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An Intertemporal Applied General Equilibrium Model Based On ORANI

by

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The Impact Project is a cooperative venture between the Australian Federal Government and Monash University, La Trobe University, and the Australian National University. By researching the structure of the Australian economy the Project is building a policy information system to assist others to carry out independent analysis. The Project is convened by the Industry Commission on behalf of the participating Commonwealth agencies (the Industry Commission, the Australian Bureau of Agricultural and Resource Economics, the Bureau of Industry Economics, the Department of Employment, Education and Training, the Department of Immigration, Local Government and Ethnic Affairs, and the Department of the Arts, Sport, the Environment, Tourism and Territories). The views expressed herein do not necessarily represent those of any government agency or government.

ABSTRACT

The ORANI model of the Australian economy [Dixon, Parmenter, Sutton and Vincent (1982)] has been used extensively to analyse issues involving the composition of the economy. ORANI is essentially comparative static in nature; it does not project the trajectory of the economy between the initial point at which a shock occurs and the new equilibrium configuration. Even in the forecasting version of ORANI, ORANI-F [Parmenter (1988)], results show only average annual growth rates over the length of run simulated.

The usefulness of intertemporal applied general equilibrium models is well documented (see for example Dixon (1990), Dixon and Parmenter (1990) and Dixon, Parmenter, Powell and Wilcoxen (forthcoming)). The purpose of this paper is to describe a computational approach to an intertemporal elaboration of ORANI. This implementation is motivated by the work of Dixon and Parmenter (1990). The prototype model described here offers the prospect of comparative dynamic simulations with a detailed applied general equilibrium model.

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*AN INTERTEMPORAL APPLIED GENERAL EQUILIBRIUM MODEL BASED ON ORANI : INITIAL CALIBRATION**

by

Micheal MALAKELLIS

1. *Introduction*

The attraction of Computable General Equilibrium (CGE) analysis is its potential to quantify the implications of sectoral interdependencies which are often crucial for understanding the effects of economic shocks on particular sectors and on the economy as a whole (see Parmenter (1982)). In analysing sectoral interdependence CGE models emphasise both direct and indirect sectoral linkages. Direct interactions may occur between users and producers via the demand and supply of goods and services. Indirect interactions arise mainly via economy-wide constraints such as limitations on the availability of factors of production. In either case, relative prices exert a crucial influence on the responses of economic agents. Most CGE models, however, deal with a single period only.

Intertemporal economic models recognize that the behaviour of economic agents may be influenced by trade-offs between the present and the future. An intertemporal model provides the scope for enforcing inter-temporal resource constraints on all agents (including governments). In addition, intertemporal models may allow the consequences of expected future events to affect current decision making by economic agents. Rather few intertemporal models incorporate significant multisectoral detail.

Due to their focus on sectoral linkages, CGE models are well suited to dealing with issues concerning the composition of the economy at a particular point in time. On the other hand, intertemporal models have great potential for analysing economic problems which involve trade-offs across time. The incorporation of intertemporal behaviour by economic agents into the CGE framework combines the strengths of the two modelling approaches. An intertemporal CGE model which incorporates intertemporal optimization for some agents and which imposes economy-wide constraints that operate at all points in time is therefore amenable to addressing a wide range of policy issues.

This paper presents an intertemporal CGE model based on ORANI, which is a large-scale single-period CGE model that has been used extensively

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* Without implicating them in any remaining errors the author would like to thank J. Mark Horridge and Alan A. Powell for their helpful comments and assistance. This paper has also benefitted from a review by Geoffrey Kingston of an earlier draft.

to analyse issues concerning the composition of the Australian economy (Dixon, Parmenter, Sutton and Vincent (1982))[1]. As explained in section 2.1, ORANI is a strictly neo-classical multisectoral model in the tradition of Johansen (1960).

Normally an ORANI simulation compares two possible states of the economy at a particular point in time. The effect of an exogenous shock is assessed as the difference between the values of the endogenous variables in the unshocked (control) equilibrium and the value of these variables in the shocked (new) equilibrium, all other exogenous variables remaining at their control values. As with all comparative-static models, ORANI's structural equations are agnostic about the amount of calendar time that it takes to move from one equilibrium to another following some perturbation[2]. Rather, timing is inferred from the degree of adjustment that factors of production are allowed in reaching a new 'equilibrium'. For short-run applications, the point of interest is in the near future where it is assumed that sector-specific capital stocks currently in use cannot respond to exogenous shocks. Long-run applications focus on a point in time in the future which is distant enough for market forces to produce new levels of sectoral capital stocks which are compatible with an exogenous configuration of rates-of-return.

In both short and long-run closures of ORANI only a single point of time is captured by the model and no information about how the economy evolves over time in the absence of an exogenous shock is provided. This lack led to the development of ORANI-F, a forecasting version of ORANI described in Parmenter (1988). ORANI-F includes accumulation relations between investment and capital stocks and between borrowing and foreign debt. This specification of minimal dynamics allows the model to be used in forecasting mode. Applications compare the forecast state of the economy at some future point in time (typically, 5-6 years ahead) with its state in the base year. Results are presented in the form of forecasts of the average annual growth rates of the endogenous variables over the specified period of time.

Although ORANI-F is a movement towards a full intertemporal model, it is, like its predecessor, a single-period model. In ORANI the period is typically a calendar year, while in ORANI-F the period is the interval of time between the base year and the forecast year.[3]

ORANI and ORANI-F have proved very useful to policy-makers in the analysis of issues involving the composition of the economy. As with any single

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1 Hereafter referred to as DPSV (1982).

2 With the aid of a dynamic model with which ORANI has been interfaced (Cooper and McLaren (1983), Cooper, McLaren and Powell (1985) and more recently, Breece *et al.* (1991)), it has proved possible to infer that the length of the ORANI short-run corresponds to about two years in calendar time.

3 Note that the period referred to here is the interval of time to which simulation results refer; it should not be confused with the length of run (i.e. the time that elapses between the injection of a shock and the consequent new equilibrium).

period model however, they are not so well suited for coping with issues dealing with allocation over time. The model presented in this paper is motivated by Dixon and Parmenter's (1990) description of a stylized intertemporal CGE model which is designed to preserve the detail incorporated in existing versions of ORANI whilst at the same time equipping the model to deal with temporal issues.

The remainder of this paper is organised as follows: Section 2 outlines the theoretical structure of an intertemporal CGE model which consists of a sequence of identical single-period CGE models linked over time by forward-looking investment and consumption behaviour; Section 3 outlines the solution procedure used for the intertemporal CGE model; Section 4 describes the construction of the database; Section 5 gives the results of an illustrative calibration experiment. Finally, Section 6 presents some concluding remarks.

2. An Intertemporal CGE Model

The intertemporal CGE model consists of a sequence of identical single-period CGE sub-models, each of which represents the economy at a different 'point' in time.[4] The single-period CGE sub-models are linked via the specification of intertemporal optimizing behaviour on the part of investors and households. Implicit in this intertemporal CGE model is the assumption that investors and consumers have model-consistent expectations. In the absence of any unanticipated exogenous shocks, agents have perfect knowledge about the values of all variables in the model.

The single-period CGE sub-models account for the bulk of the equations in the total model. The equations within each of the single-period CGE sub-models may be categorized as atemporal since all variables carry the same time subscript. The remainder of the equations express relationships among variables at different points in time. These are the intertemporal equations which link the single-period CGE sub-models over time.

2.1 The Atemporal Equations

Each of the sequence of identical single-period CGE sub-models which forms the atemporal core of the total model strongly resembles standard ORANI (as described by DPSV (1982) and extended by Codsi, Horridge and Pearson (1988)).

In the following sub-sections, the salient features of ORANI are briefly described. The theoretical differences between standard ORANI and the version of the model actually used in developing the intertemporal CGE model

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4 Below it will sometimes be crucial to distinguish between continuous-time and discrete-time concepts. Strictly, the sub-models relate to discrete intervals (perhaps one or two years apart); when the expressions time point, or point in time, are used to indicate such an interval, the word 'point' is enclosed within single inverted commas; when it refers literally to an instant in time, the inverted commas are suppressed.

are then discussed. Finally, the structure of the single-period CGE sub-models is summarized.

2.1.1 Summary of Standard ORANI

Standard ORANI is strictly neo-classical in that optimizing behaviour on the part of producers (i.e., cost minimisation) and consumers (i.e., utility maximisation) is assumed in the context of competitive markets. ORANI consists of six main equation blocks:

I. a series of equations describing household and other final demands for commodities ;
II. a series of equations describing industry demands for primary factors and intermediate inputs ;
III. a series of equations describing the demand for goods and services needed to facilitate the transfer of commodities from the producers to the users;
IV. a series of pricing equations which set pure profits from all activities to zero ;
V. a series of market clearing equations for primary factors and commodities ; and
VI. a series of definitional equations.

These equations are briefly described in the following sub-sections.

2.1.1(I) Final Demands

ORANI distinguishes 228 commodities of which 114 are domestically produced and 114 are imported. Four sources of final demand are identified, namely, household consumption, capital creation, 'other' demands and exports.

2.1.1(I(a)) Household Consumption

It is assumed that the household sector (represented by a single "typical" household) allocates its aggregate budget across composite commodities according to the Linear Expenditure System (which is consistent with a Stone-Geary or Klein-Rubin specification of the single-period utility function)[5]. The composite commodities are defined by a CES aggregate in which households may substitute between domestic and imported sources in response to changes in the relative prices of domestic and foreign goods of the same input-output category.

2.1.1(I(b)) Capital Creation

In ORANI the composition of fixed capital varies across industries so that capital is sector-specific. A unit of fixed capital for use in industry j can be constructed according to a two-tiered technology. At the top level industry j

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5 As will be explained in section 2.2.4, the intertemporal equations extend the Linear Expenditure System to accord with intertemporal optimizing behaviour by households. However, the ORANI equation which governs household demands for commodities undifferentiated by source (i.e. equation (14.24) is DPSV (1982)), is respecified to facilitate computation. This issue is addressed in Appendix A.

chooses effective inputs of produced (i.e., non-primary) goods to minimize costs subject to a Leontief production function (i.e., inputs of effective units of goods are used in fixed proportions). At the next level, effective inputs of each good i are assembled from domestically produced and imported good i according to a CES (Armington) technology so as to minimize unit costs.

In typical short-run closures, ORANI makes no attempt to explain aggregate investment — rather, it focuses on how investment is allocated across industries. Aggregate investment is set exogenously taking into account macroeconomic phenomena (e.g., monetary and fiscal policy) which are not explicitly modelled. ORANI then determines how this investment is to be allocated across industries essentially by requiring that expected future (one-period-ahead) rates-of-return[6] are equated across industries. For those industries in which the rate-of-return theory is considered inappropriate (e.g., government dominated industries), investment is handled by indexation to some suitable aggregate.

In long-run closures of ORANI sectoral, and hence aggregate investment, is governed by an exogenous terminal condition under which the growth rates of capital stocks in the terminal year are set by the user. The terminal (post-shock) *levels* of sectoral capital stocks however, are determined by the requirement that exogenously set sectoral rates-of-return are achieved.

2.1.1(I(c)) 'Other' Demands

This category of final demand consists mainly of government demands for both imported and domestically produced goods and services. There is no formal theory explaining 'other' demands in ORANI. Instead, 'other' demands are modelled in a flexible manner where the user is free to introduce alternative scenarios. The default setting in ORANI is for 'other' demands to move in line with real household expenditure.

2.1.1(I(d)) Export Demands

Demands for Australian exports are assumed to be functions of their foreign currency prices. For most commodities the small country assumption is adopted (i.e., the foreign elasticity of demand is very high). For commodities of which Australia is a major seller on world markets some influence on foreign currency prices is assumed (i.e., foreign elasticities of demand are lower)[7].

2.1.1(II) Industry Demands

In ORANI producers are competitive in the sense that they are price takers in all input and output markets. They are also assumed to be efficient in

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6 Expected rates-of-return in each industry are assumed to be negatively related to the corresponding investment levels. Note, however, that nothing in ORANI ensures or verifies that these expectations are fulfilled.

7 Typical absolute values for the demand elasticities of exports where the small country assumption is adopted are 20, whilst for exports of which Australia exerts some influence on world prices, the values range upward from about 1.2 for wool.

that input mixes are chosen to minimize costs and output bundles to maximize revenue.

2.1.1(II(a)) Demand for Inputs

Producers in ORANI choose inputs for current production by minimising costs subject to a three-tiered constant-returns-to-scale input technology. At the top level the production function is assumed to be Leontief. Effective intermediate input i (i.e., produced good i) is used in fixed proportion with effective intermediate input j, effective primary input k, and 'other cost' tickets. The last-mentioned input allows for production taxes and other miscellaneous production costs.

At the next level, effective intermediate input i is defined by a cost-minimising CES aggregate of imported and domestic good i. Effective primary inputs are defined by a cost-minimising CRESH[8] aggregate of agricultural land, fixed capital and effective labour inputs. In common closures of ORANI, agricultural land and fixed capital are sector-specific whilst labour of any given occupation is mobile between sectors.

The third level of the input technology only applies to labour. Effective labour units are defined according to a CRESH aggregate which allows substitution possibilities between the 10 different skill groups identified in ORANI.

2.1.1(II(b)) Supply of Outputs

On the output side, ORANI uses multiproduct production functions which means that all industries may in principle produce all types of domestic commodities. The problem for producers is to choose an output mix which maximizes revenue subject to a CRETH[9] production transformation frontier. In practice, data limitations confine all non-agricultural industries in ORANI to be treated as single product industries.

2.1.1(III) Demand for Margins

The direct demand for commodities as intermediate inputs and for final usage generates indirect demands for margins (i.e., goods and services used to facilitate the transfer of commodities from producers to users). Whilst the theory allows all commodities to be used as margin services, the input-output database shows only 9 types of margins (including commodity taxes). The demand for a margin is assumed to be proportional to the direct demand for the commodity to which it applies.

2.1.1(IV) Zero Pure Profits Conditions

ORANI defines four sets of commodity prices; namely, basic values (i.e., the price received by producers before sales taxes and margin costs),

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8 CRESH (constant ratios of elasticities of substitution, homothetic) (see Hanoch (1971)).

9 CRETH (constant ratios of elasticities of transformation, homothetic) (see Dixon, Vincent and Powell (1980)).

purchasers' prices (which includes these items), f.o.b foreign currency export prices and c.i.f foreign currency import prices.

The relationship between the different sets of prices reflects the following three assumptions: (1) there are no pure profits in production, capital creation, importing, exporting or distribution (i.e., prices are set to just cover costs including a (possibly imputed) return on capital and land in all activities); (2) basic values are uniform across users and producing industries. Differences across users in purchasers' prices are accounted for entirely by taxes and payments for margins; and (3) there are no margins on margins.

2.1.1(V) Market Clearing Conditions

The market clearing equations ensure that demand equals supply for domestically produced commodities and for the industry-specific primary factors capital and agricultural land. With respect to labour, in ORANI's standard short-run closure it is assumed that the demand for labour of each skill category across all industries is satisfied. This means that employment is demand-determined and that there is no presumption of full employment.

2.1.1(VI) Miscellaneous Equations

The final category of equations consists mainly of accounting relationships which serve to define several key aggregates (e.g., aggregate imports, exports and the balance of trade) as well as several key macro-economic indices (e.g., the consumer and capital-goods price indexes, aggregate employment and aggregate capital stocks).

Equations indexing 'other cost' tickets and money wages to the consumer price index are also included in this category. The degree of indexation is user specified.

2.1.2 Changes made to the Theoretical Structure of Standard ORANI

Several changes are made to the theoretical structure of standard ORANI in order to simplify computation and to facilitate the imposition of intertemporal optimization by investors.

2.1.2(a) Changes made to Simplify Computation

The standard version of ORANI is very large and quite demanding on computing resources. This size factor is exacerbated when ORANI must be replicated over time. For development purposes it is desirable to use a smaller, slightly simpler version of ORANI which is still capable of producing realistic empirical results.

Accordingly, as is explained in more detail in section 5.2, standard ORANI is aggregated to 13 single-product industries.[10] In addition, only one type of labour is recognized and margins are treated as if they were direct demands.

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10 See Table 5.2(a) for the industry classifications.

2.1.2(b) *Treatment of Investment*

In the intertemporal CGE model investment decisions are to be made by intertemporal optimizing agents. The equations which govern the distribution of investment in standard ORANI are not compatible with this approach. Consequently, these equations (specifically, equations (19.7), (19.8) and (19.9) in DPSV (1982)) are removed so that investment in each industry becomes exogenous to each of the sequence of single-period CGE sub-models.

2.1.2(c) *New Atemporal Equations*

Two new equations are added to each member of the sequence of single-period CGE sub-models. The first is an arbitrage condition which reflects the assumption of competitive capital markets. The second new equation defines the market rate of interest at which domestic households can borrow money. These equations are described in more detail in sections 2.2.3 and 2.2.4 respectively.

2.1.3 *Summary of the Structure of the CGE Sub-models*

The modified version of ORANI described above is the single-period CGE sub-model which is to be replicated over time. Replication of this sub-model is achieved by indexing all the equations and variables in the model with respect to time. Since each CGE sub-model pertains to a single period, the time index on all variables and equations within each of these sub-models is the same. Note that the distribution of investment between industries and over time is exogenous to the CGE sub-model as is the distribution of aggregate household expenditure over time. The role of the intertemporal equations is to link the single-period CGE sub-models by governing these investment and consumption flows over time.

2.2 *Intertemporal Equations*

The intertemporal equations relate variables at different points in time; they provide the links between periods in the intertemporal CGE model. Three major links between the periods are established by the intertemporal equations.

The first link is purely a physical accounting one in which capital stocks and investment flows are related by an accumulation relation. The second and third links are established via the specification of forward-looking investment and consumer behaviour respectively.

Before discussing these links in detail, a brief digression into the nomenclature adopted for the intertemporal CGE model is necessary.

2.2.1 *Nomenclature for Intertemporal Modelling*

The nomenclature for intertemporal modelling adopted in this paper is set out in Figure 1. The model interrelates the values of a set of variables X at successive time points $t = 0, 1, 2, ..., T$. The time points correspond to particular calendar dates and are separated by intervals of time (measured in years) L_t, where L_t is the number of years between time points t and $t + 1$.

Figure 1: *Notation for Intertemporal Modelling*

Time point	t =	0	1	2	3	4	T
Interval length[a]	L_t =	← L_0=2 →	← L_1=2 →	← L_2=2 →	← L_3=5 →	← $L_t = S_t - S_{t-1}$ →	
Relative time	S_t =	0	2	4	6	11	$S=\sum_{t=0}^{T} L_t$
Levels variable value	X =	X_0	X_1	X_2	X_3	X_4	X_T
Change variable value	x =	0	x_1	x_2	x_3	x_4	x_T
Calendar date	CD =	1991 (now)	1993	1995	1997	2002	now+S
Data year	DY=	1990-91	1992-93	1994-95	1996-97	2001-02	(CD-1)-CD

[a] The values shown here for L_t are chosen purely to illustrate the idea that L_t is a varying function of t.

Variables with subscript t=0 refer to the most recent date for which historical data is available. All such variables are therefore predetermined.

In this model, agents can only make and implement decisions at the chosen set of time points. The information set upon which agents make decisions is assumed to be revised instantaneously at each time point. Thus, new information has no effect on behaviour in the preceeding interval.

The intervals between time points need not be equal to one year. Since the size of the model is a function of the number of time points identified, this approach has important efficiency implications for the computation of the model. The number of years over which the model is solved may be quite large (say 50 years or more) whilst the number of time points identified may be significantly less.

Scope for further savings in computation is provided by allowing for the possibility that the L_t need not all be equal. Strategic location of the time points will ensure that the loss of descriptive economic detail will be minimised. For instance, time points may be densely concentrated where the dynamic variables of the model are expected to change significantly (or where there is a particular interest from a policy viewpoint). Conversely, where the dynamic variables are expected to change more slowly (or where policy interest is less pronounced — perhaps far into the future) the time points may be more sparsely concentrated.

Each variable X can be mapped to a time point or to an interval. A stock variable X_t is the level of the relevant stock at the instant t (e.g., midnight on June 30th 1991). In the case of a flow variable, X_t is the average annual flow rate during the interval [t, t+1), a period of L_t years.[11]

2.2.2 *Accumulation Relation between Investment and Capital Stocks*

Accumulation relationships between capital stocks and investment are necessary in multi-period models. In this implementation the following accumulation relation is used

$$K_{j,t+1} = K_{j,t}(1 - \delta_j)^{L_t} + L_t I_{j,t} \quad . \qquad t=0, \ldots T. \qquad (2.2.1)$$

This formulation assumes that sector j's capital stock at time point t+1 ($K_{j,t+1}$) is equal to the depreciated (where δ is the rate of depreciation) level of the capital stock inherited from the previous period plus any investment ($L_t I_{j,t}$) undertaken between time points t and t+1. Implicit in equation (2.2.1) is the assumption of a one-period gestation lag.[12]

---

11 In practical situations of stock-flow accounting it is necessary to ensure that the measurement of average flow rates X_t is consistent with assumptions made about growth rates within the intervals L_t.

12 In this context, the term gestation lag refers to the interval between the point in time when the decision to invest is made and the point in time when a productive unit of capital comes on stream. In this implementation of the model, it is

2.2.3 *Forward-looking Investment Behaviour*

It is assumed that current investment decisions take into account both future opportunities and the rate of depreciation of existing capital stocks. The capital accumulation relation presented above implies that at least some investment is required in each period in order to maintain the capital stock at its initial level.

In the multi-period context producers are efficient in the sense that for any given time path of output they minimize the present value of their expected future cost stream. In order to calculate this present value an appropriate discount factor is required.

The assumption of competitive markets means that producers will only choose to invest in fixed capital if the rate of return on that investment is at least as high (after allowing for any risk differential) as the rate of return on any alternative investment option (including bonds). In equilibrium, the rates of return on any assets (net of any risk factors) will be equal.

This assumption is captured by the following arbitrage condition which holds at all time points and relates sectoral rates of return RR, to the safe domestic bond rate B^d, and a sector specific risk factor:

$$RR_{j,t} = B_t^D + Risk_{j,t} \ . \qquad (2.2.2)$$

Assuming uncovered interest parity,

$$B_t^D = [1 + B_t^F] \frac{\Phi_{t+1}}{\Phi_t} - 1 \qquad (2.2.3)$$

where B^F is the exogenously given foreign safe bond rate and Φ is the exchange rate ($Australian/$foreign).

Producers are competitive in that they take the time paths of all prices as given. Thus the typical producer in the j^{th} sector takes the time paths of the sectoral rate of return, the sectoral price of capital Π, and the prices of all other factors W, as given. The producer then chooses the paths of factor inputs (e.g. labour and capital) and gross investment I, to minimise the present value of the expected future stream of costs PVC, given by:

$$PVC_j = \sum_{t=1}^{T} \left\{ \frac{L_t}{\prod_{u=1}^{t} (1 + RR_{j,u})^{L_u}} \right\} [W_{j,t} \bullet F_{j,t} + \Pi_{j,t} I_{j,t}] \qquad (2.2.4)$$

---

assumed for simplicity that the length of the gestation lag for all sectors is one period. Note that this assumption implies that the model results are affected by the choice of values for L_t. Consistent behaviour over time then requires that L_t is in fact uniform over t. For L_t to vary, (the purpose of which is outlined in section 2.2.1), the theory would have to be generalised to allow for gestation lags of more or less than one period.

subject to the following constraints:

$$K_{j,t+1} = K_{j,t}(1 - \delta_j)^{L_t} + L_t I_{j,t} \qquad t=0, \ldots T, \qquad (2.2.5)$$

$$\overline{Q}_{j,t} = f(K_{j,t}, F_{j,t}) \qquad t=1, \ldots T, \qquad (2.2.6)$$

$$K_{j,0} = \overline{K}_{j,0} \quad , \qquad (2.2.7)$$

$$I_{j,0} = \overline{I}_{j,0} \quad , \qquad (2.2.8)$$

and

$$K_{j,T+1} = (1+G_{j,T})^{L_T} K_{j,T} \quad . \qquad (2.2.9)$$

Here, F $\Leftrightarrow$ vector of factor inputs other than capital

Q $\Leftrightarrow$ the level of output

f $\Leftrightarrow$ production function,

G_j $\Leftrightarrow$ growth rate towards which sector j is assumed to tend for timepoints beyond the explicit planning horizon (i.e. $t > T$) (provided that T is chosen sufficiently large),

and all other variables are as previously defined. The • operator takes the inner product of two vectors having the same number of components and superscript bars indicate that variables are exogenous.

Equation (2.2.5) is the capital accumulation constraint which has been explained in section (2.2.2) above. The constraint represented by equation (2.2.6) specifies the production technology available to producers and the path of output which must be produced. Equation (2.2.7) and (2.2.8) are initial conditions relating to capital stocks. and investment respectively. Finally, terminal capital stocks are given by equation (2.2.9). The terminal condition (2.2.9) is chosen because it provides maximum flexibility for specifying the growth path to which the economy is assumed to tend beyond the terminal time point.[13]

The first-order conditions derived from this problem may be manipulated to give the following equilibrium conditions for sectoral capital demands:[14]

$$\Pi_{j,t} \left[1 + RR_{j,t+1}\right]^{L_{t+1}} = MRPK_{j,t+1} L_{t+1} + \Pi_{j,t+1} (1 - \delta_j)^{L_{t+1}} \qquad t=1, \ldots T \quad (2.2.10)$$

Here all variables are as previously defined and MRPK is the marginal revenue product of capital.

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13 Note that when $G_{j,T} = 0 \ \forall_j$, the economy is assumed to reach a steady state.

14 See Appendix B for the derivation.

Equilibrium condition (2.2.10) implies that producers will desire to hold that level of capital stocks which equates the marginal revenue product of capital with the *net* cost of creating and using that capital. The rate of investment can be deduced from equation (2.2.5) as the difference between the level of capital stock desired in period t+1 and the level of capital stock inherited from the previous period. It is assumed that a unit of capital takes one period to be brought on stream (i.e. one period gestation lag). Thus, in order to obtain a productive unit of capital in the next period producers must undertake an appropriate investment program in the current period. Condition (2.2.9) is forward-looking in the sense that when producers make current investment decisions they relate the current opportunity cost of a unit of capital (i.e. LHS of equation (2.2.9)) to the sum of the rent that the unit of capital will earn over the next period when it comes on stream and the resale value of the unit of capital (net of depreciation) at the end of the next period.[15]

2.2.4 *Distribution of Consumption through Time*[16]

It is assumed that the aggregate utility function of the representative (denoted hereafter by ^) household $\hat{U}$, is additively separable over time so that

$$\hat{U} = \sum_{t=1}^{T} \left[\frac{\hat{U}_t L_t}{\prod_{u=1}^{t} [1 + \hat{RHO}]^{L_u}} \right] \qquad (2.2.11)$$

where $\hat{RHO}$ is the pure time preference rate and $\hat{U}_t$ is the sub-utility function that holds in period t. Note that $\hat{U}_t$ has dimensions utils per unit time period as seen during that time period, while $\hat{U}$ is measured in utils as seen at t=0.

The representative household's sub-utility functions have the Klein-Rubin form

---

15 In this implementation of the model the length of the interval between adjacent time points simultaneously determines the gestation lag in investment as well as the period of time over which agents cannot revise their behaviour. The choice of L_t is therefore an important part of the model specification. In particular, it would be invalid to choose very small interval lengths as the model would then approximate a continuous time specification. As is well known, (see McLaren (1990), Brechling (1975), Nickell (1978)), the continuous time analogue of the investment problem presented above does not yield multi-period decision rules. In continuous time, capital goods can be brought and sold at each instant in time so that in the absence of adjustment costs, the values of exogenous variables in future periods are of no relevance to current investment decisions. Note that in the model presented here, current investment decisions are influenced by the values of exogenous variables (i.e. the price and rental value of a unit of capital) in the next period.

16 The forward-looking consumption model developed in this section is a discrete-time elaboration of Lluch's (1973) Extended Linear Expenditure System (ELES).

$$\hat{U}_t = \sum_{i=1}^{n} \beta_i \ln [\hat{H}_{i,t} - \hat{\gamma}_i]$$

Here the β_i are marginal budget shares (which sum to one) and the γ_i are parameters denoting the subsistence quantity of good i consumed. $H_{i,t}$ is the total quantity of good i consumed over period t.

As suggested in section 2.1.1(I(a)), the Linear Expenditure System is consistent with the Klein-Rubin form of the one-period sub-utility function. An implication of the LES is that the above subsistence (i.e., supernumerary) expenditure on good i is a constant share of total supernumerary expenditure. That is,

$$P_{i,t} \; [\hat{H}_{i,t} - \hat{\gamma}_i] = \beta_i \sum_{k=1}^{n} P_{k,t} \; [\hat{H}_{k,t} - \hat{\gamma}_k] \quad . \qquad (2.2.12)$$

Equation (2.2.12) may be expressed more compactly as

$$\hat{D}_{i,t} = \beta_i \hat{E}_t \,/\, P_{i,t} \quad , \qquad (2.2.13)$$

where $\hat{D}_{i,t} = [\hat{H}_{i,t} - \hat{\gamma}_i]$ and $\hat{E}_t = \sum_{i=1}^{n} \hat{D}_{i,t} P_{i,t}$. (2.2.14)

i.e., the variable $\hat{D}_{i,t}$ denotes supernumerary expenditure on good i at time point t; $\hat{E}_t$ is the total supernumerary expenditure at time point t.

The representative household's indirect utility function holding for period t, $\hat{V}_t$, may be deduced as follows: from equation (2.2.14), total supernumerary expenditure by the representative household in period t, $\hat{E}_t$, may be rewritten as

$$\hat{E}_t = \hat{C}_t - \sum_{i=1}^{n} P_{i,t} \gamma_i \quad , \qquad (2.2.15)$$

where $\hat{C}_t$ denotes total expenditure by the representative household in period t. Substituting equation (2.2.15) into equation (2.2.13) and rearranging gives:

$$\hat{D}_{i,t} = \frac{\beta_i (\hat{C}_t - \sum_{i=1}^{n} P_{i,t} \hat{\gamma}_i)}{P_{i,t}}$$

so that the indirect utility function is:

$$\hat{V}_t = \sum_{i=1}^{n} \beta_i \ln \left[\frac{\beta_i (\hat{C}_t - \sum_{i=1}^{n} P_{i,t} \hat{\gamma}_i)}{P_{i,t}} \right].$$

The present value of the utility stream (2.2.11) may therefore be expressed as a function of the $\hat{D}_{i,t}$ (i.e., supernumerary expenditures on good i at time point t) which in turn are a function of $\hat{C}_t$ (i.e., total expenditure) and prices. That is,

$$\hat{U} = \sum_{t=1}^{T} \left[\frac{\hat{V}_t L_t}{\prod_{k=1}^{t} [1 + RHO]^{L_k}} \right]. \qquad (2.2.16)$$

The remaining problem for the representative household is to choose the levels of total expenditure, $\hat{C}_t$ (t = 1, ... T) to maximize (2.2.16) subject to the following budget constraint which holds over the plan terminating in T:

$$\sum_{t=1}^{T} \left[\frac{\hat{C}_t L_t}{\prod_{k=1}^{t} [1 + I_k^d]^{L_k}} \right] = \hat{\Omega} \qquad (2.2.17)$$

where the present value of the consumption stream $\hat{\Omega}$, is exogenous to the household and I^d is the domestic market rate of interest faced by the representative household. The domestic market rate of interest is given by:

$$I_t^d = B_t^d + R_t^m \qquad (2.2.18)$$

where R^m is the retail bank margin.

The first-order conditions which are derived from the consumer's intertemporal optimization problem are:

$$\frac{1}{\hat{E}_t} = \Lambda \frac{\prod_{k=1}^{t} [1 + RHO]^{L_k}}{\prod_{k=1}^{t} [1 + I_k^d]^{L_k}} \qquad (t = 1, ..., T), \quad (2.2.19)$$

and the budget constraint (i.e., equation (2.2.17). The time-invariant scalar Λ in equation (2.2.19) is a Lagrange multiplier.

The first-order conditions given by equation (2.2.19) imply that:

$$\frac{\hat{MUC}_t}{\hat{MUC}_{t+1}} = \frac{[1 + RHO]}{[1 + I^d_{t+1}]}$$

where $\hat{MUC}_t$, the marginal utility of consumption in time period t, is given by:

$$\frac{d\hat{V}_t}{d\hat{E}_t} = \frac{1}{\hat{E}_t}$$

The present value $\hat{\Omega}$ of the consumption stream is determined via a constraint on national debt. The negative of national debt, net foreign assets, grows according to the rule:

$$\hat{NFA}_{t+1} = \frac{\Phi_{t+1}}{\Phi_t} \hat{NFA}_t \ (1 + I^d_t)^{L_t} - L_t(\hat{X}_t - \hat{M}_t) + \hat{TRANSOS}_t \, . \qquad (2.2.20)$$

where Φ is the exchange rate ($Australian/$Foreign), $\hat{NFA}$ is the nominal value of net foreign assets valued in Australian dollars (but denominated in foreign currency), $\hat{X}$ and $\hat{M}$ are the nominal values of exports and imports, and $\hat{TRANSOS}$ represents exogenous unrequited transfers (e.g. aid). An implication of the accumulation relation (2.2.20) is that foreign interests in Australia (or *vice-versa*) are held as debt rather than equity. The ratio of the exchange rate at time point t+1 to its value at t captures the revaluation effect on foreign debt.

The constraint on national debt (and hence on the value of $\hat{\Omega}$) is imposed via a terminal condition that requires the ratio of debt to GDP to stabilize by the end of the planning period.

Following DPSV (1982) it is assumed that aggregation across households is legitimate so that the per-typical-household values above may be converted into total values by multiplying the per-typical-household values by the number Q of households in the economy (e.g. $E_t = \hat{E}_t \, Q_t$).

3. Solution Procedure

Following the Johansen solution procedure adopted in solving ORANI, the intertemporal CGE model is solved in a linearized form using the method expounded by Wilcoxen (1989) as implemented in the GEMPACK software (see Codsi, Pearson and Wilcoxen (1991)).

In this section the Johansen solution procedure is briefly outlined. The method adopted in GEMPACK for eliminating linearization errors which may arise from the Johansen procedure is then described. Finally, a method for providing an initial solution to the intertemporal CGE model is presented.

3.1 The Johansen Solution Method

The intertemporal CGE model presented in section 2 may be represented by

$$
\left.
\begin{array}{l}
F_1\,(X_t) = 0 \\
\vdots \\
F_h\,(X_t) = 0 \\
\hline
F_{h+1}\,(X_o, X_1, ..., X_{T+1}) = 0 \\
\vdots \\
F_{h+m}\,(X_o, X_1, ..., X_{T+1}) = 0
\end{array}
\right. \quad . \; (t = 1, ..., T) \qquad (3.1.1)
$$

Here, the F_i (i=1, ..., h+m) are h+m differentiable functions. The first h of these functions represent the atemporal equations which interrelate variables at the same 'point' in time (i.e., the single-period CGE submodels). The remaining m functions represent the intertemporal equations which relate variables at adjacent 'points' in time (i.e., link the single-period CGE sub-models through time).

Following Johansen (1960) the equations in system (3.1.1) are expressed in proportional changes so that the model is represented as a system of linear equations which may be solved at least to first-order accuracy by relatively simple matrix manipulations.

In its linearized form the intertemporal CGE model may be represented as

$$
\begin{bmatrix}
0 & A_1 & 0 & 0 & 0 & ... & 0 \\
0 & 0 & A_2 & 0 & 0 & ... & 0 \\
... & ... & ... & ... & ... & ... & 0 \\
0 & 0 & 0 & 0 & ... & A_T & 0 \\
B_0 & B_1 & B_2 & B_3 & ... & B_T & B_{T+1}
\end{bmatrix}
\begin{bmatrix}
x_0 \\ x_1 \\ \vdots \\ x_{T+1}
\end{bmatrix}
= 0 \qquad (3.1.2)
$$

where x_t for t=0,1, ..., T+1 is an (n × 1) vector of proportional changes in all of the model's variables in period t; A_t for t=1, ..., T is an (h × n) matrix of coefficients corresponding to the atemporal equations at 'point' t; and, B_t for t=0,1, ..., T+1 is an (m × n) matrix of coefficients corresponding to the variables indexed by t in the intertemporal equations.

To illustrate the solution procedure, equation (3.1.2) may be written more compactly as

$$\Phi z = 0 \qquad (3.1.3)$$

Here, z is a vector of proportional changes in all the model's variables and Φ is a matrix of coefficients of all equations in the model.[17] In the intertemporal CGE model, the number of variables V exceeds the number of equations E. Hence (E-V) of the variables must be set exogenously. Equation (3.1.3) may then be expressed as

$$\Phi_1 z_1 + \Phi_2 z_2 = 0 \quad , \tag{3.1.4}$$

where z_1 is a vector of proportional changes in the endogenous variables, z_2 is a vector of proportional changes in the exogenous variables, Φ_1 is a sub-matrix of coefficients corresponding to the endogenous variables and, Φ_2 is a sub-matrix of coefficients corresponding to the exogenous variables.

A Johansen solution for the proportional changes in the endogenous variables brought about by the changes in z_2, the exogenous variables, is then obtained as follows

$$z_1 = -\Phi_1^{-1} \Phi_2 z_2 \quad . \tag{3.1.5}$$

The Johansen solution procedure relies only on matrix inversion and matrix multiplication. Constraints on model size and flexibility are consequently less severe than for non-linear solution methods. As argued in DPSV (1982), the Johansen approach provides maximum scope for model modifications (e.g., changes in the equations, additions and deletions of equations, and respecification of the closure) without requiring that the solution algorithm be re-written.

In the case of the intertemporal CGE model developed in this paper, the computational advantages of the Johansen approach are particularly attractive, since in moving from a single-period model to a multi-period model there is less need to sacrifice economic detail in terms of either the description of the economy at a point in time or of the specification of inter-period linkages. In addition, the flexibility of the solution procedure (especially in specifying the split between endogenous and exogenous variables) means that the model is amenable to addressing a diverse range of policy issues.

3.2 Eliminating Johansen Linearization Errors

The Johansen solution procedure described in the previous section assumes that the coefficient matrices Φ in equation (3.1.4) are constant. This means that equation (3.1.3) is only a local representation of the non-linear equations suggested by economic theory (i.e., system (3.1.1)) and that the results obtained from equation (3.1.3) are only valid for "small" changes in the exogenous variables (i.e., the z_2 are "small").

There is no formal guidance as to what constitutes a "small" change in an exogenous variable. The "small" change concept is relative to both the particular model to be solved and to the particular application (i.e., which variables are exogenous and which of the exogenous variables are shocked).

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17 Note that these proportional changes are all relative to a control solution of the model; thus a typical element of z is the proportional difference between the value of a variable at 'point' t in a shocked solution and the value of the same variable at the same 'point' in the control solution.

There is no simple way therefore, of determining *a priori* whether the degree of approximation error introduced by the Johansen solution procedure will be significant.

Solutions which are free of significant linearization errors may be obtained by using the Euler solution method (DPSV (1982)). The Euler method involves dividing the shocks to the exogenous variables into a sequence of n smaller shocks. The first part of the shock is then applied and the effects on the endogenous variables are calculated. The database is then updated to account for the exogenous and endogenous consequences of the first part of the shock. In this process, the Φ matrix in equation (3.1.3) is re-evaluated using the updated data as is the $-\Phi_1^{-1}\Phi_2$ matrix in equation (3.1.4). This procedure, after repetition n times, is known as an n-step simulation.

Under reasonable assumptions on the continuity of the original functions and their derivatives (see Pearson (1991)), results which are as close to the exact solution as machine accuracy allows may be obtained by choosing n large enough. Note that when n = 1 the Euler method is equivalent to the Johansen approach.

A potential problem of the Euler method is that convergence to a result which is free of significant linearization errors may require a large number of steps. The procedure may therefore be very taxing on computer resources. However, Euler's method may be supplemented by an extrapolation technique so that results which are free of significant linearization errors may be obtained with a small number of steps (see DPSV (1982)).

In GEMPACK extrapolated results based on two or three different Euler solutions are routinely available. The extrapolation procedure used is that of Richardson (see Pearson (1991)). In moving from single-step to multi-step solutions in GEMPACK, details regarding how the data base is to be updated between steps must be provided.

3.3 *Solving Models When No Initial Solution Is Available*

Under the multi-step (Euler) solution method the values of all endogenous variables are simultaneously determined in a single computational process. This method has the advantage of allowing complete freedom in the structure of intertemporal links with terminal conditions appearing explicitly as part of the equation system. An apparent difficulty with the Euler solution method is that an initial (time series) solution of the model is required. The initial solution must consist of a sequence of databases 0 to T + 1 which satisfy all the non-linear equations of the model. Having found an initial solution to the intertemporal model the next step is to construct a Control Path Solution (CPS). The CPS consists of the expected or preferred values of all variables in the model and should be a suitable reference scenario for performing policy simulations or a good starting point for performing forecasting simulations.[18] Due to the insights of Horridge (1990), obtaining an initial solution and constructing a suitable CPS is relatively straightforward. In practice, the

18 See Dixon, Parmenter and Horridge (1986) for a discussion on the importance of the control path scenario and on the problems of constructing such a scenario.

processes of obtaining an initial solution and a CPS are combined into a single operation.

Rather than starting by attempting to find a sequence of databases which satisfy the given non-linear equations of the intertemporal model, the Horridge method first modifies the equation system to satisfy a sequence of databases which are easily constructed. The latter sequence may be labelled the *quasi-scenario*. One quasi-scenario which is a convenient starting point for computations is[19]:

$$X_t = X_0 , \qquad t = 1, 2, ..., T + 1 \tag{3.3.1}$$

where X_0 is the set of current variable values obtained from the most recent input-output tables. This scenario of cloned databases satisfies all the atemporal equations of the model. However, the intertemporal equations will not necessarily be satisfied. The method then is to augment these intertemporal equations by adding to each a unique slack variable (hereafter referred to as a *calibration variable*) which adopts a value that allows these equations to be satisfied in the levels by the sequence of cloned data bases.[20]

Having obtained one solution for the augmented equation system it is then possible to find other solutions using Euler's method. The calibration variables can be shocked to any values, so, if appropriate values (zero if the slack variable enters the equation additively and one if it is multiplicative) are chosen, the augmented equation system can be made equivalent to the original equation system. That is, the solution of the augmented equation system with these values of the calibration variables is also a solution of the original equation system (3.1.1). If in addition all other exogenous variables are shocked to their preferred or expected values, a CPS is obtained.

To illustrate this approach consider a capital accumulation equation of the following form:

$$K_{t+1} = K_t (1 - \delta) + I_t \tag{3.3.2}$$

where K, I and δ are the capital stock, investment and the rate of depreciation respectively. This equation will not be consistent with the clonal quasi-scenario unless the base-period data represent the economy in a steady state where the rate of gross investment is equal to the rate of depreciation.

Following the Horridge method, each such accumulation equation is augmented with a calibration variable S_t, as follows:

$$K_{t+1} = [K_t (1 - \delta) + I_t] S_t \quad . \tag{3.3.3}$$

The next step is to calculate the levels value of S_t in the clonal quasi-scenario. Since each calibration variable S is unique and only appears in one equation the resulting value is consistent with all other equations. Having obtained a solution to the augmented equation, S_t is shocked to unity so that a solution to the original unaugmented accumulation equation (3.3.2) is obtained.

---

19 Other quasi-solutions are possible, e.g., balanced growth sequences of the form: $X(t) = \alpha(t) X(0)$ $t = 1, 2, ..., N + 1$.

20 A sequence of identical databases is referred to below as a clonal *quasi-scenario*.

As long as the calibration variables are exogenously set to unity, any closure of the model (and any shocks to the other exogenous variables) could be used to construct some arbitrary solution to the original equation system.

4. Construction of Database

4.1 Sources of Data and Levels of Aggregation

The base period database D_0 used to construct the quasi-scenario is a slightly modified version of the so-called Urbino database which is a 13 sector aggregation of the 1980-81 ORANI database (see Higgs, Meagher and Parmenter (1991) and Meagher (forthcoming)). The sectors correspond with the ASIC divisions used by the ABS in the Australian National Accounts.

4.1.1 Additional Atemporal Data Requirements

The modifications and additions made to the atemporal part of the model (i.e. ORANI) introduce several new data requirements.

1. The Urbino database (as with the ORANI database) identifies 10 occupations. In this implementation the 10 occupational categories are aggregated into one as follows:

$$\tilde{U}_j = \sum_{m=1}^{10} \tilde{U}_{m,j} \quad .$$

where $\tilde{U}_{m,j}$ is expenditure by industry j on labour of skill m.

2. The Urbino database identifies four margin commodities (6, 7, 8, 12). In this implementation these margins are treated as if they were direct demands as follows:

For final consumption,

$$\tilde{C}_i^* = \tilde{C}_i + \sum_{j=1}^{4} \tilde{M}_{j,i} + \sum_{j=1}^{4} \tilde{R}_{j,i} \quad .$$

where $\tilde{C}_i$ is the final consumption flow of domestic good i; $\tilde{M}_{j,i}$ is the flow of domestic good j used as a margin in delivery of domestically produced domestic good i to final consumption; $\tilde{R}_{j,i}$ is the flow of good j used as a margin in delivery of imported good i to final consumption.

For exports,

$$\tilde{D}_i^* = \tilde{D}_i + \sum_{j=1}^{13} \tilde{N}_{j,i} \quad .$$

where $\tilde{D}_i$ is the export flow of domestic good i and $\tilde{N}_{j,i}$ is the flow of domestic good j used as a margin to facilitate the export of good i.

For intermediate inputs,

$$\tilde{A}^*_{j,i} = \tilde{A}_{j,i} + \sum_{k=1}^{13} \tilde{K}_{k,j,i} + \sum_{k=1}^{13} \tilde{P}_{k,j,i} \ .$$

where $\tilde{A}_{j,i}$ is the direct flow of domestic commodity j into the production process of domestic industry i; $K_{k,j,i}$ is the use of domestic good k to facilitate the transfer domestically produced intermediate input j to industry i; $P_{k,j,i}$ is a similar flow of domestic good k in the transfer of imported intermediate inputs.

For inputs into capital creation,

$$\tilde{B}^*_{j,i} = \tilde{B}_{j,i} + \sum_{k=1}^{13} \tilde{L}_{k,j,i} + \sum_{k=1}^{13} \tilde{Q}_{k,j,i} \ .$$

where $\tilde{B}_{j,i}$ is the direct flow of domestic good j into capital formation in industry i; $\tilde{L}$ and $\tilde{Q}$ denote the usage of margin good k in the delivery of respectively domestic and imported good j for use in capital creation in industry i.

3. As described in Appendix A, the respecification of the equation which governs household demands for commodities undifferentiated by source (i.e. equation 14.24 in DPSV) means that the above-subsistence share of expenditure on good i (i.e. the A_i's in Appendix A) must be calculated. In the LES initial values of the A_i's may be deduced from estimates of the Frisch parameter (which is already held in the ORANI database) and the expenditure elasticities (which are easily calculated using information already held on the ORANI database). This issue is addressed explicitly in Appendix A.

4. The initial values of net transfers paid overseas TRANSOS, domestic retail bank margins R^m, and sector specific risk premium $Risk_j$ are set as follows:

TRANSOS	=	\$A80 mill.	
R^m	=	0.0	
$Risk_j$	=	0.0	$j \neq 8, 9$
$Risk_j$	=	0.04	$j = 8, 9$

4.1.2 Intertemporal Data Requirements

The intertemporal equations introduce several new data requirements for the clonal quasi scenario. The initial settings of these variables are listed below:

net holdings of Australian assets by foreigners

$$NFA_t = \$A14581.5 \text{ million} \qquad t=0, \ldots, T+1$$

pure time-preference discount rate

$$RHO = 0.03$$

Lagrange multiplier (see equation (2.2.19))

$$\Lambda = {}^1/_{45,000}$$

safe foreign bond rate	B_t^F	=	0.03	t=0, ..., T+1
rate of growth of the economy	G	=	0.0	
prices	P_t^*	=	1	t=0, ..., T+1

where P_t^* denotes the vector of *all* prices (including the exchange rate).

As will be explained in section 5.1, attempts to construct the CPS using the above data led to some sectors having a negative rate of investment at time point 1. In the absence of non-negativity constraints, the data was manipulated to ensure that sectoral investment rates never became negative. This data manipulation exercise, however, is only a temporary solution. A more adequate solution will involve the incorporation of non-negativity constraints into the model (with perhaps some limited manipulation of the data).

The data relating to sectoral rates of return (especially, the rental and replacement values of sectoral fixed capital) appears suspect (in particular where they relate to government dominated industries). A more thorough understanding about how the Australian Statistician measures these variables is needed so that adjustments can be made where appropriate.

Table 4.1 summarizes the changes made to the original data base. The values in Table 4.1 are interpreted as $A million.

Table 4.1: *Modifications Made to the Original Data Base*[1]

Variable →	Rental Value of Fixed Capital		Replacement Cost of Fixed Capital		Sales to 'Other' Demands	
Sector ↓	$\tilde{V}$	$\tilde{V}^*$	KS	KS*	$\tilde{E}$	$\tilde{E}^{*2}$
1 Agriculture	1365	2500			4007	5142
4 Electricity, Gas and Water	2131	2536			14	419
5 Construction	818	3400	38407	42000	0	2582
7 Transport and Storage	1498	3911	42642	39110	3427	5840
12 Recreation	851	1050	8154	10000	43	242
13 Non-Competing Imports	0	0.003			0.001	0.004

1. Variables whose values have been modified are denoted by *. Variables which are unstarred denote the initial values observed in the original data base. Blank entries indicate that the original data have been changed.

2. To ensure that the data base remained balanced $\tilde{E}^*$, the modified values of sales to 'other' demands was calculated as follows: $\tilde{E}^* = \tilde{E} + \tilde{V}^* - \tilde{V}$.

5. Calibration Results from the Model

This section describes the construction of a control path solution. The data base outlined in section 4 is used in conjunction with the method described in section 3.3 for obtaining an initial solution to the model. The process involves computing simultaneously a solution in which: a) all of the equations of the intertemporal CGE model are satisfied, and b) all exogenous variables are set to their "preferred" values.

Finally, some comments are made about the performance of the Euler solution method.

5.1 The Control Path Solution

To obtain a solution using the method described in section 3.3 each intertemporal equation of the model must be augmented with a calibration variable S so that system (3.1.1) which represents the original equation system of the intertemporal CGE model becomes:

$$
\begin{array}{c}
F_1 (X_t) = 0 \\
\vdots \\
F_h (X_t) = 0 \\
\hline
\end{array} \quad . \; (t = 1, \ldots, T) \quad (5.1.1)
$$

$$
\begin{array}{c}
F_{h+1} (X_0 , X_1 \ldots, X_{T+1} : S_0 , S_1 \ldots, S_{T+1}) = 0 \\
\vdots \\
F_{h+m} (X_0 , X_1 \ldots, X_{T+1} : S_0 , S_1 \ldots, S_{T+1}) = 0
\end{array}
$$

5.1.1 Intertemporal Closure

The exogenous variables of the intertemporal CGE model fall into four main categories:

1. Initial Conditions: the values of all variables at time point 0 are historical observations and so are naturally exogenous.
2. Calibration Variables: the calibration variables (denoted S in equation (5.1.1)) are exogenous because they must be shocked to a specific value in order for the augmented equation system (5.1.1) to be equivalent to the original system (3.1.1).
3. Variables for which the model has no formal theory: these include among others technical change, world prices, indirect taxes, non-traditional exports, overseas transfers, risk factors, government expenditure, population, labour supply, capital stocks in the government dominated industries, the supply of agricultural land and all shift variables (including those representing foreign demand conditions).
4. A Numeraire Sequence: to tie down the absolute price level at least one price must be exogenously specified at all time 'points'. In this application the nominal exchange rate is set exogenously at all time 'points' and acts as the numeraire.

5.1.2 Generation of Control Path Solution

This process starts from the clonal quasi-scenario and ends with a control path solution specifically designed to demonstrate the ability of the intertemporal CGE model to converge to a balanced steady-growth path.[21] Establishing this property of the model serves two purposes: First, it provides a useful reference point by identifying the model within the class of neo-classical growth models. Second, it allows an infinite period solution to be approximated by computing only the solution to the first T+1 periods, where T is an integer chosen by the modeller. The exogenous variables converge to steady growth by design. If the endogenous variables also show strong evidence of convergence to steady growth, then it is plausible to assume that their values at time points $t > T+1$, would, if actually computed, conform to the steady growth path. In other words, the choice of a new T (T*, say) arbitrarily larger than the originally chosen T would be expected to lead to solutions in the interval [T+1, T*+1] which conform to the balanced growth rule.

In this simulation, the aim was to construct a CPS which converged to a balanced steady-growth path where all real variables were growing at 2.5 percent per annum. Accordingly, the values of the exogenous variables in this computation conformed to a predictable pattern: the percentage changes in all variables at time point 0 were set to zero; the percentage changes in all exogenous nominal variables were also set to zero; all exogenous real variables between time points 1 and T were assumed to grow by 2.5 per cent compounding annually; and finally, the growth factor in the terminal conditions for sectoral capital stocks (i.e., G_j in equation (2.2.9)) was assumed to be 2.5 percent.

In this implementation of the model, the absence of non-negativity constraints proved to be a problem. Attempts to construct a CPS using a clonal quasi-scenario based on the data outlined in section 4, but without the modifications summarized in Table 4.1, proved to be problematic because some sectors recorded negative investment rates.

Moving from the clonal quasi-scenario to the CPS required some rather large shocks to the multiplicative calibration variables with which both sectoral capital accumulation equations (i.e., equation (2.2.5)) and equilibrium conditions for sectoral capital demands (i.e., equation (2.2.10)) had been augmented. Several industries responded to these shocks by having a negative rate of investment in the first period (i.e., the percentage fall in investment in period 1 exceeded 100 per cent). Ideally, the solution should involve the

---

21 The balanced growth CPS constructed here differs in one respect from the CPS described in section 3.3. Whilst the closure is the same in both cases, not all exogenous variables conform to their "expected" values in the balanced growth CPS. For example, the balanced growth CPS assumes no technological change and steadily growing endowments of agricultural land. These unrealistic assumptions are made purely to ensure that the model converges to balanced growth. (A non-zero uniform and constant rate of Harrod-neutral technological change would also have led to a balanced growth path.) More realistic settings of the exogenous variables would imply that agents' expectations of variable values beyond time point T are based on some rule more complex than the balanced growth hypothesis. The form of this rule is a matter for further study.

imposition of non-negativity constraints on investment. Although new versions of the GEMPACK software will handle such constraints, the problem was handled here by manipulating the data to avoid negative investment rates (see section 4).

A selection of the macro and industry results from the control path simulation are presented in Tables 5.1 and 5.2 respectively.

Overall, the results show that all real variables tend to converge towards steady growth of 2.5 percent per annum. A feature of the industry results is the tendency for significant adjustments to sectoral demands for capital (and hence sectoral investment) in the first few periods. The degree of adjustment largely depends on the extent to which the clonal quasi-scenario represents a disequilibrium: in particular, the extent to which the sectoral rates of return observed in the clonal quasi-scenario differ from the rates of return required by the imposition of the arbitrage condition (2.2.2). Industries which had a rate of return that was too low in the clonal quasi-scenario attempted to rapidly reduce their holdings of capital stocks when the arbitrage condition was imposed (and *vice-versa* for industries with a high initial rate of return).

5.2 *Performance of the Solution Method*

The speed and accuracy of the solution method is of crucial importance when dealing with models which are as large and complex as the one described in this paper.

The construction of the CPS may involve some very large shocks to variables. This means that in 1-step Johansen solutions, large linearisation errors are to be expected. This expectation was in fact realized. These linearisation errors were largely overcome by obtaining extrapolated results based on 1, 2 and 4- step Euler solutions. Even more accurate results were obtained by an extrapolation based on 2, 4 and 8-step Euler solutions.

The gain in accuracy however comes at a cost in terms of computing time. The 1-step (Johansen) solution takes about 1 minute of VAX CPU time to compute whilst the extrapolated results based on the 1, 2 and 4- step Euler solutions take about 5 minutes of CPU time. The extrapolated results based on the 2, 4 and 8-step Euler solutions take about 10 minutes of CPU time to compute.

The CPU times reported above were achieved with an experimental version of the GEMPACK software implemented by Ken Pearson. With the current (publicly available) version of GEMPACK, the CPU times for multi-step solutions are quite large. For example, an extrapolation based on 2, 4, and 8-step Euler solutions takes over an hour of CPU time to compute (compared with about 10 minutes in the experimental version).

Each step in the Euler solution process involves the computation of coefficients and the inversion of a matrix. Most computing resources are used by the matrix inversion process. The experimental version of GEMPACK is much more efficient because the matrix inversion process has been speeded

Table 5.2(c): *Projections of Macroeconomic Variables**

Variable	Interval (L_t): t =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1.	Net foreign debt/GDP	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.15
2.	Average Annual change in balance of trade as % of GDP	1.09	1.11	1.07	1.03	0.99	0.95	0.92	0.89	0.86	0.84	0.81	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62
3.	Real devaluation	-0.50	0.82	0.62	0.52	0.41	0.34	0.28	0.23	0.19	0.15	0.13	0.01	0.09	0.07	0.06	0.05	0.04	0.04	0.03	0.03
4.	Capital stock	4.41	4.01	3.67	3.44	3.27	3.13	3.01	2.92	2.84	2.78	2.73	2.68	2.65	2.62	2.60	2.57	2.56	2.54	2.53	2.52
5.	Employment	2.5	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
6.	Real GDP	3.02	3.05	2.85	2.78	2.72	2.68	2.65	2.62	2.60	2.58	2.57	2.56	2.55	2.53	2.53	2.52	2.52	2.51	2.51	2.51
7.	Real Consumption (Private)	4.86	2.95	2.93	2.85	2.77	2.73	2.69	2.65	2.63	2.60	2.59	2.57	2.56	2.55	2.54	2.53	2.53	2.52	2.52	2.52
8.	Real Consumption (Public)	2.5	2.50	2.50	2.50	2.50	2.50	2.50	2.50	.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
9.	Real Investment	-1.92	5.75	1.80	1.77	1.84	1.95	2.04	2.12	2.18	2.23	2.28	2.30	2.35	2.33	2.37	2.38	2.39	2.38	2.37	2.40
10.	Exports	1.62	2.40	3.95	3.77	3.54	3.36	3.20	3.08	2.97	2.89	2.82	2.77	2.72	2.68	2.65	2.63	2.61	2.60	2.59	2.59
11.	Imports	0.02	5.33	2.39	2.33	2.28	2.33	2.37	2.39	2.41	2.42	2.45	2.43	2.47	2.44	2.46	2.47	2.46	2.47	2.45	2.50
12.	CPI	0.59	-0.75	-0.67	-0.55	-0.43	-0.36	-0.29	-0.24	-0.24	-0.20	-0.16	-0.13	-0.11	-0.09	-0.07	-0.05	-0.04	-0.04	-0.03	-0.03
13.	GDP deflator	0.50	-0.81	-0.62	-0.52	-0.41	-0.34	-0.28	-0.23	-0.19	-0.15	-0.13	-0.10	-0.09	-0.07	-0.06	-0.05	-0.04	-0.04	-0.03	-0.03
14.	Capital Goods Price Index	0.34	-0.59	-0.48	-0.39	-0.32	-0.26	-0.21	-0.17	-0.14	-0.12	-0.10	-0.08	-0.07	-0.05	-0.05	-0.04	-0.03	-0.03	-0.03	-0.02
15.	Price Index for Imports	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16.	Price Index for Exports	0.12	0.02	-0.23	-0.20	-0.17	-0.14	-0.11	-0.09	-0.08	-0.06	-0.05	-0.04	-0.04	-0.03	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01

* Apart from rows 1 and 2, the numbers in this table refer to the percentage rate of growth of the relevant variables over the interval between instants t and (t+1).

Table 5.2(a): *Projections of Sectoral Rates of Investment**

Sector	Interval (L_t): t =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1.	Agriculture, Forestry, Fishing and Hunting	-23.53	82.00	2.76	2.00	2.20	2.20	2.22	2.29	2.29	2.37	2.29	2.48	2.39	2.42	2.44	2.44	2.47	2.44	2.52	1.96
2.	Mining	35.92	-38.18	0.40	2.13	2.29	2.32	2.34	2.38	2.39	2.43	2.38	2.47	2.47	2.42	2.45	2.44	2.45	2.45	2.51	2.43
3.	Manufacturing	-65.57	191.81	1.50	2.11	2.06	2.17	2.16	2.25	2.26	2.39	2.19	2.48	2.37	2.36	2.41	2.38	2.43	2.34	2.60	2.24
4.	Electricity, Gas and Water	-21.40	25.97	11.56	1.62	1.86	1.85	1.94	2.08	2.10	2.22	2.15	2.42	2.21	2.37	2.34	2.37	2.41	2.35	2.47	2.23
5.	Construction	-75.29	281.79	-2.09	5.55	1.30	2.34	2.64	2.06	2.60	2.01	3.52	0.83	3.54	1.94	2.46	2.51	1.89	2.60	1.18	4.46
6.	Wholesale and retail Trade	-29.99	38.86	-2.57	1.54	2.08	2.14	2.14	2.27	2.26	2.39	2.18	2.51	2.35	2.36	2.40	2.27	2.65	2.23	2.56	2.30
7.	Transport and Storage	52.49	-36.85	12.57	1.71	1.70	1.75	1.84	2.00	2.04	2.17	2.08	2.39	2.19	2.29	2.33	2.33	2.37	2.31	2.46	2.27
8.	Finance, Property and Business Services	127.48	-55.24	-3.01	1.41	1.99	2.02	2.10	2.18	2.24	2.24	2.18	2.52	2.39	2.28	2.37	2.38	2.42	2.34	2.41	2.44
9.	Ownership of Dwellings	-14.45	23.57	-0.38	-0.24	1.31	1.21	1.42	1.70	1.72	1.94	1.89	2.39	1.81	2.36	2.23	2.31	2.36	2.31	2.42	1.86
10.	Public Administration and Defence	180.34	-28.68	2.49	2.51	2.49	2.51	2.48	2.51	2.50	2.51	2.49	2.50	2.50	2.51	2.50	2.50	2.50	2.50	2.50	2.50
11.	Community Services	15.25	-2.86	2.49	2.51	2.49	2.51	2.49	2.51	2.50	2.51	2.49	2.50	2.50	2.51	2.50	2.50	2.50	2.50	2.50	2.50
12.	Recreation, Personal and Other Services	-59.92	189.67	-5.09	0.99	1.87	1.85	1.96	2.10	2.10	2.22	2.17	2.42	2.19	2.42	2.25	2.40	2.40	2.37	2.43	2.19
13.	Non-Competing Imports	5737.12	-45.26	2.49	2.51	2.49	2.51	.49	2.51	2.50	2.51	2.49	2.50	2.50	2.51	2.50	2.50	2.50	2.50	2.50	2.50

Table 5.2(b): *Sectoral Growth Rates**

Sector	Interval (L_t) t =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1.	Agriculture, Forestry, Fishing and Hunting	0	2.53	3.67	3.50	3.31	3.17	3.05	2.95	2.88	2.80	2.75	2.70	2.68	2.64	2.62	2.60	2.58	2.57	2.56	2.56
2.	Mining	0	-0.18	2.78	2.80	2.73	2.69	2.66	2.63	2.61	2.59	2.57	2.55	2.55	2.54	2.53	2.52	2.52	2.51	2.51	2.51
3.	Manufacturing	0	4.28	3.16	3.03	2.90	2.84	2.78	2.72	2.68	2.65	2.63	2.59	2.59	2.56	2.56	2.55	2.53	2.53	2.52	2.53
4.	Electricity, Gas and Water	0	2.87	2.89	2.89	2.81	2.75	2.71	2.67	2.64	2.61	2.60	2.57	2.57	2.55	2.54	2.53	2.53	2.52	2.52	2.52
5.	Construction	0	0.11	1.64	1.62	1.95	1.99	2.07	2.16	2.20	2.26	2.27	2.39	2.31	2.38	2.38	2.40	2.42	2.39	2.42	2.34
6.	Wholesale and retail Trade	0	4.18	2.80	2.72	2.65	2.63	2.61	2.58	2.57	2.56	2.55	2.53	2.53	2.52	2.52	2.52	2.50	2.52	2.50	2.52
7.	Transport and Storage	0	3.56	2.71	2.71	2.66	2.64	2.61	2.59	2.58	2.56	2.55	2.54	2.53	2.52	2.52	2.52	2.51	2.51	2.51	2.51
8.	Finance, Property and Business Services	0	3.54	2.66	2.68	2.65	2.62	2.60	2.58	2.57	2.55	2.54	2.53	2.52	2.54	2.52	2.51	2.51	2.50	2.50	2.50
9.	Ownership of Dwellings	0	2.61	3.82	3.53	3.28	3.16	3.04	2.94	2.86	2.79	2.74	2.69	2.68	2.62	2.61	2.59	2.57	2.56	2.55	2.54
10.	Public Administration and Defence	0	2.57	2.53	2.52	2.52	2.51	2.51	2.51	2.51	2.51	2.51	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
11.	Community Services	0	2.79	2.60	2.59	2.57	2.56	2.55	2.54	2.53	2.53	2.52	2.52	2.52	2.51	2.51	2.51	2.51	2.51	2.51	2.51
12.	Recreation, Personal and Other Services	0	2.87	2.89	2.80	2.73	2.69	2.66	2.63	2.61	2.59	2.57	2.56	2.56	2.53	2.52	2.52	2.52	2.52	2.52	2.52
13.	Non-Competing Imports	0	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50

* The numbers in these tables refer to the percentage rates of growth of the relevant variables over the interval between instants t and (t+1).

Table 5.2(c): *Projections of Sectoral Demands for Capital* *

Sector	Interval (L_t): t =	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1.	Agriculture, Forestry, Fishing and Hunting	-0.16	-2.78	4.71	4.41	4.05	3.77	3.54	3.35	3.20	3.07	2.97	2.88	2.82	2.76	2.71	2.68	2.64	2.62	2.59	2.58	2.50
2.	Mining	9.60	14.14	3.38	3.01	2.91	2.83	2.77	2.72	2.68	2.65	2.62	2.59	2.58	2.57	2.55	2.54	2.53	2.52	2.51	2.51	2.50
3.	Manufacturing	3.64	-5.04	3.91	3.60	3.41	3.25	3.11	3.00	2.91	2.83	2.78	2.71	2.68	2.65	2.61	2.59	2.57	2.55	2.53	2.53	2.50
4.	Electricity, Gas and Water	3.08	0.93	2.64	3.39	3.23	3.11	3.00	2.91	2.83	2.77	2.72	2.68	2.65	2.62	2.60	2.57	2.56	2.55	2.53	2.52	2.50
5.	Construction	3.28	-3.06	2.80	2.43	2.66	2.55	2.54	2.54	2.51	2.52	2.48	2.56	2.43	2.51	2.47	2.47	2.47	2.43	2.44	2.35	2.50
6.	Wholesale and retail Trade	5.87	0.80	4.54	3.61	3.36	3.20	3.08	2.97	2.88	2.81	2.76	2.69	2.67	2.64	2.60	2.58	2.55	2.56	2.52	2.53	2.50
7.	Transport and Storage	4.18	8.82	2.68	3.49	3.33	3.19	3.06	2.96	2.88	2.81	2.75	2.70	2.67	2.63	2.60	2.58	2.56	2.55	2.53	2.52	2.50
8.	Finance, Property and Business Services	8.05	25.11	4.23	3.43	3.21	3.09	2.98	2.89	2.82	2.76	2.71	2.66	2.64	2.62	2.59	2.57	2.55	2.54	2.52	2.51	2.50
9.	Ownership of Dwellings	3.88	2.61	3.82	3.53	3.28	3.16	3.04	2.94	2.86	2.79	2.74	2.69	2.68	2.62	2.61	2.59	2.57	2.56	2.55	2.54	2.50
10.	Public Administration and Defence	-1.30	6.45	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
11.	Community Services	1.85	3.15	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
12.	Recreation, Personal and Other Services	3.32	-3.33	5.08	3.86	3.54	3.36	3.20	3.07	2.97	2.88	2.82	2.75	2.72	2.67	2.64	2.60	2.58	2.56	2.55	2.53	2.50
13.	Non-Competing Imports	-7.21	13.23	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50

* The numbers in this table refer to the percentage rates of growth of the relevant variables over the interval between instants t and (t+1).

up. This is achieved by saving the pivot points identified in the first step of a multi-step solution and re-using these points in all subsequent steps.

It should be noted that the multi-step solution procedure is typically required only for constructing the CPS. For policy experiments which use the CPS as the starting point, results which are free of significant linearisation errors may often be obtained from a single-step (Johansen) solution.

6. Conclusion

6.1 Summary

This paper describes the construction of an intertemporal CGE model. The construction of this model consisted of three key aspects:

1. specification of the theoretical structure,
2. construction of a data base, and
3. choice of a solution method which allows the model to be solved numerically.

The theoretical structure of the model is based on the ORANI model which is extended into an intertemporal framework via the specification of forward-looking investment and consumption behaviour.

In moving to an intertemporal framework a major challenge was to keep the model computationally tractable without compromising significantly on the type and degree of detail modelled. This challenge was met by adopting the Euler method which exploits linear mathematical techniques in the solution procedure. Thus, unlike most comparable intertemporal CGE models the model presented in this paper was able to handle sector-specific capital stocks without any significant computational problems.

The generation of a Control Path Scenario (CPS) demonstrated several important points: firstly, that the solution method chosen was capable of producing accurate results relatively quickly; secondly, that the model is well behaved in the sense that it posses the convergence properties expected of aneoclassical intertemporal model; and finally, that the Horridge method for constructing an intertemporal data base from the latest published input-output data is viable. A benefit of the Horridge method was shown to be that there is no necessity to adopt the typical practice of constructing a synthetic data base which represents the economy at a steady-state or in balanced growth.

6.2 The Agenda for Future Research

This paper clearly presents work in progress. The directions for future research listed below are by no means intended to be exhaustive. Rather, they are an agenda that is motivated by the desire to build an intertemporal CGE model which is both practical and capable of addressing a wide range of issues in a plausible manner. This goal places priority on research in the following areas:

a) Treatment of Expectations: currently, all agents are assumed to have model-consistent expectations. It is intended that the model be set up in a manner which provides maximum flexibility in specifying alternative views on how agents form their expectations.

b) Non-negativity Constraints: in this implementation of the model inequality constraints were not imposed formally, necessitating ad hoc adjustments to the initial data base. In future versions investment should be constrained to be non-negative.

c) The Overspecialisation ("flip-flop") Problem: In long-run models of open economies with constant returns to scale technology and an absence of sectoral fixed factors there is a tendency for unrealistic degrees of sectoral specialisation (see DPSV (1982)). Some suggestions for overcoming this problem are proposed in Dixon and Parmenter (1987), Horridge, Powell and Wilcoxen (1990) and McDougall (1991). These proposals need further investigation with particular emphasis on the empirical aspects.

d) Costs of Adjustment: much of the recent literature on investment behaviour recognises adjustment costs explicitly. Observed lags in investment behaviour are often rationalised in terms of adjustment costs. More work is required in identifying the exact sources and magnitudes of such costs before they can be convincingly incorporated into the CGE framework.

e) Investment Technology: the investment technology specified in this model is overly restrictive in terms of the length of the gestation period and does not allow for lumpiness in the capital creation process. Recent work in the area of public utility economics suggests that a more flexible specification of the investment technology which allows for lumpiness and gestation lengths which vary across industries may be necessary (see Dixon (1989)).

f) The Value of Flow Variables between Time Points: the use of average rates of investment within periods in accummulation equations like (2.2.5) is a simplification which is less valid as the interval lengths L_t increase (this may be true of any flow variable which is accumulated into a stock). In such cases it may be necessary to respecify the accumulation relationships to take into account the time profile of such variables within intervals L_t.

g) Variable Period Lengths: As foreshadowed in section 2.2.1, considerable gains in computing can be achieved (without significant loss of economic detail) by using variable period lengths. In conjunction with developments under category (f) above, it remains to demonstrate these gains.

Appendix A The Linear Expenditure System

In the intertemporal CGE model the typical household has two basic decisions to make. The first is an intertemporal choice about how to allocate its budget over time (see section 2.2.4). The second, which is the focus of this Appendix, is an atemporal choice about how to allocate its budget across commodities at a given point in time.

In the version of ORANI used to construct the intertemporal CGE model the Linear Expenditure System (LES) governs the allocation of the household budget across commodities. Since the LES is consistent with intertemporal optimising behaviour by households, it is conveniently retained. However, for purely computational purposes, the ORANI equation which governs household demands for commodities undifferentiated by source (i.e. equation (14.24) in DPSV (1982)) is reformulated along the lines suggested by Horridge (1991).

The problem is that the price and expenditure elasticities derived from the LES are treated as parameters in the standard implementation of ORANI. Whilst DPSV (1982) acknowledge that these elasticities are functions of variables in the model rather than parameters, they do not make these functional relationships explicit.

When ORANI is solved using a one-step Johansen linearization the treatment of these elasticities as constants is not objectionable since it is consonant with the approximation errors involved in the method. A problem arises however, when a multi-step solution method is adopted. The Euler solution method for example requires that the values of all variables be updated to reflect the state of the economic model after each of the n shocks of the simulation have been administered. Under this solution method the functional relationships between the price and expenditure elasticities and the variables in the model must be made explicit to facilitate the updating procedure.

The equation governing the distribution of the household budget across commodities may be derived from the following household optimisation problem: Choose the level $\hat{H}_i$ of consumption of good i to maximise the level $\hat{U}$ of Stone-Geary utility:

$$\hat{U} = \sum_{i=1}^{n} \beta_i \ln [\hat{H}_i - \hat{\gamma}_i] \qquad \text{(A.1)}$$

subject to the constraint:

$$\sum_{i=1}^{n} P_i \hat{H}_i = \hat{C} \ . \qquad \text{(A2)}$$

Here the β_i are marginal budget shares (which sum to one) and the γ_i are parameters denoting the subsistence quantity of good i consumed. The variables P_i and $\hat{C}$ denote the price of good i and the household's expenditure respectively.

The first order conditions derived from this problem are:

$$\frac{\beta_i}{p_i [\hat{H}_i - \hat{\gamma}_i]} = \frac{\beta_k}{P_k [\hat{H}_k - \hat{\gamma}_k]} \qquad i,k=1, \ldots, n \qquad \text{(A.3)}$$

and the budget constraint (A.2).

Summing equation (A.3) over k results in a demand system consistent with the LES where the above subsistence $[\hat{H}_i - \hat{\gamma}_i]$, quantity of good i purchased is a constant share of the total amount spent on above-subsistence goods. That is:

$$P_i [\hat{H}_i - \hat{\gamma}_i] = \beta_i \sum_{k=1}^{n} P_k [\hat{H}_k - \hat{\gamma}_k] \ . \tag{A.4}$$

Aggregating equation (A.4) over Q identical households gives:

$$QP_i [\hat{H}_i - \hat{\gamma}_i] = \beta_i \sum_{k=1}^{n} P_k [\hat{H}_k - \hat{\gamma}_k] Q \tag{A.5}$$

which may be expressed more compactly as:

$$D_i = \frac{\beta_i E}{P_i} \tag{A.6}$$

where $D_i = Q [\hat{H}_i - \hat{\gamma}_i]$, and

$$E = \sum_{k=1}^{n} P_k [\hat{H}_k - \hat{\gamma}_k] Q \ .$$

The variable D_i denotes the total above-subsistence *quantity* of good i demanded and E denotes the total above-subsistence (i.e. supernumerary) expenditure on all goods.

Total demand X_i for good i may therefore be expressed as:

$$X_i = Q\hat{H}_i = D_i + Q\hat{\gamma}_i \ . \tag{A.7}$$

Using (A.6) to eliminate D_i from (A.7) gives:

$$X_i = \frac{\beta_i E}{P_i} + Q\hat{\gamma}_i \ . \tag{A.8}$$

To tie down the value of E equation (A.8) is multiplied by P_i giving total expenditure on good i:

$$P_i X_i = \beta_i E + P_i \hat{\gamma}_i Q \ . \tag{A.9}$$

Using equation (A.7) the level C of total household expenditures is given by:

$$C = \sum_{i=1}^{n} P_i X_i \tag{A.10}$$

so that

$$E = C - \sum_{i=1}^{n} P_i \hat{\gamma}_i Q \quad . \qquad \text{(A.11)}$$

In this implementation of the model the linearised forms of equations (A.9) and (A.10) replace equation (14.24) in DPSV (1982). Thus the new equations are:

$$x_i = A_i (e - p_i) + (1 - A_i)q \quad . \qquad \text{(A.9)}'$$

and

$$c = \sum_{i=1}^{n} S_i (x_i + p_i) \quad . \qquad \text{(A.10)}'$$

The lower-case representations in equations (A.9)′ and (A.10)′ denote percent changes in the variables previously defined whilst upper-case representations denote shares with

$$A_i = \frac{D_i P_i}{X_i P_i} \quad ,$$

and

$$S_i = \frac{X_i P_i}{C} \quad .$$

Here, A_i is the above-subsistence share of expenditure on good i and S_i is the share of expenditure on good i in total expenditure.

Calculation of the A_i's at first sight appears to be problematic because it requires division of the input-output consumption flows into separate subsistence and above-subsistence components (i.e. $QP_i \hat{\gamma}_i$ and $D_i P_i$ respectively). However, since it is assumed that domestic/import and basic/purchasers' price ratios are the same for both subsistence and above-subsistence consumption it is possible (and more efficient) to deduce the A_i's using estimates of ω, the Frisch parameter, and ε_i, the expenditure elasticities (see Powell (1974)) as follows:

$$\varepsilon_i = \frac{\beta_i C}{P_i X_i}$$

and

$$-\omega = \frac{C}{E} \ ;$$

thus

$$-\frac{\varepsilon_i}{\omega} = \frac{\beta_i E}{P_i X_i} = \frac{D_i P_i}{P_i X_i} = A_i \quad .$$

The A_i's must now be recognized as coefficients that vary rather than parameters and updated according to the formula

$$A_{i}\_updated = A_i [1 + (e - p_i - x_i)/100] \quad .$$

Appendix B Derivation of the Equilibrium Condition for Sectoral Capital Demands

This appendix is concerned with the derivation of the equilibrium conditions for sectoral capital demands (i.e. equation (2.2.10)) described in section 2.2.3.

The Lagrangian expression from which these equilibrium conditions are obtained is formed by combining the firm's objective function given by equation (2.2.4) with the constraints (2.2.5) and (2.2.6) as follows:

$$\mathcal{L} = \sum_{t=1}^{T} \left\{ \frac{L_t}{\prod_{u=1}^{t} [1+RR_{j,u}]^{L_u}} (W_{j,t} \bullet F_{j,t} + \Pi_{j,t}\ I_{j,t}) + \Gamma_{j,t} (K_{j,t+1} - K_{j,t} (1-\delta_j)^{L^t} - L_t I_{j,t}) + \Xi_{j,t} (\overline{Q}_{j,t} - f(K_{j,t}, F_{j,t})) \right\} \qquad \text{(B.1)}$$

Partially differentiating expression (B.1) with respect to $I_{j,t}$, $K_{j,t}$ and $F_{j,t}$ and setting the partial derivatives to zero gives:

$$\frac{\partial \mathcal{L}}{\partial I_{j,t}} = \frac{\Pi_{j,t} L_t}{\prod_{u=1}^{t} [1+RR_{j,u}]^{L_u}} - L_t\ \Gamma_{j,t} = 0 \qquad \text{(B.2)}$$

$t=1, \ldots, T$
$j=1, \ldots, J$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t}} = \Gamma_{j,t-1} - \Gamma_{j,t} (1-\delta_j)^{L_t} - \Xi_{j,t} \frac{\partial f}{\partial K_{j,t}} (K_{j,t}, F_{j,t}) = 0 \qquad \text{(B.3)}$$

$t=1, \ldots, T$
$j=1, \ldots, J$

$$\frac{\partial \mathcal{L}}{\partial F_{j,t}} = \frac{L_t W_{j,t}}{\prod_{u=1}^{t} [1+RR_{j,u}]^{L_u}} - \Xi_{j,t} \frac{\partial f}{\partial F_{j,t}} (K_{j,t}, F_{j,t}) = 0 \qquad \text{(B.4)}$$

$t=1, \ldots, T$
$j=1, \ldots, J$

Here, Γ and Ξ denote sequences of Lagrange multipliers and all other variables are as previously defined.

The equilibrium conditions for sectoral capital stocks (B.3) may be solved using the values of the Lagrange multipliers from (B.2) and (B.4). From equation (B.2),

$$\Gamma_{j,t} = \frac{\Pi_{j,t}}{\prod_{u=1}^{t} [1+RR_{j,u}]^{L_u}} \qquad \text{(B.5)}$$

and

$$\Gamma_{j,t-1} = \frac{\Pi_{j,t-1}}{\prod_{u=1}^{t-1} [1 + RR_{j,u}]^{L_u}} . \qquad (B.6)$$

Similarly, using equation (B.4)

$$\Xi_{j,t} = \frac{L_t W_{j,t}}{\prod_{u=1}^{t} [1 + RR_{j,u}]^{L_u} \frac{\partial f}{\partial F_{j,t}} (K_{j,t}, F_{j,t})} . \qquad (B.7)$$

Substituting equations (B.5) — (B.7) into (B.3), making appropriate cancellations and rearranging gives:

$$\Pi_{j,t-1} [1 + RR_{j,t}]^{L_t} - \Pi_{j,t} (1- \delta_j)^{L_t} = \frac{L_t \frac{\partial f}{\partial K_{j,t}} (K_{j,t}, F_{j,t}) W_{j,t}}{\frac{\partial f}{\partial F_{j,t}} (K_{j,t}, F_{j,t})} . \qquad (B.9)$$

The numerator and denominator on the RHS of equation (B.9(may be multiplied by $P_{j,t}$, the price of sector j's output to convert the respective marginal physical products into value terms. That is,

$$\Pi_{j,t-1} [1 + RR_{j,t}]^{L_t} - \Pi_{j,t} (1- \delta_j)^{L_t} = \frac{L_t \, MRPK_{j,t} W_{j,t}}{MRPF_{j,t}} . \qquad (B.10)$$

where,

$$MRPK_{j,t} = P_{j,t} \frac{\partial f}{\partial K_{j,t}} (K_{j,t}, F_{j,t})$$

and

$$MRPF_{j,t} = P_{j,t} \frac{\partial f}{\partial F_{j,t}} (K_{j,t}, F_{j,t}) .$$

The variables MRPK and MRPF denote the marginal revenue products of capital and "other" factors respectively.

Since the typical firm in sector j rents "other" factors (rather than owning them as is the case for capital) it is implicitly assumed that the firm hires units of these "other" factors for a single period. Together with the assumption of competitive factor markets this means that in each period the firm will hire "other" factors up to the point where the marginal revenue product of "other" factors is equal to their price. That is,

$$MRPF_{j,t} = W_{j,t} . \qquad (B.11)$$

Using (B.11), equation (B.10) may be re-written as:

$$\Pi_{j,t-1} [1 + RR_{j,t}]^{L_t} - \Pi_{j,t} (1- \delta_j)^{L_t} = L_t \; MRPK_{j,t} . \qquad (B.12)$$

Shifting the time subscripts ahead by one period and rearranging, equation (B.12) becomes:

$$\Pi_{j,t} [1 + RR_{j,t+1}]^{L_{t+1}} = L_{t+1} \, MRPK_{j,t+1} + \Pi_{j,t+1} (1- \delta_j)^{L_{t+1}} . \qquad (B.13)$$

which corresponds to equation (2.2.10) in section 2.2.3.

Bibliography

Brechling, F. (1975) *Investment and Employment Decisions*, Manchester University Press.

Codsi, G., M. Horridge, K.R. Pearson (1988) "An Implementation of ORANI Using the GEMPACK Program TABLO", *Impact Project Computing Document* No. C8-01, September.

Codsi, George, Ken Pearson and Peter J. Wilcoxen (1990), "Dynamic Models in GEMPACK," *Impact Project Preliminary Working Paper*, University of Melbourne.

Dixon, P.B., B.R. Parmenter and M. Horridge (1986) "Forecasting Versus Policy Analysis with the ORANI Model", *IAESR Working Paper* No. 4/1986.

Dixon, P.B., B.R. Parmenter, A.A. Powell and P.J. Wilcoxen (forthcoming), *Notes and Problems in Applied Equilibrium Economics*, North-Holland, Amsterdam.

Dixon, P.B., B. R. Parmenter, J. Sutton and D.P. Vincent (1982) *ORANI: A Multisectoral Model of the Australian Economy*, North-Holland, Amsterdam.

Dixon, P.B. and B.R. Parmenter (1990) 'Dynamics in ORANI, A Large Scale General Equilibrium Model', *IAESR Working Paper* No. 1/1990. University of Melbourne.

Higgs, P.J., G.A. Meagher, and B.R. Parmenter (1991) "How Accurately can the ORANI-F Model Forecast Industry Performance? A Validation Exercise over the 1980s", Draft Working Paper Graduate School of Management, University of Melbourne.

Horridge, J.M. (1990) "Proposal for Stripped-Down Intertemporal Version of ORANI" Mimeo, Monash University.

Johansen, L. (1960) *A Multisectoral Study of Economic Growth*, North-Holland, Amsterdam.

Lluch, C. (1973) "The Extended Linear Expenditure System", *European Economic Review* 4, No.1.

McLaren, K. (1990) "A Reappraisal of the Neoclassical Approach to Modelling Business Investment", Working Paper 5/90, Department of Econometrics, Monash University.

Meagher, G.A. (forthcoming) "Construction of the 13-Sector Aggregation of the ORANI Database". Draft Research Memorandum, Monash University.

Nickell, S.J., (1978) *The Investment Decisions of Firms*, Cambridge University Press, Cambridge.

Parmenter, B.R. (1988) 'ORANI-F: User's Manual', IAESR Working Paper No. 7, University of Melbourne.

Parmenter, B.R. (1982) "Inter-Industry Analysis" in L.R. Webb and R.H. Allan (eds) *Industrial Economics, Australian Studies*, George Allen and Unwin, Sydney.

Pearson, K.R. (1991) "Solving Nonlinear Economic Models Accurately Via a Linear Representation", *Impact Project Preliminary Working Paper* No. IP-55, University of Melbourne.

Powell, A.A. (1974) *Empirical Analytics of Demand Systems*, Lexington Books, D.C. Heath.

Vincent, D.P., P.B. Dixon and A.A. Powell (1980) "The Estimation of Supply Response in Australian Agriculture: The CRESH/CRETH Production System", *International Economic Review*, Vol. 21, No. 1.

Wilcoxen, P.J. (1989) "Intertemporal Optimization in General Equilibrium: A Practical Introduction", *Impact Project Preliminary Working Paper* No. IP-45. University of Melbourne (December).