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INSTABILITY IN
RURAL-URBAN MIGRATION

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ABSTRACT

Newly observed instances of reverse migration, i.e., migration from urban to rural areas, suggest the possibility that the labor market adjustment process could be unstable. We provide a theoretical analysis of this conjecture using a Harris-Todaro-Lewis model. A bifurcation analysis is presented that shows how various stable and unstable adjustments are possible including chaotic fluctuations and how these possibilities are related to adjustment speed, the productivity of industry and agriculture, and the relative importance of industry and population size. Examples of unstable adjustments in a hypothetical LDC and DC are given using "plausible" parameter values.

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Key words: labor market dynamics, rural-urban migration, economic development.

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After decades of massive rural-urban migration in the developed countries, numerous instances of reverse migration from urban areas to suburban and rural areas have been observed.¹ In developing countries, especially those in Africa and Asia, "circular" and "temporary" migration patterns have been identified.² Even for those countries where migratory reversals have not yet actually taken place, analysts have frequently characterized rural-urban migration as being excessive, suggesting that such reversals will or at least should take place.³ Unstable migratory movements and migration reversals are also evident at an international level, especially to and from the Persian Gulf states. Not surprisingly, in most cases the unstable migratory flows are related to unstable movements in the terms of trade.

The standard model relating migration to the terms of trade is the Harris, Todaro model [Todaro (1969) and Harris and Todaro (1970)]. Although there has been some controversy on specific points, the model has been supported in empirical tests and widely applied to investigate various development issues and to evaluate alternative policies usually using comparative static analyses which have assumed rapid convergence of the underlying market adjustments.⁴ The phenomenon of reverse migration suggests a lack of monotonic convergence and possible fluctuations. Unfortunately, the stability of equilibrium and dynamic properties of migration processes have been given only cursory treatment. Where such

analyses have been undertaken, theoretical modifications make the exact source of instability in the Harris-Todaro model itself unclear.⁵

It is our purpose here to investigate the question of stability using some recent developments in nonlinear dynamic analysis and to do so within a framework which closely resembles the original Harris-Todaro model. We find that instability is indeed a possibility, and that regular or irregular, non-periodic fluctuations can be propagated -- even in the absence of any exogenous shocks. The source of these results is the lag in adjustment on labor markets combined with the nonlinearity of the labor supply function. We emphasize that they arise from the most conventional or "normal" specifications of technology and of competitive market adjustments.

To summarize our main findings, we can assert the following "causes" of unstable oscillations in labor migration:

- (1) *ceteris paribus*, a sufficiently large speed of adjustment to disequilibrium (λ);
- (2) *ceteris paribus*, a very small or a very large elasticity of production with respect to labor in industry (β);
- (3) *ceteris paribus*, a very small or very large ratio of industry to agriculture (m);
- (4) *ceteris paribus*, a sufficiently large agricultural productivity multiplier (A);
- (5) *ceteris paribus*, a sufficiently sparse population (N).

Because some of them could well be satisfied in both less developed and highly developed countries, these sources of instability could be in-

volved in the cases of reverse, circular, and temporary migration that have been noted in a variety of development situations.

THE MODEL

Following the order of the original two-sector formulation, with capital, land and technology fixed, the agricultural and industrial production functions are given respectively by

$$X_t = f(N_{At}) \quad (1)$$

$$Y_t = g(N_{It}) \quad (2)$$

where X_t and Y_t are the outputs in agriculture and industry respectively and where N_{At} and N_{It} are employment in agriculture and industry respectively, all relating to time period t .

Defining the agricultural good as the numeraire, the relative price of the industrial good in time period t , P_t , is determined by the relative outputs of X and Y according to a function $P_t = \rho(X_t/Y_t)$, which depends implicitly on the aggregate demand functions for both goods. For simplicity we assume as did Harris and Todaro in their original analysis that

$$\rho(X_t/Y_t) = mX_t/Y_t \quad (3)$$

where m is a positive constant representing the value ratio of industrial goods to agricultural goods.⁶ In the spirit of Lewis (1954) the agricultural wage w_{At} in our version of the model is equated to the average product of labor:

$$w_{At} = X_t/N_A, \quad (4)$$

and the manufacturing wage rate w_I is treated as fixed in terms of food.

The overall endowment of labor, N , is allocated between agricultural employment, N_{At} , and the industrial labor force, R_t , which in turn is divided between the employed, N_{It} , and unemployed, U_t . Thus, $N_{At} + R_t = N_{At} + N_{It} + U_t = N$. With these definitions the expected urban wage rate is

$$w_{Et} = w_I N_{It}/R_t \quad (5)$$

where the ratio N_{It}/R_t can be thought of as the probability of employment for a person drawn at random from the industrial labor force.

Industrial employment is assumed to be determined by the profit-maximizing marginal productivity condition, which, when combined with the preceding equations, yields the implicit function

$$\rho[f(N-R_t)/g(N_{It})]g'(N_{It}) - w_I = 0. \quad (6)$$

It can be shown that under the standard assumptions on (1) and (2) and given equations (3), - (5) the conditions of the implicit function theorem are satisfied.⁷ This implies that N_{It} is a function of the industrial labor force R_t which depends on total population N and the parameters of the production functions of the two sectors. Thus,

$$N_{It} = h(R_t). \quad (7)$$

We shall refer to $h(\cdot)$ as the (industrial) employment function.

The Harris-Todaro migration hypothesis is that rural-urban migration is a function of the gap between the expected urban wage rate and the agricultural wage rate, i.e., that

$$R_{t+1} - R_t = \lambda[w_{Et} - w_{At}] . \quad (8)$$

Upon substitution using (4), (5) and (7) we obtain

$$R_{t+1} = \theta(R_t) = R_t + \lambda[w_I h(R_t)/R_t - f(N-R_t)/(N-R_t)] . \quad (9)$$

It is possible to work directly with equation (9) and to show that all the results of this note hold for this "general" case. Stronger and more specific results, however, can be obtained by specifying the standard power production functions for agriculture

$$X = f(N_A) = AN_A^\alpha = A(N-R)^\alpha \quad (10)$$

and for industry

$$Y = g(N_I) = BN_I^\beta . \quad (11)$$

A little calculation shows that the industrial wage function is

$$w_{Et} = m\beta A(N-R_t)^{\alpha}/R_t \quad (12)$$

while that of agriculture is

$$w_{At} = A(N-R_t)^{\alpha-1} \quad (13)$$

yielding the profiles shown in Figure 1a.⁸ The wage differential, which is simply the vertical difference between these two curves, therefore looks like Figure 1b. The adjustment equation for the industrial labor force (9), which is now

$$R_{t+1} = R_t + \lambda [m\beta A(N-R_t)^\alpha / R_t - A(N-R_t)^{\alpha-1}] \quad (14)$$

therefore, has the qualitative shapes shown in Figure 1c, a diagram obtained by adding the wage differential of Figure 1b weighted by λ to the 45° line.⁹

The analysis is simplified if we transform (14) to get the dynamic structure in terms of the proportion $r = R/N$ of the total population in the industrial labor force. Dividing both sides of (14) by N we get

$$r_{t+1} = T(r_t) := r_t + \lambda N^{\alpha-2} [m\beta(1-r_t)^\alpha / r_t - (1-r_t)^{\alpha-1}] . \quad (15).$$

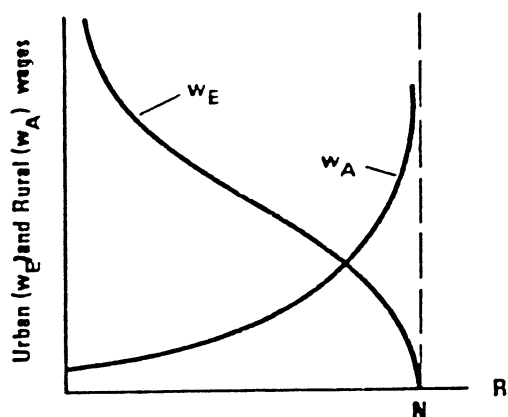
This is the difference equation we will be studying to determine (a) when reverse migration can occur, (b) if and when equilibrium can be unstable, and (c) the conditions under which irregular migratory fluctuations can emerge.¹⁰

There is a single stationary state

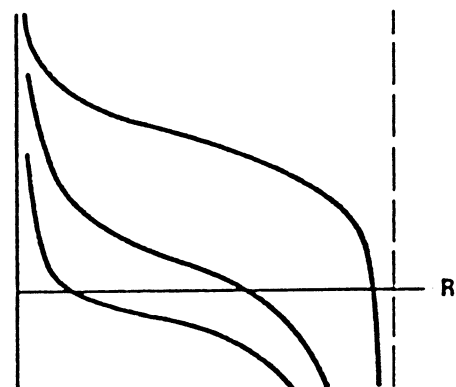
$$\bar{r} = m\beta / (1+m\beta) \quad (16)$$

which is the equilibrium fraction of the population in industry. The derivative of $T(\cdot)$ evaluated at \bar{r} is

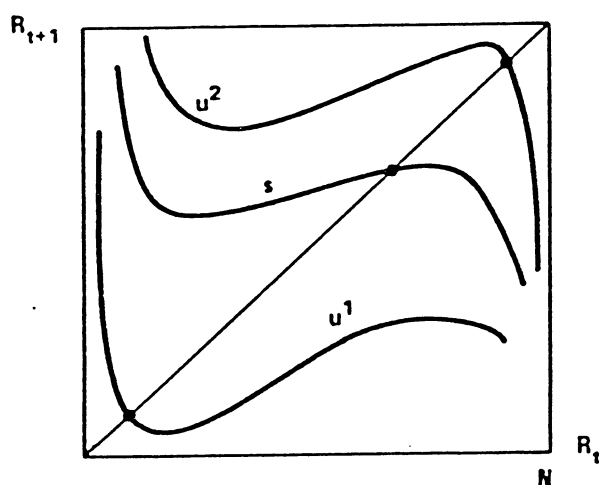
$$T'(\bar{r}) = 1 - \lambda N^{\alpha-2} \bar{r}^{-1} (1-\bar{r})^{\alpha-2} = 1 - \lambda N^{\alpha-2} (1-m\beta)^{3-\alpha} / m\beta. \quad (17)$$



(a) The wage adjustment functions



(b) Wage differentials. Three alternative functions shown.



(c) Phase portraits for different parameter values.

FIGURE 1: WAGES, WAGE DIFFERENTIALS AND INDUSTRIAL LABOR FORCE EQUATIONS

This expression must be less than -1 to get locally unstable oscillations around the stationary state, a condition equivalent to

$$\lambda A N^{\alpha-2} > 2\bar{r}(1-\bar{r})^{2-\alpha} = h(\bar{r}) \quad (18)$$

or, using (16),

$$\lambda A N^{\alpha-2} > 2m\beta(1+m\beta)^{\alpha-3} = g(m\beta) . \quad (19)$$

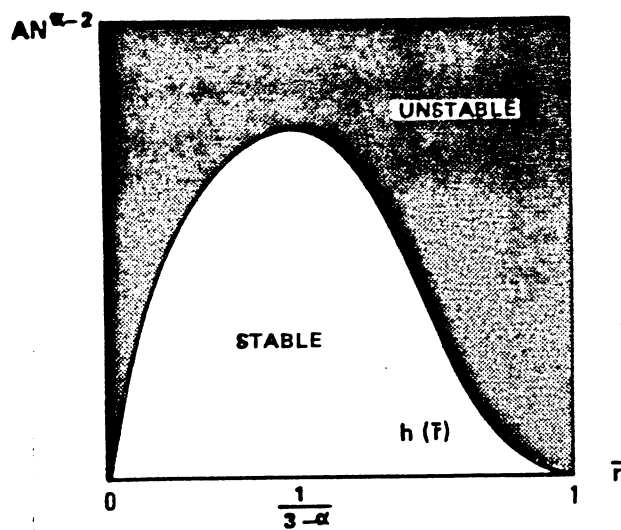
Evidently a wide range of parameter values are compatible with instability as is shown in Figure 2a which illustrates (18) and in Figure 2b which illustrates the equivalent condition (19). Suppose α and \bar{r} (or equivalently $m\beta$) are fixed. From (18) we find that the expression

$$\lambda A > h(\bar{r})N^{2-\alpha} \quad (20)$$

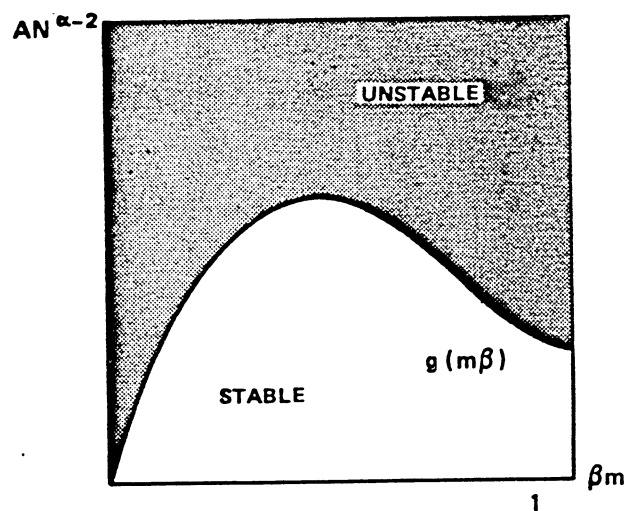
gives the region of stability and instability of λA and N as shown in Figure 2c.

From the inequality expression and the diagrams it is easy to derive the comparative dynamic statements listed at the beginning of our discussion. It is worth emphasizing that, while decreases (increases) in the adjustment speed λ stabilize (destabilize) the system as is to be expected, for any given λ there exist parameter combinations m , β , α , N and A satisfying the conditions for unstable migratory behavior. Whether such parameter values are realistic is, of course, an empirical question.

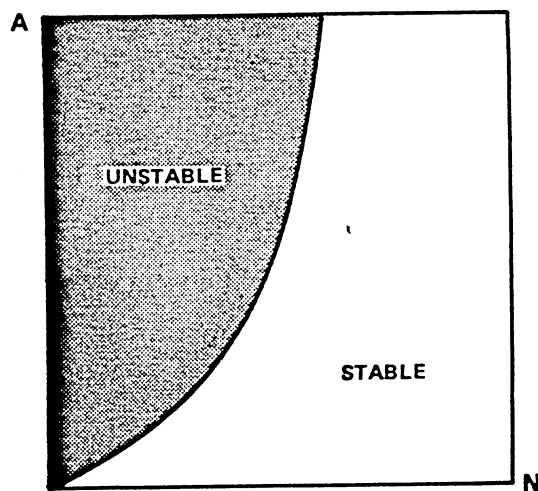
In the unstable situation oscillations are bounded because of the essential nonlinearity of the adjustment equation. Using methods outlined for example in Day and Shafer (1985, 1986) it can be shown that these can converge to cycles of some finite periodicity, or what is perhaps more interesting, to irregular migratory movements that display



(a) STABILITY IN TERMS OF \bar{r}



(b) STABILITY IN TERMS OF β_m



(c) STABLE AND UNSTABLE REGIONS GIVEN IN α AND \bar{r}

FIGURE 2: SUFFICIENT CONDITIONS FOR UNSTABLE MIGRATION FLUCTUATIONS

"chaotic", more or less random, sawtooth patterns that behave like a stochastic process.

NUMERICAL EXAMPLE

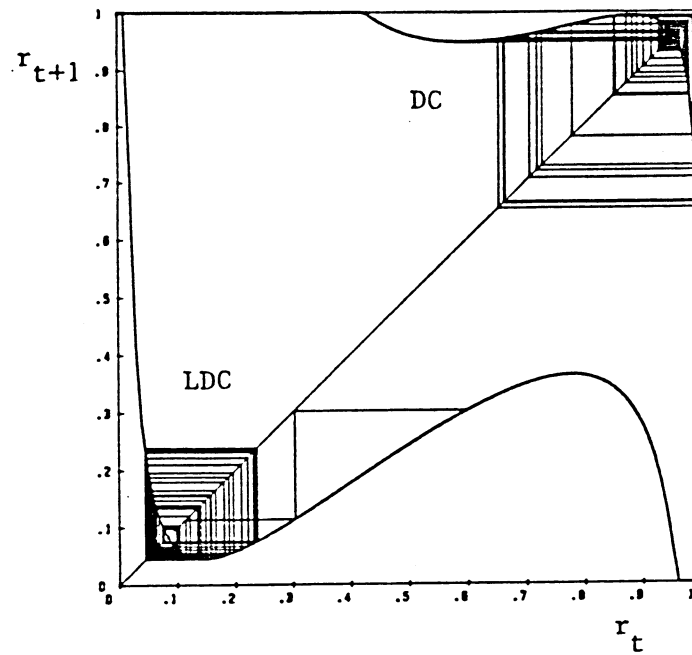
It may be useful to consider the situation for specific parameter values. Too much should not be made of such examples because the model itself, simplified as it is to make complete analysis possible, is rather too simple to be suitable for direct empirical application, which should require attention to a host of additional endogenous and exogenous variables. But it is interesting to see what type of dynamics occur for various parameter settings within the range of plausibility. To this end let us consider examples corresponding to developed and less developed cases. First, consider a DC with $\bar{r} = .95$. The unit of agricultural output is implicitly chosen if we fix the wage rate in industry in terms of food (w_I) at some arbitrary number, say, 10. Similarly, let $N = 100$ (million). Since 5% of the population is in agriculture, we can use the steady state condition $w_A = w_E = w_I = 10$ together with (12) to pin down A . If $\alpha = 1/3$, as would seem quite reasonable, $A \cong 30$. With these parameters (19) is satisfied for $\lambda > .9$. A value of λ close to 1 should be "reasonable" because it implies that over a long period of time, a 10% differential between w_E and w_A would cause 20% of the rural population to migrate (this is, however, only about 1% of the total population). Given $\bar{r} = .95$ and using (16) a value of $\beta = 2/3$ gives $m = 32.8$, i.e., only about 3% of the GDP originates in agriculture.

Suppose on the other hand that we have an LDC with $\bar{r} = .1$, $N = 100$ (million), $w_1 = 1$, $A = 10$, $\alpha = .5$. Now some 90% of the population would be in agriculture, with a standard of living for industrial workers that is 1/10th that of the DC discussed above. Retaining $\beta = .667$, $m = .136$ so that the ratio of GDP originating in industry to that of agriculture is 14%. $\lambda > 5.6$ gives unstable (and possibly chaotic) trajectories. A value of 20 for λ for example would imply that about 2% of the rural population migrates in response to a 10% wage differential. The lower relative speed of adjustment implied by these figures could reflect the greater importance of cultural and financial barriers to migration in the LDC's compared to the DC's.

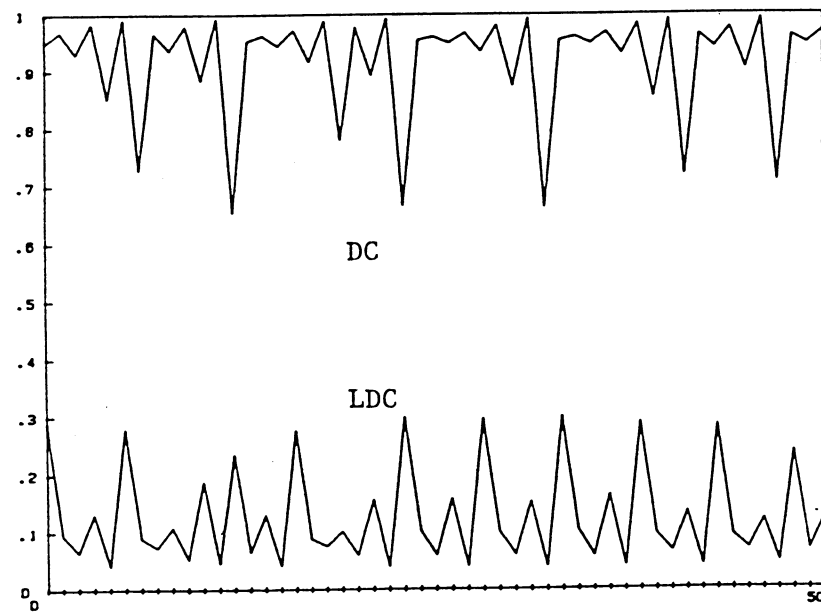
Simulation for the two examples are shown in Figure 3. In the DC case a stable cycle emerges while irregular or high order cyclic fluctuations appear in the LDC. Panel 3a gives the trajectories plotted on the "phase diagram" while panel 3b gives the implied time series data.

FINAL COMMENTS

Obviously, rural-urban migration in the real world will be influenced by many variables that are omitted from the present analysis. Moreover, the values of parameters that are held fixed (capital, population, productivity) will all change over time in response to investment, technological change and demographic forces. Nonetheless, the analysis and the examples clearly indicate the possible complexity of rural-urban movements. In particular, they suggest that in response to broader dynamics in the economy a point in the parameter space (Figures 2a and 2b) might move from the unstable regime with low \bar{r} to a stable regime with increasing \bar{r} and then, as changes (exogenous to the present model)



(a) The "phase" diagram



(b) The implied trajectories

FIGURE 3
 FLUCTUATIONS IN RURAL-URBAN MIGRATION FOR
 DEVELOPED AND LESS DEVELOPED ECONOMY EXAMPLES

continue, back into the unstable regime with high \bar{r} . In such a world rural urban migration might fluctuate for a time and then approach an upward trend with unstable fluctuations reemerging at an advanced stage of development.

In the real world such developments would very likely trigger measures intended to counteract instabilities, thus adding another parametric change to the process of market adjustment. As policymakers faced with the phenomenon of reverse migration consider instruments for this purpose their counterparts in countries still experiencing forward migration may want to anticipate a possible need to do the same.

Evidently reverse migration and instability are not exclusively characteristic of countries at low levels of development; they can also occur at high levels of development. In this sense one may conjecture that rapid enough improvements in technology can destabilize labor market adjustments. In this sense the policy issues concerning reverse migration may be intrinsic to "successful" technological advance.

Obviously, the whole issue of labor market stability needs further study using models which include more of the relevant variables and forces than the Harris-Todaro framework provides. Even within its narrow confines, however, the model and its possibilities for instability could be interpreted so as to apply to a relatively broad range of situations. For example, noting that migrants take with them their human capital and frequently also significant amounts of real capital, we could interpret N as bundles of mobile resources and R as their intersectoral flow. Also, the two sectors could be thought of as different countries or regions of countries, e.g., the North and the

South, with the South exporting the agricultural good and the North the industrial good. As before, the terms of trade P and the (international) allocation of labor between the regions would be determined endogenously, once again with the possibility of unstable patterns not unlike those experienced in recent years at the international level.¹¹

NOTES

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1. See, for example, Vining, Pallone and Plune (1981), DaVanzo and Morrison (1981, 1982), DaVanzo (1983) and the references therein.

2. On circular migration see Elkan (1967), Fan (1982), Fan and Lee (1983), Goldstein (1978), Hugo (1975, 1978), Mitchell (1969), Stretton (1981) and VanderKamp (1972). On temporary migration see Barnum and Sabot (1975), Breman (1979), Connell et al. (1976) and especially Nelson (1976) and the numerous references to village studies therein.

3. See especially Todaro (1976a, 1976b) and World Bank (1983). Several countries, such as Tanzania, Indonesia, the Philippines, South Africa, China and Cuba, have taken strong measures to prevent rural-urban migration and/or to promote migration reversals [Simmons (1981)].

4. Most of this controversy has centered on the precise conditions under which the "Todaro paradox," wherein an increase in the rate of new urban job generation would increase the level or the rate of urban unemployment, would be fulfilled. See especially Blomqvist (1978) and Arellano (1981). For the most part, these conditions depend on the specification of the migration function. For empirical work see especially Barnum and Sabot (1975), Todaro (1976a, 1976b) and Cole and Sanders (1983). On the policy side see, e.g., Bhagwati and Srinivasan (1974), Bhatia (1979), Corden and Findlay (1975), Das (1982), Jha and Lachler (1981) and Robertson and Wellisz (1977).

5. Some exceptions are the studies of Neary (1981), Amano (1983) and Bartlett (1983) who have shown that stability of equilibrium is not guaranteed. In all these cases, however, the simple Harris-Todaro framework is modified substantially and other elements appended. In the case of Neary, it is by bringing in the mobility of capital; in Bartlett it is by bringing in population and capital growth and by using the more complicated version of the model (i.e., Todaro (1969)); in the case of

Amano it is by introducing adjustment costs (with respect to employment by urban firms and of wage rates to unemployment rates and other determinants), and by opening up the product markets to trade with the rest of the world, thereby pegging the prices of sectoral outputs.

6. Algebraic convenience is perhaps the main reason for the popularity of the particular functional form of the price equation in (3), but two economic justifications for its use can be suggested. First, we show that the assumptions of (i) homothetic preferences for rural and urban consumers and (ii) zero cross price-elasticity of demand between industrial and agricultural goods imply the particular form of (4).

Let us suppose that the endowment of urban consumers is given in terms of industrial goods (Y), and that of rural consumers in terms of agricultural goods (X). Homotheticity implies at any given value of P_A/P_I , urban consumers always spend the same fraction of total income Y on industrial goods. The zero cross elasticity assumption implies that at any given Y , the consumption of industrial goods remains unchanged, irrespective of the value of P_A/P_I . These two properties together imply that consumption of industrial goods by urban consumers is of the form $C_1 Y$, where C_1 is a constant. Similar arguments suggest that the consumption of agricultural goods by rural consumers is of the form $C_2 X$, where C_2 is a constant. The demand for agricultural goods by urban consumers, expressed in terms of agricultural goods, is $[P_I(1-C_1)Y]/P_A$. The supply of agricultural goods to urban consumers is $(1-C_2)X$. Equating demand and supply, we get $[P_I(1-C_1)Y]/P_A = (1-C_2)X$ or $P_I/P_A = (1-C_2)X/[(1-C_1)Y] = mX/Y$.

An alternative economic interpretation of (3) is to think of the economy as a monetary one in which money consists of food stamps backed 100% by the availability of food. The equation of exchange for such an economy can be written as $(P_I/P_A) Y + X = VX$, where V is the velocity of circulation of food stamps. From this it follows that $P_I/P_A = (V-1)X/Y = mX/Y$. Even if money is only partially backed by food, as long as the proportionality between the quantity of money (in terms of agricultural output) and agricultural output is constant, the same result would obtain.

7. It is shown in Appendix A of Day, Dasgupta, Datta and Nugent (1986) that, under the standard assumptions in the production functions,

the implicit function theorem holds and the industrial employment function exists.

8. The shapes of all curves in Figure 1 are derived in Appendix B of Day, Dasgupta, Datta and Nugent.

9. The reason why w_I does not appear in (14) is the specific form assumed for $p(\cdot)$ in (3). Using (3) one can show that (6) reduces to: $m\beta A(N-R_t)^\alpha/N_{It} = w_I$. Substituting in (5) we get (12) and hence (14).

10. It is interesting to consider alternative forms for the basic adjustment equation (10) such as the proportional form $(R_{t+1}-R_t)/R_t = \lambda[w_{It}-w_{At}]$. We have done this and virtually all of the results remain unaffected.

11. We have not incorporated capital primarily because another state variable would complicate the model and obscure the main issue. However, capital accumulation can be introduced in a somewhat ad hoc manner, as follows. Since the issue here is the instability of rural-urban migration, the assumption of over-all steady-state growth in capital does not bias the model in favor of the conclusion. If the capital stocks in industry and agriculture and total population grow at the same rate, and production functions exhibit constant returns to scale, a slight modification of equation (10), with the number of people migrating now being expressed as a percentage of total population, gives us the following reduced form:

$$r_{t+1} = [1/(1+g)]r_t + [\lambda n/(1+g)][w_I h(r_t)/(r_t) - f(1, n-r_t)/(n-r_t)]$$

where $r_t = R_t/K_{At}$, K_{At} = the capital stock in agriculture at time t , g = the steady state rate of growth, $n = N_t/K_{At}$, a constant and the properties of the function $h(\cdot)$ are as in the text. Although not identical to (13); the analysis in the text also applies.

References

- Amano, Masanori (1983) "On the Harris-Todaro Model with Intersectoral Migration of Labour," Economica 50 (August): 311-325.
- Arellano, J.-P. (1981) "Do More Jobs in the Modern Sector Increase Urban Unemployment?" Journal of Development Economics 8: 241-247.
- Barnum, Howard N. and R.H. Sabot (1975) Migration, Education and Urban Surplus Labour, Paris: OECD Development Center.
- Bartlett, William (1983) "On the Dynamic Instability of Induced-Migration Unemployment in a Dual Economy," Journal of Development Economics 13 (August-October): 85-96.
- Bhagwati, J.N. and T.N. Srinivasan (1974) "On Reanalyzing the Harris-Todaro Model: Policy Rankings in the Case of Sector-Specific Sticky Wages," American Economic Review (June).
- Bhatia, Kul B. (1979) "Rural-Urban Migration and Surplus Labour," Oxford Economic Papers 31, No. 3 (November).
- Blomqvist, A.G. (1978) "Urban Job Creation and Unemployment in LDCs: Todaro vs. Harris and Todaro," Journal of Development Economics 5, No. 1 (March): 3-18.
- Breman, Jan C. (1979) "Seasonal Migration and Cooperative Capitalism: The Crushing of Cane and Labor by the Sugar Factories of Bardoli, South Gujarat," Paper presented to the Conference on Adjustment Mechanisms in Rural Labor Markets in Developing Areas ICRISAT, Hyderabad, India, August 22-24.
- Cole, William E. and Richard D. Sanders (1983) "Interstate Migration in Mexico: Variations on the Todaro Theme," Journal of Development Economics 12, No. 3 (June): 341-355.
- Connell, J., Biplap Dasgupta, R. Laishley and Michael Lipton (1976) Migration from Rural Areas: The Evidence from Village Studies, Delhi: Oxford University Press.
- Corden, W.M. and R. Findlay (1975) "Urban Unemployment, Intersectoral Capital Mobility and Development Policy," Economica (February): 59-78.
- Das, Satya P. (1982) "Sector-specific Minimum Wages, Economic Growth and Some Policy Implications," Journal of Development Economics 10 (February): 127-131.
- DaVanzo, Julie (1983) "Repeat Migration in the U.S.: Who Moved Back and Who Moved On?" Review of Economics and Statistics (May).
- DaVanzo, Julie and Peter A. Morrison (1981) "Return and Other Sequences of Migration in the U.S.," Demography 18 (February): 85-101.
- _____, (1982) Migration Sequences: Who Moves Back and Who Moves On? Santa Monica: Rand Corporation, R-2548-NICHD.
- Day, Richard H. and Wayne Shafer (1985) "Keynesian Chaos" Journal of Macroeconomics 7:277-295.
- _____, (1986) "Ergodic Fluctuations in Deterministic Economic Models" Journal of Economic Behavior and Organization (forthcoming). MRG Working Paper #8631, University of Southern California, Los Angeles, CA.
- _____, Sudipto Dasgupta, Samar K. Datta and Jeffrey B. Nugent (1986) "A Note on Instability in Rural-Urban Migration," MRG Working Paper #8614, University of Southern California, Los Angeles, CA.

- Elkan, W. (1967) "Circular Migration and the Growth of Towns in East Africa," International Labor Review 96: 581-589.
- Fan, Yiu-Kwan (1982) "Risk Aversion and Migration Behavior: A Theoretical Note on Circular Migration in South East Asia," Paper presented to the Midwest Economics Association, Chicago, March-April.
- Fan, Yiu-Kwan and Bun Song Lee (1983) "Risk Behavior and Sex-Differential Patterns of Migration among Rural-Born Koreans," Paper presented to the 35th Annual Meeting of the Association of Asian Studies, San Francisco.
- Goldstein, S. (1978) Circulation in the Context of Total Mobility in Southeast Asia, Honolulu: East-West Center, East-West Population Institute No. 53.
- Harris, John R. and M.P. Todaro (1970) "Migration, Unemployment and Development: A Two Sector Analysis," American Economic Review 60: 126-142.
- Hugo, G.J. (1975) "Population Mobility in West Java, Indonesia," Canberra: Ph.D. Dissertation, Department of Demography, Australian National University.
- _____ (1978) "New Conceptual Approaches to Migration in the Context of Urbanization: A Discussion Based on Indonesian Experience," Paper presented to the Seminar on New Conceptual Approaches to Migration in the Context of Urbanization, Bellgio, Italy.
- Jha, R. and U. Lächler (1981) "Optimum Taxation and Public Production in a Dynamic Harris-Todaro World," Journal of Development Economics 9, No. 3: 357-374.
- Lewis, W.A. (1954) "Economic Development with Unlimited Supplies of Labour," Manchester School of Economic and Social Studies 22 (May): 139-191.
- Mitchell, J. Clyde (1969) "Structural Plurality, Urbanization and Circulation in Southern Rhodesia," in J.A. Jackson, ed., Migration, Cambridge: Cambridge University Press.
- Neary, J.P. (1981) "On the Harris-Todaro Model with Intersectoral Capital Mobility," Economica 48: 219-234.
- Nelson, Joan M. (1976) "Sojourners versus New Urbanites: Causes and Consequences of Temporary versus Permanent Cityward Migration in Developing Countries," Economic Development and Cultural Change 24: 721-757.
- Robertson, P. and S. Wellisz (1979) "Steady-State Growth of an Economy with Intersectoral Migration," Oxford Economic Papers 31: 370-388.
- Simmons, Alan B. (1981) "A Review and Evaluation of Attempts to Constrain Migration to Selected Urban Centres and Regions," Population Distribution Policies in Development Planning, New York: United Nations, Department of Economic and Social Affairs, Population Studies No. 75.
- Stretton, A.W. (1981) "The Building Industry and Urbanization in Third World Countries: A Philippines Case Study," Economic Development and Cultural Change 29: 325-339.
- Todaro, Michael P. (1969) "A Model of Labor Migration and Urban Unemployment in Less Developed Countries," American Economic Review 59: 138-148.
- _____ (1976a) Migration in Developing Countries: A Review of Theory Evidence, Methodology and Research Priorities, Geneva: International Labour Organization.

- ____ (1976b) "Urban Job Expansion, Induced Migration and Rising Unemployment: A Formulation and Simplified Empirical Test for LDCs," Journal of Development Economics 3: 211-225.
- Vanderkamp, J. (1972) "Return Migration: Its Significance and Behavior," Western Economic Journal 10 (December): 460-465.
- Vining, D., R. Pallone and D. Plune (1981) "Recent Migration Patterns in the Developed World: A Clarification of Some Differences between Our and IIASA's Findings," Environment and Planning 13: 243-250.
- World Bank (1983) Growth and Employment in Rural Thailand, Washington, D.C., World Bank, Country Programs Department, East Asian and Pacific Regional Office, Report No. 3906-TH.

A NOTE ON
INSTABILITY IN RURAL-URBAN MIGRATION
by

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After decades of massive rural-urban migration in the developed countries, numerous instances of reverse migration from urban areas to suburban and rural areas have been observed.¹ In developing countries, especially those in Africa and Asia, "circular"² and "temporary"³ migration patterns have been identified. Even for those countries where migratory reversals have not yet actually taken place, analysts⁴ have frequently characterized rural-urban migration as being excessive, suggesting that such reversals will or at least should take place.⁵ Unstable migratory movements and migration reversals are also evident at an international level, especially to and from the Persian Gulf states. Not surprisingly, in most cases the unstable migratory flows are related to unstable movements in the terms of trade.

The standard model relating migration to the terms of trade is the Harris, Todaro model [Todaro (1969) and Harris and Todaro (1970)]. Although there has been some controversy on specific points⁶, the model has been supported in empirical tests⁷ and widely applied to investigate various development issues and to evaluate alternative policies⁸ usually using comparative static analyses which have assumed rapid convergence of the underlying market adjustments. The phenomenon of reverse migration suggests a certain lack of stability, implying at least a lack of monotonic convergence and possibly cyclic or nonperiodic patterns in behavior. Unfortunately, the stability of equilibrium and dynamic pro-

perties of migration processes have been given only cursory treatment. Where such analyses have been undertaken, theoretical modifications make the exact source of instability in the Harris-Todaro model itself unclear.⁹

It is our purpose here to investigate the question of stability using some recent developments in nonlinear dynamic analysis and to do so within a framework which closely resembles the original Harris-Todaro model. We find that instability is indeed a possibility, and that regular or irregular, non-periodic fluctuations can be propagated -- even in the absence of any exogenous shocks. The source of these results is the lag in adjustment on labor markets combined with the nonlinearity of the labor supply function. We emphasize that they arise from the most conventional or "normal" specifications of technology and of competitive market adjustments.

To summarize our main findings, we can assert the following "causes" of unstable oscillations in labor migration:

- (1) *ceteris paribus*, a sufficiently large speed of adjustment to disequilibrium (λ);
- (2) *ceteris paribus*, a very small or a very large elasticity of production with respect to labor in industry (β);
- (3) *ceteris paribus*, a very small or very large ratio of industry to agriculture (m);
- (4) *ceteris paribus*, a sufficiently large agricultural productivity multiplier (A);
- (5) *ceteris paribus*, a sufficiently sparse population (N).

Because some of these conditions could well be satisfied in both less developed and highly developed countries, the relevance of the sources of instability identified here could have relevance to the aforementioned cases of reverse, circular, and temporary migration in a variety of less developed and advanced countries.

THE MODEL

Following the order of the original two-sector formulation, with capital, land and technology fixed, the agricultural and industrial production functions are given respectively by

$$X_t = f(N_{At}) \quad (1)$$

$$Y_t = g(N_{It}) \quad (2)$$

where X_t and Y_t are the outputs in agriculture and industry respectively and where N_{At} and N_{It} are employment in agriculture and industry respectively, all relating to time period t .

Defining the agricultural good as the numeraire, the relative price of the industrial good in time period t , P_t , is determined by the relative outputs of X and Y according to a function

$$P_t = \rho(X_t/Y_t) , \quad (3)$$

which depends implicitly on the aggregate demand functions for both goods.

In the spirit of Lewis (1954) the agricultural wage w_{At} in our version of the model is equated to the average product of labor:

$$w_{At} = X_t/N_A, \quad (4)$$

and the manufacturing wage rate w_I is treated as fixed.

The overall endowment of labor, N , is allocated between agricultural employment, N_{At} , and the industrial labor force, R_t , which in turn is divided between the employed, N_{It} , and unemployed, U_t .¹⁰ Thus,

$$N_{At} + R_t = N_{At} + N_{It} + U_t = N. \quad (5)$$

With these definitions the expected urban wage rate is

$$w_{Et} = w_I N_{It}/R_t \quad (6)$$

where the ratio N_{It}/R_t can be thought of as the probability of employment for a person drawn at random from the industrial labor force.

Industrial employment is assumed to be determined by the profit-maximizing marginal productivity condition

$$P_t g'(N_{It}) - w_I = 0. \quad (7)$$

Using the preceding equations with this one, an implicit function is obtained,

$$\rho[f(N-R_t)/g(N_{It})]g'(N_{It}) - w_I = 0 \quad (8)$$

which, given that the conditions of the implicit function theorem are satisfied, implies that N_{It} is a function of the industrial labor force

R_t which depends on total population N and the parameters of the production functions of the two sectors. Thus,

$$N_{It} = h(R_t) . \quad (9)$$

We shall refer to $h(\cdot)$ as the (industrial) employment function.

The Harris-Todaro migration hypothesis is that rural-urban migration is a function of the gap between the expected urban wage rate given by (6) and the agricultural wage rate, i.e., that

$$R_{t+1} - R_t = \lambda[w_{Et} - w_{At}] . \quad (10)$$

Because the expected urban wage is

$$w_{Et} = w_I h(R_t)/R_t \quad (11)$$

and the agricultural wage is

$$w_{At} = f(N-R_t)/(N-R_t) , \quad (12)$$

we obtain

$$R_{t+1} = \theta(R_t) = R_t + \lambda[w_I h(R_t)/R_t - f(N-R_t)/(N-R_t)] . \quad (13)$$

This is the difference equation we will be studying to determine (a) when reverse migration can occur, (b) if and when equilibrium can be unstable, and (c) under what conditions irregular migratory fluctuations can emerge.¹¹

COMPARATIVE DYNAMICS

Everything depends on the "profile" of the function θ . First, we must make sure that the industrial employment function $h(\cdot)$ exists. This is done in Appendix A for quite general assumptions about technology. In Appendix B we show that the components of wage adjustment have the shapes shown in Figure 1a and that the implied expected urban-rural wage rate differential as a function of the industrial labor force is like that shown in Figure 1b. From these we infer that the profile of (13) can be either stable as in curve S of Figure 1c or unstable as the curve labelled U. Which of these cases would prevail depends on the underlying parameters of the model. It is shown in Appendix B, that instability is made more likely the more rapid is the responsiveness of migration and, for any given adjustment speed, the more the elasticity of output with respect to labor in agriculture is high in relatively urban economies or low in relatively rural ones.

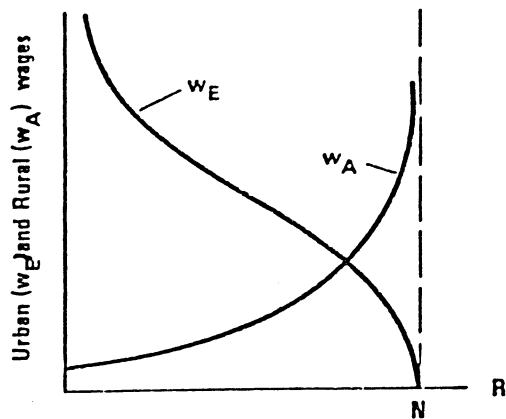
In the unstable situation we get bounded oscillations. These can converge to cycles of some finite periodicity, or what is perhaps more interesting, to irregular migratory movements that display "chaotic", more or less random, sawtooth patterns.¹²

It is not difficult to produce specific examples that satisfy both the sufficient conditions for unstable fluctuations and the standard qualitative properties of technology that we have assumed. For this purpose consider the standard power production functions for agriculture

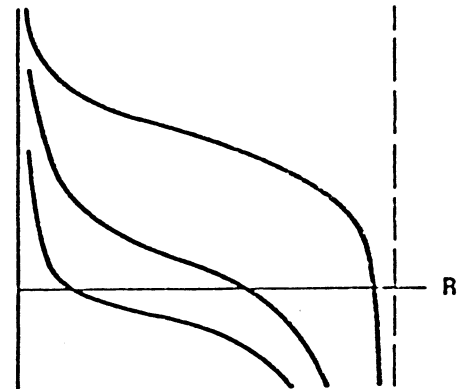
$$X = f(N_A) = AN_A^\alpha = A(N-R)^\alpha \quad (14)$$

and for industry

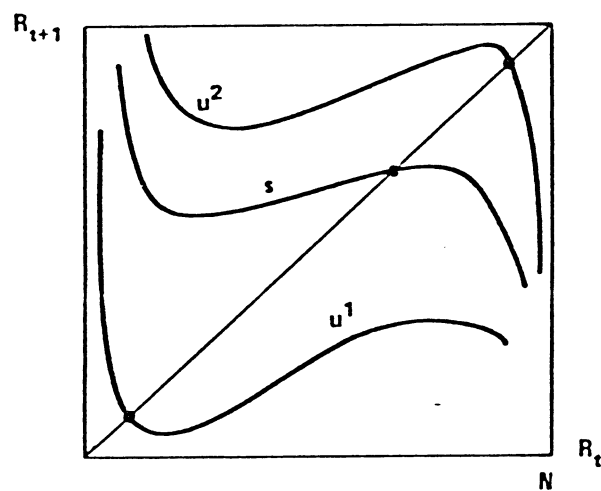
$$Y = g(N_I) = BN_I^\beta \quad (15)$$



(a) The wage adjustment functions



(b) Wage differentials. Three alternative functions shown.



(c) Phase portraits for different parameter values.

FIGURE 1: WAGES, WAGE DIFFERENTIALS AND INDUSTRIAL LABOR FORCE EQUATIONS

A little calculation shows that the industrial wage function is

$$w_{Et} = m\beta A(N-R_t)^\alpha / R_t \quad (16)$$

while that of agriculture is

$$w_{At} = A(N-R_t)^{\alpha-1} \quad (17)$$

which give the profiles shown in Figure 1a and which we have already discussed. The wage differential therefore looks like Figure 1b. The adjustment equation for the industrial labor force (13), which is

$$R_{t+1} = R_t + \lambda [m\beta A(N-R_t)^\alpha / R_t - A(N-R_t)^{\alpha-1}] \quad (18)$$

therefore, has the qualitative shapes shown in Figure 1c.

The analysis is simplified if we transform (18) to get the dynamic structure in terms of the proportion $r = R/N$ of the total population in the industrial labor force. In these terms we get

$$r_{t+1} = T(r_t) := r_t + \lambda AN^{\alpha-2} [m\beta(1-r_t)^\alpha / r_t - (1-r_t)^{\alpha-1}] \quad (20)$$

There is a single stationary state

$$\bar{r} = m\beta / (1+m\beta) \quad (21)$$

which is a positive function of $m\beta$, as one would expect. The derivative of this function is

$$T'(r) = 1 - \lambda AN^{\alpha-2}(1-m\beta)^{3-\alpha}/m\beta \quad (22)$$

This function, evaluated at \bar{r} , must be less than -1 to get locally unstable oscillations around the stationary state, i.e., local instability requires

$$T'(\bar{r}) < -1, \quad (23)$$

a condition equivalent to

$$\lambda AN^{\alpha-2} > 2m\beta(1+m\beta)^{\alpha-3} = g(m\beta). \quad (24)$$

Let σ_1 and σ_2 denote the values of $m\beta$ which equate the two sides of (24). Then, given values of λ , A , α , N such that positive σ^1 , σ^2 exist, unstable reversals in migration would occur for all $m\beta$ such that

$$0 < m\beta < \sigma^1 \quad \text{or} \quad \sigma^2 < m\beta < 1. \quad (25)$$

Thus, if $m\beta$ is sufficiently low or sufficiently high migratory movements will be unstable. Otherwise, labor market adjustments will converge. These are the possibilities illustrated in Figure 1c. The sufficient conditions are illustrated by plotting the left and right sides of (24) as shown in Figure 2.

From (21) it is easy to see how changes in $m\beta$ shift the equilibrium industrial labor force. For given m an increase (decrease) in the elasticity of production with respect to labor in industry, β , would shift \bar{r} to the right (left). Likewise, it can be seen that either an increase in A or a decrease in N would reduce the stable zone

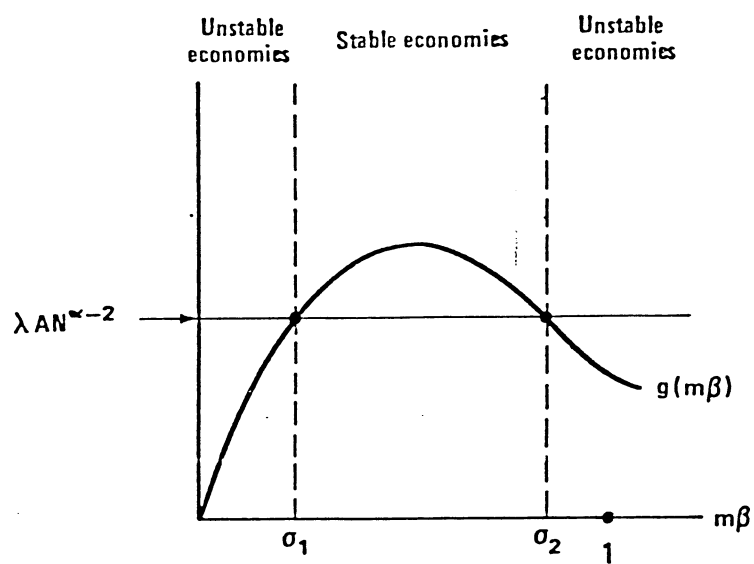


FIGURE 2: SUFFICIENT CONDITIONS FOR UNSTABLE MIGRATION FLUCTUATIONS

$[\sigma^1, \sigma^2]$ and increase the unstable zone for $m\beta$. Hence, for given $m\beta$, a sufficiently large agricultural production multiplier, A , or sufficiently small population would make instability likely.

It is worth emphasizing that, while decreases (increases) in the adjustment speed λ stabilize (destabilize) the system as is to be expected, for any given λ there exist parameter combinations satisfying the conditions for unstable migratory behavior. Whether such parameter values are realistic is, of course, an empirical question.

Evidently reverse migration and instability are not exclusively characteristic of countries at low levels of development; they can also occur at high levels of development. In this sense one may conjecture that rapid enough improvements in productivity can destabilize labor market adjustments. In this sense the policy issues concerning reverse migration may be intrinsic to "successful" technological advance. It should be emphasized that instabilities in adjustments can arise in both underdeveloped and highly developed countries when they are characterized by technologies and behavioral response functions of the type that are frequently used in discussions of rural-urban migration. This calls attention to the possible importance of identifying policies capable of treating such instabilities.

APPENDIX A

Existence of the Industrial Employment Function $h(\cdot)$

Assume that

$$f' > 0, \quad f'' < 0, \quad g' > 0, \quad g'' < 0, \quad (A1)$$

$$\lim_{N_A \rightarrow 0} f' = \infty, \quad \lim_{N_A \rightarrow N} f' \geq 0, \quad (A2)$$

and

$$f(0) = 0, \quad g(0) = 0. \quad (A3)$$

Also, for simplicity assume as did Harris and Todaro that

$$P = \rho(X/Y) = mX/Y, \quad (A4)$$

where P is the price of industrial goods in terms of agricultural goods and where m is a positive constant.

Using (A4) and taking the total differentials for (8), we find that

$$\frac{dN_I}{dR} = \frac{g'(N_I)f'(N-R)}{f(N-R)g''(N_I) - kg'(N_I)} : = h'(R) \quad (A5)$$

where $k = \bar{w}_I/m$. As $N_I \leq R \leq N < \infty$, it follows from (A1) and (A2) that (14) is finite except at $R = N$. The existence of (9) within the semi-open interval $0 \leq R < N$ follows.

APPENDIX B
 Characterization of the Components
 of Wage Rate Adjustment

We have to characterize the two "wage rate components", $h(R)/R$ and $f(N-R)/(N-R)$ which appear in (13). On considering (A5) together with assumptions (A1)-(A3), we find that

$$\lim_{R \rightarrow 0} h(R)/R = \infty \quad (B1)$$

$$\lim_{R \rightarrow N} h(R)/R = 0 \quad (B2)$$

$$\lim_{R \rightarrow 0} \frac{d(h(R)/R)}{dR} = -\infty \quad (B3)$$

$$\lim_{R \rightarrow N} \frac{d(h(R)/R)}{dR} = -\infty \quad (B4)$$

$$\lim_{R \rightarrow N} f(N-R)/(N-R) = \infty \quad (B5)$$

$$\frac{df(N-R)/(N-R)}{dR} > 0 \quad (B6)$$

Proof

Consider equation (B1). Taking the derivative

$$\frac{d(h(R)/R)}{dR} = \frac{h'(R)R - h(R)}{R^2}.$$

Now, $\lim_{R \rightarrow 0} h'(R) = 0$, as $-\infty < \lim_{R \rightarrow 0} h' < \infty$.

Also, $h(0) > 0$ because we have

$$g'(N) \cdot f(N-R) = kg(N) \quad \text{and} \quad g(0) = 0, \quad g'(0) \neq 0.$$

Hence,

$$\lim_{R \rightarrow 0} \frac{d(h(R)/R)}{dR} = -\infty.$$

Again,

$$\lim_{R \rightarrow N} \frac{d(h(R)/(R))}{dR} = \lim_{R \rightarrow N} \frac{h'(R) - h(R)}{R^2} = -\infty$$

because $\lim_{R \rightarrow N} h' = -\infty$.

Moreover,

$$\lim_{R \rightarrow 0} \frac{h(R)}{R} = \infty, \quad \lim_{R \rightarrow N} \frac{h(R)}{R} = 0.$$

From these we conclude that $h(R)/R$ is as shown in Fig. 1a. Turning now to the shape of $f(N-R)/(N-R)$, we find that both numerator and denominator $\rightarrow 0$ as $R \rightarrow N$. Hence, by applying L'Hospital's Rule,

$$\lim_{R \rightarrow N} \frac{f(N-R)}{N-R} = \frac{-f'(0)}{-1} = f'(0) = \infty$$

Also, $f(N-R)/(N-R)$ takes the value $f(N)/N$ at $R_t = 0$. The slope of $f(N-R)/(N-R)$ is given by $[f - f'x(N-R)]/(N-R)^2$, and is positive given the concavity of f . These findings yield the curve shown in Figure 1a for $f(N-R)/(N-R)$.

To find the shape of the function (9), which we may call $\theta^I(R; \lambda, A)$, we first have to bring together the functions $h(R)/R$ and $f(N-R)/(N-R)$, multiplied by constants w_I and A , respectively, and subtracting $A(f(N-R)/(N-R))$ from $w_I(h(R)/R)$ which gives the graph shown in Figure 1b. Multiplying by λ does not produce any qualitative change in the graph except to flatten it in midrange and make its approach to the

limiting asymptotic sharper. Adding R to it yields a graph like the ones shown in Figs. 1c and 1d. This is the graph of the function $\theta^I(R, \lambda, A)$.

Let R^* denote the value of R for which θ^I attains a local maxima, the \bar{R} denotes the steady-state of R . Then we have

$$\left. \frac{\partial \theta^I}{\partial R^*} \right|_{R^*} = \lambda \left[-A \frac{f - f' \times (N - R^*)}{(N - R^*)^2} + w_I \frac{h' \times R^* - h}{R^{*2}} \right] + 1 = 0 \quad (B7)$$

and

$$-A \frac{f(N - \bar{R})}{N - \bar{R}} + w_I \frac{h(\bar{R})}{\bar{R}} = 0 \quad (B8)$$

Comparative statics on \bar{R} :

\bar{R} is given by

$$- \frac{Af(N - \bar{R})}{N - \bar{R}} + w_I \frac{h(\bar{R}, A)}{\bar{R}} = 0$$

from this we get

$$\frac{\partial \bar{R}}{\partial A} = \frac{\frac{f(N - \bar{R})}{N - \bar{R}} - \frac{w_I}{\bar{R}} \cdot h_A(\bar{R}, A)}{\frac{w_I h_R(\bar{R}, A) \bar{R} - h(\bar{R}, A)}{\bar{R}^2} - \frac{A \{f(N - \bar{R}) - f'(N - \bar{R})(N - \bar{R})\}}{(N - \bar{R})^2}}$$

The denominator is clearly negative, since $f(N - \bar{R})/N - \bar{R} > f'(N - \bar{R})$ by the concavity of f , and $h_R < 0$. However, the sign of the numerator is ambiguous. As can be readily verified,

$$h_A(\bar{R}, A) = \frac{g'(N_I) \cdot g(N_I)}{A[-g''(N_I)g(N_I) - g'(N_I)^2]} > 0.$$

Substitution from the expression for \bar{R} shows that the numerator is $w_I / \bar{R}[(h(\bar{R}, A)/A) - h_A]$.

Remark 1. This expression is zero whenever $h(\bar{R}, A)$ is homogeneous of degree one in A . If we have the power function $Bg(N_I) = BN_I^\beta$, then it may be readily verified that this condition is satisfied.

Remark 2. Substitution from the expression for $h_A(\bar{R}, A)$ shows that the numerator can also be written as

$$\frac{1}{A} \left[N_I - \frac{g'(N_I)}{-\frac{g''(N_I)}{g'(N_I)} \cdot g(N_I) + g'(N_I)} \right]$$

Since by concavity of g we have $N_I < g(N_I)/g'(N_I)$, it is seen that the expression is positive for and g which is "sufficiently concave" and $(-g''(N_I)/g'(N_I))$ which is sufficiently high).

Clearly, $\frac{\partial \bar{R}}{\partial \lambda} = 0$; $\frac{\partial \bar{R}}{\partial B} = 0$.

Comparative statics on $\theta^I(R^*, \lambda, A)$.

$$\theta^I(R^*, \lambda, A) = R^* + \lambda \left[\frac{w_I h(R, A)}{R^*} - \frac{Af(N-R^*)}{N-R^*} \right]$$

$$\begin{aligned} \frac{\partial \theta^I}{\partial \lambda} &= \frac{\partial \theta^I}{\partial R^*} \frac{\partial R^*}{\partial \lambda} + \frac{w_I h(R^*, A)}{R^*} - \frac{Af(N-R^*)}{N-R^*} \\ &= 0 + \frac{w_I h(R^*, A)}{R^*} - \frac{Af(N-R^*)}{N-R^*} \end{aligned}$$

This is positive if R^* lies to the left of \bar{R} , and negative if R^* lies to the right of \bar{R} . Similarly, $\partial\theta^I/\partial A$ may be calculated. We have

$$\begin{aligned}\frac{\partial\theta^I}{\partial A} &= \lambda \left[w_I \frac{h_A(R^*, A)}{R^*} - \frac{f(N-R^*)}{N-R^*} \right] \\ &= \lambda \left[h_A - \frac{h(R^*, A)}{A} \right]\end{aligned}$$

We immediately note that Remarks 1 and 2 apply to this case as well. In particular, $\partial\theta^I/\partial A$ is zero if $h_A(R, A)$ is homogeneous in A (e.g., in the case of the power function); and $\partial\theta^I/\partial A$ is negative for g "sufficiently concave."

Comparative statics on R^* :

R^* is given by

$$\lambda \left[w_I \frac{h_R(R^*, A)R^* - h(R^*, A)}{R^{*2}} - A \frac{f(N-R^*) - f'(N-R^*) \cdot (N-R^*)}{(N-R^*)^2} \right] + 1 = 0$$

Taking the total differential of the above expression gives the following complicated expressions for $\partial R^*/\partial \lambda$ and $\partial R^*/\partial A$.

$$\frac{\partial R^*}{\partial \lambda} = \frac{A \frac{f(N-R^*) - f'(N-R^*) \cdot (N-R^*)}{(N-R^*)^2} - w_I \frac{h_R(R^*, A)R^* - h(R^*, A)}{R^{*2}}}{\lambda H}$$

where

$$\begin{aligned}H &= \left[-A \frac{\{f''(N-R^*)(N-R^*)^2 + 2(f(N-R^*) - f'(N-R^*)(N-R^*))\}}{(N-R^*)^3} \right. \\ &\quad \left. + w_I \frac{\{h_{RR}(R^*, A)R^{*2} - 2(h_R(R^*, A)R^* - h(R^*, A))\}}{R^{*3}} \right]\end{aligned}$$

Similarly,

$$\frac{\partial R^*}{\partial A} = \frac{1}{H} \left[\frac{f(N-R^*) - f'(N-R^*)(N-R^*)}{(N-R^*)^2} - w_I \frac{(h_{RA}(R^*,A) - h_A(R^*,A))}{R^{*2}} \right]$$

The sign of H is ambiguous. However, $h_R < 0$, and from equation (14), if we realize that $k = w_I/mA$ nw , it is clear that $g''' > 0$ is sufficient to ensure $h_{RA} < 0$. Then we can see that $\partial R^*/\partial \lambda$ and $\partial R^*/\partial A$ would have the same sign.

The upshot of all this is to highlight that the signs of the partial derivatives can tell us in which direction the parameters need to be manipulated for us to get chaos. Thus, for instance, let us suppose that $g(N_I)$ is sufficiently concave so that $\partial \bar{R}/\partial A < 0$ and $\partial \theta^I/\partial A < 0$, and $g''' > 0$. If the initial parameter values are such that $R^* < \bar{R}$, then, as we have seen, $\partial \theta^I/\partial \lambda > 0$. Thus, if $\partial R^*/\partial \lambda > 0$, then $\partial R^*/\partial A > 0$; so that lowering A will achieve the objective of shifting both $\theta^I(R^*, \lambda, A)$ up and R^* to the left. If $\partial R^*/\partial \lambda < 0$, then raising λ achieves both objectives. If $R^* > \bar{R}$, the $\partial \theta^I/\partial \lambda < 0$, but one can reduce A to shift θ^I up. If $\partial R^*/\partial A > 0$, the R^* is also reduced and eventually one will have $R^* < \bar{R}$.

NOTES

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1. See, for example, Vining, Pallone and Plune (1981), DaVanzo and Morrison (1981, 1982), Da Vanzo (1982) and the references therein.

2. See Elkan (1967), Fan (1982), Fan and Lee (1983), Goldstein (1978), Hugo (1975, 1978), Mitchell (1969), Stretton (1981) and VanderKamp (1972).

3. See Barnum and Sabot (1975), Bremen (1979), Connell et al. (1976) and especially Nelson (1976) and the numerous references to village studies therein.

4. See especially Todaro (1976a, 1976b) and World Bank (1983).

5. Several countries, such as Tanzania, Indonesia, the Philippines, South Africa, China and Cuba, have taken strong measures to prevent rural-urban migration and/or to promote migration reversals [Simmons (1981)].

6. Most of this controversy has centered on the precise conditions under which the "Todaro paradox," wherein an increase in the rate of new urban job generation would increase the level or the rate of urban unemployment, would be fulfilled. See especially Blomqvist (1978) and Arellano (1981). For the most part, these conditions depend on the specification of the migration function.

7. See especially Barnum and Sabot (1975), Todaro (1976a, 1976b) and Cole and Sanders (1983).

8. See, e.g., Bhagwati and Srinivasan (1974), Bhatia (1979), Corden and Findlay (1975), Das (1982), Jha and Lachler (1981) and Robertson and Wellisz (1977).

9. Some exceptions are the studies of Neary (1981), Amano (1983) and Bartlett (1983) who have shown that stability of equilibrium is not guaranteed. In all these cases, however, the simple Harris-Todaro framework is modified substantially and other elements appended. In the case of Neary, it is by bringing in the mobility of capital; in Bartlett it is by bringing in population and capital growth and by using the more complicated version of the model (i.e., Todaro (1969)); in the case of Amano it is by introducing adjustment costs (with respect to employment by urban firms and of wage rates to unemployment rates and other determinants), and by opening up the product markets to trade with the rest of the world, thereby pegging the prices of sectoral outputs.

10. Alternatively, we could interpret N_A as bundles of mobile resources in general and R as the intersectoral flow of these resources. One should bear in mind that migrants take with them their human capital and frequently also significant amounts of real capital.

11. It is interesting to consider alternative forms for the basic adjustment equation (10) such as the proportional form $(R_{t+1} - R_t)/R_t = \lambda[w_{It} - w_{At}]$. We have done this and virtually all of the analysis applies.

We therefore doubt that our results depend significantly on the specific adjustment equation used.

12. A sufficient condition for this possibility derived by Li et al. (1982) is the existence of a point, say R^C such that

$$\theta^n(R^C) \leq R^C < \theta(R^C)$$

where n assumes the value of an odd number, where $\theta(\cdot)$ is represented by equation (13) and where $\theta^n(R^C)$ is the n^{th} iterate of the equation beginning with the initial condition R^C . For an exegesis of this condition for $n = 3$ see Day (1982) and (1983). These papers contain various references to the related literature.

References

- Amano, Masanori (1983) "On the Harris-Todaro Model with Intersectoral Migration of Labour," Economica 50 (August): 311-325.
- Arellano, J.-P. (1981) "Do More Jobs in the Modern Sector Increase Urban Unemployment?" Journal of Development Economics 8: 241-247.
- Barnum, Howard N. and R.H. Sabot (1975) Migration, Education and Urban Surplus Labour, Paris: OECD Development Center.
- Bartlett, William (1983) "On the Dynamic Instability of Induced-Migration Unemployment in a Dual Economy," Journal of Development Economics 13 (August-October): 85-96.
- Benhabib, Jess and Kazuo Nishimura (forthcoming) "Competitive Equilibrium Cycles," Journal of Economic Theory.
- Bhagwati, J.N. and T.N. Srinivasan (1974) "On Reanalyzing the Harris-Todaro Model: Policy Rankings in the Case of Sector-Specific Sticky Wages," American Economic Review (June).
- Bhatia, Kul B. (1979) "Rural-Urban Migration and Surplus Labour," Oxford Economic Papers 31, No. 3 (November).
- Blomqvist, A.G. (1978) "Urban Job Creation and Unemployment in LDCs: Todaro vs. Harris and Todaro," Journal of Development Economics 5, No. 1 (March): 3-18.
- Breman, Jan C. (1979) "Seasonal Migration and Cooperative Capitalism: The Crushing of Cane and Labor by the Sugar Factories of Bardoli, South Gujarat," Paper presented to the Conference on Adjustment Mechanisms in Rural Labor Markets in Developing Areas ICRISAT, Hyderabad, India, August 22-24.
- Cole, William E. and Richard D. Sanders (1983) "Interstate Migration in Mexico: Variations on the Todaro Theme," Journal of Development Economics 12, No. 3 (June): 341-355.
- Connell, J., Biplap Dasgupta, R. Laishley and Michael Lipton (1976) Migration from Rural Areas: The Evidence from Village Studies, Delhi: Oxford University Press.
- Corden, W.M. and R. Findlay (1975) "Urban Unemployment, Intersectoral Capital Mobility and Development Policy," Economica (February): 59-78.
- Das, Satya P. (1982) "Sector-specific Minimum Wages, Economic Growth and Some Policy Implications," Journal of Development Economics 10 (February): 127-131.
- Day, R.H. (1982) "Irregular Growth Cycles," Amer. Econ. Rev. 72 (June): 406-414.
- _____ (1983) "The Emergence of Chaos from Classical Economic Growth," Quart. Jour. of Econ.
- Elkan, W. (1967) "Circular Migration and the Growth of Towns in East Africa," International Labor Review 96: 581-589.
- Fan, Yiu-Kwan (1982) "Risk Aversion and Migration Behavior: A Theoretical Note on Circular Migration in South East Asia," Paper presented to the Midwest Economics Association, Chicago, March-April.
- Fan, Yiu-Kwan and Bun Song Lee (1983) "Risk Behavior and Sex-Differential Patterns of Migration among Rural-Born Koreans," Paper presented to the 35th Annual Meeting of the Association of Asian Studies, San Francisco.
- Goldstein, S. (1978) Circulation in the Context of Total Mobility in Southeast Asia, Honolulu: East-West Center, East-West Population Institute No. 53.

- Harris, John R. and M.P. Todaro (1970) "Migration, Unemployment and Development: A Two Sector Analysis," American Economic Review 60: 126-142.
- Hugo, G.J. (1975) "Population Mobility in West Java, Indonesia," Canberra: Ph.D. Dissertation, Department of Demography, Australian National University.
- _____ (1978) "New Conceptual Approaches to Migration in the Context of Urbanization: A Discussion Based on Indonesian Experience," Paper presented to the Seminar on New Conceptual Approaches to Migration in the Context of Urbanization, Bellgio, Italy.
- Jha, R. and U. Lächler (1981) "Optimum Taxation and Public Production in a Dynamic Harris-Todaro World," Journal of Development Economics 9, No. 3: 357-374.
- Lewis, W.A. (1954) "Economic Development with Unlimited Supplies of Labour," Manchester School of Economic and Social Studies 22 (May): 139-191.
- Li, T.Y., M. Misiurewicz, G. Pranigiani, and J. Yorke (1982), "Odd Chaos," Physics Letters 87A (January 18): 271-273.
- Mitchell, J. Clyde (1969) "Structural Plurality, Urbanization and Circulation in Southern Rhodesia," in J.A. Jackson, ed., Migration, Cambridge: Cambridge University Press.
- Neary, J.P. (1981) "On the Harris-Todaro Model with Intersectoral Capital Mobility," Economica 48: 219-234.
- Nelson, Joan M. (1976) "Sojourners versus New Urbanites: Causes and Consequences of Temporary versus Permanent Cityward Migration in Developing Countries," Economic Development and Cultural Change 24: 721-757.
- Robertson, P. and S. Wellisz (1979) "Steady-State Growth of an Economy with Intersectoral Migration," Oxford Economic Papers 31: 370-388.
- Simmons, Alan B. (1981) "A Review and Evaluation of Attempts to Constrain Migration to Selected Urban Centres and Regions," Population Distribution Policies in Development Planning, New York: United Nations, Department of Economic and Social Affairs, Population Studies No. 75.
- Stretton, A.W. (1981) "The Building Industry and Urbanization in Third World Countries: A Philippines Case Study," Economic Development and Cultural Change 29: 325-339.
- Todaro, Michael P. (1969) "A Model of Labor Migration and Urban Unemployment in Less Developed Countries," American Economic Review 59: 138-148.
- _____ (1976a) Migration in Developing Countries: A Review of Theory Evidence, Methodology and Research Priorities, Geneva: International Labour Organization.
- _____ (1976b) "Urban Job Expansion, Induced Migration and Rising Unemployment: A Formulation and Simplified Empirical Test for LDCs," Journal of Development Economics 3: 211-225.
- Vanderkamp, J. (1972) "Return Migration: Its Significance and Behavior," Western Economic Journal 10 (December): 460-465.
- Vining, D., R. Pallone and D. Plune (1981) "Recent Migration Patterns in the Developed World: A Clarification of Some Differences between Our and IIASA's Findings," Environment and Planning 13: 243-250.
- World Bank (1983) Growth and Employment in Rural Thailand, Washington, D.C., World Bank, Country Programs Department, East Asian and Pacific Regional Office, Report No. 3906-TH.

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