.68[b] | 2.71 |
| Median RMSE | 2.62 | 2.34 | 2.22 | 2.21 | 2.16 | 2.13[b] | 2.14 |
| Range | 1.62-3.68 | 1.54-3.08 | 1.54-2.83 | 1.56-2.71 | 1.58-2.68 | 1.61-2.68 | 1.65-2.79 |

a. The coefficient estimate in (2.13) was employed to compute forecasts. When $\gamma=0$, the forecasts are least squares forecasts and when $\gamma=10^6$, they are produced using the coefficient estimate in (2.8).

b. Minimum of entries in each row.

c. Based on 10 forecasts, 1974-83.

RMSEs for selected countries, e.g. Belgium, the Netherlands, the U.S., and Norway. Also combinations of $\gamma$-forecast and $\eta$-forecast procedures are under consideration as well as other shrinkage techniques.

## 4.2 Forecasting Using a World Output Growth Rate Variable

As mentioned previously, countries' exports are influenced by world income or output. Thus a world output growth rate variable, denoted by $w_t$, was added to the AR(3)LI model as shown in equation (2.15). The variable $w_t$ is taken to be the median of the 18 countries' output growth rates for the year t--see Fig. 3.1 for a plot of $w_t$ for the years in our sample.

To use (2.15) in forecasting, it is necessary to forecast $w_t$. The model for $w_t$ in (2.16) was fitted by least squares, using data 1954-73, and used to forecast the 1974 value and subsequent values with coefficient estimates updated year-by-year for the years 1974-84. The RMSEs of these one-year-ahead forecasts and of those yielded by an AR(3) model for $w_t$ are reported, along with MAEs in Table 4.6. It is seen that the AR(3)LI model for $w_t$ produced a RMSE of 1.48 and a MAE of 1.24, values much smaller than those associated with forecasts from an AR(3) model, 2.74 and 2.24, respectively. The forecasts from these two models for $w_t$, denoted by $\hat{w}_t(1)$ for the AR(3) forecasts and $\hat{w}_t(2)$ for the AR(3)LI model forecasts were used to generate one-year-ahead forecasts from (2.15) for individual countries' output growth rates, 1974-84 using the $\gamma$-forecast with a very large value of $\gamma$. Also, for comparative purposes, a "perfect foresight" model, one in which it is assumed that $w_t$ is known exactly was employed to generate forecasts with results given in the first column of Table 4.7. With $w_t$'s value assumed known in each forecast period, the median RMSE of the annual forecasts for the 18 countries, 1974-84, is 1.82, a value much lower than those reported in Table 4.3 and for the AR(3)LI model without the $w_t$ variable, 2.14, shown in the last column of Table 4.7. In column 2 of Table 4.7 are shown the forecast RMSEs when $w_t$ was forecasted using an AR(3) model. Since the AR(3) forecasts of $w_t$ are not very good--see Table 4.6 the forecasts of country output growth rates based on them are not in general as good as those based on known

Table 4.6

RMSEs and MAEs of One-Year-Ahead Forecasts of the Medians of 18 Countries' Output Growth Rates, 1974-84[a]

| Model | RMSE | MAE |
|---|---|---|
| | (Percentage points) | |
| AR(3) | 2.74 | 2.24 |
| AR(3)LI[b] | 1.48 | 1.24 |

a. The initial estimation period is 1954-73, 20 years. Estimates are updated year by year in the forecast period.

b. With $w_t$ = median output growth rate in year t, the AR(3)LI model is $w_t = \pi_0 + \pi_1 w_{t-1} + \pi_2 w_{t-2} + \pi_3 w_{t-3} + \pi_4 x_{t-1} + \pi_5 z_{t-1} + \varepsilon_t$, where, for year t, $x_t$ = median of countries' growth rates of real stock prices and $z_t$ = median of countries' growth rates of real money. This equation was employed to generate one year ahead least squares forecasts for each year, 1974-84.

values of the $w_t$ variable. When the AR(3)LI model in (2.16) was used to produce forecasts of $w_t$, denoted by $\hat{w}_t(2)$ and these were used to forecast individual countries' output growth rates, the results, as shown in column 3 of Table 4.7, were much better. The median RMSE associated with these forecasts is 1.90, not far different from the "perfect foresight" median RMSE of 1.82. Also when $\hat{w}_t(2)$ was employed, 16 of 18 countries' RMSEs were reduced relative to the RMSEs for the AR(3)LI model without the world growth variable, shown in column 4 of Table 4.7. Further, the median RMSE of 1.90, associated with the AR(3)LI-world-growth-rate model is smaller than all of those shown in Table 4.3.

### 4.3 Comparisons With OECD Forecast RMSEs

Smyth (1983) has presented a description of the forecasting procedures employed by the Organization for Economic Cooperation and Development (OECD) to produce annual forecasts of seven countries' annual rates of growth of output, 1968-79. The OECD forecasts are derived from

Table 4.7
**RMSEs of One-Year-Ahead Forecasts of Annual Real Output Growth Rates Employing an AR(3)LI Model Including the World Growth Rate, 1974-84[a]**

| Country | Models: AR(3)LI with $w_t$[b] | AR(3)LI with $\hat{w}_t(1)$[c] | AR(3)LI with $\hat{w}_t(2)$[d] | AR(3)LI without $w_t$[e] |
|---|---|---|---|---|
| | (Percentage points) | | | |
| Belgium | 1.54 | 2.80 | 1.79 | 1.96 |
| Denmark | 1.77 | 2.74 | 2.04 | 2.26 |
| France | 1.03 | 1.90 | 1.36 | 1.66 |
| Germany | 0.90 | 2.39 | 1.35 | 2.00 |
| Ireland | 2.82 | 1.57 | 2.54 | 2.14 |
| Italy | 1.57 | 3.08 | 1.82 | 2.45 |
| Netherlands | 1.86 | 2.52 | 2.27 | 2.53 |
| U.K. | 1.87 | 2.67 | 2.21 | 2.39 |
| U.S. | 2.56 | 2.95 | 2.36 | 2.79 |
| Australia | 2.24 | 2.39 | 1.96 | 2.01 |
| Austria | 1.23 | 2.06 | 1.57 | 1.77 |
| Canada | 2.27 | 2.81 | 2.15 | 2.39 |
| Finland | 2.14 | 2.13 | 1.83 | 2.01 |
| Japan | 2.85 | 2.30 | 2.58 | 2.40 |
| Norway | 1.66 | 1.79 | 1.45 | 1.68 |
| Spain | 1.12 | 2.03 | 1.22 | 1.65 |
| Sweden[f] | 1.62 | 2.58 | 1.70 | 2.01 |
| Switzerland | 2.20 | 3.17 | 2.42 | 2.71 |
| Median RMSE | 1.82 | 2.46 | 1.90 | 2.14 |
| Range | 0.90-2.85 | 1.57-3.17 | 1.22-2.58 | 1.65-2.71 |

a. The model employed is shown in (2.15) and estimated under the assumption that coefficients are the same for all countries.

b. The value of $w_t$, the median output growth rate or world growth rate is assumed known in the forecast period, a "perfect foresight" assumption.

c. $\hat{w}_t(1)$ is a forecast of $w_t$ from an AR(3) model for $w_t$--see Table 4.6.

d. $\hat{w}_t(2)$ is a forecast of $w_t$ from the AR(3)LI model described in footnote b of Table 4.6.

e. Least squares forecasts using AR(3)LI model in (2.1).

f. Based on 10 forecasts, 1974-83.

elaborate country econometric models and are subjected to judgmental adjustments by individuals not associated with the modeling process, that is by individuals from the OECD's various country desks--see Smyth (1983, p. 37). Earlier in Garcia-Ferrer et al. (1987, p. 61ff.) comparisons of RMSEs of OECD forecasts for five countries with those provided by AR(3)LI models were presented and discussed. In Table 4.8, OECD RMSEs of forecast for seven countries, 1968-79 are presented along with forecast RMSEs for the same countries, 1974-84, computed in the present study. While the forecast periods, 1968-79 and 1974-84 are somewhat different and different methodologies were employed, it is still of interest to consider the relative forecasting performance of OECD and our forecasts.

From Table 4.8, it is seen that the OECD forecast RMSEs have a median of 2.12 percentage points with a range of 1.45-4.40. The OECD's RMSE for Japan, 4.40 is quite large. The median RMSEs in our study range from 2.52 for least squares forecasts from our AR(3)LI model in (2.1) to 2.15 for the forecasts obtained from our "AR(3)LI world income" model in (2.15) using a forecasted value of $w_t$ from (2.16) and a $\gamma$-forecast with $\gamma=10^6$. The range of the forecast RMSEs in this latter case is 1.35-2.58. The RMSEs in line 3 of Table 4.8 are smaller than the corresponding OECD RMSEs in five of seven cases. Large reductions were encountered for Germany, Italy and Japan while smaller reductions appeared for France and the U.K. In the cases of the U.S. and Canada, the OECD RMSEs were smaller, much smaller for the U.S.. These comparisons, however, must be qualified since the forecast period and data used by OECD are different from those employed in this study.

## 5. Summary and Concluding Remarks

We have presented the results of forecasting experiments for nine countries' annual output growth rates for the periods 1974-81 and 1974-84 and similar results for an additional sample of nine countries for the period 1974-84. In general, the forecasting experiments revealed that methods and models employed earlier in Garcia-Ferrer et al. (1987) worked reasonably well when applied to an extended sample of data and countries. The shrinkage forecasting procedures produced larger reductions in RMSEs of forecast for the nine additional countries than for the original sam-

**Table 4.8**
**Comparison With OECD Forecast RMSEs**

| Forecasts | Canada | France | Germany | Italy | Japan | U.K. | U.S. | Median | Range |
|---|---|---|---|---|---|---|---|---|---|
| | (Percentage points) | | | | | | | | |
| 1. OECD, 1968-79 | 1.71 | 1.45 | 2.12 | 2.86 | 4.40 | 2.26 | 1.38 | 2.12 | 1.38-4.40 |
| 2. AR(3)LI, 1974-84 | | | | | | | | | |
| a. Least squares | 3.68 | 2.52 | 2.28 | 3.40 | 3.33 | 2.32 | 2.14 | 2.52 | 2.14-3.68 |
| b. $\gamma$-forecasts[a] | 2.39 | 1.66 | 2.00 | 2.45 | 2.40 | 2.39 | 2.79 | 2.39 | 1.66-2.79 |
| c. $\eta$-forecasts[b] | 2.92 | 2.07 | 1.94 | 2.73 | 2.51 | 2.63 | 2.03 | 2.51 | 1.94-2.92 |
| 3. AR(3)LI with forecasted $w_t$, 1974-84[c] | 2.15 | 1.36 | 1.35 | 1.82 | 2.58 | 2.21 | 2.36 | 2.15 | 1.35-2.58 |

a. $\gamma=10^6$ in model for 18 countries.
b. $\eta=.5$ in model for 18 countries.
c. $w_t$, the world output growth rate was forecasted from (2.16) and used in (2.15) to produce $\gamma$-forecasts for 18 countries with $\gamma=10^6$.

ple of nine countries. The AR(3)LI model, incorporating a world income growth rate variable, forecasted the best and provided forecast RMSEs that compared favorably with the RMSEs of OECD forecasts for a sub-sample of seven countries that were produced using complex country models and judgmental adjustments.

In future work, we shall extend our forecasting experiments to include forecasts derived from time-varying parameter Bayesian state-space models--see García-Ferrer et al. (1987) and Highfield (1986) for macroeconomic forecasting results obtained with such models. In addition, as stated in Section 1, macroeconomic structural equation systems are under consideration that have reduced form equations for the rate of growth of output that are similar in form to the forecasting equations used in our present and past work. These structural macroeconomic equation systems also yield reduced form equations for several additional variables that will be appraised in future forecasting experiments. With several forecasting equations per country, there will be an opportunity to experiment with a broader range of shrinkage forecasting techniques.

Hopefully, this work will yield a set of forecasting equations for each country that yield reasonably good forecasts for a broad sample of countries and for a temporally expanded data set. Further, disaggregation will be studied by modeling components of GNP as fractions or log-fractions of relevant aggregates. In this way, we hope to "iterate in" to satisfactory structural models for countries in the structural econometric modeling, time series analysis (SEMTSA) approach put forward in our past work--see Zellner and Palm (1974, 1975), Palm (1983) and Zellner (1979, 1984, 1986b).

## References

Bowman, H.W. and A.M. Laporte (1975), "Stochastic Optimization in Recursive Equation Systems with Random Parameters with an Application to Control of the Money Supply," in S.E. Fienberg and A. Zellner (eds.), Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage, Amsterdam: North-Holland Publishing Co., 441-462.

Garcia-Ferrer, A., R.A. Highfield, F. Palm, and A. Zellner (1987), "Macroeconomic Forecasting Using Pooled International Data," Journal of Business and Economic Statistics, 5, 53-67.

Highfield, R.A. (1986), "Forecasting with Bayesian State Space Models," unpublished doctoral dissertation, Graduate School of Business, University of Chicago.

Lindley, D.V. and A.F.M. Smith (1972), "Bayes' Estimates for the Linear Model," Journal of the Royal Statistical Society, Ser. B, 34, 1-41.

Miller, R.B. and W.G. Fortney (1984), "Industry-wide Expense Standards using Random Coefficient Regression," Insurance: Mathematics and Economics, 3, 19-33.

Palm, F.C. (1983), "Structural Econometric Modeling and Time Series Analysis: An Integrated Approach," in A. Zellner (ed.), Applied Time Series Analysis of Economic Data, Proceedings of the Conference on Applied Time Series Analysis of Economic Data, Oct. 13-15, 1981, Washington, DC: U.S. Government Printing Office, 199-233.

Smyth, D.J. (1983), "Short-Run Macroeconomic Forecasting: The OECD Performance," Journal of Forecasting, 2, 37-49.

Zellner, A. (1971), An Introduction to Bayesian Inference in Econometrics, New York: John Wiley & Sons, Inc., reprinted by Krieger Publishing Co., Melbourne, FL, 1987.

Zellner, A. (1979), "Statistical Analysis of Econometric Models," Journal of the American Statistical Association, 74, 628-651.

Zellner, A. (1983), "Applications of Bayesian Analysis in Econometrics," The Statistician, 32, 23-34.

Zellner, A. (1984), Basic Issues in Econometrics, Chicago: University of Chicago Press.

Zellner, A. (1986a), "On Assessing Prior Distributions and Bayesian Regression Analysis with g-Prior Distributions," in P.K. Goel and A. Zellner (eds.), Bayesian Inference and Decision Techniques: Essays in Honor of Bruno de Finetti, Amsterdam: North-Holland Publishing Co., 233-243.

Zellner, A. (1986b), "Macroeconomics, Econometrics and Time Series Analysis," presented at a Bank of Spain Seminar, Madrid, Dec. 15, 1986, to be published in Spanish in a journal edited by A. Maravall.

Zellner, A. and F.C. Palm (1974), "Time Series Analysis and Simultaneous Equation Econometric Models," Journal of Econometrics, 2, 17-54.

Zellner, A. and F.C. Palm (1975), "Time Series and Structural Analysis of Monetary Models of the U.S. Economy," Sankhyā, 37, 12-56.

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