AN E-V ANALYSIS OF BEEF CALF BACKGROUNDING SYSTEMS IN GEORGIA

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Markowitz developed the theoretical background for analysis of investments in reference to the mean and variance of returns of the portfolio of all investments (E-V analysis). Subsequently, Freund adapted the E-V model to farm enterprise decisions, and agricultural economists have applied this model to various agricultural firm problems. The studies of Kliebenstein and Scott, Brink and McCarl, Heifner, Buccola and French, and Raikes, Sieck, and Miller are examples of applications to farm enterprise organization, farm commodity marketing, and procurement of farm commodities by agribusiness firms. These studies are concerned largely with either production or marketing decisions; applications to joint production and marketing decisions are more limited. Exceptions include Whitson, Barry, and Lacewell's study of vertical integration and Barry and Willmann's analysis of forward contracting. In addition, Lutgen and Helmers and Persuad and Mapp recently analyzed a limited number of marketing alternatives in conjunction with different enterprise alternatives.

The interaction between production and marketing in the beef backgrounding enterprise in the Southeast is a situation for which E-V analysis could be useful. Beef calves are bought in the fall, fed growing rations, and sold in the winter and spring for finishing in feedlots or on grass. Winter small grain pastures usually are the basis for the growing rations. The feeding period, however, can be varied with different amounts of supplemental feeding so that different purchase and sell months are feasible. Also, animals of different grade, sex, and weight can be fed.[1] Unless the purchase and sale prices of different classes of calves are perfectly correlated, combinations of different feeding plans and/or different types of calves may allow portfolio tradeoffs between risk and returns. The purpose of our article is to present an empirical analysis of the portfolio effects of selected backgrounding activities in Georgia. The analytical focus of the article is a quadratic programming model from which E-V frontiers are estimated under different marketing and production situations.

DATA AND METHOD

The basic formulation of a quadratic program for agricultural decisions is:

(1) maximize $\underline{c}'\underline{x} - \alpha/2\ \underline{x}'V\underline{x}$

subject to

(2) $A\underline{x} \leqslant \underline{b}$, and

(3) $\underline{x} \geqslant \underline{0}$

where $\underline{x}$ is a vector of farm activity levels, $\underline{c}$ is a vector of expected net revenues from $\underline{x}$, $\bar{V}$ is a variance-covariance matrix of net revenues from $\underline{x}$, α is a scaler risk aversion coefficient, A is a matrix of input-output coefficients, and $\underline{b}$ is a vector of resource constraints. If α is known or can be estimated, a specific solution can be derived (Freund). A more general approach is to estimate an E-V frontier with the optimal parametric solutions derived by varying α. This approach is consistent with a range of risk preferences (Markowitz) and was used in our study.

In specifying the quadratic programming model for the study, we made some assumptions about the production situation.[2] Previous research has documented that beef cattle enterprises are common on Georgia farms (Allison) and that they are present in profit-maximizing farm plans (Wise and Saunders). To limit the size of the model in our research, we assumed that the enterprise organization was to include a beef backgrounding enterprise so that the farmer had allocated sufficient land and labor for the activities included in the model and drylot facilities were available. With

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[1]Bradford et al. recently discussed and analyzed the beef backgrounding enterprise in some detail within a production economics framework.

[2]Shurley has more detailed information on the data and method used in our model.

the further assumption of no economies of size in marketing costs from variable number of marketing transactions, we did not specify the resource allocation to the beef backgrounding operation. We used one acre of pasture as the basic constraint of the model to minimize computational problems; the appropriate adjustment for a particular situation could be made by multiplying the solutions by the desired number of acres of pasture.

The activities in the model were defined as one calf of a particular sex and grade bought on the fifteenth day of a specific month, fed a specific ration, and sold on the fifteenth day of a future month. Because of limited time series data on market activity for some sex-grade classifications in Georgia, the study was limited to good and choice steers and good heifers. Purchase and sale weights were assumed to be 400-500 pounds and 700-800 pounds for steers and 300-400 pounds and 500-600 pounds for heifers. For each grade-sex combination, activities were defined for five purchase months, September through January. Because each purchase month had three different sell months four, five, or six months later, 15 activities were defined for each sex-grade combination for a total of 45 production-marketing activities in the model.

In the theory of agricultural production under risk, the standard assumption is that input costs are fixed parameters and that gross revenue is stochastic. This assumption is based on the proposition that input costs are known with certainty at the beginning of the production period but output and output prices are random variables which will not be realized until the production period is completed. Though admittedly all inputs are not utilized at the beginning of the production period, they can be purchased after a production plan is determined so that their quantities and prices are fixed (Dillion). For our study, this standard assumption was slightly altered. Though feed costs and most other input costs can be fixed at the beginning of the production period, the purchase costs of calves that can be purchased at different dates during the production period are random variables under the same logic whereby gross revenue is a random variable.[3] Thus, the stochastic variable for our study was assumed to be the gross profit on a calf—the difference between gross sale revenue and purchase cost—defined as:

$$P_{jt} = P_{sjt}W_{sjt} - P_{bjt}W_{bjt} \tag{4}$$

where P_{jt} is gross profit for activity j in year t, P_{sjt} is sale price for month s in year t for activity j, P_{bjt} is purchase price for month b in year t for activity j, W_{sjt} is sale weight in month s in year t for activity j, and W_{bjt} is purchase weight in month b in year t for activity j. W_{bjt} can logically be assumed to be constant because standard marketing weights were assumed in the study. In contrast, W_{sjt} is likely to be stochastic in a pasture feeding system with fixed rates of supplemental feeding. As a suitable time series of gain in backgrounding systems was unavailable for the study, W_{sjt} was also considered constant.[4]

To estimate the variance-covariance matrix for the study, a time series of gross profits was estimated for each activity with equation 4. Secondary data on prices collected from Georgia auction markets were used for the P_{jt} time series. Secular and cyclical index numbers were constructed and used to detrend the price data before they were used in equation 4 (Shurley and Williams). The variance-covariance matrix, V, was calculated from the time series created with the detrended price data.

The assumptions of the model considerably simplified the expected returns portion of the objective function. Because of the assumption that the firm would have a backgrounding operation, costs associated with the pasture and drylot facilities were fixed as were labor costs and did not need to be specified in the model. Because the activities involved animals of similar size and age, veterinary, marketing, and other miscellaneous costs should be similar for each activity and were also excluded from the analysis.[5] Though supplemental feeding costs and interest on the investment animals would be fixed for each activity, they would vary among activities. Feeding costs varied because of differences in the amount and length of supplemental feeding among activities, and interest varied because of differences in expected purchase cost of calves and length of investment period. The relevant expected returns for each activity were therefore obtained by subtracting feed and interest costs from the gross profits defined in equation 4. Because mean beef prices during the period of analysis were approximated by 1969 prices, the expected returns for 1969 were used in the analysis.

[3]Technically, interest costs are also stochastic because the purchase cost is not known with certainty at the beginning of the production period. However, the assumption that they are fixed is similar to the assumption that harvesting costs are fixed and should create small biases.

[4]The unavailability of a time series for gain is a classic problem in risk analyses of livestock production. Freund makes an assumption similar to ours. Unlike prices for agricultural commodities and yields of crops, livestock gain in weight is difficult if not impossible to estimate from secondary data. Experimental data are generally not available for a sufficiently long time period to be useful. Another potential source is farm record data; Eidman, Dean, and Carter used such data to estimate production uncertainty for turkeys. For our study, some support for the fixed gain assumption was provided by a three-year experiment that showed no significant differences in gain for forage fed beef in Georgia (Utley, Lowry, and McCormick).

[5]This assumption was justified empirically by the similar numbers of animals in the various solutions.

The costs that did not vary among activities would not affect the quadratic program solution and therefore did not have to be included in the expected returns component of the objective function. However, it must be stressed that fixed costs do affect expected returns and therefore affect the utility maximization solution in risky production situations (Dillion). Because of lack of data to estimate these costs for 1969, we did not include them in the results of our study. In the next section, we discuss further the inclusion of fixed costs in the results that would be presented to decision makers.

TABLE 1. PARTIAL LINEAR TABLEAU OF MODEL OF BEEF CALF BACKGROUNDING ACTIVITIES ON WINTER PASTURE

Restrictions	Activities: Choice Steers Sept. - Jan.	Good Steers Sept.-Jan.	Good Heifers Sept. - Jan.	Choice Steers Sept. - Jan.	Constraint level
Pasture 11-12	.33	.33	.33	.50	≤1
Pasture 12-1	.33	.33	.33	.50	≤1
Pasture 1-2				.50	≤1
Pasture 2-3					≤1
Pasture 3-4					≤1
Steer Ratio-9	1.00	- .5		1.00	=0
Steer-Heifer Ratio-9		1.5	- .5		=0
Block			1.00		=0

The constraint matrix and constraint levels used in the model are illustrated in Table 1. A constraint was included for each month of pasture usage; for example, Pasture 11-12 was a constraint for pasture utilization for November 15 to December 15. The stocking rate for each activity was specified with the entry in the respective row in which pasture was required. The amount of supplemental feeding varied directly with the length of feeding period to allow for the assumed constant weight gain. Therefore, the stocking rate varied to reflect the amount of supplemental feeding—0.33 acres per animal for four-month activities, 0.5 acres per animal for five-month activities, and 0.8 acres per animal for six-month activities.

The rest of the constraints in Table 1 were used to model various marketing situations. With no constraints on sex and grade of animals, these rows would be deleted. If the farmer had to purchase mixed lots of good and choice steer calves, Steer Ratio and Block would be used. The particular situation illustrated involves purchases of two thirds good and one third choice steers and no heifers. With Steer-Heifer Ratio and Block, the purchase situation of three quarters good steers and one quarter good heifers could be analyzed; however, the entry in Block would have to be moved to Choice Steer activities to program this situation. A separate Steer Ratio and Steer-Heifer Ratio was included for each purchase month. In Table 1, the inclusion of 9 in the rows reflects September purchases. Purchases in other months would have entries in similar constraints. Though these constraints reflect particular purchase assumptions, alternative assumptions could be incorporated with similar constraints.

RESULTS

The optimal parametric solutions for the three model specifications are presented in Table 2. The E-V frontiers defined by these solutions are shown in Figure 1—the frontier

TABLE 2. OPTIMAL LONG-TERM COMBINED WINTER PASTURE AND DRYLOT BEEF CALF BACKGROUNDING PROGRAMS UNDER VARIOUS HERD COMPOSITION CONSTRAINTS, GEORGIA SOUTHERN COASTAL PLAINS

Unconstrained Herd Composition

Activity[a]: Type of Calf	Interval	Unit	Activity Usage Per Acre for Each Portfolio: L_1	L_2	L_3	L_4	L_5
Choice Steer	Sept. 15 - Jan. 15	hd.	2.11	3.03	3.03	3.03	3.03
Choice Steer	Jan. 15 - May 15	hd.					3.03
Choice Steer	Jan. 15 - June 15	hd.				2.00	
Good Steer	Sept. 15 - Jan. 15	hd.	.92				
Good Steer	Jan. 15 - June 15	hd.	.28	.36	2.00		
Net Return Per Acre[b]		$	68.06	71.79	133.61	137.69	149.76
Variance of Returns		$(\$)^2$	.000061	.000085	3.238149	5.449612	28.400826

67 Percent Good and 33 Percent Choice Steers

Activity[a]: Type of Calf	Interval	Unit	Activity Usage Per Acre for Each Portfolio: LV_1	LV_2	LV_3	LV_4	LV_5
Choice Steer	Sept. 15 - Jan. 15	hd.	1.01	1.01	1.01	1.01	1.01
Choice Steer	Jan. 15 - May 15	hd.				.75	1.01
Choice Steer	Jan. 15 - June 15	hd.		.06	.67		
Choice Steer	Jan. 15 - July 15	hd.	.06				
Good Steer	Sept. 15 - Jan. 15	hd.	2.02	2.02	2.02	2.02	2.02
Good Steer	Jan. 15 - May 15	hd.					2.02
Good Steer	Jan. 15 - June 15	hd.	.11	.11	1.33	1.50	
Net Return Per Acre[b]		$	62.71	63.22	133.34	135.98	142.63
Variance of Returns		$(\$)^2$	.000075	.000078	4.699129	8.209549	22.607823

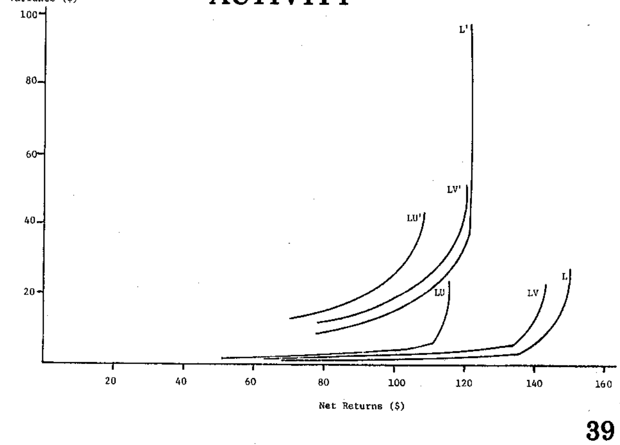
75 Percent Good Steers and 25 Percent Good Heifers

Activity[a]: Type of Calf	Interval	Unit	Activity Usage Per Acre for Each Portfolio: LU_1	LU_2	LU_3	LU_4
Good Steer	Sept. 15 - Jan. 15	hd.	2.27	2.27	2.27	2.27
Good Steer	Jan. 15 - May 15	hd.				2.27
Good Steer	Jan. 15 - June 15	hd.		1.50	1.64	
Good Heifer	Sept. 15 - Jan. 15	hd.	.76	.76	.76	.76
Good Heifer	Jan. 15 - May 15	hd.			.55	.76
Good Heifer	Jan. 15 - June 15	hd.		.50		
Net Return Per Acre[b]		$	45.69	110.58	112.79	115.56
Variance of Returns		$(\$)^2$	.174845	5.592129	8.818622	23.973874

[a]Steers purchased at 450 pounds and sold at 750 pounds, and heifers purchased at 375 pounds and sold at 575 pounds.

[b]Returns to land, labor, management, pasture, and general farm overhead at 1969 prices and feed costs.

FIGURE 1. E-V FRONTIERS FOR THE BEEF BACKGROUNDING ACTIVITY



for the unconstrained solution is labeled L, for the good and choice steer herd LV, and for the steer and heifer herd LU. In general, the activities included in the solutions reflect two time periods of production—purchases in September and January. However, lower risk-returns solutions never include full production in the second period; LU_1 includes no production in that period. Another general observation is that the heifer activities are relatively less favorable than the steer activities with no heifers in the unconstrained solution. Not surprisingly, L dominates LU and LV; LV also dominates LU because of the unfavorable heifer activities in LU.

The results in Table 2 indicate a considerable range in risk-returns tradeoffs in all three situations. These tradeoffs do not arise from purchases and/or marketings in sequential months such as September and October. Rather, the tradeoffs arise from different activities as in L_4 and L_5 and from different activity levels as in L_3 and L_4. In L_1, L_2, LV_1, and LV_2, negative covariances between activities allow achievement of the unusual situation of nearly zero variance.

Comparisons among the different marketing situations suggest an important implication for grading of feeder cattle in the marketing system. Being able to purchase graded calves allows the producer to take advantage of portfolio effects available from seasonal supply and demand differences. The gains can be measured by the higher income available for a given level of variance in L solutions in comparison with LV or LU solutions. Portfolios L_2 and LV_2, which have about the same variance, illustrate these gains; the value of the opportunity to buy graded rather than mixed grade calves can be measured by the difference in net return of $8.57 at the variance level.

The prominence of early purchases and late sales in all the solutions in Table 2 suggests a limitation in the analysis. Both of these periods are important times for crop production in Georgia when the opportunity cost of labor is likely to be higher. The assumption of labor availability from September through June therefore may not be consistent for the diversified farms which would have cropland available for temporary winter pasture. To consider the situation with less competition from crops, the activities with September purchases and May and June sales were deleted from the model and different solutions were obtained for the three marketing situations.[6]

The solutions with deleted activities are presented in Table 3 and Figure 1. The frontiers for the shorter production period are identified with a prime: L′ is the same as L, LU′ the

TABLE 3. OPTIMAL LONG-TERM WINTER BEEF CALF BACKGROUNDING PROGRAMS UNDER VARIOUS HERD COMPOSITION CONSTRAINTS, GEORGIA SOUTHERN COASTAL PLAINS

Activity[a] Type of Calf	Interval	Unit	Activity Usage Per Acre for Each Portfolio		
Unconstrained Herd Composition			L_1'	L_2'	L_3'
Choice Steer	Oct. 15 - April 15	hd.	1.25		
Choice Steer	Nov. 15 - March 15	hd.			3.03
Choice Steer	Nov. 15 - April 15	hd.		2.00	
Net Return Per Acre[b]		$	77.51	121.18	121.76
Variance of Returns		($)^2	9.282422	32.448000	98.458678
67 Percent Good and 33 Percent Choice Steers			LV_1'	LV_2'	LV_3'
Choice Steer	Oct. 15 - April 15	hd.	.42		
Choice Steer	Nov. 15 - March 15	hd.			.75
Choice Steer	Nov. 15 - April 15	hd.		.67	
Good Steer	Oct. 15 - April 15	hd.	.83		
Good Steer	Nov. 15 - April 15	hd.		1.33	1.50
Net Return Per Acre[b]		$	77.75	120.47	120.53
Variance of Returns		($)^2	11.640755	36.520346	50.925082
75 Percent Good Steers and 25 Percent Good Heifers			LU_1'	LU_2'	LU_3'
Good Steer	Oct. 15 - April 15	hd.	.94		
Good Steer	Nov. 15 - April 15	hd.		1.5	1.64
Good Heifer	Oct. 15 - April 15	hd.	.31		
Good Heifer	Nov. 15 - March 15	hd.			.55
Good Heifer	Nov. 15 - April 15	hd.		.5	
Net Return Per Acre[b]		$	70.40	107.83	108.70
Variance of Returns		($)^2	12.580724	36.481750	43.821912

[a]Steers purchased at 450 pounds and sold at 750 pounds, and heifers purchased at 375 pounds and sold at 575 pounds.

[b]Returns to land, labor, management, pasture, and general farm overhead at 1969 prices and feed costs.

same as LU, and LV′ the same as LU′ except for length of production period. In this model, two production periods are infeasible and each solution has purchases in October and November with sales in March or April. The solutions also do not have combination of activities beyond those required to meet the constraints; the October-April activities do not have covariances which allow risk-reducing diversification. However, risk-returns tradeoffs are found among activities as illustrated in Figure 1. The results in Table 3 are less favorable in an E-V context than those in Table 2. This inferiority is apparent in Figure 1 and can also be deduced from the solutions. For example, LU_2 and LU'_2 have similar expected returns but the variance of the latter is more than six times that of the former.

We must stress that the differences in results between the larger and smaller models reflect only risk and returns tradeoffs available from different beef backgrounding activities. The consideration of the portfolio effects of these beef activities with other activities on the diversified farm could result in different conclusions. Expansion of the model to include other farm enterprises would be necessary to evaluate these broader portfolio effects.

For presentation of this analysis to farmers, the results in Tables 1 and 2 and Figure 1 would need modification. First, it would be advisable to convert the results from the one-acre

[6]Similar sensitivity analysis has been used in other recent portfolio research (Levy and Sarnat; Schurle and Erven).

unit to a more realistic pasture acreage. The activity levels and expected returns would be multiplied by the pasture acreage and the variance of returns by the square of pasture acreage. In addition, the costs that were fixed in the analysis—pasture, feedlot, labor, and miscellaneous annual costs—should be subtracted from the expected returns before the E-V results are presented. This process would shift the E-V frontier in Figure 1 to the left and may cause some portfolios to have negative expected returns.

CONCLUSIONS

Our study demonstrates risk-returns tradeoffs in the beef backgrounding systems in Georgia. The most prominent cause of these tradeoffs involves selection of different feeding periods rather than combinations of feeding periods. Further, the study demonstrates portfolio gains from grading feeder calves. Though the risk-returns tradeoffs were more pronounced when the backgrounding activity could be extended over a longer period, tradeoffs still existed for the October-April period. In selecting the appropriate backgrounding enterprise for a farm, consideration therefore must be given to both risk and returns.

Extensions of the study could focus on relaxing the assumptions embodied in the model. The fixed costs in the model should be estimated and included in the results. Given availability of adequate experimental data, the gain variability could be incorporated into the analysis. If data were available, activities with stochastic supplemental feeding plans could also be included. In addition, a larger model which would consider other production and marketing activities would provide a more favorable consideration of the opportunity cost of labor and possible portfolio effects between beef backgrounding and other enterprises. However, such a large model can have undesirable methodological problems in computation and interpretation. In previous studies, which included marketing and production activity for a larger number of enterprises, the size of the model was limited by restricting the set of marketing activities in comparison with that used in our study (Lutgen and Helmers; Persuad and Mapp). A tradeoff in empirical research therefore appears to be between including all appropriate production activities and using a rich set of marketing alternatives for each production activity. Deciding which alternative to select requires consideration of the particular empirical problem of interest.

REFERENCES

Allison, John R. *Beef Production in Georgia. Resources Used and Operator Characteristics.* Georgia Agricultural Experiment Station Res. Bull. No. 131, February 1974.

Barry, Peter J. and D. R. Willmann. "A Risk Programming Analysis of Forward Contracting with Credit Constraints." *Amer. J. Agr. Econ.* 58(1976):62-70.

Bradford, Garnett L., et al. "Comparing Management Systems for Beef Cattle Backgrounding: A Multidisciplinary Approach." *S. J. Agr. Econ.* 10(1978):57-62.

Brink, Lars and Bruce McCarl. "The Tradeoff Between Expected Returns and Risk Among Cornbelt Farmers." *Amer. J. Agr. Econ.* 60(1978):259-63.

Buccola, Steven T. and Ben C. French. "An E-V Analysis of Pricing Alternatives for Long-Term Marketing Contracts." *S. J. Agr. Econ.* 9(1977):17-24.

Dillion, John L. *The Analysis of Response in Crop and Livestock Production,* second edition. New York: Pergamon Press, 1977.

Eidman, V. R., G. W. Dean, and H. O. Carter. "An Application of Statistical Decision Theory to Commercial Turkey Production." *J. Farm Econ.* 49(1967):852-68.

Freund, Rudolf J. "The Introduction of Risk into a Programming Model." *Econometrica* 24(1956): 253-63.

Heifner, R. L. "Determining Efficient Seasonal Grain Inventories: An Application of Quadratic Programming." *J. Farm Econ.* 48(1966):648-60.

Kliebenstein, James B. and John T. Scott, Jr. "Assessment of Risk When Contract Crops are Included Among Other Alternatives." *S. J. Agr. Econ.* 7(1975):105-110.

Levy, Haim and Marshall Sarnat. "International Diversification of Investment Portfolios." *Amer. Econ. Rev.* 60(1970):668-75.

Lutgen, Lynn H. and Glen A. Helmers. "Simulation of Production-Marketing Alternatives for Grain Farms Under Uncertainty." *N. C. J. Agr. Econ.* 1(1979):23-30.

Markowitz, Harry M. "Portfolio Selection." *J. Finan.* 12(1952):77-91.

Persuad, Tillak and Harry P. Mapp, Jr. "Analysis of Alternative Production and Marketing Strategies in Southwestern Oklahoma: A *MOTAD* Approach." Paper presented at annual meeting of Technical Committee of Western Regional Project W-149, Tucson, Arizona, January 16-18, 1980.

Raikes, Ronald, Gail M. Sieck, and Katherine S. Miller. "A Risk Programming Analysis of Cattle Procurement by Beef Packers." *S. J. Agr. Econ.* 10(1978):43-8.

Schurle, Bryan and Bernard L. Erven. "Sensitivity of Efficient Frontiers Developed for Farm Enterprise Choice Decisions." *Amer. J. Agr. Econ.* 61(1979):506-11.

Shurley, W. D. "Determining Optimal Marketing Strategies for Backgrounding Calves in the Georgia Middle Coastal Plain: An Application of Quadratic Programming." Unpublished M.S. thesis, University of Georgia, 1977.

______ and F. W. Williams. *Seasonal Relationships of Feeder Cattle and Calf Prices in Georgia.* Georgia Agricultural Experiment Station Res. Rep. No. 312, May 1979.

Utley, Phillip R., Robert S. Lowry, and William C. McCormick. "Corn Silage Plus Small Grain Pasture for Finishing Steers." *J. An. Sci.* 36(1973):423-7.

Whitson, Robert E., Peter J. Barry, and Ronald D. Lacewell. "Vertical Integration for Risk Management: An Application to a Cattle Ranch." *S. J. Agr. Econ.* 8(1976):1-4.

Wise, James O. and Fred B. Saunders. *Maximizing Operator Returns on Large Commercial Farms in the Beef Producing Areas of the South.* Southern Cooperative Series Bull. 217, June 1977.