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MAIDS UNDER ADDITIVE PREFERENCES: SOME EARLY ESTIMATES

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ABSTRACT

Working's model (1943) yields Engel curves which are a good approximation to observed consumer behaviour over substantial variations in real income; these Engel curves cannot be globally applicable, however, since budget shares are driven outside the [0, 1] interval at large values of real income. Computable general equilibrium (CGE) modelling often needs demand specifications which remain regular over wide variations in variables. Cooper and McLaren's MAIDS demand system (1987, 1988) is such a specification. Here we describe a special case of MAIDS in differential form which synthesises ideas from the Rotterdam School (e.g., Theil (1967)) and from Additive Preferences (Houthakker (1960)) to yield a demand system which is parsimonious in the use of parameters yet regular over a very wide range of variation in real income. The system is successfully fitted to a five-commodity sub-system of Australian consumer expenditure over the period 1953-54 through 1985-86. The Frisch 'parameter' is reinterpreted (along the lines suggested by Sato (1972)) in terms of the average elasticity of substitution.

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MAIDS UNDER ADDITIVE PREFERENCES: SOME EARLY ESTIMATES

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1. *Introduction*

Working's Law (1943) postulates, as an empirical generalization, that the share W_i of a commodity i $(i=1, \ldots, n)$ in the consumption budget is linear in the logarithm of real total expenditure. This insight, revived by Leser (1963) and by Theil and Clements (1987), is important because it supersedes the constancy of marginal budget shares inherent in the Linear Expenditure System (Stone, 1954) and in the Rotterdam model (Barten (1966), Theil (1967)), and has been incorporated into modern approaches to demand analysis; in particular, into the AIDS (Almost Ideal Demand System) model of Deaton and Muellbauer (1980a,b), and into the differential, additive preferences, model of Theil and Clements (1987). These approaches ensure that the budget shares W_i globally add across i to unity; they do not, however, guarantee that individual shares globally lie in the unit interval; that is, that

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$0 \leq W_i \leq 1$ for all i at any positive value of real total expenditure. Indeed, this is one reason why AIDS is only *almost* ideal.

Recent work by Cooper and McLaren (1987, 1988) offers some generalizations of AIDS which do not suffer from the above defect. These are examples of the fractional demand systems first explored by Lewbel (1987). The Modified AIDS, or MAIDS, system of Cooper and McLaren

> "allows the imposition of all regularity conditions [i.e., those listed below] over an extensive region of expenditure-price space..." (Cooper and McLaren (1988), p. 3).

The regularity conditions in question require that the indirect utility function be:

(i) homogeneous of degree zero in nominal total consumption expenditure and in the vector of commodity prices;

(ii) non-decreasing in nominal total consumption expenditure (at fixed prices);

(iii) non-increasing in every price argument (at a fixed value of consumption);

(iv) quasiconvex in prices (at a fixed consumption level).

It is clear from Roy's Identity that satisfaction of (ii) and (iii) is sufficient to guarantee that budget shares never become negative.[1] Such a guarantee is crucial for computable general equilibrium simulations, particularly in situations where a model may be subjected to large shocks.

In this paper we fit MAIDS to Australian national time-series data spanning 1953-54 through 1985-86 (documented in Adams, Chung and Powell (1988)). To keep the number of estimated parameters down, we invoke *additive preferences*, which effectively reduces the number of substitution parameters to be estimated to just one (the Frisch 'parameter'). Also, the model is estimated in first difference form (rather than the levels) -- this allows us to avoid estimating one subset of MAIDS parameters.

The remainder of this paper is structured as follows. In Section 2 we describe the version of MAIDS which we fit. In Section 3 we give some brief background material on the data, which are tabulated in Adams, Chung and Powell (1988). In Section 4 the econometric estimates, obtained by full-information-maximum-likelihood (FIML) methods, are reported. In Section 5 we offer concluding remarks and a perspective for future research.

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1 Of course, condition (ii) needs to be strengthened to "increasing in nominal total expenditure" to make Roy's Identity operational.

2. *A Differential Form of MAIDS Under Additive Preferences*

2.1 *The MAIDS System*

MAIDS (Cooper and McLaren (1988), p. 8) is generated from an underlying cost or total expenditure function $M(u,P)$ which satisfies:

$$(2.1) \qquad \ln M(u,P) = \ln \Pi_1 + u\,\Pi_2/M(u,P) \quad ,$$

in which M is the household's total spending on commodities; u is the utility level attained; P is the n-vector of commodity prices; and Π_1 and Π_2 are arbitrary concave functions, both homogeneous of first degree in P. The indirect utility function dual to (2.1) is:

$$(2.2) \qquad U(M,P) = [\ln(M/\Pi_1)] \; [M/\Pi_2] \quad .$$

As Cooper and McLaren point out, property (i) listed above is guaranteed by the first-degree homogeneity of Π_1 and Π_2, while (ii) and (iii) will be satisfied provided:

$$(2.3) \qquad M \geq \Pi_1 \quad .$$

(In our work below, we will insist that (2.3) be satisfied.) Moreover, if:

$$(2.4) \qquad \Pi_1, \Pi_2 \text{ are concave in } P \quad ,$$

then, over the region defined by (2.3), property (iv) above also holds (see Cooper and McLaren (1988), p. 8). Applying Roy's Identity to (2.2), they obtain the following equation for the i^{th} budget share:

$$(2.5) \qquad W_i = \frac{\varepsilon_{1i} + \varepsilon_{2i} \ln (M/\Pi_1)}{1 + \ln (M/\Pi_1)} ,$$

where the ε_{1i} and ε_{2i} are the following elasticities:

$$(2.6) \qquad \varepsilon_{ji} = \partial \ln \Pi_j / \partial \ln P_i \;. \qquad (j=1,\ 2;\ i=1,\dots,\ n)$$

Notice that the first degree homogeneity of Π_1 and Π_2 implies:

$$(2.7) \qquad \sum_{i=1}^{n} \varepsilon_{1i} = 1 = \sum_{i=1}^{n} \varepsilon_{2i} \;.$$

Moreover, as Cooper and McLaren point out (1988, p. 9), restricting the elasticities ε_{ji} to be non-negative is sufficient, via (2.5), to guarantee that each budget share lies in the unit interval (0, 1).

2.2 *Special Assumptions*

For our application below, we will work always in per capita terms. Thus, from now on, M denotes per capita nominal total expenditure by the representative household. We make the following four assumptions:

(2.8a) Apart from scaling, Π_1 is the implicit price deflator for consumption. Its use to deflate nominal per capita consumption expenditure produces an index Q of real consumption expenditure per head; thus

$$Q = M/(\Pi_1) \quad , \quad \text{where} \quad \Pi_1 = \overset{\circ}{\Pi}_1 \; e^{\alpha}$$

in which α is a scaling parameter and M, Π_1°, and Q, respectively, are nominal per capita consumption, the implicit price deflator and (a rescaled version of) per capita consumption in constant prices. (The raw data on real per capita consumption below will be written Q°.)

(2.8b) $\Pi_2 = \gamma \prod_{i=1}^{n} P_i^{\varepsilon_{2i}}$ [i.e., Π_2 is Cobb-Douglas];

(2.8c) the direct utility function is directly additive; and

(2.8d) the Frisch parameter ϕ is an absolute constant.

The last of these assumptions is almost tantamount to assuming that the average elasticity of substitution (over different pairs of commodities) σ does not vary in response to changes in income and prices. [2]

We note that the definition of Π_1 implies that its log differential is:

$$(2.9) \qquad \pi_1 = m - q,$$

where in (2.9) (and from now on) lower case letters indicate log differentials of the variables denoted by the corresponding upper case letters. Next, using Q_i to denote the quantity of i consumed, and following the differential approach to demand analysis (e.g., Theil (1967)), we note that the change in the i^{th} budget share is:

$$(2.10) \qquad w_i = p_i + q_i - m \ .$$

Now

$$(2.11) \qquad q_i = \sum_{j=1}^{n} \frac{\partial \ln Q_i}{\partial \ln P_j} p_j + \frac{\partial \ln Q_i}{\partial \ln M} m \ .$$

Partitioning the price elasticity $\partial \ln Q_i / \partial \ln P_j$ into substitution and income effects, (2.11) becomes:

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2 See the Appendix.

$$(2.12) \qquad q_i = \sum_{j=1}^{n} (\eta_{ij}^{subst} - W_j E_i) p_j + E_i m \ .$$

The term on the right of (2.12) carrying the superscript "subst" is the utility-compensated derivative of $ln\ Q_i$ with respect to $ln\ P_j$, while E_i is the i^{th} total expenditure elasticity $\partial ln\ Q_i / \partial ln\ M$. Using (2.9) and substituting from (2.12) into (2.10), we obtain:

$$(2.13) \qquad w_i = (p_i - \pi_1) + \sum_{j=1}^{n} (\eta_{ij}^{subst} - W_j E_i) p_j$$

$$+ (\pi_1 + q) E_i - q \ .$$

Under directly additive preferences, the substitution term may be written (Theil (1967), pp. 197-198):

$$(2.14) \qquad \sum_{j=1}^{n} \eta_{ij}^{subst} p_j = \phi E_i (p_i - \sum_{j=1}^{n} W_j E_j p_j),$$

where ϕ is the reciprocal of the elasticity of the marginal utility of total expenditure with respect to total expenditure (the Frisch 'parameter'). Under MAIDS, we find from (2.5) that the total expenditure elasticities E_i are :

$$(2.15) \qquad E_i = (W_i R + \varepsilon_{2i}) / [(1 + R) W_i] \ ,$$

where

$$(2.16) \qquad R \equiv \ln (M/\Pi_1) \equiv \ln Q = \ln M - \ln \overset{\circ}{\Pi}_1 - \ln \alpha \ .$$

Noting that if $(p_i - \pi_1 - q)$ is subtracted from both sides of (2.13), the left-hand variable of the new equation is just:

$$(2.17) \qquad q_i \equiv w_i - (p_i - \pi_1) - q,$$

and substituting from (2.14) and (2.15) into the new equation, we obtain:

$$(2.18) \qquad q_i = \frac{\phi(W_i R + \varepsilon_{2i})}{(1+R)\, W_i} \left\{ p_i - \sum_{j=1}^{n} \frac{W_j R + \varepsilon_{2j}}{(1+R)} p_j \right\}$$

$$- \frac{W_i R + \varepsilon_{2i}}{(1+R)\, W_i} \left\{ \sum_{j=1}^{n} W_j p_j - \pi_1 \right\} + \frac{W_i R + \varepsilon_{2i}}{(1+R)\, W_i} q \ .$$

$$(i = 1,\ldots,n)$$

The three right hand terms of (2.18) may be interpreted as follows: the first term captures substitution effects at a fixed real level Q of consumption per head, and the last measures the response of consumption of i to a change in real per capita income at fixed relative prices. The middle term is of second order in comparison to the first and last -- it is a correction due to the divergences (presumably small) between movements in the implicit price deflator Π_1 and in the Divisia index P^{Div} defined by:

$$(2.19) \qquad d \ln P^{Div} \equiv \sum_{j=1}^{n} W_j \, d \ln P_j \equiv \sum_{j=1}^{n} W_j \, p_j \, .$$

Notice that the system (2.18) is not subject to an exact linear constraint of the type that usually leads, in demand systems, to the dropping of one equation before estimation. For suppose that we add an explicit error term u_i to the right of (2.18), then multiply each side by the budget share W_i and sum over all commodities. The resultant equation, after restoring time subscripts, is:

$$(2.20) \qquad q_t^{Div} = \pi_{1t} - p_t^{Div} + q_t + \sum_{i=1}^{n} W_{it} \, u_{it}$$

in which q_t^{Div} and p_t^{Div} are the Divisia quantity and price indexes, respectively. It is usual at this point in the analysis of a demand system to find that all variables in the analogue of (2.20) are predetermined except for the sum $\sum_{i=1}^{n} W_{it} \; u_{it}$: it is then concluded that this sum must similarly be predetermined (and usually with a value of zero); and so, that the rank of the covariance matrix of the u's can be at most $(n\text{-}1)$. Equation (2.20) differs from the standard case, however, because the Divisia quantity index q_t^{Div}

can in principle be regarded as a current endogenous variable. This is because no accounting identity is violated if we assume that:

$\sum_{i=1}^{n} W_{it}\, u_{it}$ is non-zero.

To make this a little clearer, write q_{it} as the sum of a systematic part and its random error component u_{it}:

$$(2.21) \qquad q_{it} = q_{it}^{sys} + u_{it} \;;$$

then, with levels of W_{it} treated as predetermined in our differential demand system, we have:

$$(2.22) \qquad q_t^{Div} \equiv \sum_{i=1}^{n} W_i\, q_{it} \equiv \sum_{i=1}^{n} W_i\, q_{it}^{sys} + \sum_{i=1}^{n} W_{it}\, u_{it} \,.$$

Comparing (2.20) with (2.22), we find the following identity for the systematic part of the Divisia quantity index:

$$(2.23) \qquad q_t^{Div(sys)} = \pi_{it} - p_t^{Div} + q_t \,.$$

Note, however, that our system of n equations does not contain the variable $q_t^{Div(sys)}$. Provided we make the following additional assumption:

(2.8e) Π_1 is *not* such that $d \ln Q$ has the Divisia form, then we may treat the variance-covariance matrix of the u's as having full rank. In practice the above assumption seems very likely to apply (since the official Statistician does not, as a rule, seek to produce a Divisia index when compiling statistics of consumption at constant prices).

2.3 *Discrete-time Analogue*

The discrete-time analogues of the variables appearing in (2.18) selected for empirical work were as follows:

(2.24a)	q_i	$\leftrightarrow$	$\ln(\dot{Q}_{it} / \dot{Q}_{it-1})$
(2.24b)	$W_i R$	$\leftrightarrow$	$\bar{W}_{it} \ \frac{1}{2}(\ln \dot{Q}_{t-1} + \ln \dot{Q}_t) - \bar{W}_{it}\, \alpha$
(2.24c)	W_i	$\leftrightarrow$	$\bar{W}_{it} \equiv \frac{1}{2}(W_{it-1} + W_{it})$
(2.24d)	$(1 + R)\, W_i$	$\leftrightarrow$	$\bar{W}_{it}$ + RHS of (2.24b) above)
(2.24e)	p_j	$\leftrightarrow$	$\ln(P_{jt}/P_{jt-1})$
(2.24f)	π_1	$\leftrightarrow$	$\ln(\dot{\Pi}_{1t} / \dot{\Pi}_{1t-1})$
(2.24g)	$(1 + R)$	$\leftrightarrow$	$1 + \frac{1}{2}(\ln \dot{Q}_{t-1} + \ln \dot{Q}_t) - \alpha$
(2.24h)	q	$=$	$\ln(\dot{Q}_t / \dot{Q}_{t-1})$.

In these expressions, the subscript t denotes the value of the variable in year t, while the $\overset{\circ}{Q}$'s relate to unscaled values of per capita consumption in constant prices.

After the substitution of these discrete analogues into (2.18) and the appendage of zero mean stochastic terms, this system of equations may be estimated by maximum likelihood. The parameters are ϕ, α, and the $\{\varepsilon_{2i}\}$. Since the last-mentioned add to unity, there are $(n + 1)$ free parameters. Notice that beyond just first degree homogeneity, we have maintained an agnostic stance on the functional form of Π_1 -- its elasticities $\{\varepsilon_{1i}\}$ are in principle *functions* of P (unlike the ε_{2i} which, by virtue of the Cobb-Douglas functional form (2.8b), are constants): we make no attempt in our estimations, however, to specify these functions.

3. *Brief Remarks about the Data*

The national time-series data span fiscal years 1953-54 through 1985-86. A considerable amount of splicing and editing was required to obtain consistent series covering so long a period. This is fully documented in Adams, Chung and Powell (1988). Hence, we confine ourselves to a brief recapitulation of the principles followed in this exercise; namely: (A) all index numbers should respect the (price) × (quantity) = (value) identity. (B) Price indexes, to be computed as implicit deflators from data on expenditures in current prices, and in constant prices, should be computed *only* from strictly matched series; i.e., from data for these variables published in the same issue of the same publication. (This is required because substantial revisions of these data are made over time. Mismatched series would produce spurious apparent price variations.) (C) Where the relevant price information can be inferred from more than one matched pair of series, the most recently published matched data are used. (D) Series requiring linking are spliced using an OLS regression through the origin which utilizes all the available overlapping observations. (E) The current-price expenditure data in the final data base is taken from the most recently published statistics. (F) Where accuracy in the value of the level of a variable, and accuracy in its percentage change over time, become competing goals, the latter objective is given precedence (after all, our model is in log changes).

Consistent with these principles it proved possible to obtain a relatively long (33-year) sample on per capita expenditures in current and in constant prices for the following six-commodity split-up of consumption expenditures:

1. Food
2. Tobacco, cigarettes, alcoholic drinks
3. Clothing, footwear
4. Household durables
5. Rent
6. All other expenditures .

Because Rent is largely an imputed item and because, in any event, there are strong arguments for treating its price, rather than its quantity, as endogenous in the short run, in this paper we confine ourselves to the subsystem excluding Rent. Hence, M_t becomes per capita consumption in year t, excluding Rent.

For further details of the manipulations performed, citation of primary sources, and tabulations of the data, see Adams, Chung and Powell (1988).

4. *Econometric Estimates*

The five commodity expenditure system (2.18) was estimated by full-information-maximum-likelihood (FIML) using TSP (version 4.1b) on a VAX computer. The commodities 1 through 5 are identified by the order in which they are listed above in Section 3 (but with Rent removed). The parameters to be estimated were:

(i) the Frisch parameter ϕ ;

(ii) the scaling parameter α ;

(iii) four of the elasticities ε_{21}, ε_{22}, ε_{23}, ε_{24} and ε_{25}

(with the remaining elasticity to be recovered from property (2.7)).

Some difficulty was experienced in locating suitable initial values. To find a good starting point (i.e., one from which convergence could be achieved), we proceeded in four stages:

1. Treating the values of ϕ and $\varepsilon_{21}, \ldots, \varepsilon_{24}$ as 'known', we estimated α, using $\alpha = 7$ as initial value.
2. Using the estimate of α so obtained and the same 'known' values of $\varepsilon_{21}, \ldots, \varepsilon_{24}$ as starting values, we reestimated α and simultaneously estimated ϕ.
3. Taking the estimated values of ϕ and α from the last round as starting values, we estimated simultaneously all of the parameters listed above. This led to a negative estimate of ε_{21}.
4. The value of ε_{21} was set to zero and the remaining parameters reestimated. (A likelihood ratio test revealed that ε_{21} did not differ significantly from zero, the values of the log likelihood function with and without the constraint being 492.031 and 492.141, respectively).

Where did the 'known' values of α and the ε's come from? We took $\alpha = 7$. This value of α was chosen in the light of the requirement (2.3). Using (2.8a), we see that (2.3) implies that:

$$Q^{\circ} e^{-\alpha} > 1 \, , \tag{4.1}$$

where Q° is *unscaled* real per capita expenditure (i.e., $Q^\circ = M/\Pi_1^\circ$);

(4.1) in turn implies that:

(4.2) $$Q^o_{min} > e^\alpha,$$

where Q^o_{min} is the smallest value of total real per capita consumption expenditure (excluding Rent) for which the model is expected to 'work' (in the sense of staying in a regular region). Then we could take Q^o_{min} equal to the smallest sample value of Q° (namely, the value in 1953-54, which was 2,428 in 1979-80 dollars). From (4.2) this yields $\alpha = 7.795$ as a maximum value; in practice we used the smaller value $\alpha = 7.00$ (corresponding to a Q^o_{min} of $\$_{1979\text{-}80}$ 1096.6).

The 'known' values of ε_{21}, ..., ε_{24} were calculated from Chung and Powell's (1987) estimates of Working's Law from the same data. These values were:

$$\varepsilon_{21} = 0.0214 \quad \varepsilon_{22} = 0.0455 \quad \varepsilon_{23} = 0.0528 \quad \varepsilon_{24} = 0.1686;$$

they were chosen so as to make the total expenditure elasticities in our system at the sample mid-point (1969-70) the same as those reported by Chung and Powell in column 4 of their Table 8.1.

The results in Table 4.1 demonstrate that there may be practical limitations on the applicability of MAIDS; however, the

Table 4.1

ESTIMATED PARAMETER VALUES FOR 5-COMMODITY SUB-SYSTEM EXCLUDING RENT(a)

Stage(b)	Frisch parameter	Elasticities with respect to individual commodity prices of the price index Π_2					Scaling parameter	Minimum real expenditure for which estimated model is regular(c)
	ϕ	ε_{21}	ε_{22}	ε_{23}	ε_{24}	ε_{25}	α	e^{α}
0	[-0.587]	[.0214]	[.0455]	[.0528]	[.1686]	[.7117]	7.000(d)	[1097]
1	[-0.587]	[.0214]	[.0455]	[.0528]	[.1686]	[.7117]	7.439 (195.4)	1702
2	-0.606 (4.41)	[.0214]	[.0455]	[.0528]	[.1686]	[.7117]	7.436 (541.0)	1695
3	-0.627 (8.54)	-.0422 (0.49)	.0178 (0.55)	.0333 (0.86)	.1919 (4.98)	.7992 (6.20)	7.028 (13.53)	1127
4	-0.626 (2.75)	[.0000]	.0300 (0.44)	.0452 (0.54)	.1783 (4.03)	.7464 (4.05)	7.287 (13.79)	1461

(a) Numbers in round parentheses are absolute values of ratios of maximum likelihood estimate to estimated asympt standard error. Numbers in square parentheses are treated as 'known' constants at the stage of estimation shown.

(b) 'Stage 0' simply allows us to note our starting value of α at stage 1.

(c) Expressed in 1979-80 dollars per head (excluding Rent).

(d) Starting value.

fitted system represented by the last row of Table 4.1 will not cause any regularity problems in CGE simulations involving real total per capita expenditures (excluding rent) exceeding $\$_{1979-80}$ 1461 per annum. Other versions of MAIDS (for example, generalized MAIDS (GMAIDS) -- Cooper and McLaren (1988)) may exhibit regularity over a wider range of variation in *Q*.

Next, we should note that the serial properties of the residuals from our fitted equations are far from ideal. Using the Durbin-Watson statistic (DW) as a descriptive device, our equations yielded:

	Commodity	*DW*
1.	Food	1.24
2.	Tobac., cigs, alcohol	1.09
3.	Clothing and footwear	1.09
4.	Household durables	0.98
5.	Other (excluding Rent)	1.08 .

These results (not surprisingly) are nevertheless much less pathological than those obtained for a version of generalized MAIDS (GMAIDS) fitted *in the levels* to much the same data by Cooper and McLaren (1987, Table 2), where the DWs had the following values:

	Commodity	*DW*
1.	Food	0.58
2.	Tobac., cigs, alcohol	0.15
3.	Clothing and footwear	0.36

4.	Rent	0.18
5.	Other (excluding durables)	0.32 .

How do our estimated 'total expenditure' elasticities (we are referring only to expenditure in the subsystem) compare with those obtained using the same data, the same additive preference specification, but using Working's Law instead of MAIDS? In Table 4.2 we contrast our estimated values at the middle and end of the sample with those obtained by Chung and Powell (1987). Under Working's Law total expenditure elasticities take the form:

$$(4.3) \qquad E_i^{WL} = 1 + \beta_i / W_i \ , \qquad (i=1, \ldots, n)$$

where the β_i are parameters which sum across commodities to zero. Commodities with an elasticity exceeding unity gain share as real total expenditure increases at fixed relative prices; hence, for such 'luxury' goods, elasticities E_i^W approach unity from above as total real expenditure grows without limit. Necessities (with $\beta_i < 0$) experience declining shares as total real expenditure grows; hence, the absolute value of the negative term β_i/W_i gets larger with increasing affluence, and E_i^W for these commodities declines without limit (a defect of Working's Law).

In MAIDS the total expenditure elasticities take the form:

Table 4.2

ESTIMATED ELASTICITIES OF DEMAND WITH RESPECT TO 'CASH'[(a)]

TOTAL EXPENDITURE -- SELECTED IN-SAMPLE YEARS

Year	Elasticity of demand for:				
	Food	Tobacco, cigarettes, alcohol	Clothing, footwear	Household durables	Other (non-rent)
	1	2	3	4	5
1955	.348	.521	.552	1.701	1.654
1970[(b)]	.461 [.37]	.615 [.60]	.693 [.65]	1.548 [1.6]	1.299 [1.3]
1986[(b)]	.522 [.33]	.688 [.52]	.796 [.54]	1.503 [1.7]	1.175 [1.3]

(a) 'Cash' expenditure refers to consumption expenditure on items excluding housing. (A very large component of 'Rent' in the national accounts data is imputed.)

(b) Values shown in square parentheses are comparable values from Chung and Powell (1987), Table 8.1, for 1969-70 and 1985-86, respectively.

$$(4.4) \qquad E_i^{MAIDS} = (R + \varepsilon_{2i}/W_i)/(1 + R) \quad .$$

As total real expenditure grows without limit, so does R, while W_i converges to ε_{2i} (see (2.5)). As a result, E_i^{MAIDS} tends towards unity (as affluence increases) for *every* commodity.

This differential behaviour of the two systems is quite important in explaining the differing time-series behaviour of the estimated total expenditure elasticities: while the differences between E_i^{WL} and E_i^{MAIDS} are not great at the sample midpoint, they increase to substantial size by the end of the sample period. Between these two points of time, real total expenditure per head (for the subsystem) increased by 27 per cent.

Finally, we note that the Frisch parameter once again has demonstrated its characteristic robustness -- the value found by Chung and Powell (1987), namely -0.587, is in general agreement with the value found in Table 4.1 (-0.626). At the sample mid-point this corresponds to an average elasticity of substitution of 0.714 (see the Appendix).

5. *Concluding Remarks*

It has proved possible to estimate a version of MAIDS containing just $n + 1$ free parameters (where n is the number of commodities) from time-series data. This required the maintenance of strong assumptions; given the quality of the data, however, it is doubtful whether a more demanding informational load could reasonably be put on them.

Many areas are ripe for further explorations with MAIDS. With richer time-series data sources (especially panel data) it may prove possible to go beyond the Cobb-Douglas specification for Π_2 and to adopt an explicit functional form for Π_1. However, with cross-sectional data, the lack of price variation and/or availability of price data is likely to mean that the framework adopted in this paper (or some variant of it) is optimal, at least in the case of broad commodity groupings. In particular, the variation of consumption patterns across households is likely to enable the ε_{2i} elasticities to be estimated, since these magnitudes hold the key to the responsiveness of commodity demand to changes in income (see (4.4)). To be sure, an extraneous estimate of the Frisch parameter will be needed, but as we have seen above, such estimates are easily obtained from time-series estimation.

In the cross-section work planned for the 1984 Australian Household Expenditure Survey there is scope to reformulate the $\varepsilon_{2i}s$ as functions of demographic variables. Approaches to this kind of problem are given in the pioneering work of Barten (1964), in Jorgenson, Lau and Stoker (1982) and Jorgenson, Slesnick and Stoker (1983). A particularly promising approach is the recent work of Chung (1987).

As far as the current thread of time-series work goes, four items are on the agenda:

1. Restoring Rent to the overall system, probably along the lines investigated by Adams, Chung and Powell (1988), who treat the rental price as an endogenous variable and the flow of rental services as predetermined.

2. Some further disaggregation of commodities, probably to the 16-item level for which official data are available from 1969-70 on.

3. In this world where Engel curves are both more flexible than in Working's model and better integrated with demand theory, it would be desirable to reexamine the Frisch (1959) conjecture (which was confirmed in international comparisons work by Lluch, Powell and Williams (1977), but refuted by more recent work by Theil and Clements (1987, pp. 68-70)). That is, ϕ should be allowed to vary with real income and the strength of the estimated empirical relationship (if any) between ϕ and Q should be investigated.

4. Investigating slightly more flexible versions of MAIDS, such as GMAIDS (Cooper and McLaren (1988)).

APPENDIX: THE SATO INSIGHT

Sato (1972) reinterpreted $\phi = -1/\omega$ as the average elasticity of substitution. This avoids the cardinality inherent in the interpretation of ω as a welfare indicator and so may be a preferred option.

We have not been able to establish Sato's conjecture exactly; what we can show is that $-\phi$ is, to a first approximation, a weighted average, over all pairs of commodities, of the Allen-Uzawa partial substitution elasticities σ_{ij}. To establish this we proceed as follows. First, we note that

$$(A1) \qquad \sigma_{ij} = -\phi \, E_i E_j \; , \qquad (i \neq j)$$

where E_i is the total expenditure elasticity of i, and that

$$(A2) \qquad \sum_{j=1}^{n} W_j \, E_j = 1.$$

The share-weighted average of substitution elasticities over all pairs involving commodity i is:

$$S^i = \sum_{j \neq i} \frac{W_j}{(1 - W_i)} \, \sigma_{ij}$$

$$= \frac{-\phi E_i}{(1 - W_i)} \sum_{j \neq i} W_j E_j$$

$$\text{(A3)} \qquad = \frac{-\phi E_i}{(1 - W_i)} (1 - W_i E_i) \quad .$$

[Notice that the weights $W_j/(1 - W_i)$ in the first line of (A3) do sum to unity over $i \neq j$.] We would like to show that a weighted average of the S^i is equal to $-\phi$. We have not been able to do this. However, consider the following positive linear combination of the S^i:

$$\text{(A4)} \qquad \sum_{i=1}^{n} \frac{W_i(1 - W_i)}{(1 - E_i W_i)} S^i$$

$$= \sum_{i=1}^{n} \sum_{j \neq i} \frac{W_i W_j}{(1 - E_i W_i)} \sigma_{ij}$$

$$= \sum_{i=1}^{n} \frac{W_i}{(1 - E_i W_i)} \sum_{j \neq i} W_j \sigma_{ij}$$

[from (A1) and (A2)]

$$= -\phi \sum_{i=1}^{n} E_i W_i = -\phi \quad .$$

Thus $-\phi$ is indeed a weighted *sum* over all i of the weighted average of substitution elasticities of other commodities with i. The weights, however, do not (quite) sum to unity; rather, they sum to ψ, where:

$$(A5) \qquad \psi = \sum_{i=1}^{n} \frac{W_i(1 - W_i)}{(1 - E_i W_i)} .$$

Consider the following two (arbitrary) examples (with $n = 3$):

Example 1				Example 2			
W_i	$\sum_{j \neq i} W_j$	E_i	$W_i E_i$	W_i	$\sum_{j \neq i} W_j$	E_i	$W_i E_i$
0.3	0.7	0.46*	0.14	0.3	0.7	1.63*	0.49
0.5	0.5	1.00	0.50	0.5	0.5	0.30	0.15
0.2	0.8	1.80	0.36	0.2	0.8	1.80	0.36
1.0	2.0		1.00	1.00	2.00		1.00
$\sum_{i=1}^{3} \frac{W_i(1 - W_i)}{(1 - E_i W_i)} = 0.9942$				$\sum_{i=1}^{3} \frac{W_i(1 - W_i)}{(1 - E_i W_i)} = 0.9559$			

Whilst we have not established the upper and lower bounds for

$$\sum_{i=1}^{n} \frac{W_i (1 - W_i)}{(1 - E_i W_i)} \quad ,$$

in most examples the sum is likely to be close to 1.

Ex post one could compute

$$\text{(A4)} \qquad \sigma = -\phi \Big/ \sum_{i=1}^{n} \frac{W_i (1 - W_i)}{(1 - E_i W_i)}$$

at some set of co-ordinates, and report this estimate as "the average substitution elasticity" at these coordinates. The weights implicit in this definition would be:

$$\text{(A5)} \qquad N_{ij} = \frac{W_i W_j}{(1 - E_i W_i)} \Big/ \sum_{k=1}^{n} \frac{W_k (1 - W_k)}{(1 - E_k W_k)} \quad ,$$

so that

$$\text{(A6)} \qquad \sigma = \sum_{i=1}^{n} \sum_{j \neq i} N_{ij} \, \sigma_{ij} \quad .$$

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