TIME VARYING PARAMETERS WITH RANDOM COMPONENTS: THE ORANGE JUICE INDUSTRY

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The assumption of nonstochastic parameters has long been recognized as restrictive to the solution of many marketing problems and to economic modeling in general. Parameter variation historically has been treated with the use of nonstochastic adjustments through interaction variables and the use of proxy dummy and trend variables. Though these empirical techniques in many cases give reasonable results, they presuppose that the researcher can specify the nature of the parameter change. In fact, it may not be obvious that random parameters are part of the estimation problem. Furthermore, specification of structural shifts in parameters is usually difficult. Comparison of parameter changes through techniques such as grouping of data and using various F-tests is most often dependent on the criteria for grouping (Maddala, p. 390-404). Also, the procedure fails to identify the dynamic path of adjustments that must have occurred when various F-tests indicate that parameters have changed. Other approaches to determining structural shifts in parameters may require elaborate search procedures. To limit the extent to which the search is required, restrictive assumptions about many of the parameters are sometimes made (Simon).

Random coefficients can be a particularly important problem in many marketing studies drawing on both cross-sectional and time series data. Demand studies, for example, are often used to evaluate various pricing strategies. Similarly, such studies can be of particular importance for assessing product substitutability. If the pricing parameters are estimated with aggregate data or if they are from a cross-sectional sample, such parameters may have a stochastic component due to differences in the cross-sections that are not measurable. Furthermore, if the same parameters are estimated with time series data, the parameters may change over time, especially for those markets where significant market adjustments have taken place. In both circumstances, policies drawn from fixed estimates may be misleading and statistically questionable.

In the subsequent discussions, we illustrate the problems that can occur when parameters that have changed are, in fact, ignored in the specification of the econometric model. A relatively new set of estimation procedures that explicitly account for adjustments in the parameters over time are considered. Random and systematic variation in parameters has increasing importance to many marketing problems, as we illustrate through an application of the time varying parameters procedures to the processed orange juice industry. Retail demand equations for three forms of processed orange juice are used to illustrate a situation in which parameters have changed over time.

TIME VARYING PARAMETERS

Parameters may be stochastic or nonstochastic as well as time varying. Stochastic time varying parameters further compound the problem in that the dynamic properties of the parameters must be considered (Cooley and Prescott, Oct. 1973; Ward and Myers). Cooley and Prescott's model explicitly accounts for parameters that are both random and time varying. Define the model $Y_t = X_t B_t$ where X_t is a $1 \times (K + 1)$ matrix of K explanatory variables in the time period t and B_t is a $(K + 1) \times 1$ vector of parameters for period t. B may have both stochastic (transitory) and time varying (permanent) components. The Cooley-Prescott model is the most general of the set of time varying models where the permanent component is assumed to follow a moving average process. Define

(1) $B_t = B_t^p + U_t$

(2) $B_t^p = B_{t-1}^p + V_t$

where B_t^p is the permanent component. Error U_t measures the transitory component and with this definition alone (i.e., equation 1) the problem is simply the well-known random coefficient model (Swamy). Equation 2 gives the

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dynamic path of the parameters showing the permanent adjustments over time and V_t is the error associated with the permanent change.

Given a sample size T and using equations 1 and 2, we find that $B^P_{T+1} = B^P_t + \sum_{s=t+1}^{T+1} V_s$ and the vector of unknown parameters in period t can be calculated in terms of a fixed reference period T + 1.

$$(3) \quad B_t = B^P_{T+1} + U_t - \sum_{s=t+1}^{T+1} V_s$$

B^P_{T+1} is estimated for one period beyond the sample period and all parameter estimates for earlier periods are calculated in relation to the parameters estimated for T + 1 (Ward and Myers). The model's error structure immediately follows after substitution of equation 3 back into the base equation where $Y_t = X_tB_t$. After this substitution the base equation has the error term W shown in equation 4.

$$(4) \quad W_t = X_tU_t - X_t \sum_{s=t+1}^{T+1} V_s$$

The covariance for W is obviously complicated because $E(\sum_{s=t+1}^{T+1} V_s \sum_{s=j+1}^{T+1} V_s) \neq 0$ where $t \geqslant j$. The errors (U_t and V_t) are assumed to be independent random variables and the specification of the covariance matrix Cov(W) becomes the most crucial aspect for calculating the time varying parameters. Define

$$(5) \quad \mathrm{Cov}(W) = \sigma^2[(1-\gamma)R+\gamma C]$$

where R is a T × T diagonal matrix with the diagonal element t defined as $X_t \Sigma_u X_t{}'$ and $1 < t < T$. Note that the error structure (i.e., Σ_u) for the transitory component is constant across the sample because each u_{it} is assumed independent, whereas the elements of matrix C (i.e., the permanent component error terms V across the sample size) are not independent. Define C_{ij} as that element in C for time periods i and j; then $C_{ij} = \min(T - i + 1, T - j + 1)\, X_i \Sigma_v X_j{}'$ (see Appendix). The error structure Σ_v is constant across the sample, but the relative weighting changes according to the time periods. Finally, γ gives a direct measure of the importance of the permanent and transitory effects. If $\gamma = 0$ the model is simply transitory and the parameters are not time varying, whereas if $\gamma > 0$ some evidence of time varying adjustments is noted. Once R and C are specified, maximized likelihood procedures can be used to estimate all parameters over values of γ.

Specification of Σ_u and Σ_v is necessary for the estimation. Usually these values are not known *a priori.* Without prior information on U or V, $\Sigma_u = \Sigma_v$ is assumed and the elements are estimated from the Cov (B) by using OLS. Several studies show the time varying results to be robust with respect to changes in the elements of Σ_u (Cooley and Prescott, May 1973; Hsiao; Ward and Myers).

Inclusion of zero elements in Σ_u or Σ_v gives special meaning to the analysis. For example, if $\Sigma_u=\Sigma_v$ and all off-diagonal elements are zero, it is implicitly assumed that there is no covariance structure among the parameters. Also, for covenience all elements can be normalized in relation to the intercept. If all elements in Σ_u and Σ_v were zero except the firt, the estimates would give an adaptive regression model (Cooley and Prescott, June 1973). The traditional random coefficient model occurs when all elements of Σ_v are zero and at least some positive element is present in Σ_u. Finally, for those B parameters not suspected to be time varying, zero values would be inserted in the diagonal matrix elements corresponding to those particular parameters.

Estimation of B_T reduces to a generalized least squares problem with the nonlinear restriction on γ in equation 5. If the primary interest is with the last-period parameter estimate, B_T is calculated with both R and C as defined heretofore. Alternatively, if the time path for the parameters is needed, B_t is estimated with correction for the Cov (W) where $C_{ij} = \min(t-1, t-j)\, X_i \Sigma_v X_j'$. Repeating C_{ij} for each period and correcting for the Cov (W) will give the estimates for each period (see Appendix).

The time varying parameters procedures developed by Cooley and Prescott offer substantial improvements for addressing several economic problems. We illustrate the usefulness of these procedures with an application to the processed orange juice market.

PROCESSED ORANGE JUICE INDUSTRY

The processed orange juice industry includes three major products that are ultimately consumed as orange juice—frozen concentrate (FCOJ), canned single strength (CSS), and chilled orange juice (COJ). Each of these products requires special distribution and marketing functions and they are often consumed by identifiable types of users according to consumer demographic characteristics (Ward and Kilmer).

The three product forms differ considerably with respect to levels of market development. For COJ, in the past 10 years several new brands have been introduced and a shift from glass to paper and plastic packaging has occurred. The packaging shift has facilitated reprocessing of bulk FCOJ into COJ in locations close to population centers. Before 1970 the

COJ market was very small. FCOJ was introduced in the 1950s and was subject to a lower level of market development activity during the data period used in our study. CSS is the oldest product of the three and has not been undergoing significant market changes in the past 10 years.

Two broad economic policy areas are of particular importance to the market development of these products. First, questions relating to pricing and allocation of product to each of the markets require that consumer price responsiveness be understood for each. Furthermore, because of the long-run commitments to marketing programs necessary for developing these markets, the degree of market stability (or dynamics) must be known. Hence, an understanding of the pricing and related parameters is essential to the development and implementation of various marketing programs.

Second, each product provides the consumer a supply of orange juice marketed in a particular form. Processors of these products operate within a basically oligopolistic market structure and the degree of industry concentration depends on which product form is measured and how the industry is defined according to product form (Ward and Kilmer). The vertical linkages between retailers and wholesalers also differ among the processed forms. FCOJ is marketed predominantly under private label contracts with large retail chains. COJ is frequently processed by firms such as large dairies which then distribute to the retail sector. Bulk FCOJ is commonly shipped from the point of initial processing to be reprocessed into COJ. The CSS market is small in relation to the FCOJ and COJ markets. CSS is marketed both under private labels and as brands.

Empirical measurement of the pricing parameters is particularly germane to antitrust issues arising when mergers or acquisitions occur in oligopolistic markets. Specifically, measurements of cross-price elasticities are extremely important. Furthermore, empirically establishing whether or not dynamic parameter adjustments are taking place may become paramount to addressing the entire antitrust problem. Recent merger activity between distributors and processors has increased the attention given to these issues. If COJ and FCOJ are close substitutes, market concentration, say in the COJ market, may not lead to excess market power in that market because consumers can readily switch to the substitute juice. Recent merger activity between a large predominantly COJ processor and a retail distributor suggests the potential for change in market power. If COJ and FCOJ are close substitutes, a merger between the COJ processor and the distributor may be inconsequential to the industry and consumers. If COJ and the other products are no more than weak substitutes, however, the antitrust issues take on new meaning. In the following section the retail demand for FCOJ, COJ, and CSS is estimated by means of the Cooley-Prescott model.

OLS MODEL FOR ORANGE JUICE

Monthly retail data on aggregate consumption of each processed orange juice product are available through the MRCA,[1] 1971-79. With these data, the demand models specified hereafter can be estimated. The orange juice product forms are hypothesized to be substitutes. Each product market should respond to income changes and each product has historically shown some degree of seasonality in consumption. Also, product promotions and introduction of competing brands, especially in the COJ market, may have led to increased per capita consumption. These increases will be reflected in intercept adjustments initially using a time proxy variable.

The demand equation for each product is defined in equation 6 and is estimated in the nonlinear form.[2]

$$\text{(6)}\quad Q_{it} = p_{1t}^{\beta_{i1}}\, p_{2t}^{\beta_{i2}}\, p_{t}^{\beta_{i3}}\, I_t^{\Gamma_{i1}}\, S_t^{\Gamma_{i2}}\, M^{\Gamma_{i3}} \exp(\Gamma_{io} + \varepsilon_{it})$$

where Q_{it} = per capita consumption of product i in period t (ounces per 1000 population), i = product group (i = 1, FCOJ; i = 2, COJ; i = 3, CSS), p_{it} = real price for product i in period t, I_t = real per capita income, S_t = seasonality index, and M = monthly time period (M = 33, Mar. 1971;...M = 34 Apr. 1971;...M = 132, June 1979).

Several econometric as well as model specification problems are apparent with equation 6. Each price parameter β_{ij} is fixed across the sample and the model does not facilitate any adjustments. The same would be true if the model were initially in a linear form. Furthermore, if an adjustment were suspected, no prior information is readily evident for

[1]Market Research Corp. of America (MRCA) is a private organization maintaining a panel of 9500 consumers who report their weekly consumption information and selected commodities.

[2]Several linear and nonlinear specifications of model 6 were evaluated and the log model shown in equation 6 was selected as the best among those initially considered. Furthermore, the consumption data represent actual demand and do not necessarily reflect total supplies. The three equations are estimated with each price assumed to be exogenous and, hence, each equation can be estimated independently of any simultaneity problems. The price parameters in equation 2 give a direct measure of the elasticities. Any subsequent discussion of parameter changes over time gives a direct measure of changes in the elasticities. Elasticities would obviously change in a fixed linear model versus the log specification. However, the change would be due to different levels of prices and quantities for the OLS and not the parameter values. In contrast, the VC model measures the parameter change directly.

specifying a fixed change via time or dummy shifters. As is often the case, income parameters and trend variables tend to be correlated and an interpretation of the resulting parameters becomes suspect. If the intercept could be adjusted without inclusion of the trend variable, part of the statistical problems relating to the correlation between income and time could be alleviated. Inclusion of a seasonality variable presupposes that the cyclical adjustments are predetermined and fixed over time. Only the amplitude of the cycle is estimated. Again, the Cooley-Prescott procedure offers an alternative to this fixed cyclical pattern. As a final consideration, the errors ε_{it} are likely to be related across the three equations in 6, thus suggesting a seemingly unrelated regression problem. Because each Q_{it} is shown to be related to the same data set, the use of seemingly unrelated regression procedures for solving these error problems would be identical to using OLS (Kmenta, p. 519).

OLS estimates for equation 6 are reported in Table 1. For each market the direct price effects are significant and have the correct signs. The cross-elasticities are mixed with no statistically significant substitution evident between FCOJ and COJ. CSS and COJ are asymmetrically substitutable, the COJ prices having the greater effect on the CSS market. The income parameters vary considerably across the three equations and, obviously, raise questions about the parameter validity. The Durbin-Watson statistic suggests some autocorrelation, particularly in the COJ equation.

The preceding results are fixed across the sample size. As indicated before, there is reason to suspect that adjustments in these parameters should be considered in light of the marketing developments for each product.

TIME VARYING MODEL

Equation 6 can be respecified in the time varying parameter framework with equation 7.

$$(7)\quad Q_{it} = p_{1t}^{\lambda_{i1(t)}}\, p_{2t}^{\lambda_{i2(t)}}\, p_{3t}^{\lambda_{i3(t)}}\, I_t^{\lambda_{i4(t)}} \exp^{\lambda_{i0(t)}}$$

where

$$(8a)\quad \lambda_{ij(t)} = \lambda^{p}_{ij(t)} + \mu_{ij(t)}$$

$$(8b)\quad \lambda^{p}_{ij(t)} = \lambda^{p}_{ij(t-1)} + \nu_{ij(t)}$$

and $j = 0,1,2,3,4$.

Equation 7 differs from equation 6 in that all parameters are allowed to change over time. Both seasonality and time are dropped from the equation and are reflected by $\lambda_{i0(t)}$. With this method no prior specification of the seasonality or time path is required except for that of the Markovian process shown in equations 8a and 8b which are restatements of equations 1 and 2.

In the absence of prior information for Σ_u and Σ_v, the Cov (B) from Table 1 is used to approximate both. Note that Cov (B) is estimated with time and seasonality included. Ommission of these variables in the OLS model would lead to a greater misspecification and a larger bias in the OLS estimates of Cov (B). Using Cov (B) and normalizing on the intercept value, we show Σ_u in equation 9. Both the diagonal and off-diagonal values are nonzero, implying adjustments in all parameters and some degree of association among parameters.

$$(9)\quad \Sigma_u = \Sigma_v =$$

$$\begin{array}{cc}
\begin{array}{ccccc} \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{array} & \\
\begin{bmatrix}
1.0000 & .0080 & -.0532 & .0212 & .2627 \\
.0080 & .0059 & .0066 & .0008 & -.0001 \\
-.0532 & .0066 & .0112 & -.0045 & -.0112 \\
.0212 & .0008 & -.0045 & .0067 & .0042 \\
.2627 & -.0001 & -.0012 & .0042 & .0702
\end{bmatrix} & \begin{array}{c} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{array}
\end{array}$$

Price Adjustments

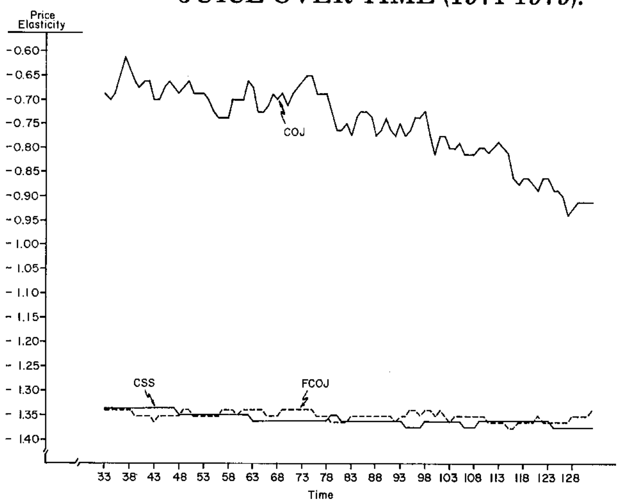
The new direct elasticities over time are shown in Figure 1 for each product (see Appendix). At the outset, it is obvious that

TABLE 1. OLS ESTIMATES OF THE DEMAND FOR ORANGE JUICE (SEE EQUATION 6)[a]

Quantity	Parameter Values: FCOJ price	COJ price	CSS price	Income	Seasonality	Time	Intercept	R^2	DW	F	DF
FCOJ	-.7863	.1481	-.3409	-.6342	.0467	.2459	5.2643	.8412	1.4743	82.11	93
	(.1368)	(.1880)	(.1451)	(.4705)	(.0081)	(.0729)	(1.7753)				
COJ	.0295	-.6108	.4417	1.8013	.0425	.3551	12.583	.9496	.9290	292.12	93
	(.1504)	(.2067)	(.1595)	(.5174)	(.0089)	(.0801)	(1.9519)				
CSS	.2103	.5825	-1.3419	.0219	.0348	-.0938	7.2366	.5152	1.2482	16.47	93
	(.2258)	(.3102)	(.2394)	(.7764)	(.0134)	(.1203)	(2.9293)				

[a]Estimates are based on using a double log model for all variables. Standard errors are shown in parentheses.

FIGURE 1. ADJUSTMENTS IN THE PRICE ELASTICITIES OF DEMAND FOR ORANGE JUICE OVER TIME (1971-1979).



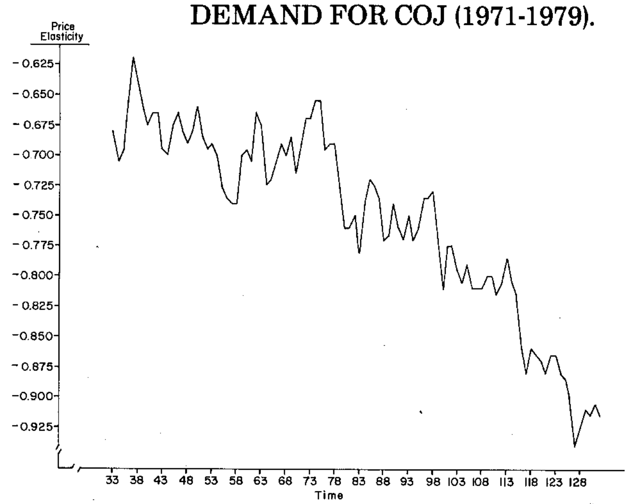
both FCOJ and CSS elasticities have been very stable over the sample period, as well as within a season. There is no evidence that these elasticity parameters show any cyclical change within a season. The small change in both FCOJ and CSS price parameters indicates that they have become slightly more elastic. Numerically, however, the change is extremely small.

Comparing the OLS with the VC results for the FCOJ market shows that the time varying model consistently yielded a larger elasticity than the OLS estimate. The difference is important in that the OLS suggests an inelastic market whereas the VC model shows the market to be elastic. These estimates are consistent with earlier findings by Ward and Myers (p.7).[3] The OLS price estimate must have been influenced by the rigidity of the other parameter specifications in the OLS model. Though this effect on stable parameters is not readily apparent from the initial specification, it is a potential problem that must be recognized when any of the other parameters are suspected to be dynamic.

Historically, the COJ market has shown the greatest propensity to change during the 1970s. Estimates of the price elasticity show that considerable adjustment has, in fact, taken place with consumers' responsiveness to COJ prices. The COJ market has consistently remained inelastic in contrast to the markets of the other orange juices. However, the market elasticity has increased from −.60 in May 1971 to −.93 in June 1979. This difference clearly shows the empirical problem that can occur when the time varying parameters have been ignored (i.e., the OLS model indicated an elasticity of −.61 in contrast to the current estimate of −.93). Finally, the CSS direct price elasticities remained essentially the same for both OLS and VC.

In terms of pricing policy, the relative elasticities provide reasonable guidelines for evaluating the economic consequences of alternative pricing strategies. Price increases (or declines) clearly have different effects on aggregate expenditure changes for each product. The economic consequences from price changes would have been evaluated very differently under the OLS model in contrast to the results shown in Figure 1.

FIGURE 2. ADJUSTMENTS IN THE PRICE ELASTICITY OF DEMAND FOR COJ (1971-1979).



In Figure 2, which shows the COJ price elasticity in greater detail, a strong seasonal pattern is evident. Net of the adjustment over the years, the price parameter tends to become less inelastic in the winter months and more inelastic during the summer. COJ is a chilled product that can be used as a refresher drink in the summer and the consumers may be somewhat less sensitive to price changes during the summer season because of a change in product preference. This adjustment would not have been evident under the fixed estimates nor was there strong reason to specify a seasonal parameter adjustment.

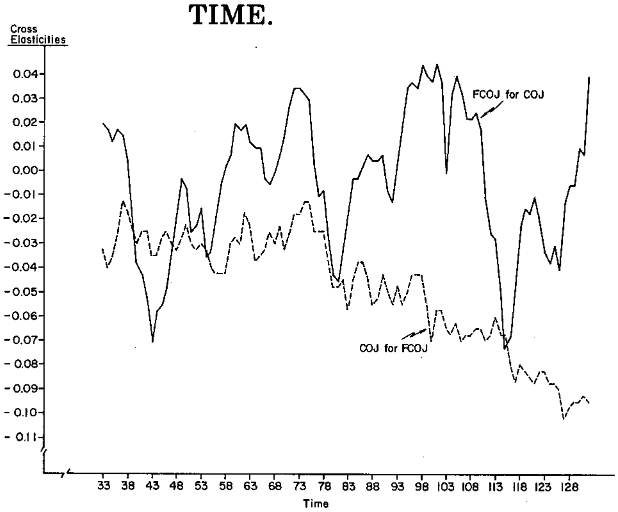
Cross-Elasticities

Substitutability between FCOJ and COJ is of major importance to structural issues within

[3]The reader is cautioned about comparing the results from our study with those of Ward and Myers. Though the conclusions about parameter change are totally consistent, the elasticities are likely to differ because the current analysis is based on a monthly model and Ward and Myers used a quarterly model. One would generally expect higher direct elasticities with the monthly data. Second, the Ward and Myers model was estimated with a difference equation (which could have been approximated with a log specification) and hence the elasticities were not estimated directly as they are in our study. The elasticity value shown by Ward and Myers (p. 8) is for one time period assuming the average values for all variables in the equation. Such elasticities would obviously change with different levels for these variables.

the industry. Again, the basic question is whether or not the markets for each product are reaching unique customers with little or no substitutability between the two products. The initial OLS estimates show that statistically there is no substitution between FCOJ and COJ. Comparison with estimates from the VC model further substantiates that within the range of pricing data there is no evidence of substitutability. The cross-elasticities are not statistically different from zero for every point in the sample period (see Figure 3).

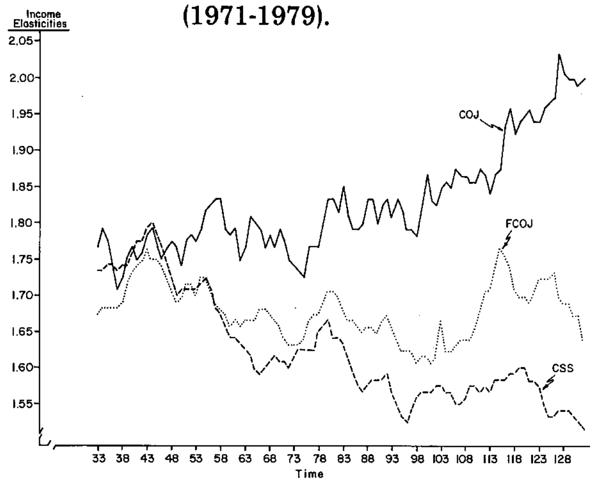
FIGURE 3. ADJUSTMENTS IN THE CROSS ELASTICITIES OVER TIME.



These results are especially important in that, given sufficient time to adjust to new information and/or product forms, consumers may show signs of willingness to substitute within a product group. This is clearly not the case for FCOJ and COJ. FCOJ requires additional preparation prior to consumption where as COJ is ready to drink. The failure to show substitution between the two forms of orange juice is likely to be related not to the container content but rather to the container and additional preparation required. Consumers are purchasing convenience versus storability and show no statistical evidence of substituting these product forms. Application of various linear and nonlinear forms of equations 6 and 7 leads to the same conclusion about substitutability.

As a final comment on substitutability, though both $\lambda_{12(t)}$ and $\lambda_{21(t)}$ are statistically not different from zero, $\lambda_{21(t)}$ does appear to be much more volatile than $\lambda_{12(t)}$ (see Figure 3). $\lambda_{12(t)}$ declined slightly over the sample whereas $\lambda_{21(t)}$ ranged from +.05 to −.10. Interpretation of this difference between $\lambda_{21(t)}$ and $\lambda_{12(t)}$ is not clear except to provide an indication that additional monitoring of both markets is needed.

FIGURE 4. INCOME ELASTICITIES ADJUSTED OVER TIME (1971-1979).

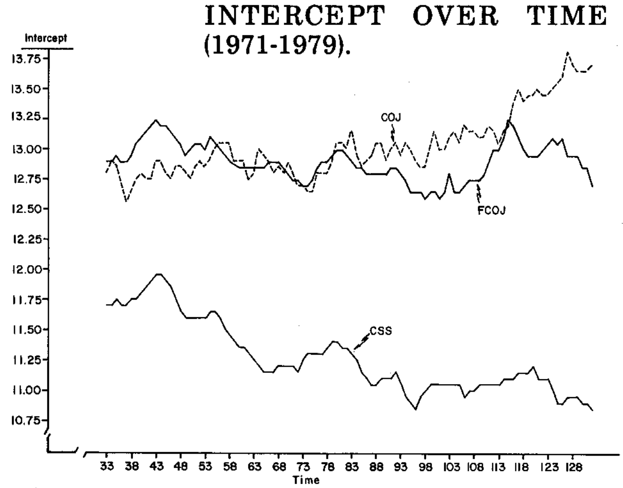


Income Elasticities

The income effects were estimated with time removed from the model and the resulting income elasticity for each product is shown in Figure 4. The initial OLS estimates gave considerable variation in income responses and a questionable sign in the case of FCOJ. With the random coefficient model, income was shown to be positive and statistically significant for all three products. The income elasticities for both FCOJ and COJ have increased over time, COJ showing the larger gain. The income response to CSS declined over the data periods.

Clearly, the initial OLS model was misspecified with the fixed income parameters. Also, the increasing importance of COJ to larger incomes would not have been evident. The growth in $\lambda_{24(t)}$ in relation to the other income parameters suggests that as income grows, consumers increase their consumption of the more convenient form of orange juice in relation to FCOJ and CSS.

FIGURE 5. DYNAMIC PATH OF THE INTERCEPT OVER TIME (1971-1979).



Intercept Adjustments Over Time

The time varying model specifically excluded both the seasonality and trend variables initially in equation 6. These adjustments are proxy for a number of variables influencing the markets. The effects from those excluded variables are, however, reflected in $\lambda_{i0(t)}$ and are illustrated in Figure 5.

Comparison of the parameters reveals distinctive differences in stages of market development. The chilled juice market has shown rapid increases in consumption subsequent to mid-1974 (i.e., period 73). Also, the seasonality in consumption is most apparent for each market. Again, these adjustments are net of the effects of income and several of the statistical problems with OLS are resolved.

The time varying adjustments for FCOJ and COJ are somewhat different from those for CSS. In Figure 5, $\lambda_{10(t)}$ has risen but tends to be very unstable within season. Greater seasonality in concentrate consumption in comparison with chilled juice consumption is most evident even though both show seasonal peaks during the winter months. The OLS seasonal parameters would not have shown the difference in seasonality seen in Figure 5 because of the fixed nature of OLS specifications.

The results for the CSS market are of unique interest in that the OLS estimates indicated insignificant income and time effects, suggesting that this market has matured and is stable. Yet the evidence for CSS in Figure 5 shows a market that continues to decline over time. The time varying model directly captures the structural decline not evident in the fixed model.

Within any industry product life cycles occur but frequently are not measured. A multitude of factors influencing markets are present simultaneously and the long-run cycles can become lost in the modeling. The Cooley-Prescott model has facilitated the measurement of these cycles for orange juice where CSS is in the declining phase of the cycle, COJ in the rising stage, and FCOJ rising but highly volatile. Obviously, there is nothing inherent in these products indicating that demand must eventually decline. Rather, the time varying methods simply allow measurement of such change if it is present.

The cumulative effect of all parameters changing is reflected in the degree of permanent to transitory change via γ (see equation 5). γ differs with the three products where $\gamma_{FCOJ} = .66$, $\gamma_{COJ} = .98$, and $\gamma_{CSS} = .34$. Because γ_{COJ} is near 1, the COJ market shows the strongest trend toward permanent change as calculated with equation 5, whereas the CSS market is mostly transitory. The concentrate market includes a combination of both effects. This value of γ_{COJ} further serves to reinforce the dynamic properties of the chilled juice market and the need to use VC models.

Statistical Considerations

Time varying models were developed initially to improve forecasting because the most recent parameter values should better represent the current and future markets. As is evident from the preceding figures the estimates can also be extremely useful for respecifying a nonstochastic model.

Though the intent of our analysis was not forecasting, the performance of the time varying model for estimating the demand is superior to that of the OLS. Table 2 includes the RMSE for each model using both OLS and VC. The RMSE for each product was consistently smaller under the ramdom parameter model. Because the COJ parameters showed the greatest propensity for change, it is not unexpected that the RMSE for the COJ estimates led to the largest improvements. The estimated values over time for both models were very accurate. However, the VC procedure did tend to predict the extreme points better. Given the reasonably close predictive values from the two procedures, the most important information from our application of time varying methods is related to the identification and measurement of structural drift over time.

TABLE 2. ROOT MEAN SQUARE ERROR (RMSE) FOR FIXED AND TIME VARYING MODELS[a]

	RMSE		$\frac{RMSE_{vc}}{RMSE_{ols}}$
	OLS	VC	
FCOJ	1478.10	592.14	.401
COJ	352.12	12.99	.045
CSS	102.00	56.02	.549

[a]The RMSE were calculated for the original values of Q_{it} and not the log form.

CONCLUSION

OLS and VC models are estimated for the demand for orange juice products and the estimates clearly show that structural drift is an important component of demand over time.

Asymmetric parameter changes show the three orange juice markets to be somewhat autonomous with little substitution evident between frozen concentrate and chilled orange juice. These markets appear to be in substantially different stages of development and the VC procedures clearly identify these differences.

The intercept adjustment in COJ and the insignificance of the cross-elasticities indicate that the growth in the chilled juice market represents real market expansion rather than substitution for FCOJ. Though some substitution with CSS has occurred, the growth differential in COJ could not have been reflected totally by the decline of CSS.

Time varying parameters have many marketing applications when the models draw from time series data. The results lead to model respecification and frequently provide an alternative method for addressing econometric problems encountered with both cross-sectional and time series data. Adjustments estimated with proxy variables that are based on limited theoretical models can be handled without the need for such proxies in many cases. In general, if dynamic adjustments are suspected and no prior information for specifying the time path is available, the VC provides the researcher with an alternative. As a final note, when the VC models show a dynamic path as calculated for the chilled orange juice market, consistently using the parameters from the last period (T+1) will be unsatisfactory. If the analysis is for period T+Z where Z > 1, the additional adjustments should be incorporated into subsequent analysis. The models could be updated and in some cases the dynamic path can be easily expressed in equation form.

APPENDIX

The time varying parameters procedure outlined is one of a family of such procedures, each having its own specific restrictions on the parameter changes. At the outset if parameters change with a clearly identifiable pattern and are nonstochastic, trend variables and dummy adjustments could possibly be used. Such restrictions are, however, very demanding on the model. The VC procedure is less restrictive in terms of the parameter values than that with the OLS estimates with interaction terms. Also, the fixed models ignore the statistical problems arising when parameters are stochastic. Frequently, the VC model suggests structural drifts that can be remodeled with the use of trend and dummy variables, depending on how well behaved the changes are with the VC results (Belsley, p. 495-500; Ward and Myers, p. 9-10).

Given the substitution outlined in equation 3, the VC model reduces to an estimation problem dealing with heteroskedasticity as evident in equation 4 when $E(W_t W_s)$ is calculated. The value of the covariance elements differs with each time period t because one component of W_t includes the summation over t+1 to T+1. The covariance is partitioned into two components, i.e., R and C. R arises from the transitory adjustments whereas C results from permanent changes and the elements of C change with time.

To estimate first the parameters for the end period T+1, some assumption regarding Σ_u and Σ_v found in R and C must be made. One solution to this problem is to estimate first the covariance of the parameter estimates derived with OLS and use this covariance as an approximation to Σ_u and Σ_v. If $\Sigma_v = 0$ and all elements of Σ_u are zero except for the first element, the model is the OLS estimate. If Σ_u and Σ_v are diagonal, the parameter adjustments are assumed to be unrelated. For practical applications, one can explore the sensitivity of the estimates to slight variations in the values of Σ_u and Σ_v.

Given the choice for Σ_u and Σ_v, the estimated vector B_{T+1} gives the parameter values for one period beyond the sample period T. These parameter estimates represent the most current values to use for policy analysis and forecasting and the B_{T+1} is calculated by using GLS procedures (Ward and Myers, p. 3).

The element C_{ij} from C has a crucial role in the derivations of B_{T+1} and the path of change. In the calculation of the covariance of W, the cross-products $E(\sum_{s=t+1}^{T+1} V_s)(\sum_{m=j+1}^{T+1} V_m)$ will be nonzero for all values where the time periods are common to the two summations. The elements C_{ij} will be $X_i \Sigma_v X_j'$ multiplied by the minimum of (T+1−i) or (T+1−j) because all time periods other than this minimum would not be common to the two summations and, hence, the cross-products for those noncommon periods would be zero. If the time paths of the parameters are to be derived, the C_{ij} are weighted by the minimum of (t − i) or (t − j) for (i,j) < t, and $C_{ij} = 0$ for (i,j) > t. $C_{ij} = 0$ precludes inclusion of X_i on the permanent component when i > t. Intuitively, the permanent values for period t should depend only on current and prior information and not on subsequent values. The covariance matrix is changed for period t and the GLS estimates are derived for that period. This process is repeated for each period t until t=T and the resulting paths for the parameters are revealed (Cooley and Prescott 1976, p. 170). The procedure is by no means simple and does require considerable computer time and relatively large data sets.

To operationalize the model one must select Σ_u and Σ_v, determine the increment for parameter adjustments, and determine γ. Once these elements are known, the parameter sets follow through a systematic application of GLS given the respecification of Cov (W_t) for each time period.

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