

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

A NON-LINEAR PROGRAMMING APPROACH TO FLOODPLAIN MANAGEMENT WITH NON-STRUCTURAL ALTERNATIVES*

Jawahar L. Kaul and Cleve E. Willis

INTRODUCTION

There is apparently a growing awareness of the role of non-structural measures as an important part of an overall flood damage reduction program. This awareness has come in part with the realization that structural measures often provide a false sense of security to floodplain occupants and can, then, result in increased food damages — contrary to their intended purpose. To be sure, restrictions prohibiting all development in flood prone areas is a polar case (which will eliminate all damages). There are no a priori reasons to believe that all uses should be prohibited from all floodplain areas. Some of these areas can, in fact, be put to economic use by land use management such that benefits derived outweigh costs associated with such development.

The objectives underlying this investigation, then, are (i) to develop a methodology useful to planners at several levels for efficient floodplain management, considering both structural and non-structural measures and (ii) to demonstrate the usefulness of the methodology by applying it to a selected floodplain in the Connecticut River Basin.

Floodplain Management Methodologies

Few empirical efforts have been directed at providing a comprehensive methodology for floodplain management. James [1967] made the first such attempt. His approach sought the least cost combination of flood control measures

by systematically comparing totals of measured costs and residual damages for a number of discrete combinations of alternatives defined by kind and designed level of protection.

Day [2, 3] provided the first application of operations research methods to the problem of floodplain land use management. His efforts took the form of a recursive linear programming solution to "optimal" land use management of a flood prone area. His framework, however, made no explicit reference to structural measures (other than flood proofing).

Following Day's formulation, Smiarowski, et. al. [8] applied a mathematical (linear) programming technique to provide (conditional) normative decisions regarding choice of land use alternatives ranging over a 25 year planning horizon for a community on the Connecticut River floodplain.

All of the formulations above are subject to some (in some cases, rather severe) shortcomings, most of which are pointed out by the authors. Each formulation was cast in a deterministic mode and hence some rather important aspects of risk and uncertainty were all but ignored. In addition, the demand for land in various uses was presumed in each case to be price inflexible — at least over the relevant range. That is, land values were presumed constant regardless of the quantity developed. The formulation presented here addresses these and other limitations.

Farmington, Connecticut

The floodplain in Farmington (the selected

^{*}The work upon which this report is based was supported in part by funds provided by the USDI Office of Water Research and Technology, as authorized under the Water Research Act of 1964 (Public Law 88-379 as amended). Partial support was also provided by Experiment Station Project No. 344, Paper No. 1060, Massachusetts Agricultural Experiment Station, University of Massachusetts at Amherst.

floodplain for this study) comprises almost 3,000 acres of which approximately 2,176 are undeveloped. Despite the threat of floods, demand for urban development on the floodplain is rather strong. Indeed, part of an industrial park is situated on the floodplain and demand for industrial and commercial uses is intense.

The floodplain was divided into three basic regions on the basis of demand for various land uses. That is, each of these three regions was presumed homogeneous with respect to demand for (and price of) land for the various uses. Each of these basic regions is further subdivided, on the basis of flood frequency, into three zones. These zones provide information regarding probabilities of a flood occurring in a particular year and suggest the relative risk involved if development is permitted. The lowest frequency (risk) zone corresponds to the maximum flood of record [1955], the second reflects land which is expected to be flooded once every hundred years, and the highest risk zone is land which is flooded, on the average, every fifty years.

DECISION FRAMEWORK

A general formulation for floodplain management including some special features to be recognized in any given regional application is provided below.

Activities

The control variables (x_{ijt}) denote the portion of an area of floodplain and non-floodplain land (not necessarily contiguous) i to be devoted to a particular use j in period t (and beyond);

$$i = 1, 2, ..., I; j = 1, 2, ..., J; t = 1, 2, ..., T$$

All land in category i is homogeneous with respect to value and expected flood damages in each use j for all t. For simplicity, further assume that demands for a particular use j in i are unrelated with demands for the same use j in i'; i' ϵ I and i' \dagger i.

Examples of uses j are: residential, industrial, commercial, agricultural and open space. The designation of land to be contained in a particular category i is conditional upon not only location within the floodplain with respect to flood risk but also upon relevant factors 1 accounting for differences in the derived demand for such parcels.

Finally, each variable $(x_{\mbox{ijt}})$ presumes some (optimal) level of flood proofing of structures as well as a given level of structural protection through dams, etc. Willis and Aklilu [1973] provide the methodology for determining the optimal level or amount of flood proofing for a given set of conditions (regarding probabilities of floods of various intensities and market values of structures) such that for any given ijt circumstance, there need be considered only one level of flood proofing. Thus, the flood proofing decisions can be considered as separable. Similarly, each dam (or set of structural measures), in the presence of land use constraints, brings with it an associated benefit in terms of expected damage reductions. These are reflected in the objective function below. Hence, each problem is solved for a particular dam specification, the expected costs of such an undertaking are subtracted from the respective objective function values and the final decisions are made by inspection.

Objective Function

The objective is to select x_{ijt} so as to achieve maximum expected economic rent from the land. The criterion function is expressed, then, as:

(1) Maximize
$$Z = Z (x_{iit})$$
.

More specifically, the objective may be rewritten as:

(1.a) Maximize
$$Z = \sum_{i,j,t} \left(r_{ijt} \sum_{t'=1}^{t} x_{ijt'} \right)$$

where r_{ijt} are measures of economic rent on a per acre basis. The r_{ijt} can be considered as a set of constants (as in Day [1973] and Smiarowski, et. al.[1974]) or as functions of x_{ijt} . The latter approach is more realistic and is adopted in the sections below.

In dealing generally with a less than infinite time horizon (T), terminal conditions must be considered. One means of accomplishing this is to replace the annual rent functions in the terminal period $(r_{ij}T)$ by an inverse demand relationship using expected land price as a proxy variable for the stream of expected economic rents given the conditions on $\sum_{t} x_{ij}t'$

¹Lindsay and Willis [1974], for example, provide an econometric estimation of the influence of such factors as distance to the nearest central city, parcel size, and income levels on the market values of a parcel in a region of Massachusetts.

at T. That is, in the absence of serious market imperfections, the price of land reflects a buyer's expected net returns attributable to the land. This ignores the problems of consumers' surplus and alternative buyers' motives, of course. Support to this measure, however, is given by Day [3] and Gaffney [4].

Constraints

The maximization of the objective function (1) is subject to a set of physical constraints:

$$(2) \quad \sum_{j,t} x_{ijt} \leq b_i \quad , \quad \forall_i$$

That is, each of these I restraints simply requires that no more land is devoted to uses in region i than is physically available (b_i) in that location.

Other constraints may be added, depending upon the specifics of the community preferences in the floodplain under investigation. Minimum restrictions on open space in certain locations, for example, may be added to reflect alternative (non-efficiency) goals.

Special Features

Attention is focused here on several features to be developed more fully in succeeding sections.

Non-linear Objective Function. Since any reasonable representation of the r_{ijt} in the objective function would recognize the likelihood that r_{ijt} are variables which are functions of the decision variables or activities, this representation should appear in the objective functional. The quadratic programming approach provided below presumes the relation-

ship between \mathbf{r}_{ijt} and $\sum\limits_{t'=1}^{t}\mathbf{x}_{ijt'}$ is linear and

continuous over the relevant range.

Externalities. In a larger regional context, the framework provided above and variations to follow permit the internalization of the externality value commonly associated with development of floodplain lands. That is, development along a particular reach of a floodplain may increase damages both above and below its site. For these individuals, such costs would be considered external and hence would be ignored. In a larger regional context, however, these costs are internal. It is

important, then, that these frameworks permit a regional decision-maker to internalize these costs with respect to the zoning decision.

Uncertainty. Especially in such areas as floodplain planning and management, aspects of risk and uncertainty assume considerable importance. While problems associated with providing a comprehensive characterization of all aspects of risk aversion and risk taking preferences and behavior - and characterizing all levels and types of uncertainty - assume monumental proportions, some attempts should be made to recognize and define at least the major areas of uncertainty. These should be formally incorporated into the decision framework. For the quadratic programming framework, for example, this takes the form of entering the estimated covariance matrix of the uncertain r_{ijt} directly into the objective function, weighted by various assumed or estimated risk aversion (or taking) factors.

QUADRATIC PROGRAMMING APPROACH

Emphasis is placed on special considerations named above in setting out a quadratic programming approach to the more general framework outlined in the preceding section.

Objective Function

As indicated earlier, a basic objective is selection of land use patterns and structural measures which maximize present value of economic rents, less costs of structural measures and appropriate penalties for assuming different levels of risk. Economic rent associated with the devotion of one acre of land in i to use j in period t is denoted by r_{ijt} (t = 1, 2, ..., T-1). The present value of the expected stream of economic rents² beginning in the terminal planning period T is denoted by r_{ijT} .

With this designation, one representation of rent (and price) views r_{ijt} as linear functions

of
$$\sum_{t'=1}^{t} x_{ijt'}$$
 Ignoring the stochastic residual

for the moment, this translates to:

(3)
$$r_{ijt} = a_{ijt} + b_{jt} \sum_{t'=1}^{t} x_{ijt'}; t = 1, 2, ..., T,$$

²Recall that we argued earlier that land value (price) can serve as a reasonable proxy for this concept.

where $b_{jt} = b_{jt}$ for t, t' = 1, 2, ..., T-1, and b_{jT} is the slope of the inverse demand function. The assumptions inherent in this representation appear not to be overly restrictive or unrealistic. The formulations presume, for example, that both the intercept and slope of the rent and price functions are specific to use j. The intercept of each depends on t, while slopes are presumed constant over time and across regions (i) for a given use. The influence (for a given j and t) of the investment in various structural measures on rent (price) in some region i is felt via the intercept (aiit). That is, the magnitude of riit reflects in part expected flood damages which, of course, vary across i. We assume, finally, that demands (and rents) for a particular use in i are unrelated with demands (rents) for the same use in i'; $i \neq i'$.

The basic objective function can then be stated rather simply as:

(4) Maximize

$$Z = \sum_{ijt} \left(a_{ijt} + b_{jt} \sum_{t'=1}^{t} x_{ijt'} \right) \sum_{t'=1}^{t} x_{ijt'}$$

or in matrix notation,

(4.a) Maximize
$$Z = Ax + x'Bx$$

where A is the 1 x IJT row vector.

 $\{a_{111} a_{211} \dots a_{I11} a_{121} \dots a_{IJ1} a_{112} \dots a_{IJT}\},$ and x is conformable.

Equation (4) can, of course, be replaced by:

Maximize

$$Z = \sum_{ijt} \left[a_{ijt} \sum_{t'=1}^{t} x_{ijt'} + b_{jt} \left(\sum_{t'=1}^{t} x_{ijt'} \right)^{2} \right]$$

The IJT dimensional symmetric matrix B of (4.a) is, therefore, given as:

$$\mathbf{B} \; = \left[\begin{array}{ccccc} \mathbf{Q_{11}} & \mathbf{Q_{12}} & & \dots & \mathbf{Q_{1T}} \\ \mathbf{Q_{12}} & \mathbf{Q_{22}} & & & & \\ & \ddots & & & & \\ \vdots & & & \ddots & & \\ \mathbf{Q_{1T}} & & \dots & \mathbf{Q_{TT}} \end{array} \right]$$

where Q_{tt} , is the IJ dimensional diagonal matrix defined as:

and $q_{jtt'}$ is the I dimensional scalar matrix of slopes given by $q_{jtt'} = \sum\limits_{t=t}^{T} b_{jt} \quad [^{\delta}_{I}]$ for t=t' and $q_{jtt'} = 2 \sum\limits_{t=t}^{\Sigma} b_{jt} \quad [^{\delta}_{I}]$ for t=t', where $[^{\delta}_{I}]$ is the I dimensional unit matrix. Assuming rent slopes to be independent of planning period $t, \ b_{it} = b_{it'}$ for $t, \ t'=1, \ \ldots, \ T\text{-}1.$

Economic variables are generally stochastic in nature and it is useful to incorporate degrees of uncertainty with respect to unknown prices and rents directly into the decision model. This can be accomplished by respecifying (3) as:

(3.a)
$$\mathbf{r}_{ijt} = \mathbf{a}_{ijt} + \mathbf{b}_{jt} \sum_{t'=1}^{t} \mathbf{x}_{ijt'} + \mathbf{U}_{ijt'}$$

where U_{ijt} are the elements (random disturbances) of the row vector U having a multivariate distribution with E(U) = O and $E(U'U) = \Omega$, where $\Omega = [\sigma_{ij}]$.

In a stochastic context, then (4.a) becomes:

(4.b) Maximize Z = Ax + x'Bx + Ux, whose mathematical expectation remains as Ax + x'Bx, since E(Ux) = xE(U) = O, by assumption. The uncertainty as measured by the variance of Z -- i.e., $(x'\Omega x)$ -- can be incorporated into the decision framework in several ways.

One is to maximize the expectation of (4.b) with a side condition on the maximum allowable variance set at some level V, i.e.,

(4.c) Maximize E(Z)Subject to: $x'\Omega x \le V$.

Although the assumed levels of V can be varied to reveal the trade-offs between expected value (of Z) and risk, a more satisfying approach (and the one used in this investigation) involves entering the risk component directly as an arguement of the objective function. For

example, (4.b) can again be modified as:

(4.d) Maximize
$$Z = Ax + X'(B + \Phi\Omega)x$$
,

where Φ is a scalar which expresses an aversion or attraction to risk. Since there is more evidence of risk averse than risk-taking preferences in these types of decisions, Φ is likely to be negative.

From an individual standpoint, there are grounds (Arrow [1965], Pratt [1964]) to expect aversion to risk to be an increasing function of wealth. However, since in floodplain management it is the "community preferences" which are important, the assumption of some average (scalar) Φ is probably not unwarranted.

Constraints

The objective function (4) is constrained by a number of considerations, some of which vary with the specifics of regional preferences (and legal, political, institutional and social considerations). Some are basic to all regional applications. Physical limitations constitute this latter category.

The most obvious set of restrictions is that for each area (i) and period (t), not more land than is available should be developed. This restraint can be expressed simply as:

$$(5) \qquad \sum_{j,t} x_{ijt} \leq p_{it}, \qquad \forall_i.$$

Second, to insure that not more land is developed for the jth use in t than is available throughout the planning region, we can further require:

(6)
$$\sum x_{ijt} \leq p_{jt}$$
, $U_{j,t}$.

In (5) and (6) the p_{it} and p_{jt} are acre availabilities. Expressions (5) and (6), along with any other region-specific constraints (such as minimum levels of open space, maximum

industrial concentrations, etc.), can be cast in general form as:

(7)
$$Cx \leq P$$
.

EMPIRICAL RESULTS

Empirical results based on application of the model presented in the previous section to the regional situation described above are set out in Tables 1-4 and interpreted below. The whole analysis is carried out using two decision models. One considers land use planning without structural measures, the other flood proofing decisions. In each model, several different runs were made to test the sensitivity of optimal planning decisions due to changes in discount rate and the flood damage coefficient. The effect of uncertainty on the optimal planning decisions are also examined and discussed below. 4

Results of the two decision models are presented in the same table to facilitate the discussion and comparison of results. Tables 1-4 provide decisions by region, use and period for alternative discount rates of 5 and 10 percent and damage coefficients of .052 and .057. The interpretation of these results is straightforward. Assuming a 5 percent rate of discount and a damage coefficient of .052, the model suggested development of 62 acres under single family homes in the 50 year flood zone in Region 1 (without flood proofing) but 153 acres with flood proofing. Apartment complexes were completely restricted from high risk zones in all regions (without flood proofing), but this development was allowed when structures were flood proofed. Tables 2, 3 and 4 provide similar results for slightly higher damage coefficients and higher discount rates. The solution vectors were insensitive to the change in the rate of discount at least in the range considered but sensitivity was reflected when the damage coefficient was changed.

 $^{^3}For$ a given flood depth d, the flood damage to a structure bears a constant proportion to its market value M. It is calculated by the relation $D_j = a_h M_j d$, where the constant of proportionality a_h is called the damage coefficient. See Willis and Aklilu [1973].

⁴The results of these various runs of the two models, as well as data for all these models, are available on request.

Table 1. LAND USE MANAGEMENT DECISIONS (ACRES ZONED) BY REGION, USE, AND PERIOD ASSUMING 5 PERCENT DISCOUNT RATES, DAMAGE COEFFICIENT (a_h) OF 0.052 AND RISK AVERSION (Φ) = 0.0*

		Planning Period							
		1			2				
	Land Use	50 Year	100 Year	500 Year	50 Year	100 Year	500 Year		
Region	Activity	Flood	Flood	Flood	Flood	Flood	Flood		
i	j	Zone	Zone	Zone	Zone	Zone	Zone		
	1	62.38		59.54	42.08				
		(152.85)	(8.57)	(2.58)					
	2		39.12	92.03		5.88	19.02		
		(125.46)			(42.54)				
1	3			15.87					
•	J				(27.30)				
	4			14.13			6.41		
	4	(15.00)			(15.00)				
	5	1057.54							
		(783.85)	(36.43)	(204.42)					
	1		4.00		11.30				
		(73.00)							
	2		2.00	18.00					
		(20.00)							
2	3								
- 1		(16.10)			(27.70)				
	4								
	5 1 2 3	458.70							
		(333,20)	(6.00)	(18.00)					
			101.63	7.81	17.56				
3			(25.77) 6.37	(2.69)		(54.86)	(43.69)		
		(41.10)	0.37	39.63		(4,00)			
		(41.10)		23.09		(4.90)			
		(38.90)							
	4 .	(30.90)		.88					
		268.00							
	5	(214.98)				(19.40)			
		(214.90)				(19.40)			

^{*}Numbers in parentheses indicate decisions assuming flood proofing of structures and contents.

Table 2. LAND USE MANAGEMENT DECISIONS (ACRES ZONED) BY REGION, USE, AND PERIOD, ASSUMING 5 PERCENT DISCOUNT RATES, DAMAGE COEFFICIENT (a_h) OF 0.057 AND RISK AVERSION $(\Phi) = 0.0*$

		Planning Period							
			1		2				
	Land Use	50 Year	100 Year	500 Year	50 Year	100 Year	500 Yea:		
Region	Activity	F1ood	Flood	Flood	Flood	Flood	Flood		
i	j	Zone	Zone	Zone	Zone	Zone	Zone		
	1	108.09 (106.96)	(12.59)	55.91 (44.45)					
	2	(51.70)	34.13 (19.50)	60.66	(43.12)	10.87 (12.91)	62.34 (26.10)		
1	3				(23.69)		14.13		
	4	(11.67)		13.96	(8.67)				
-	5	1053.91 (916.19)		(121.77)					
	1	45.62 (52.67)	4.00 (6.00)	(14.33)	23.00				
	2	(20.00)	2.00	18.00					
2	3	55.00 (1.86)			(28.18)				
	4								
	5	346.00 (367.29)		(3.67)					
3	1	39.95	80.33 (70.68)	4.75 (56.32)	1.96				
	2	(20,41)	19.75 (18.31)	26.25 (7.29)					
	3	(53.14)			(3,13)		40.87		
	4	(3.33)		1.04		7.92 (6.33)	7.08		
	5	268.00 (238.93)		(16.40)		(12.68)			

^{*}Numbers in parentheses indicate decisions assuming flood proofing of structures and contents.

The model was respecified as (4.d) which incorporated the uncertainty component via Ω . This was done to study the effect of uncertainty on the decision vector. In this model, Φ was assumed to take different values indicating various levels of risk aversion. Some sensitivity did appear when Φ was assumed to take values above 0.02 (a reasonably high level of risk aversion in comparison with related empirical frameworks).

CONCLUSIONS

The purpose of this paper was to identify an area of decision-making which is, or can

reasonably be, in the realm of at least a quasipublic domain. This area has received remarkably little attention with respect to formal decision frameworks and virtually none in the area of mathematical programming procedures. The framework developed could provide information for improved decisions. In some regional situations, the variables designated as controls are not controllable under current institutional arrangements. In these cases, solutions⁵ can provide implications (opportunity costs) associated with the maintenance or alteration of these institutional structures. Finally, the applicability of the framework for an existing regional situation was demonstrated.

⁵For the sizes of problems with which we have been working, computer solution is relatively simple. Actual solution time for the problems reported varied from under six seconds to a maximum of seventeen seconds.

Table 3. LAND USE MANAGEMENT DECISIONS (ACRES ZONED) BY REGION, USE, AND PERIOD, ASSUMING 10 PERCENT DISCOUNT RATES, DAMAGE COEFFICIENT (a_h) OF 0.052 AND RISK AVERSION (Φ) = 0.0*

		Planning Period						
		1			2			
	Land Use	50 Year	100 Year	500 Year	50 Year	100 Year	500 Year	
Region	Activity	Flood	Flood	Flood	Flood	Flood	Flood	
i	j	Zone	Zone	Zone	Zone	Zone	Zone	
		45.08	5,53	65.92	47.47			
	1	(151.59)	(1.51)		(10.90)			
	2		35.27	93.43		4.20	16,46	
		(136.39)			(31.61)			
1	3			9.82				
1	3				(28,23)			
	4			14.17			7.20	
		(15.00)			(15.00)			
	5	1069.44						
			(43.49)	(207.00)				
	1		4.00					
		(71.43)			(1.57)			
	2		2.00	18.00				
		(20.00)			==-			
2	3	(19.37)			(26.77)			
	4							
	5	470.00 (330.85)	(6.00)	(18.00)				
		(330.83)	105.47	16.32	5.21			
	1		(.58)			(81.84)	(44.58)	
	2		2.53	43.47				
3		(44.37)				(1.62)		
	3			11.57				
		(35.62)			~-			
	4			0.83			7.80	
	5	268.00		(75, 42)		(18.45)		
	5	268.00 (214.12)		(35.42)		(18.45)		

^{*}Numbers in parentheses indicate decisions assuming flood proofing of structures and contents.

Table 4. LAND USE MANAGEMENT DECISIONS (ACRES ZONED) BY REGION, USE, AND PERIOD, ASSUMING 10 PERCENT DISCOUNT RATES, DAMAGE COEFFICIENT (a_h) OF 0.057 AND RISK AVERSION (Φ) = 0.0*

		Planning Period						
		1			2			
	Land Use	50 Year	100 Year	500 Year	50 Year	100 Year	500 Year	
Region	Activity	Flood	Flood	F1ood	Flood .	Flood	Flood	
i	j `	Zone	Zone	Zone	Zone	Zone	Zone	
	1	125.79		28.63	9.58			
	1		(16.30)	(43.22)	(104.49)			
	2		39.49	68.57		5.51	54.43	
		(76.06)	(26.98)	(19.61)	(30.13)	(1.72)	(13.50)	
1	3			25.50			16.27	
•					(24.07)			
	4			13.61				
		(13.90)			(11.95)			
	5	1026.63						
	ļ	(901.41)		(130.67)				
	1	44.01 (57.17)			28.99			
	2	(5/.1/)	6.00	14.00				
		(20.00)	(6.00)	(9.83)				
		(20.00)	(0.00)	4.00				
2	3	(.40)			(30.93)			
	4							
		'						
	5	397.00						
		(361.50)		(8.17)		- -		
	1 2 3	34.30	70.01		22.70			
3			(71.55)	(55.45)				
			25.02	20.36		0.62		
		(24.30)	(18.24)	(3.46)				
				25.50			30.09	
		(54.60)		,,-				
	4	;=		1.40		12.35	2.65	
	L	(1.10)		'		(3.05)		
	5	268.00		(24 00)		(15.4)		
		(231.75)		(21.09)		(15.16)	L	

^{*}Numbers in parentheses indicate decisions assuming flood proofing of structures and contents.

REFERENCES

- 11 Arrow, K. J. Aspects of the Theory of Risk Bearing, Helsinki: Yrjo Johnssonin Saatio, 1965.
- [2] Day, John C. "A Recursive Programming Model for Non-Structural Flood Damage Control," Water Resources Research, 6:1262-1271, October 1970.
- [3] Day, John C. "A Linear Programming Approach to Floodplain Land Use Planning in Urban Areas," American Journal of Agricultural Economics, 55:2:165-173, May 1973.
- [4] Gaffney, M. "Ground Rent and the Allocation of Land Among Firms," in *Rent Theory, Problems, and Practices*, North Central Regional Research Publication, 139:30-49, 1962.
- [5] James L. Douglas. "Economic Analysis of Alternative Flood Control Measures," Water Resources Research, 3:333-343, 1967.
- [6] Lindsay, Bruce E. and Cleve E. Willis. "Factors Influencing Land Values in the Presence of Suburban Sprawl," *Journal of the Northeastern Agricultural Economics Council*, 3:1:112-124, May 1974.
- [7] Pratt, J. V. "Risk Aversion in the Small and the Large," Econometrica, 32:1-2:122-136, 1964.
- [8] Smiarowski, Joseph, Cleve E. Willis and John H. Foster. "An Application of Mathematical Programming to Floodplain Land Use Planning," A paper presented at the Annual Meeting of Northeastern Agricultural Economics Council at Rhode Island, June 25, 1974.
- [9] Willis, Cleve E. and Petros Aklilu. "Flood Proofing Decisions Under Uncertainty," Journal of the Northeastern Agricultural Economics Council, 2:2:235-249, October 1973.