USC

MRG WP # 8634

MODELLING RESEARCH GROUP

DEPARTMENT OF ECONOMICS
UNIVERSITY OF SOUTHERN CALIFORNIA
UNIVERSITY PARK
LOS ANGELES, CALIFORNIA
90089-0152

db

USC

MRG WP # 8634

MODELLING RESEARCH GROUP

COMPARATIVE STATICS FOR OLIGOPOLY: DEMAND SHIFT EFFECTS

HERMAN C. QUIRMBACH*

MRG WORKING PAPER #M8634

DEPARTMENT OF ECONOMICS
UNIVERSITY OF SOUTHERN CALIFORNIA
UNIVERSITY PARK
LOS ANGELES, CALIFORNIA
90089-0152

db

COMPARATIVE STATICS FOR OLIGOPOLY: DEMAND SHIFT EFFECTS

HERMAN C. QUIRMBACH*

MRG WORKING PAPER #M8634

July 1986

ABSTRACT

When a demand curve shifts in an imperfectly competitive industry, price, quantity, consumer surplus, and welfare may individually rise or fall. This breadth of possibilities contrasts sharply with the narrower and more predictable effects of either a demand shift in a competitive industry or a cost shift in either type of industry. Even under imperfect competition, however, the pattern of demand shift effects cannot be entirely arbitrary. A system of necessary and sufficient relationships among the changes is established.

How profits change when demand shifts is of particular note. When the demand curve rises, profits may actually fall. As shown here, this possibility is unique to imperfect competition: profits always rise with demand under both perfect competition and perfect collusion. The demand shift case again also contrasts with the cost shift case, where imperfect competition is not a necessary ingredient in generating perverse profit effects.

1. INTRODUCTION

In a recent paper, Dixit [1986] examines the comparative statics of an oligopoly, using conjectural variations to capture the degree of industry collusion. Dixit's treatment is at a very general level: the shift parameter is simply modelled as any parameter of the profit function. Several earlier papers (*e.g.*, Seade [1983], Katz and Rosen [1983], Salop and Scheffman [1983], and others cited by Dixit) have examined the more specific case of a shift parameter in the cost function. One interesting finding of those papers is that cost *increases* can lead to *higher* profits. This can happen with either perfect or imperfect competition, although not with perfect collusion. When costs increase, however, most of the other results for the symmetric case are quite predictable: price always rises, and quantity, consumer surplus, and welfare always fall. Thus, for a cost shift, imperfect competition offers no qualitative change from perfect competition.

This paper presents the comparative statics of a demand curve shift. With a demand shift, imperfect competition introduces a variety of possibilities not found in the perfectly competitive case. Moreover, the set of possible effects is generally richer than with a cost shift. In particular, when demand shifts, an imperfectly competitive industry's quantity, price, consumer surplus, and welfare can each either rise or fall — although not in any arbitrary combination. I establish a system of necessary and sufficient relationships among these various effects.

Perhaps the most interesting findings concern profits. As with a cost shift, profits can change perversely: profits can actually fall when the demand curve rises. However, unlike a cost shift, a demand shift can produce such a perverse profit effect *only* in the middle ground between competition and complete collusion. Thus, the profit effect of a demand shift in an imperfectly competitive industry may be qualitatively different from the effects in *both* perfectly competitive and perfectly collusive industries.

Section 2 presents the formal model and reviews the cost shift results. The mathematical formulation here is somewhat different than that used by the papers cited above. My formulation allows a clear analytical and graphical interpretation of comparative static signs in terms of standard demand, marginal revenue, marginal cost, and average cost curves.

In Sec. 3, I derive the comparative statics of a demand shift and present a specific functional form example of a perverse change in profits. I then explore the reasons for the differences in the cost and demand shift effects. Finally, I identify some common policy situations in which perverse profit effects can easily be ruled out and other cases where perverse profit effects could plausibly arise.

Section 4 briefly summarizes the results and concludes by presenting an alternative welfare interpretation of a conjectural variations equilibrium.

2. THE MODEL AND COST SHIFT RESULTS

This paper treats the case of a homogeneous product industry consisting of n identical firms. Firm i produces output level q_i at a cost $C(q_i)$, for $i = 1, \ldots, n$. The (inverse) demand curve for the industry's output is $P(Q)$, where $Q \equiv \sum_{i=1}^{n} q_i$ is the total industry supply.

Firm i chooses q_i to maximize its profits π_i, where

$$\pi_i \equiv P(Q) \cdot q_i - C(q_i). \tag{1}$$

The firm's first order condition for this maximization is

$$d\pi_i/dq_i = P(Q) + \beta \cdot Q \cdot P_Q(Q) - MC(q_i) = 0, \tag{2}$$

where subscripts (except i, j) will indicate partial derivatives, MC is the firm's marginal cost function, and $\beta_i \equiv (q_i/Q)(dQ/dq_i)$ is firm i's conjecture about the elasticity of total industry supply with respect to the firm's output.[1] We limit consideration here to the case of identical conjectures: $\beta_i = \beta_j = \beta$ for all i, j. A little rearranging of (2) then yields, for $i = 1, \ldots, n$,

$$(1 - \beta) \cdot P(Q) + \beta \cdot MR(Q) = MC(q_i), \tag{3}$$

where $MR(Q) = P(Q) + Q \cdot P_Q(Q)$ is the industry marginal revenue curve. Throughout I will assume that both P_Q and MR_Q are negative at the equilibrium Q, that MC_q is nonnegative at equilibrium, and that all three functions are finite. These assumptions guarantee (but are not necessary for) the second order conditions for the firm's maximization problem.[2]

If MC is globally strictly increasing, then any solution to the n equations (3) is symmetric among the firms. Otherwise, I will assume the symmetric solution directly. Let Q^* denote the industry output corresponding to the symmetric solution, with each firm's output given by Q^*/n, and let $P^* \equiv P(Q^*)$ be the equilibrium price. Henceforth, unless indicated otherwise, all functions will be evaluated at Q^* and P^*. Then, (3) becomes

$$(1-\beta) \cdot P(Q^*) + \beta \cdot MR(Q^*) = MC(Q^*/n) \tag{4}$$

and the profits of a representative firm are

$$\pi^* \equiv P^* \cdot Q^*/n - C(Q^*/n). \tag{5}$$

Clearly, $\beta = 0$ corresponds to pure competition, while $\beta = 1$ represents a perfectly collusive cartel. At the symmetric Cournot solution, $\beta = 1/n$, since with Cournot $dQ/dq_i = 1$ for all i. Thus, the value of β can be interpreted as an index of industry collusion, with higher β representing greater collusion and $[0,1]$ being the reasonable range of values for β.

The lefthand side of (3) is then a convex combination of demand and marginal revenue. The natural interpretation of this expression is as the firm's "conjectural" marginal revenue curve, henceforth denoted $CMR(Q,\beta)$. The conjectural marginal revenue deviates from the true marginal revenue when the firm fails to perceive that its interests are fully parallel to those of its rivals. The above assumptions imply that CMR is downward sloping at Q^*.

In the symmetric case considered here, the solution (4) has a nice graphical representation. The case of $\beta = 1/2$ is illustrated in Fig.1.

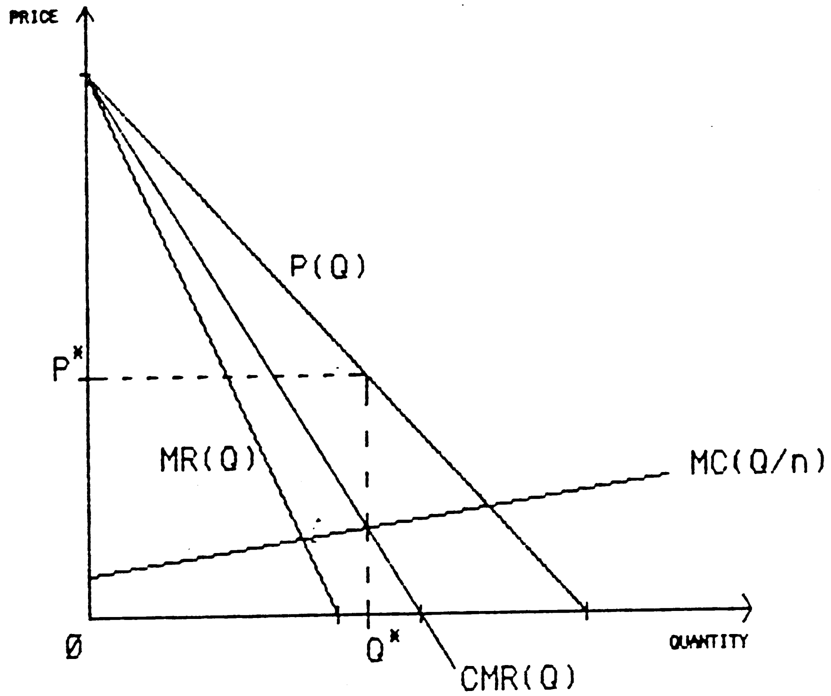

FIG.1 PRICE & QUANTITY FOR β = 1/2

Consumers' utility at Q^* is given by the standard consumer surplus:

$$CS^* \equiv \int_0^{Q^*} P(t)dt - P^* \cdot Q^*. \tag{6}$$

Welfare at Q^* will be measured by industry profit, $n \cdot \pi^*$, plus consumer surplus, CS^*; or equivalently by

$$W^* \equiv \int_0^{Q^*} P(t)dt - n \cdot C(Q^*/n). \tag{7}$$

Let us now briefly review the comparative statics of a cost function shift.[3] Throughout the paper, the shift parameter will be denoted by θ. For the cost shift case, the cost function becomes $C(q, \theta)$. The convention in this section will be that an increase in θ raises the marginal cost curve: $MC_\theta > 0$ at all q. With θ entered in the cost and marginal cost functions, equations (4) and (5) characterize industry equilibrium. The endogenous variables are now parameterized on θ: $Q^* = Q^*(\theta), P^* = P\big(Q^*(\theta)\big)$, etc.

For a cost shift, Q^*, P^*, consumer surplus, and (for the usual case) welfare (see note 6 below) move in the predictable directions, regardless of the degree of competition. Total differentiation of (4) yields[4]

$$dQ^*/d\theta = MC_\theta/\Omega < 0, \tag{8}$$

where, by the above assumptions,

$$\Omega \equiv (1-\beta)\cdot P_Q + \beta \cdot MR_Q - MC_q(Q^*/n,\theta)/n < 0. \tag{9}$$

Thus,

$$dP^*/d\theta = P_Q \cdot dQ^*/d\theta > 0, \quad \text{and} \tag{10}$$

$$dCS^*/d\theta = -Q^* \cdot dP^*/d\theta < 0. \tag{11}$$

A reasonable (but not necessary) presumption is that total costs also rise with θ : $C_\theta > 0$. If so, the effect of θ on welfare is[5]

$$dW^*/d\theta = -n \cdot C_\theta + (P^* - MC)\cdot dQ^*/d\theta < 0. \tag{12}$$

Finally, the profit effect can be derived from (5):

$$\begin{aligned} \frac{d\pi^*}{d\theta} &= -C_\theta + (1/n)\cdot[MR - MC]\cdot\frac{dQ^*}{d\theta} \\ &= \frac{-Q^*}{n}\cdot[AC_\theta - (1-\beta)\cdot(P_Q/\Omega)\cdot MC_\theta], \end{aligned} \tag{13}$$

where $MR - MC = (1-\beta)\cdot Q^* \cdot P_Q$ is used (from (4)) and $AC_\theta \equiv C_\theta(Q^*/n,\theta)/(Q^*/n)$.

Perhaps the most intuitively appealing notion is that higher costs should yield lower profits. Indeed, this is true if the firms are perfectly collusive: if $\beta = 1$, then $d\pi^*/d\theta < 0$. For less than perfect collusion, however, $d\pi^*/d\theta$ does not have a definite sign. From (9), when $\beta < 1$, $0 < (1-\beta)\cdot(P_Q/\Omega) \leq 1$. Thus, a sufficient condition for $d\pi^*/d\theta < 0$ is that average costs rise more at Q^*/n than do marginal costs. Conversely, a necessary condition for a perverse profit change — *i.e.*, for $d\pi^*/d\theta > 0$ — is that $MC_\theta > AC_\theta$.[6] If the industry is perfectly competitive and marginal costs are locally flat, this condition is both necessary and sufficient.

3. THE COMPARATIVE STATICS OF A DEMAND SHIFT

a. Analysis

This section examines how a demand shift affects quantity, price, profits, consumer surplus, and welfare. The shift parameter, which will again be θ, now appears in the inverse demand function — *i.e.,* $P = P(Q, \theta)$ — but not in the cost function. The convention will be that a rise in θ shifts the demand curve outward: $P_\theta > 0$ at all output levels. If one's intuition is based on competitive supply and demand, then the intuitively "normal" effects of an outward demand shift would be to raise P^*, Q^*, π^*, and W^* (but perhaps not CS^*). All of these signs are correct for competition, and all can be reversed with imperfect competition.

We should begin by noting that both MR and CMR now depend on θ: $MR_\theta = P_\theta + Q \cdot P_{Q\theta}$ and $CMR_\theta = (1-\beta) \cdot P_\theta + \beta \cdot MR_\theta$. Therefore, $P_{Q_\theta} > 0$ is equivalent to $MR_\theta > CMR_\theta > P_\theta$. Also, when $P_\theta > 0$, $CMR_\theta > 0$ is implied by $MR_\theta > 0$, which in turn is implied by $P_{Q\theta} \geq 0$.

The effect of θ on Q^* is found by totally differentiating (4) (*mutatis mutandis*):

$$dQ^*/d\theta = -CMR_\theta/\Omega, \tag{14}$$

One immediate result is that $dQ^*/d\theta$ is indeed positive under competition, since $CMR_\theta = P_\theta$ when $\beta = 0$. Note, however, that $dQ^*/d\theta$ will be negative if $MR_\theta < -((1-\beta)/\beta) \cdot P_\theta$ or, equivalently, if $P_{Q\theta} < -P_\theta/(\beta \cdot Q^*)$.

The effect of θ on price is determined from $dP^*/d\theta = P_\theta + P_Q \cdot (dQ^*/d\theta)$. Using (9), (14), and the above expression for CMR_θ, we get

$$\frac{dP^*}{d\theta} = \frac{\beta \cdot [P_\theta \cdot MR_Q - P_Q \cdot MR_\theta] - P_\theta \cdot MC_q/n}{\Omega} \tag{15}$$

Under competition, $dP^*/d\theta \geq 0$, with strict inequality holding if marginal cost is upward sloping. A sufficient condition for $dP^*/d\theta$ to have the normal (positive) sign is that the quantity change be perverse (negative). Conversely, for $dP^*/d\theta$ to be negative, it is necessary that $dQ^*/d\theta > 0$ and, more stringently, that $MR_\theta > P_\theta \cdot MR_Q/P_Q (> 0)$.[7]

The effect of θ on consumer surplus is

$$\frac{dCS^*}{d\theta} = \int_0^{Q^*} P_\theta(t,\theta)dt - Q^* \cdot \frac{dP^*}{d\theta}$$

(16)

$$= \left[\int_0^{Q^*} [P_\theta(t,\theta) - P_\theta(Q^*,\theta)]dt\right] - Q^* \cdot P_Q \cdot \frac{dQ^*}{d\theta}.$$

This expression may be either positive or negative, even for pure competition.[8] Expression (16) is positive either if $dP^*/d\theta < 0$ or if $P_{Q\theta} = 0$ at all outputs.

The welfare effect is

$$\frac{dW^*}{d\theta} = \int_0^{Q^*} P_\theta(t,\theta)dt + (P^* - MC) \cdot \frac{dQ^*}{d\theta}. \tag{17}$$

Clearly, (17) is positive for competition. Also, $dQ^*/d\theta > 0$ is sufficient for (17) to be positive for any β. However, $dW^*/d\theta$ may be negative if $dQ^*/d\theta < 0$ and $\beta > 0$.

Finally, the effect of θ on firm profits can be found from (5):

$$\frac{d\pi^*}{d\theta} = P_\theta \cdot Q^*/n + (1/n) \cdot [MR - MC] \cdot \frac{dQ^*}{d\theta}. \tag{18}$$

Since $MR \leq MC$ from (4), a necessary condition for $d\pi^*/d\theta < 0$ is that $dQ^*/d\theta > 0$. Thus, $d\pi^*/d\theta < 0$ is sufficient for $dW^*/d\theta > 0$ and $dCS^*/d\theta > 0$.

Substituting $MR - MC = (1-\beta) \cdot Q^* \cdot P_Q$ into (18) and using $dP^*/d\theta = P_\theta + P_Q \cdot (dQ^*/d\theta)$ yields

$$\frac{d\pi^*}{d\theta} = \frac{Q^*}{n} \cdot \left[\beta \cdot P_\theta + (1-\beta) \cdot \frac{dP^*}{d\theta}\right]. \tag{19}$$

Equation (19) shows that perverse profit effects cannot arise in either perfectly competitive or perfectly collusive markets. It is immediate that $d\pi^*/d\theta > 0$ if $\beta = 1$. Further, $\beta = 0$ also guarantees that $d\pi^*/d\theta > 0$, since $dP^*/d\theta > 0$ when $\beta = 0$. Perverse profit effects can only occur when $0 < \beta < 1$, and then only when $dP^*/d\theta < 0$, a necessary condition more stringent than $dQ^*/d\theta > 0$.

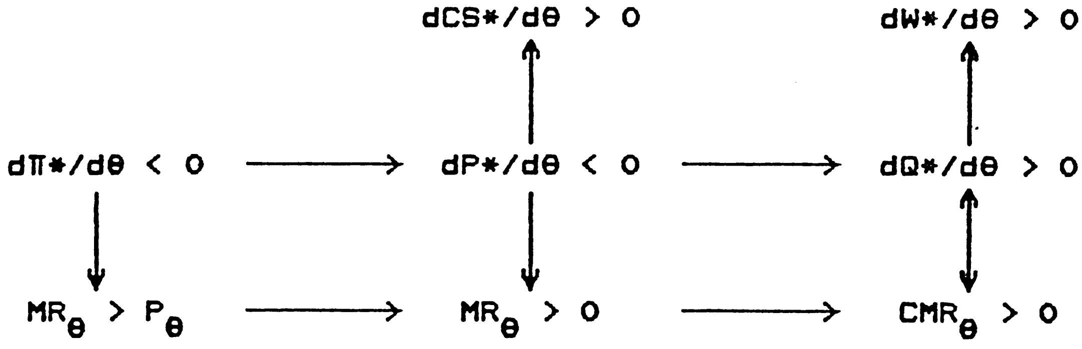

Fig. 2 Demand Shift Comparative Statics--Sufficient Conditions When $P_\theta > 0$
(Arrowheads indicate direction of implications.)

One further condition is also necessary for $d\pi^*/d\theta < 0$. Substituting $MR - MC = (1-\beta)\cdot Q^*\cdot P_Q$ and (14) into (18) and rearranging yields

(20) $$\frac{d\pi^*}{d\theta} = \frac{Q^*}{n}\cdot[P_\theta - (1-\beta)\cdot(P_Q/\Omega)\cdot CMR_\theta]$$

Since $0 < (1-\beta)\cdot(P_Q/\Omega) < 1$ when $0 < \beta < 1$, $d\pi^*/d\theta < 0$ requires $MR_\theta > CMR_\theta > P_\theta$, or equivalently $P_{Q\theta} > 0$.[9] Of course, if $P_{Q\theta} > 0$ and $P_\theta > 0$, then demand gets more elastic at the same quantity.

The network of necessary and sufficient conditions just developed is summarized in Fig. 2.

With all of these conditions being necessary to get $d\pi^*/d\theta < 0$, is it in fact possible for profits to fall as θ rises? A numerical example gives an affirmative answer. Suppose total and marginal costs are zero everywhere. For $0 \le Q < 0.9$ and for θ varying in a neighborhood of zero, let the inverse demand curve be[10]

(21) $$P(Q,\theta) = 2 + (a\theta - 2.7)\cdot Q + Q^2.$$

When $\theta = 0$, the slope assumptions on demand and marginal revenue are satisfied: $P_Q < 0$ and $MR_Q < 0$ for all Q in $[0, 0.9)$. If the industry is a Cournot duopoly (*i.e.*, $n = 2$ and

$\beta = 1/2$), direct calculation yields $dQ^*/d\theta = 2.0189 \cdot a$, $dP^*/d\theta = -1.1493 \cdot a$, and $d\pi^*/d\theta = -0.063 \cdot a$. Since $P_{Q\theta} = a$, $P_{Q\theta} > 0$ implies $dQ^*/d\theta > 0 > dP^*/d\theta$ and $d\pi^*/d\theta < 0$.[11]

b. Discussion.

When the demand curve rises, why are there so many more qualitative possibilities with imperfect competition than with perfect competition? The answer lies in the fact that with perfect competition the change in firms' marginal revenue conjectures exactly equals the shift in the demand curve, while with imperfect competition the change in firms' conjectural marginal revenue also involves the change in true marginal revenue. Of course, when demand rises, true marginal revenue may rise by either more or less than demand or may even fall. It is this extra degree of freedom that supports the perverse possibilities under imperfect competition: $MR_\theta < 0$ is needed for Q^* or W^* to fall, while $MR_\theta > P_\theta$ is needed for profits to fall. These possibilities do not arise with perfect competition because the change in true marginal revenue is then irrelevant.

It is for the reverse reason that a cost shift produces the *same* effects under imperfect competition as under perfect competition: as far as costs are involved, Q^* always depends only on MC — never on AC — and firms never misperceive this fact. Thus, for Q^*, P^*, and CS^*, there is no extra degree of freedom to cause perverse effects.[12]

The discrepancy between the profit effects of cost and demand shifts is somewhat different: it is in the competitive case that the discrepancy appears, and then it is a cost shift that yields the richer possibilities. The explanation, however, again has to do with the role — or lack of role — of the MR curve.

Specifically, equations (13) and (18) show that the profit effect of cost and demand shifts have two parts: the first part is the effect at the old Q^*, and the second is the effect through the induced change in Q^*. The profit change at the old Q^* is captured by the change in average revenue or cost. For both cost and demand shifts, this average effect provides the intuitive basis for the "normal" sign. The normal sign can be reversed only if Q^* is moved strongly enough in the appropriate direction.

The change in Q^* is in turn determined by the changes in the marginal curves — *i.e.*, by MC_θ and CMR_θ. To swamp the effect of an average cost (revenue) shift thus requires $MC_\theta > AC_\theta$ (resp., $CMR_\theta > P_\theta$). For a cost shift, this can occur even for competition, since the relative magnitudes of AC_θ and MC_θ do not depend on β. However, this is not true of a demand shift: perfectly competitive firms ignore the true marginal revenue curve and conjecture that $CMR_\theta = P_\theta$. Thus, when demand shifts in a competitive industry, the change in Q^* will never be so strong as to reverse the effect of P_θ on π^*.

These results have a variety of applications. First, demand growth in an oligopoly may not yield higher profits, especially if the new customers entering the market have more elastic demands than the continuing customers. Conversely, to be profitable, advertising even in a homogeneous product industry should not only expand demand but also make it less elastic: profits need not rise even when $P_\theta > 0$ if $P_{Q\theta} > 0$ fails to hold. Further, with imperfect competition, environmental regulation which lowers the demand for a certain product need not reduce the profitability of that product, nor need it increase the profitability of unregulated substitutes, nor need it increase the incentives to develop new nonpolluting alternative products.[13,14]

Perverse profit effects can be ruled out in some common policy situations. In particular, such effects cannot arise for either a standard per unit commodity tax or an ad valorem sales tax. Of course, when a tax θ is imposed, the demand curve shifts down rather than up: $P_\theta < 0$. A perverse profit effect would then be $d\pi^*/d\theta > 0$ — *i.e.*, firms' profits actually rising as a result of a tax on their output. If $MR_Q < 0$ as we have assumed, then (20) indicates that for profits to rise would require $P_{Q\theta} < 0$. However, imposing a tax of θ per unit of output causes a parallel downward shift in the demand curve (*i.e.*, $P_{Q\theta} = 0$), while imposing a θ percent ad valorem sales tax rotates the demand curve downward about its horizontal intercept, so that $P_{Q\theta} > 0$.

One policy option which could cause a perverse profit effect is a system of marketable pollution permits. A marketable permit system has, for instance, been proposed for regulating the output of the chlorofluorocarbon family of chemicals ("CFCs").[15] These chemicals have a wide variety of largely unrelated uses, from refrigerants to solvents to blowing agents. Roughly put, a marketable permit system would require the purchasers of CFCs to buy a permit for each pound of the chemical used. It has been proposed that total CFC production be limited by limiting the number of permits, but that specific limits not be placed on individual chemicals within the CFC family.

Consider the demand curve for any one CFC. At any given output of that chemical, a permit system would cause its demand curve to fall by the corresponding price of a permit. To expand output of that CFC would then require permits to be bid away from ever more valuable uses of the other CFCs. Thus, the higher the output of the one CFC, the higher the permit price and the more the demand curve for that one would fall. In terms of the model above, P_θ and $P_{Q\theta}$ would both be negative; and profits of the oligopolistic CFC manufacturers might actually rise under a permit system.

4. CONCLUSIONS

When a demand curve shifts in an imperfectly competitive industry, price, quantity, consumer surplus, and welfare may each individually either rise or fall. This breadth of possibilities with a demand shift under imperfect competition contrasts sharply with the much narrower and more predictable effects of either a demand shift in a competitive industry or a cost shift in either type of industry. Even under imperfect competition, however, the pattern of demand shift effects cannot be entirely arbitrary. This paper has established a system of necessary and sufficient relationships among the changes in the various variables.

How profits change when demand shifts is particularly interesting. When the demand curve rises, profits may actually fall. As I have shown, this possibility is unique to imperfect competition: profits always rise with demand under both perfect competition and perfect collusion. The demand shift case thus again contrasts with the cost shift case, where imperfect competition is not a necessary ingredient in generating perverse profit effects.

Finally, there are welfare interpretations of the conjectural supply elasticity β which are independent of any comparative statics context. Equation (4) can be rearranged to give

$$(1-\beta)\cdot[P(Q^*)-MC(Q^*/n)]+\beta\cdot[MR(Q^*)-MC(Q^*/n)]=0. \tag{22}$$

Thus, Q^* maximizes the weighted average of welfare and industry profits

$$(1-\beta)\cdot W+\beta\cdot(n\cdot\pi), \tag{23}$$

with $(1-\beta)$ and β providing the weights.[16] It has been the practice of some authors (*e.g.*, Ordover and Panzar [1982]) to use a general maximand of form (23) with arbitrary weights to study simultaneously the welfare optimal, profit optimal, and second-best (Ramsey) optimal market outcomes. Our treatment establishes that such general weights can be interpreted in terms of conjectural variations.

Conversely, when firms use conjectural variations in choosing output, β becomes directly involved in some standard welfare measures of the resulting equilibrium. For instance, (4) can be rearranged to yield

$$\frac{P^*-MC(Q^*/n)}{P^*}=\frac{\beta}{e} \tag{24}$$

where $e=-P^*/(Q^*\cdot P_Q)$ is the demand elasticity. Thus, β/e is the Lerner Index. Furthermore, it can be shown [Quirmbach, 1984b] that when an input price changes the change in the Marshallian surplus behind the *input* demand curve exactly equals the change in (23), where W and $n\cdot\pi$ refer to the consumer surplus-plus-profit and industry profit in the (possibly imperfectly competitive) industry which uses that input.

NOTES

*University of Southern California. The results presented here extend results appearing in Quirmbach [1982], a study supported by U.S. Environmental Protection Agency Contract No. 68-01-6236 to The Rand Corporation. The earlier work benefited from comments by Jonathan Cave, James Dertouzos, Adele Palmer, and Charles Phelps. I am particularly indebted to Daniel Spulber for suggesting the format for the current paper. I would like to thank Leigh Tesfatsion for comments on the current draft.

1. Most authors represent conjectural variations as firm i's conjectures about its individual rivals' outputs (dq_j/dq_i for $j \neq i$) or its conjecture about the total output of all rivals $\left(d\{\sum_{j\neq i} q_j\}/dq_i\right)$. For other comparative static uses of the conjectural elasticity format, see Quirmbach [1982, 1984a, 1984b]. The elasticity representation is also used in an empirical context by Appelbaum [1982].

2. Throughout I will also assume that β is independent of q_i and Q. This holds in the perfectly competitive, perfectly collusive, and symmetric Cournot cases. The second order necessary condition is then

$$n \cdot \beta \cdot [(1-\beta) \cdot P_Q + \beta \cdot MR_Q] - MC_q \leq 0.$$

While the three conditions $P_Q < 0$, $MR_Q < 0$, and $MC_q \geq 0$ are not individually necessary for the second order conditions for some particular values of β, there is no strictly weaker set of conditions which guarantees that the second order conditions are satisfied by a locally unique Q independent of β. For competition, $MC_q \geq 0$ is necessary for the second order conditions. If this is the only restriction on MC_q, then $P_Q < 0$ must hold to guarantee a unique competitive Q; and $MR_Q < 0$ must hold to assure a unique Q under perfect collusion.

If β were to depend on Q, then $d\beta/dQ \geq 0$ together with the previous assumptions would assure the the second order conditions. (See also note 4 below.)

3. Many of these results are available elsewhere, although not in the "β" format. See, for instance, the sources cited in the introduction.

4. As is often the practice in this literature, I will assume that conjectures are independent of the shift parameter; *i.e.*, that $d\beta/d\theta = 0$. (See, for instance, Katz and Rosen [1985, p.18, fn.5].) Were this not an appropriate assumption for a particular application, then each comparative static derivative that follows would have an extra additive term reflecting the partial effect through a change in β. The signs of such terms would be obvious in all cases: a higher β would tend to lower quantity, consumer surplus, and welfare while raising price and profits.

Note that it would be entirely reasonable for β to depend on n. Indeed, one would expect that $d\beta/dn \leq 0$ — *i.e.*, more firms lead to greater competition. This idea has been applied in Quirmbach [1982, 1984a].

5. It is, of course, possible that $C_\theta < 0$ even though $MC_\theta > 0$ locally. In that case, $dW^*/d\theta$ could be positive. For instance, if a rise in θ represented a *fall* in the price of an inferior input (*i.e.*, an input the demand for which is a *decreasing* function of the firm's output), then $C_\theta < 0 < MC_\theta$ would in fact hold. However, this case hardly meets our intuitive notion of an upward cost shift.

6. Seade [1983, p.13] gives an example of a perverse profit change in a case where $MC_\theta = AC_\theta$. However, his example requires in addition that $Q \cdot P_{QQ}/P_Q < -2$. This latter condition is equivalent to $MR_Q > 0$, so that the second order conditions may not be

satisfied if β is large. (See note 2 above.) Seade's result differs from ours because, when $MR_Q > 0$ and $MC_q = 0$ as in Seade's example, then $(1-\beta)\cdot P_Q/\Omega > 1$ for $\beta < 1$.

7. It can be shown that this more stringent condition *cannot* be satisfied by a demand curve that is linear in Q. However, it can be satisfied for a very simple nonlinear demand curve. See (21) below.

8. Suppose, for instance, that marginal costs are locally vertical, so that $dQ^*/d\theta = 0$ for all β. Then, if $P_{Q\theta}$ has the same sign at all output levels, $dCS^*/d\theta < (>)\ 0$ as $P_{Q\theta} > (<)\ 0$.

9. The necessity of this condition rests on our assumption that $MR_Q < 0$. (See note 6 above.) It may be of further interest to note that, while $P_{Q\theta} > 0$ implies $dQ^*/d\theta > 0$ from (14), $P_{Q\theta} > 0$ is in general neither necessary nor sufficient for $dP^*/d\theta < 0$. This can be shown by rearranging (15):

$$\frac{dP^*}{d\theta} = \frac{P_\theta}{\Omega}\cdot[(MR_Q - P_Q)\cdot\beta - MC_q/n] - \frac{\beta\cdot Q^*\cdot P_Q}{\Omega}\cdot P_{Q\theta}.$$

For general demand curves MR_Q may be either greater or less than P_Q.

10. Let the demand curve turn vertically downward at $Q = 0.9$.

11. As mentioned in note 7 above, the demand curve must be nonlinear to get $dP^*/d\theta < 0$, a necessary condition for $d\pi^*/d\theta < 0$. Otherwise this example is not particularly pathological: for instance, it satisfies the stability conditions specified in Dixit [1986, p.116].

12. Demand and cost shifts are modelled here somewhat asymmetrically: a demand shift is a shift in the *average* revenue curve while a cost shift is a shift in the *marginal* cost curve. This asymmetry is appropriate, since these are the two curves which determine the equilibrium in the baseline case of perfect competition. While it is tempting to try to explain the asymmetry between the cost and demand shift effects on the basis of this modelling asymmetry alone, such an explanation would be incomplete. One could, for instance, impose modelling symmetry by requiring a demand shift to satisfy both $P_\theta > 0$ and $MR_\theta > 0$ and a cost shift to satisfy both $MC_\theta > 0$ and $AC_\theta > 0$. This would take care of the differences in the effects on Q^* and W^*, but the discrepancies would nevertheless remain for P^*, consumer surplus, and profits.

13. See Quirmbach [1982].

14. Bulow, Geanakoplos, and Klemperer [1985] present an interesting example of a related effect. In a multimarket context, they show that an outward shift in the demand curve in a market where a diversified firm has a monopoly may cause a fall in the profits the firm earns in another market where it faces oligopolistic rivals. The authors confine their analysis to the case where the transmission mechanism between the markets is the multiproduct cost function (*i.e.*, the products' demands are independent), but they point out that similar effects are possible when demands are interrelated.

15. See Palmer and Quinn [1981].

16. As above, β is treated here as being independent of Q. Also, the maximization here takes the number of firms (and their symmetry) as given.

REFERENCES

Appelbaum, E., "The Estimation of the Degree of Oligopoly Power," *Journal of Econometrics*, Vol.19 (1982), 287-299.

Bulow, J.I., Geanakoplos, J.D., and Klemperer, P.D., "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, Vol. 93 (June 1985), 489-511.

Dixit, A.K., "Comparative Statics for Oligopoly," *International Economic Review*, Vol.27 (February, 1986), 107-122.

Katz, M.L. and Rosen, H.S., "Tax Analysis in an Oligopoly Model," *Public Finance Quarterly*, Vol.13 (January, 1985), 3-19.

Ordover, J.A. and Panzar, J.C., "On the Nonlinear Pricing of Inputs," *International Economic Review*, Vol.23 (October, 1982), 659-676.

Palmer, A.R. and Quinn, T.H., *Allocating Chlorofluorocarbon Permits: Who Gains, Who Loses, and What Is the Cost?*, The Rand Corporation, R-2806-EPA, (July, 1981).

Quirmbach, H.C., *Environmental Regulation and R&D Incentives*, The Rand Corporation, R-2968-EPA, (November, 1982).

______, "Project Risk, Patents, and the Profitability of R&D," MRG Working Paper No.8425, University of Southern California, (October, 1984a, revised August, 1985).

______, "Input Market Surplus: The Case of Imperfect Competition," *Economics Letters*, Vol.16 (1984b), 357-362.

Salop, S.C. and Scheffman, D.T., "Raising Rivals' Costs," *American Economic Review*, Vol. 73 (May, 1983), Papers and Proceedings, 267-271.

Seade, J.K., "Prices, Profits and Taxes," working paper, University of Warwick, (1983).

REPORTS IN THIS SERIES

M8607: JOSEPH G. HIRSCHBERG: A Comparison of Three Flexible Cost Functions Using Establishment Level Electricity Use Data.
M8608: IAN E. NOVOS and MICHAEL WALDMAN: Complementarity and Partial Non-excludability: An Analysis of the Software/Computer Market.
M8609: ARNOLD ZELLNER: Further Results on Bayesian Minimum Expected Loss (MELO) Estimates and Posterior Distributions for Structural Coefficients.
M8610: ARNOLD ZELLNER: Bayesian Analysis in Econometrics.
M8611: ARNOLD ZELLNER: A Tale of Forecasting 1001 Series: The Bayesian Knight Strikes Again.
M8612: ARNOLD ZELLNER: Biased Predictors, Rationality and the Evaluation of Forecasts.
M8613: ARNOLD ZELLNER: Bayesian Estimation and Prediction Using Asymmetric Loss Functions.
M8614: RICHARD H. DAY, SUDIPTO DASGUPTA, SAMAR K. DATTA, and JEFFREY B. NUGENT: A Note on Instability in Rural-Urban Migration.
M8615: DENNIS J. AIGNER, FRIEDRICH SCHNEIDER and DAMAYANTI GHOSH: Me and My Shadow: Estimating the Size of the U.S. Underground Economy from Time Series Data.
M8616: ROBERT KALABA and LEIGH TESFATSION: Flexible Least Squares.
M8617: JOHN M. VEITCH: Wavering Virgins, the Deficit and Gramm-Rudman.
M8618: TIMUR KURAN: Chameleon Voters and Public Choice.
M8619: RICHARD H. DAY: The Evolving Economy.
M8620: JACQUES SILBER: Factor Components, Income Classes and the Computation of the Gini Index of Inequality.
M8621: MARIO D. TELLO: Manufactured Exports from Developing Countries and Government Policy Under Imperfect International Markets: Corporate Income Taxes and Exchange Rate.
M8622: DANIEL F. SPULBER: The Second Best Core.
M8623: DANIEL F. SPULBER: Variety and Efficiency: Monopolistic Competition with Nonlinear Price Schedules.
M8624: ABDELHAMID MAHBOB and JEFFREY B. NUGENT: The North's Demand for Southern Exports and Immiserization of the South.
M8625: IAN E. NOVOS and MICHAEL WALDMAN: The Emergence of Copying Technologies: What Have We Learned.
M8626: REZAUL KHANDKER: Offer Heterogeneity in a Two State Model of Sequential Search.
M8627: ROBERT KALABA, THOMAS PLUM, and LEIGH TESFATSION: Automation of Nested Matrix and Derivative Operations.
M8628: RICHARD H. DAY and KYOO-HONG KIM: A Note on Nonperiodic Demoeconomic Fluctuations with Positive Measure.
M8629: JOHN M. VEITCH: The Optimality of Loan Commitment Contracts in the Extension of Credit.
M8630: ANDREW A. WEISS: Estimating Nonlinear Dynamic Models Using Least Absolute Error Estimation.
M8631: RICHARD H. DAY and WAYNE SHAFER: Ergodic Fluctuations in Deterministic Economic Models.
M8632: TIMUR KURAN: The Craft Guilds of Tunis and Their Amins: A Study in Institutional Atrophy.
M8633: ROBERT KALABA and LEIGH TESFATSION: Time-Varying Regression Via Flexible Least Squares.
M8634: HERMAN C. QUIRMBACH: Comparative Statics for Oligopoly: Demand Shift Effects.