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*Optimal Tax Rates and Tax Design
during Systemic Reform*

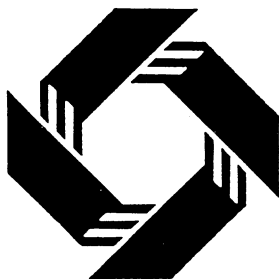
David M. Newbery

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*Optimal Tax Rates and Tax Design
during Systemic Reform*

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*Department of Applied Economics
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and

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Institute for Policy Reform*

January, 1994

Budgets in Eastern Europe are under strain as taxes are reformed and receipts fall. Should existing levels of expenditure be maintained or should personal taxes be cut at the expense of redistributive expenditures? The paper asks how optimal tax rates might respond to a transition in which the government becomes less averse to inequality, but the inequality in skills increases. These lead to roughly offsetting changes, and are less important than the fall in enterprise revenue and the decline in the efficiency of personal taxation, both of which cause a sharp fall in optimal transfers. Tax design consists in balancing the differing inefficiencies of incompleteness in indirect tax coverage against evasion-prone direct taxes.

Disclaimer

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Optimal Tax Rates and Tax Design during Systemic Reform

NON-TECHNICAL SUMMARY

The transforming countries of Central and Eastern Europe (CEE) are replacing Soviet-type institutions by those appropriate to a market economy. Soviet-type economies had little need for personal income or consumption taxes as state enterprises paid a net wage, with tax revenue collected out of profits. Private enterprise requires a shift to taxing persons rather than enterprises, so an important part of systemic transformation is tax reform. Hungary led the way in 1988 by introducing a personal income tax, a Value Added Tax, and a profits tax, modelled on western counterparts. Other countries in the region are following suit. Systemic reform therefore raises the question of the appropriate design of the tax system. This in turn requires choosing the balance between direct and indirect taxation, as well as its level and the degree of progression of personal income tax, and of the share of taxes in GDP.

The question is of considerable practical concern, as the transforming countries are subject to two offsetting pressures. On the one hand the population and external advisors argue that their tax shares are high by comparison with most western economies, and should be lowered to improve incentives and allow market forces to play a greater role. On the other hand, the costs of restructuring are high, revenues from the enterprise sector are falling sharply with increased competition and privatisation, and budgets are everywhere under pressure. Should the government recoup the lost enterprise revenue by higher personal income taxes, or should it cut transfers to individuals and hence lower the overall share of tax and public expenditure?

Questions about the desirable level of taxation and redistribution are most readily addressed using the machinery of optimal tax theory. This approach is not without its critics, but it provides the only systematic way in which the relative magnitudes of different influences on the desirable level of taxation can be estimated. It provides a coherent framework to guide the analysis, consistent with modern welfare economics, as exemplified, for example, by cost-benefit analysis.

Four factors influence the optimal tax rate. The first is the degree of inequality in skills. The higher the degree of inequality, the higher the tax rate is likely to be, other things equal. The degree of skills inequality appears to be lower in socialist economies than market economies, as measured by the wage dispersion, but is likely to increase as new skills are required for success in the market place, and many existing skills become obsolete. The second factor is the level of government expenditure required for non-redistributive purposes- the higher the fraction of GDP required, the higher the tax rate. CEE countries may have a higher non-redistributive revenue requirement for administration than comparable countries, which in the past was adequately covered by enterprise revenue. The main problem is that enterprise revenue has fallen sharply, while infrastructural investment demands (especially in transport and telecommunications) have increased, both increasing the uncovered component of required revenue.

The third factor is the elasticity of substitution between taxed and non-taxed activities (work and leisure in the benchmark case of a comprehensive tax system in which all goods

and services can be taxed). The higher is this elasticity, the lower the tax rate, provided taxes are adequate to cover the fixed expenditure requirement. Soviet-type economies have remarkably extensive control over incomes (via state enterprises) and over access to consumption and work, greatly reducing the opportunities for substituting untaxed for taxed activities. The transition threatens this in various ways. The move to a market economy with numerous smaller firms leads to a loss of information about tax liabilities, while the reform of the tax system disrupts existing procedures. Lower tax efficiency reduces the fraction of income falling within the tax net, raising the marginal cost of tax collection and reducing the optimal tax rate. Finally, the more egalitarian the government, the higher the tax rate. Countries like Hungary appear exceptionally egalitarian by western standards, but the transition in Eastern Europe is eroding political commitment to the previous degree of equality, and with it, the desired marginal tax rate.

These theoretical considerations pull in different directions. The first two arguments suggest that an increase in taxes might be justified. Wages may become more dispersed, arguing for more redistribution. Required public expenditure net of enterprise receipts will almost certainly rise. The last two arguments suggest that optimal tax rates should fall as a result of transformation, as the ability of the tax system to collect revenue falters, and public support for equality erodes. If the theoretical arguments appear balanced, the issue can only be resolved by numerical calculations showing how the optimal tax rates vary with each factor.

The paper presents a model and calculations that allow these issues to be addressed, and finds that the most potent influence on redistribution is the coverage of the tax system, primarily through its effect on the degree of substitutability between taxed and untaxed goods and services. If we ask why some taxes are hard to collect, it becomes clear that there is an important difference between direct and indirect taxes. Indirect taxes may be hard to levy on some goods (casual services, farm produce marketed on street stalls, and other products of the informal sector), but where the taxes may be levied, they are paid by all. Direct taxes may be evaded by under- or non-reporting, but where paid affect disposable income and hence all expenditure. Relying on indirect taxes means that all people are taxed on some of their expenditure, while relying on direct taxes means that some people are taxed on all their expenditure. This suggests that the optimal design of the tax system in the presence of evasion will involve both direct and indirect taxation.

Although changing attitudes to inequality and changes in the inequality of skills may lead to roughly offsetting changes in the optimal tax rate, these effects seem small compared to changes in the efficiency of tax collection as measured by tax coverage. The other striking finding is that increases in non-redistributive expenditure (net of enterprise revenue) crowd out redistributive transfers to a considerable extent, so that for example a (permanent) fall in enterprise revenue of 1 per cent of GNP should lead to a cut in transfers of between one-third and two-thirds of 1 per cent of GNP, and only a small rise in the total tax share in GNP.

Optimal Tax Rates and Tax Design during Systemic Reform

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August 1993 (revised January 1994)

Budgets in Eastern Europe are under strain as taxes are reformed and receipts fall. Should existing levels of expenditure be maintained or should personal taxes be cut at the expense of redistributive expenditures? The paper asks how optimal tax rates might respond to a transition in which the government becomes less averse to inequality, but the inequality in skills increases. These lead to roughly offsetting changes, and are less important than the fall in enterprise revenue and the decline in the efficiency of personal taxation, both of which cause a sharp fall in optimal transfers. Tax design consists in balancing the differing inefficiencies of incompleteness in indirect tax coverage against evasion-prone direct taxes.

1. Introduction

The transforming countries of Central and Eastern Europe (CEE) are replacing Soviet-type institutions by those appropriate to a market economy. Soviet-type economies had little need for personal income or consumption taxes as state enterprises paid a net wage, with tax revenue collected out of profits. Private enterprise requires a shift to taxing persons rather than enterprises, so an important part of systemic transformation is tax reform. Hungary led the way in 1988 by introducing a personal income tax, a value added tax, and a profits tax, modelled on western counterparts. Other countries in the region are following suit. Systemic reform therefore raises the question of the appropriate design of the tax system. This in turn requires choosing the balance between direct and indirect taxation, as well as its level and the degree of progression of personal income tax, and of the share of taxes in GDP.

The question is of considerable practical concern, as the transforming countries are subject to two offsetting pressures. On the one hand the population and external

* This paper was prepared under a cooperative agreement between the Institute for Policy Reform (IPR) and the Agency for International Development (AID), Cooperative Agreement No. PDC-0095-A-00-1126-00. Views expressed in this paper are those of the author and not necessarily those of IPR or AID. I am indebted to Tony Atkinson for perceptive comments on normalising the social welfare function, and to Angus Deaton, Barry Weingast, John Tirole and participants at an IPR/AID workshop on November 5, 1993, for their helpful comments. An earlier version dealing with indirect taxes was issued as a Discussion Paper on Economic Transition (DPET 9307) and presented at the *Royal Economic Society* Annual Conference at the University of York, 13-16 April, 1993.

advisors argue that their tax shares are high by comparison with most western economies, and should be lowered to improve incentives and allow market forces to play a greater role. On the other hand, the costs of restructuring are high, revenues from the enterprise sector are falling sharply with increased competition and privatisation, and budgets are everywhere under pressure. Should the government recoup the lost revenue by higher personal income taxes, or should it cut transfers to individuals and hence lower the overall share of tax and public expenditure?

2. Normative Public Economics and Budget Reform

The most coherent normative theory of public economics, as set out for example in Atkinson and Stiglitz (1980), is utilitarian, in that it concerns itself with the consequences of social choices for individuals. This ethical framework has considerable appeal, the more so if tempered by concepts of rights. The theory is most directly applicable to the revenue raising side of the budget, as a theory of optimal taxation. The problem is seen as one of raising an appropriate amount of revenue for redistribution and to pay for other public expenditures at least social cost. The expenditure side of the budget is complimentary, in that the larger part of redistribution typically occurs on the expenditure rather than tax side. In a simple world with no market failures, government activity should be purely redistributive (and to that extent directly addressed by the theory of optimal taxation). In practice, market failures are pervasive and provide an important rationale for government intervention, both on the tax side (corrective taxes such as those on road users), but again more importantly on the expenditure side.

Optimal tax theory, as set out in Atkinson and Stiglitz (1980) and summarised in Mirrlees (1979) or Stern (1987), takes a Bergson-Samuelson social welfare function as embodying utilitarian normative precepts, and a rather simple description of the economy (competitive, with no market failures) to obtain quite striking results for the choice of commodity taxation. Given some further defensible assumptions about consumer preferences it is possible to draw rather sharper conclusions for the design of a set of taxes.¹ Under these assumptions, the optimal tax involves uniform commodity taxes (or, equivalently, a proportional income tax) and an optimal lump-sum demo-grant (i.e. a grant that depends on demographic circumstances, such as the size and age structure of the family).

¹ These are that consumers are essentially identical (except for observable and unalterable characteristics, such as age) and preferences are separable (Atkinson and Stiglitz, 1980, chapter 14; Deaton and Stern, 1986).

The main arguments for non-uniform commodity taxation are that differential taxes would allow better redistribution or would reduce distortions. Neither argument is very convincing. The redistributive argument normally involves subsidising essentials such as food, housing, heating (and these goods were notably underpriced or subsidized in CEE countries). Subsidizing commodities is an inefficient way of redistributing income as almost all commodities are consumed in larger amounts by the better off, who would receive more transfers with commodity subsidies than with equal lump-sums to all. The main remaining argument for differential commodity taxation is that it may introduce smaller inefficiencies than uniform taxation. If one could clearly identify goods that were complementary to leisure, and not very substitutable for other goods, these could be taxed more heavily to reduce the incentives to take leisure, but it is hard to think of good examples. A considerable part of the efficiency case for uniform commodity taxation is the lack of any empirically convincing evidence that differential taxes could encourage more effort and less leisure (Deaton, 1987).

It is also plausible that the income tax system should be progressive, though it is remarkable how linear most tax systems seem for the overwhelming bulk of the population when considered in their entirety. At this level of abstraction the main distinction between direct and indirect taxes is that the latter are necessarily uniform, while direct taxes can be non-linear, and depend on demographic circumstances (age, retirement, marital status, number of children, etc) though much of this discrimination is more readily achieved on the expenditure side of the budget through transfers in cash and in kind. There are further important differences between direct and indirect taxes addressed below.

Ideally, taxes should fall on final consumption, not on intermediate goods, so that distortions are confined to consumption and not needlessly visited upon production. A value added tax achieves this, though arguably at considerable additional complication, which may not be justified if tax administration is weak.² The main implication of this principle is that trade taxes are hard to justify (except on tax administration grounds, or possibly as part of the transition process, providing

² Koltay (1993) argues persuasively that Hungary would have been better advised to adapt its turnover tax towards a purchase tax, and leave the move to a VAT until a later date, once the tax authorities had acquired sufficient experience. The other argument for a VAT is that it provides information to enforce tax collection, particularly from corporate taxation (Gil Díaz, 1987), though this presupposes the willingness and ability of the authorities to match up tax returns.

temporary production subsidies to otherwise exposed firms in the trade goods sector until they learn how to compete).

The main argument for taxing corporate income (as distinct from personal income) is to tax foreign incomes (Gersovitz, 1987) or rents. Natural resources like oil are typically subject to specific taxes designed to capture as much of the rent as possible in a non-distortionary way. A corporate income tax or profits tax may provide a way of taxing some personal incomes and so the income tax and profits tax need to be integrated and considered together. The main practical reason for keeping a profits tax is to avoid giving windfall gains to owners. A related practical reason in the Hungarian context is that when privatising enterprises subject to profits tax, foreigners can take advantage of foreign tax credits, while domestic entrepreneurs need pay less, as the profits tax is rather like non-voting state equity in the enterprise, reducing the difficulty of raising the required finance.

The *structure* of a tax system can be described by the kinds of taxes employed (whether falling on wages, interest, dividends, profits, commodities, imports, etc), and whether they are proportional or progressive, uniform or differentiated, etc, on value, value added, or quantity (such as excises). These general principles are useful for defining the broad structure of the tax system, but are not sufficient. The main area in which they are incomplete has to do with the tax treatment of intertemporal transactions, such as lending and borrowing. They also need to be extended to deal with such issues as the taxation of tobacco and alcohol (which interact with the provision of medical care) and the taxation of road vehicles and fuel (which can be thought of as user charges rather than taxes).

The reformed Hungarian tax system, described in more detail in Newbery (1993) has a *structure* that is a defensible approximation to the prescriptions of modern public economics. The question we wish to examine is how one might judge the *level* of taxation, measured by the size of the personal lump-sum transfer and the (marginal) tax rates, and the *balance* between direct and indirect taxes. If we consider that the enterprise sector, then profits taxes and state dividends are alike in generating public revenue to an extent that will be largely determined by the size of profits and the degree of state ownership, as the rate of profits tax will be largely dictated by international considerations. This means that we can concentrate attention on the taxation of personal income and expenditure. As a first approximation, the tax system can be represented by a proportional income tax (converting commodity taxes into their income tax equivalent), a uniform demo-grant, and a fixed revenue requirement to provide for that part of exhaustive (and non-redistributive) government expenditure

not covered by enterprise tax and dividend revenue (eg for defence, infrastructural investment, administration, and to service the foreign debt). This simplification enables us to describe the tax system by two parameters: the tax rate, and the fixed revenue requirement.

We can now post the central questions of this paper: what determines the optimal tax rate, noting that the higher the tax rate, the larger will be the redistributive benefits (given the fixed revenue requirement), and how will this rate change with systemic change? More to the point, are there good arguments suggesting that the tax rate should increase or decrease during the transition to a market economy? These last two questions are related but not identical. Suppose that the original tax system was optimally designed for the objectives and circumstances of the previous economic system, and that objectives and circumstances change in a way that the costs of raising revenue efficiently rises, and the redistributive benefits are less highly valued, then one

would expect that the optimal tax rate should fall. But if the original tax system were highly distorted, it might be possible to raise the average tax rate while reducing the distortions and lowering the overall cost enough to offset the cost-raising factors associated with systemic reform.

An example may make this clearer. Consider fig. 1, in which the marginal cost of the optimal tax under the old system is graphed as a function of the share of taxes in GDP and is shown meeting the marginal benefit of optimally targeted benefits at point B, corresponding to a tax share of 55 per cent.³ If the actual tax system is

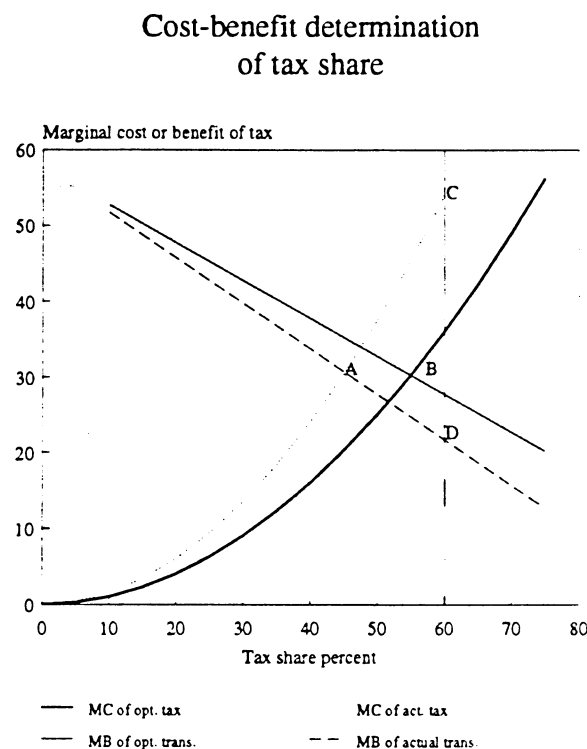


Fig. 1

³ In the notation of the model developed in the Appendix, if r is the share of tax revenue in GDP, a function of the tax rate t and the lump-sum transfer g , and if $W\{V^h(t, g), \dots V^h(t, g), \dots\}$ is social welfare, defined over individual welfare of household h , $V^h(t, g)$, and if starred values refer to the optimum, then the marginal cost of taxation is $\partial W(g^*, t^*) / \partial t \partial r(g^*, t^*) / \partial t$, and the marginal benefit is $\partial W(g, t^*) / \partial g \partial r(g, t^*) / \partial g$. The graph is

inefficient, it will be more costly (in terms of social welfare) to generate any given tax share, and its marginal cost may pass through a point such as A. Similarly, if benefits are inefficiently targeted (via subsidies instead of lump-sum transfers, for example), then the marginal benefit schedule may pass through points A and D, instead of D. Given these inefficiencies in tax design and targeting, the second-best optimal tax share may correspond to point A, at a share of 45 per cent. Systemic reform and changes in the government's objectives, reflected in the social welfare function, may shift the optimal marginal cost schedule up somewhat, and the optimally targeted marginal benefit schedule down, so they now might meet at a point between A and B (not shown), perhaps at a tax share of 50 per cent. The *optimal* tax share has fallen from 55 to 50 per cent, but if the new government introduces an optimal tax system to replace the previous inefficient system, the actual tax share will have risen from 45 to 48 per cent.

Clearly, there are other alternatives. The original system may have been doubly inefficient, in that the original tax system was not only inefficiently designed, but excessively redistributive, so that a careful balancing of the costs of raising tax revenue against the benefits of improved income distribution may suggest not only reducing the inefficiencies (and hence the cost of raising the revenue), but also reducing the tax share to bring costs into line with benefits. For example, the actual tax share might have corresponded to points C and D in fig. 1, at a tax share of 60 per cent, where the marginal cost of additional taxes at point C is well above the marginal benefit of the ensuing redistribution at point D.

It is therefore difficult to say how the *actual* tax rate will change with systemic reform without a detailed analysis of the extent to which the original and final tax systems depart from optimality. This paper has the less ambitious objective of asking how the *optimal* tax rate might change during systemic reform, though it might be possible to go somewhat further and ask how far the second best tax rate might have been below the initial optimal tax rate, given the actual pattern of taxes, subsidies and transfers.⁴

thus designed to describe a general equilibrium in a partial equilibrium supply and demand diagram.

⁴ There is a more problematic interpretation in which the initial tax structure is the best feasible equilibrium, given the instruments and institutions available, and the objectives of the participants, as exemplified by its persistence over time and prevalence across different Soviet-type economies. On this view, apparent inefficiencies arise because of information imperfections, difficulties in commitment, and the fragility of certain kinds of transfer mechanisms, which cannot be changed without a radical change in institutions and/or a change in the balance of power. Within this perspective, optimal tax theory loses most

2.1 *Determinants of the optimal tax rate*

Four factors influence the optimal tax rate (Stern, 1976). The first is the degree of inequality in skills (which is hard to observe, but is related to the degree of pre-tax earnings inequality when workers are paid their marginal product). The higher the degree of inequality, the higher the tax rate is likely to be, other things equal. The degree of skills inequality appears to be lower in socialist economies, possibly because education is more uniformly allocated than in market economies. One might argue that there are no strong reasons for expecting large differences in the underlying skill distribution between countries at European levels of education, and differences in wage inequality may reflect the operation of implicit tax systems in the more egalitarian CEE countries, rather than differences in the distribution of marginal products. Atkinson and Mickelwright (1992) provide extensive statistical comparisons of wage dispersions in CEE countries compared to Britain. They find (p94) that in the 1970s, the USSR and Poland had a higher ratio of the top to bottom decile of earnings than Britain (at about 3.0), though Hungary and Czechoslovakia were systematically lower, at about 2.5.⁵ In the 1980s, Poland's ratio fluctuated widely, but eventually fell to that of Hungary and Czechoslovakia (Hungary actually increased somewhat), while Britain's inequality rose steadily over the 1980s along with the USSR to 3.3.

The second factor is the level of required government expenditure to be financed by taxes on workers and consumers - the higher the fraction of GDP required, the higher the tax rate. Hungary may have a higher required (non-redistributive) revenue requirement for administration than comparable countries, which in the past was adequately covered by enterprise revenue. The main problem is that enterprise revenue has fallen sharply, while infrastructural investment demands (especially in transport and telecommunications) have increased, both increasing the uncovered component of required revenue. The other difference with market economies is in the level of foreign interest payments, where Hungary's ratio of foreign debt to GDP puts it among the most heavily indebted developing countries. Given Hungary's intention to retain its international credit standing, debt repayment places a heavy initial claim on the budget.

of its analytic force, and very little can be said. The approach taken here assumes that optimal tax theory (and welfare economics more generally) continues to provide a relevant benchmark against which to judge a necessarily imperfect world.

⁵ that is, the average earnings of the top 10 per cent of workers was 2.5 times the average wage of the bottom 10 per cent of workers.

The third factor is the elasticity of substitution between taxed and non-taxed activities (work and leisure in the benchmark case of a comprehensive tax system in which all goods and services can be taxed). The higher is this elasticity, the lower the tax rate, provided taxes are adequate to cover the fixed expenditure requirement. Soviet-type economies have remarkably extensive control over incomes (via state enterprises) and over access to consumption and work, greatly reducing the opportunities for substituting untaxed for taxed activities. The transition threatens this in various ways (Kornai, 1992). The move to a market economy with numerous smaller firms leads to a loss of information about tax liabilities, while the reform of the tax system disrupts existing procedures. Reduced tax efficiency reduces the fraction of income falling within the tax net, raising the marginal cost of tax collection and reducing the optimal tax rate.

Finally, the more egalitarian the government, the higher the tax rate. Hungary appears exceptionally egalitarian by western standards, comparable only to very rich and homogenous countries like Sweden. The transition in Eastern Europe is eroding political commitment to the previous degree of equality, and with it, the desired marginal tax rate.

These theoretical considerations pull in different directions. The first two arguments suggest that an increase in taxes might be justified. Marginal products of labour (and hence the correct measure of wages) may become more dispersed with a move to a more internationally exposed market economy, arguing for more redistribution. Pudney (1993) finds a significant growth in earnings inequality in Hungary after 1988, to levels comparable with western Europe. Required public expenditure on infrastructure (telecoms, transport), debt service, and support for the rapidly increasing numbers of unemployed may have to rise as the CEE countries reach the limit of prudent international borrowing and attempt to restructure their economies, again requiring tax increases. The last two arguments suggest that optimal tax rates might well fall as a result of transformation, as the ability of the tax system of collect revenue efficiently falters, and public support for equality erodes.

The theoretical arguments appear balanced, and therefore unhelpful. We need to quantify the arguments and see what factors have a large influence on the outcome. Newbery (1993) provides some quantitative evidence on taxes and expenditures for Hungary, but there is no guarantee that they are at optimal levels. Stern's (1976) article asked similar questions for the choice of an optimal linear income tax, but assumed complete tax coverage, so the critical question of how a decline in the

efficiency of tax collection affects the amount of tax to raise cannot be readily addressed.

If we ask why some taxes are hard to collect, it becomes clear that there is an important difference between direct and indirect taxes. Indirect taxes may be hard to levy on some goods (casual services, farm produce marketed on street stalls, and other products of the informal sector), but where the taxes may be levied, they are paid by all. Direct taxes may be evaded by under- or non-reporting, but where paid affect disposable income and hence all expenditure. Relying on indirect taxes means that all people are taxed on some of their expenditure, while relying on direct taxes means that some people are taxed on all their expenditure. This suggests that the optimal design of the tax system in the presence of evasion will involve both direct and indirect taxation.

There is a useful analogy to draw with food policy in developing countries. If food prices fluctuate relative to money incomes, then the poor may be highly vulnerable. The most efficient instrument to provide insurance against adverse fluctuations would be tradeable ration coupons, but Newbery (1989) proves that unless coverage is complete, additional food price stabilisation, which affects the price stability of food for *all* consumers, not just those covered by the ration scheme, will improve social welfare. The natural question to ask in the present context if indirect taxes are incomplete, is whether it is desirable to introduce an income tax even if partially evaded (or, conversely, if income taxes are partially evaded, whether incomplete indirect taxes improve matters). One natural way of doing this is to start with incomplete commodity taxation by supposing that not all goods can be taxed, and then introduce income taxes, and that is the strategy adopted here.

3. The model and calculations

In order to quantify the effect on optimal tax rates and redistribution of changes in the environment, we need a model that can be solved numerically for the optimal tax rate and which can be calibrated to the empirical evidence on labour supply elasticities and the elasticity of substitution between consumption and leisure, both of which Stern (1976) argues are important determinants of the optimal tax rate. There are few models that are analytically tractable, and even fewer that allow one to address problems of incomplete indirect and direct tax coverage. The model selected is a special case of Deaton (1983), which has several advantages. The model is described in the appendix, and is chosen so that the optimal commodity tax system is uniform

provided that lump-sum transfers can be made to individuals.⁶ The instruments available to the government are the indirect tax rate, t , and the lump-sum transfer g . If indirect tax coverage is complete, this will be equivalent to a linear income tax. Direct taxes will be introduced later when the distinction between incomplete coverage of the two types of tax becomes important.

Workers differ only in their skill level, measured by the wage rate, w , and these skills are log-normally distributed in the population, corresponding closely to observed earnings distributions. The size of the population is normalised to one, so that per-capita and total amounts coincide. The labour supply decision is responsive both to the wage rate and to the size of lump-sum transfers, and can be calibrated to give the observed low labour supply elasticities found in empirical work. Labour supply ℓ (as a fraction of total time) is

$$\ell = (1 - \delta)(1 - \gamma_0) - \frac{\delta}{w}(g - q' \gamma), \quad (1)$$

where g is the lump sum transfer.⁷ The parameter δ turns out to be critical, as it determines both the income and substitution effects of the wage rate.⁸ As labour supply is voluntary, so is unemployment (i.e. choosing to supply zero labour), and this will occur if the wage falls below w_0 , which depends on taxes and transfers (which affect the attractiveness of work vs leisure). If this is above the minimum skill in the earnings distribution, there will be unemployment.

When calibrating the model, two elasticities are critical - that between leisure and consumption, ϵ , and that of the labour supply to the wage, η . Expressions for both elasticities are given in the appendix, and both depend on the critical parameter of the labour supply equation, δ , as well as the wage rate and consumption level.

Suppose that a subset of goods $i \in \Gamma$ is subject to a Value Added Tax (VAT) at rate t , so that for these goods the after-tax or consumer price q_i is raised above the

⁶ Thus it has linear Engels' curves (graphs of commodity expenditure against income) with leisure and goods weakly separable.

⁷ γ_0 is the necessary or subsistence fraction of time needed for leisure, consumer prices are given by the vector q , and γ is the vector of subsistence consumption requirements. The term $g - q' \gamma$ is thus lump sum income net of minimum subsistence requirements, and describes the way in which income affects labour supply.

⁸ Thus δ not only measures the disincentive effects of unearned income on labour supply, but $\delta - 1$ is the compensated wage elasticity of over-subsistence leisure, $1 - \ell - \gamma_0$. Constraining the income and substitution effects to bear such a tight relationship is both highly convenient and restrictive.

before-tax or producer price p_i , thus: $q_i = (1+t)p_i$, and that the remaining goods (possibly a null set) are untaxed. The total tax revenue collected by such taxes will then be

$$T = \sum_{i \in \Gamma} t p_i X_i, \quad (2)$$

where X_i is total demand for good i . If exhaustive (ie non-redistributive) public expenditure net of enterprise revenue is R , then the amount of revenue available for redistribution as a uniform lump-sum grant, g , is $T - R$.

The (constrained) optimal indirect tax problem is now to choose a rate of VAT, t , and a feasible (balanced budget) level of lump-sum grants, g , given a specification of the set of taxable goods, Γ , and net required non-redistributive public expenditure, R , to maximise social welfare, W :

$$W = \sum_h U(V^h(g, t, w^h)) = \int_{\underline{w}}^{\bar{w}} U(V(g, t, w)) dF(w), \quad (3)$$

where $U(V^h(g, t, w^h))$ is the social value of the income enjoyed by worker h with skill w^h when the VAT rate is t and grant is g , and V^h describes the preferences of worker h . Note that any monotonic transform of V^h describes the same preferences over goods and leisure, but social valuations will depend on levels of well-being and attitudes to inequality. Thus social welfare is the sum of appropriately measured individual *welfare*, and if, as assumed, workers are completely described by their skill, w , which is continuously distributed with a distribution function $F(w)$, we can replace sums by integrals over the actual skill range $[\underline{w}, \bar{w}]$.

Society's attitudes to inequality will be reflected in the choice of the function U . The most convenient parameterisation of inequality normally employed amounts to assuming that the social value of a consumption level of c is $U(c) = c^{1-\nu}/(1-\nu)$, $\nu \neq 1$ (and $\ln c$ for $\nu = 1$), where ν is Atkinson's (1970) coefficient of inequality aversion, directly analogous to the coefficient of relative risk aversion. This has the appealing property that $\nu = 0$ corresponds to a complete disregard for inequality, where society is solely concerned with total consumption and not its distribution, with higher values of ν attaching more importance to redistributive goals. The social marginal utility of making a unit grant to a consumer enjoying consumption c is then $dU/dg = c^{-\nu} dc/dg$, and one can think of $c^{-\nu}$ as the social weight attached to consumption. Thus if $\nu = 2$, the value of an extra \$ of consumption for someone consuming twice the reference level is only one-quarter that of the reference level ($= 2^{-2}$).

Once the equations for equivalent consumption as a function of taxes and tax coverage and social welfare have been specified, as described in the appendix, the next step is to write a computer program to find the value of taxes and transfers that maximise social welfare.

3.1 *Model calibration*

Stern (1976) discusses the problem of calibrating the critical parameters of the model in considerable detail, and further discussion is relegated to the Appendix. Mirrlees (1971) computed optimal (non-linear) income taxes assuming that skills were log-normally distributed in the population with parameters (μ, σ) (the mean and standard deviation of the associated normal distribution) taken as $(-1, 0.39)$ to approximate the UK skill distribution. The log-normal distribution function (given in the appendix) has several appealing properties (Cowell, 1977) apart from fitting wage data reasonably well.⁹

Numerical experiments suggested that the optimal degree of redistribution, measured by the lump-sum transfer, g , was very sensitive to the parameter δ in the labour supply equation for values near zero, which are required for plausible values of the labour supply elasticity, and also depends fairly sensitively on the coefficient of inequality aversion, ν , the required government expenditure, R , and the degree of skill inequality, σ . The strategy adopted here is to choose a set of parameters that are roughly consistent with a plausible value for the elasticity of substitution between leisure and consumption, ϵ , with complete tax coverage, and check that the rates at which redistribution, g , varies with the parameters of interest (R, ν, σ) are reasonably robust to these initial parameters. If this hypothesis is confirmed, then one may be moderately confident about the effect of *changes* in those environmental parameters associated with the transition, even if one is less confident in the *level* of the optimal tax and expenditure levels. This seems to be borne out by the results presented in the various figures below.

3.2 *Sensitivity of optimal tax rates to the environment*

The effect of changes in the skill distribution (caused, for example, by the obsolescence of skills specifically attuned to those required in a Soviet-type economy)

⁹ The Gini coefficient, G , is $2N(\sigma/\sqrt{2})-1$, so if $\sigma = 0.39$, $G = 0.217$. Atkinson's (1970) index of inequality, $I(\nu)$, which measures the proportion by which average income could be reduced if it were costlessly and optimally redistributed to yield the same level of social welfare, using the same definition of inequality aversion as here, is $1 - \exp(-\frac{1}{2}\nu\sigma^2)$, so $I(0.5) = 0.0373$ and $I(2) = 0.141$.

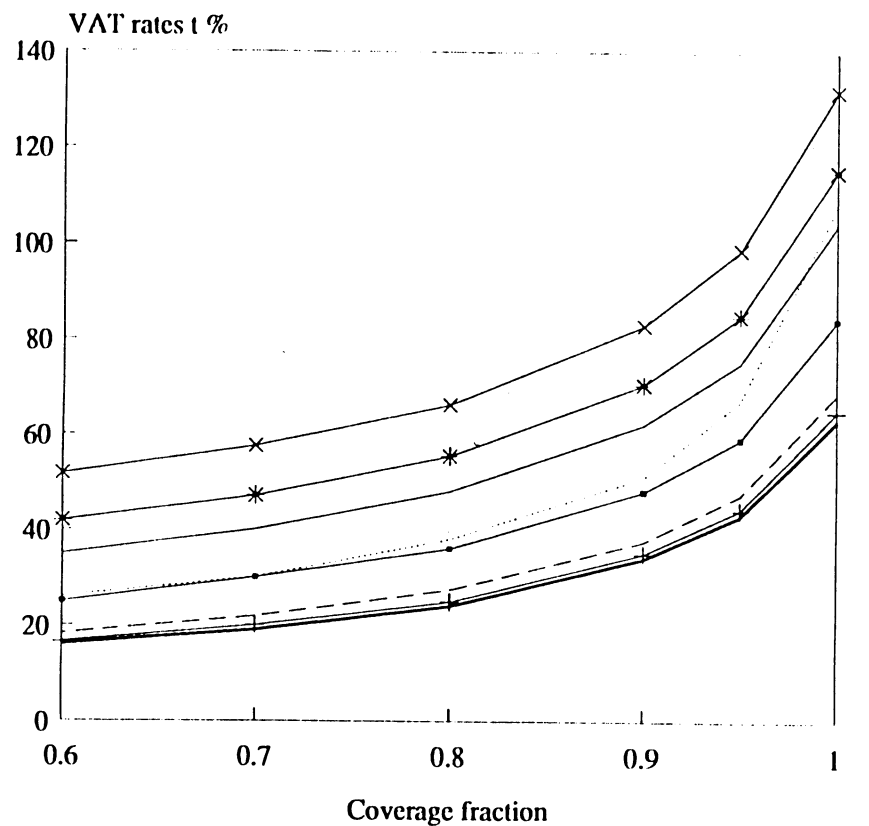
are modelled as a shift from a CEE reference case of $\sigma = 0.30$ (which has a Gini coefficient of 0.168) to the UK case, taken to be 0.39, with a Gini of 0.217. (Pudney, 1993, finds that the Gini coefficient for gross earnings for all Hungarian workers has risen from 0.267 using data from the 1988 Earnings Survey to 0.301 for 1992, compared with 0.276 in 1988 for Britain, and for 1990 of 0.283, lower than the 1990 figure for Hungary of 0.289. The Hungarian estimates are subject to considerable uncertainty as they are derived from rather coarsely grouped data.)

The main results are presented in figs. 2 and 3 and Table 2. They show the effect of reducing the coverage of commodity taxes, measured by β , from 1 down to 0.6, assuming that the taxed goods are a random subset of all goods as far as subsistence requirements are concerned. (Alternatives in which subsistence requirements varied more or less strongly with β , showed the results to be insensitive to the exact specification.) The more dramatic and revealing results are given in fig. 2, which shows the rate of VAT on those goods that are still taxed. Despite the reduced coverage, which might argue for higher rates of tax to compensate for the fall in revenue, the optimal tax decreases quite sharply. The effect of this on redistribution is shown in fig. 3, which shows the share of redistributive public expenditure for the same set of cases. Reducing the tax coverage by 20 percent roughly halves the level of redistributive expenditure, and is clearly of the first importance.

The effects of varying inequality aversion, v , from 0.5 to 0.8, 1.2, 1.5 and 2.0, can be seen by comparing the five thin continuous lines, all plotted for a skill dispersion $\sigma = 0.39$ (shown as $s = 0.39$ in the figures), the higher values of v corresponding to higher tax and transfer rates. The effect is to shift the plots almost parallel (from between 3 and 7 per cent for g going from $v = 0.5$ to 0.8, a further 5 percent in going to $v = 1.2$, and the same again going to $v = 2.0$). Again we see that the effect of increasing the skill dispersion from $\sigma = 0.3$ to $\sigma = 0.39$ is the same as moving from $v = 0.8$ to $v = 0.5$ (almost exactly) or from $v = 1.2$ to $v = 0.8$. Put another way, if wage inequality rose as much as represented by this move, then it would need a considerable decrease in concern for inequality to offset the case for higher tax rates.

Increasing the required level of public expenditure from zero to about 4 percent of GDP raises the rate of VAT by a very modest amount (between 1 and 4 per cent) and further reduces the amount of redistributive expenditure (by about 3 per cent of GDP), so that non-redistributive public expenditure crowds out redistributive expenditure almost one for one. Finally, the effect of reducing the critical value in the

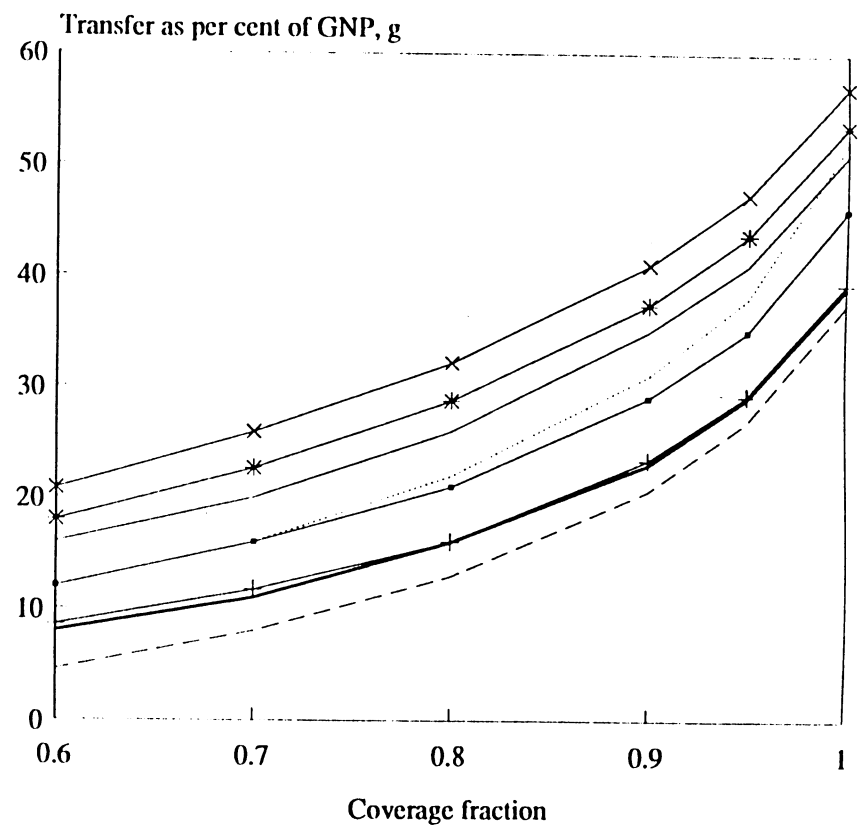
Optimal tax rates with varying tax coverage



$G0=0.5, GT=.01, GN=.01, d=.05,$

Fig. 2

Optimal Transfers with varying tax coverage



$G0=0.5, GT=.01, GN=.01, d=.05,$

Fig. 3

labour supply function, δ , (which directly affects the labour supply elasticity) from 0.05 to 0.03 has hardly any effect for a coverage of less than 0.8, but quite a high effect for full coverage.

The evidence from these and similar calculations suggest that the rate at which t and g decrease with tax coverage β , are similar across a wide range of values of v , σ , and δ (at least, for the range of values of δ that give plausible elasticities), although the level of taxes and transfers are, as predicted, sensitive to all these parameters. Just how this sensitivity manifests itself is discussed further below.

4. The balance between direct and indirect taxes

Suppose that in addition to choosing the rate of VAT for the subset of taxable goods, it is possible to impose a linear income tax at rate τ on a fraction α of the population (where in general α may be a function of τ , w and possibly also t). That is, given the tax instruments available, some fraction of the wage-earning population may evade income taxes completely.¹⁰ The after-tax wage rate of those paying income tax will now be $w_n = (1-\tau)w$, with consequent effects on labour supply and consumer demands.

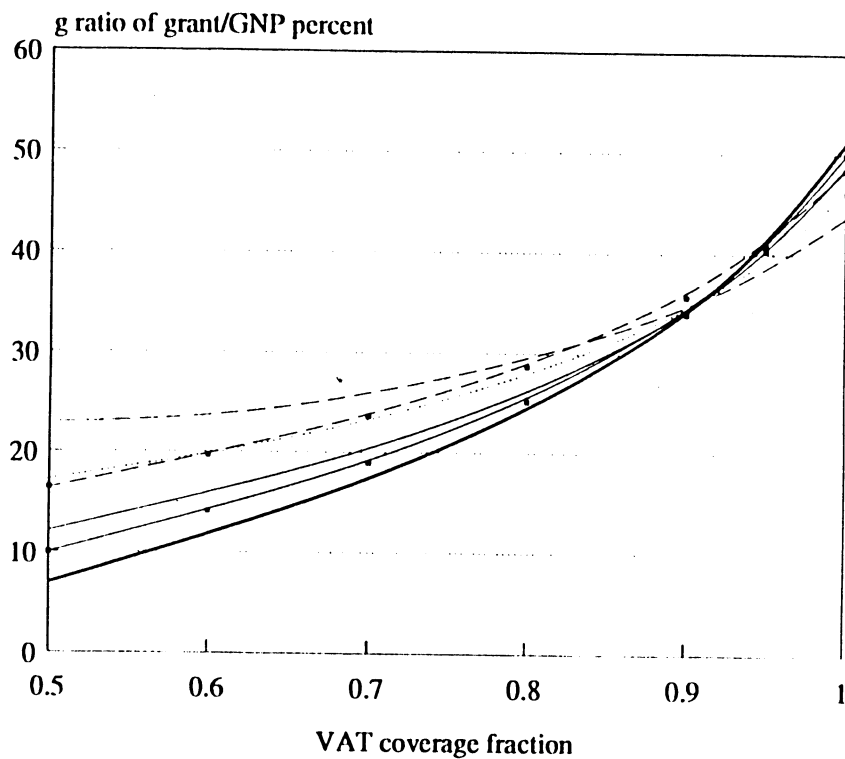
To keep matters manageable, suppose that α , the extent of income tax coverage, is a parameter independent of τ and w ,¹¹ and that the least skilled person chooses to work, (i.e. $w > w_0$), so that there is no (voluntary) unemployment. Total income tax revenue is $\alpha\tau\int w\ell_n dF$, and total tax revenue will be the sum of direct and indirect taxes, as shown in the appendix. The expressions for the size of transfers and equivalent consumption, as well as social welfare, will also need modification.

The numerical investigation specified α and β , varied τ in steps of 10 per cent from 0 to 30 per cent, and found the optimum VAT rate t given the income tax and extent of tax coverage. Sample results are given in figs. 4 and 5 for the case of rather high inequality aversion, $v = 1.5$, that tends to exaggerate the costs of tax evasion (as evasion increases the dispersion of net income). Fig. 4 shows the extent of redistribution measured by the lump-sum grant, g , after meeting the required revenue of 5 per cent of baseline GNP (measured at $\beta_t = 1.0$).

¹⁰ If some workers evade a fraction of their income tax liabilities, the effect will be similar but less socially costly than the extreme case considered here, so the formulation chosen tends to exaggerate the seriousness of tax evasion.

¹¹ The incentive to evade will increase with τ , but the effect of this can be found by examining α and τ jointly.

Effects of varying tax coverage on optimal transfers

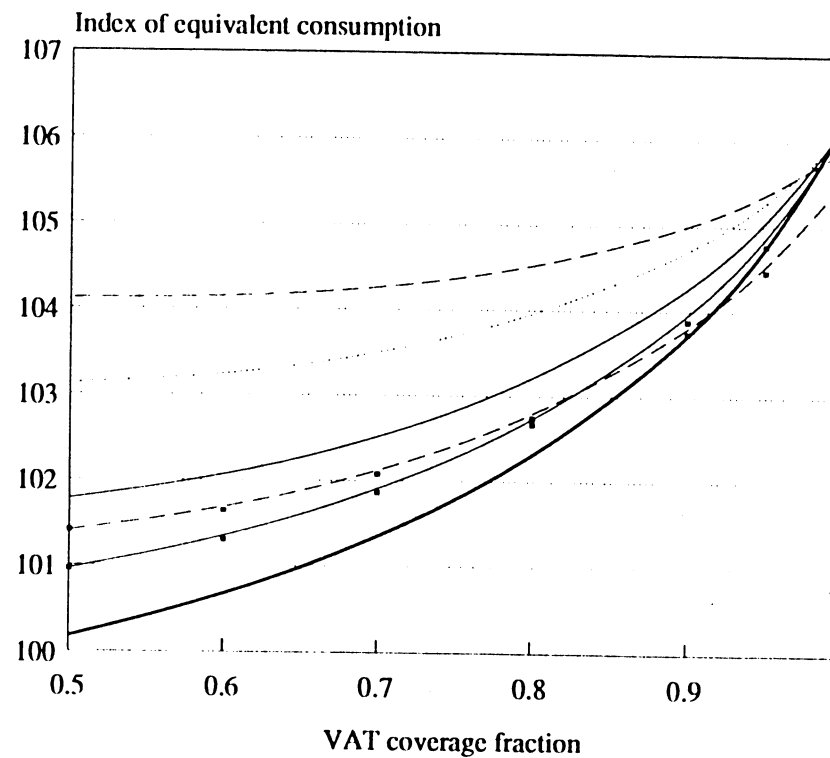


— IT = 0 - - - IT = 10% ... IT = 20%
 - . - IT = 30% - • - IT = 10% (0.5) - • - IT = 30% (0.5)

Income tax coverage 0.9 unless indicated
 in brackets eg (0.5)
 $G_0=0.5$, $G_T=GN=0.01$, $d=0.05$, $nu=1.5$, $R=5\%$

Fig. 4

Effects of varying tax coverage on social welfare



— IT=0 - - - IT=10% ... IT=20%
 - . - IT=30% - • - IT=10% (0.5) - • - IT=30% (0.5)

Income tax coverage 0.9 unless indicated
 in brackets eg (0.5)
 $G_0=0.5$, $G_T=GN=0.01$, $d=0.05$, $nu=1.5$, $R=5\%$

Fig. 5

Fig. 5 shows the index of social welfare, measured by the level of consumption equally distributed to all (with no taxes) of equal social value to that actually ruling, where 100 is the level of equivalent consumption with no taxes and transfers. In this consequentialist welfare economics, only vertical inequality counts, and income tax evasion is socially costly solely because it reduces the tax revenues deemed to be better directed at the more deserving, not because in addition it attracts feelings of horizontal inequity or unfairness. To the extent that these are morally or politically important, the relative merits of a partially evaded income tax compared with the less easily evaded but incomplete VAT would be weakened. On the other hand, the case has been biased against an income tax by restricting attention to linear income taxes (though the lump sum transfer is superior to the typical system in which income tax is levied on the excess of income above a given sum). It may be that the additional advantages of progressivity which are specific to income tax offset some of the disadvantages of perceived horizontal inequity associated with evasion, though the case for progressivity is considerably weakened by looking at taxation in a lifetime context, as lifetime incomes are considerably less dispersed than cross-section incomes (Falkingham, Hills and Lessof, 1993).

In both figs. 4 and 5 the bold line shows the effect of incomplete VAT coverage in the absence of any direct taxation, while the upper left dashed line shows the effect of a 30 per cent income tax covering 90 per cent of the workers. Notice that this is inferior (lower social welfare) to zero income taxation if the VAT coverage is greater than about 97 per cent. The two lines with small square markers show the effect of lowering the income tax coverage to 50 per cent, and are typically inferior to the higher level of coverage. As expected, the optimal VAT declines as the income tax increases, though the equivalent level of income tax of the two partial systems is now measured by $(\alpha\tau + \beta_t t)/(1 + \beta_t t)$ and this increases in τ for given VAT coverage, β_t , provided that income tax increases welfare, given α and β_t .

The problem of choosing the optimal income tax and VAT tax rates, given the extent of coverage and evasion consists in specifying β_t and α , varying τ , optimising $t(\tau)$, and then selecting the pair $(\tau, t(\tau))$ that maximises social welfare, W . The present computer program to optimise t is already cumbersome, and so a cruder grid search over τ with values of 0, 10, 20 and 30 per cent was adopted. Fig. 6 shows that if β_t falls below 97 per cent and $\alpha = 90$ per cent, then the highest rate $\tau = 30$ per cent is the best. Higher income tax evasion with $\alpha = 50$ per cent reduces the attractiveness of high income tax rates, and the best income tax rate is zero if $\beta_t > 97\%$, $\tau = 10$ per cent

if $93\% < \beta_i < 97\%$, $\tau = 20$ per cent if $60\% < \beta_i < 93\%$, and $\tau = 30$ per cent only for $\beta_i < 60\%$.

What does this mean for the desirability of introducing a income tax? The following proposition demonstrates that in this model, a non-zero income tax is not necessarily desirable.

Proposition: If VAT coverage is complete, then the optimal linear income tax rate is zero if the degree of income tax evasion is positive.

Proof: A linear income tax is at best equivalent to a uniform VAT, and if the VAT coverage is complete, then income tax will at best be equivalent if there is no evasion, and will introduce horizontal inequity if incomplete, lowering social welfare. QED

This proves that an income tax is only attractive if VAT coverage is incomplete. It does not seem feasible to derive a readily interpretable analytical condition for a positive rate of income tax to be desirable, though the form of such expressions is derived in Appendix A. Instead an exhaustive computer search was undertaken to find conditions under which introducing an income tax which is 50 per cent evaded at a rate of 1 per cent (starting from zero) improved welfare. The only cases in which this failed to improve matters was $\delta > 0.5$, $\beta_i > 0.98$, $v < 1.8$, and $R = 0$ (and values for $\epsilon > 0.8$). These correspond to cases where the optimal VAT is rather low, so the advantage of introducing additional taxes is small. Holding preferences constant, increases in v and R which increase the optimal tax rate reduce the range of values of β_i for which a small income tax is undesirable.

This supports the claim that introducing an income tax is desirable if VAT coverage is incomplete, even if the income tax is evaded (providing the administration costs are not excessive and the desired level of taxes are not too low). The quantitative evidence suggests that at high rates of income tax evasion only rather low rates of income tax may be justified, unless the VAT coverage is also rather low.

Finally, what effect does extending the range of tax instruments to include income taxes have on the degree of crowding out of redistribution by increases in essential expenditure, R (or reductions in other sources of tax revenue)? Comparing levels of g for different levels of R , for the parameter values of figs. 4-5, if R increases by 1 percentage point of GNP, then g falls by 0.6 of 1 percentage point of GNP (with a standard deviation or $SD = .07$) if $\tau = 0$, and as τ increases, so the degree of

crowding out of transfers falls, and g drops by 0.4 of 1 percent of GNP at $\tau = 30\%$ ($SD = 0.24$). The variability of the degree of crowding out increases at low rates of tax coverage ($\beta_i = 0.5-0.6$) and higher rates of income tax. This is consistent with a view that the social marginal cost of raising revenue rises quite rapidly with attempts to raise more tax revenue when the tax system is more incomplete and evaded, as one might expect.

5. The underlying determinants of redistribution

Figs 2-5 show that the optimal tax and transfers are sensitive to the underlying individual preferences, measured by δ , social preferences, measured by v , the degree of income inequality measured by σ , and the extent of tax coverage. Fig. 6 provides a different method of describing the sensitivity of the transfer system (the key element under study) to the elasticity of substitution between consumption and leisure, ϵ , given by (A5).¹² Fig. 6 suggests that the main determinant of the degree of redistribution is the elasticity of substitution between consumption and leisure, ϵ , though there remains a considerable variation in g holding ϵ constant. The figure gives the impression that points lie on rays through the point (1,0). All points on the highest such ray have income tax $\tau = 30\%$, $\delta = -0.12$, while all points on the lowest ray have $\tau = 0$, and rather higher values of δ . Otherwise there do not appear to be any obvious causes for the scatter of points.

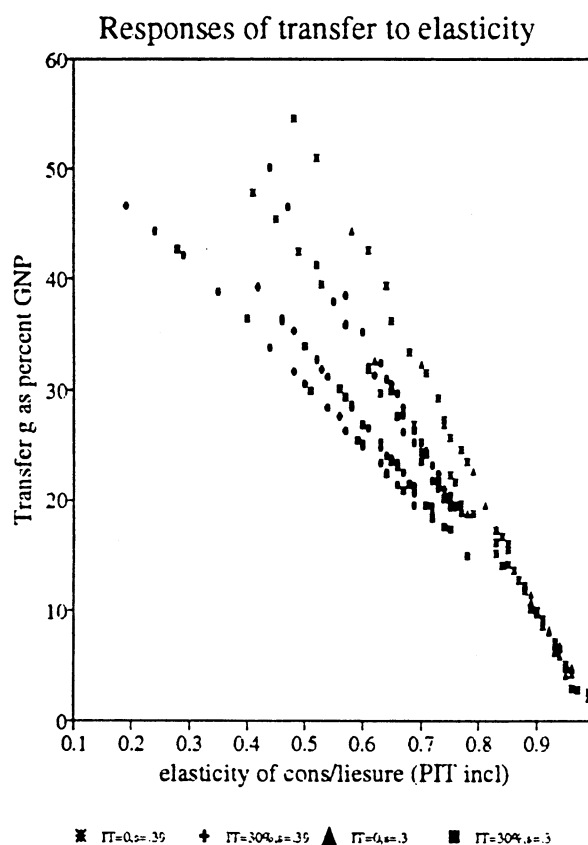


Fig. 6

¹² Fig. 6 was constructed by varying the labour supply parameter, δ from -0.12 to 0.05, inequality aversion v from 0.5 to 0.8, revenue R from 0 to 8%, VAT coverage β_i from 0.5 to 1.0, income tax rate τ from 0 to 30%, and income tax coverage α from 0.5 to 0.9, for outcomes that satisfied the various constraints.

The interpretation is that most of the environmental parameters (σ , v , R , β , α) jointly determine the optimal tax and transfer level and the elasticity of substitution, ϵ , given the structural parameters (δ , γ , γ). If the environment calls for large transfers, this will only be feasible if the taxes needed and the degree of tax coverage do not increase the elasticity of substitution unduly. Equivalently, the reason that incomplete tax coverage has such a dramatic effect on the degree of redistribution is almost entirely explained by its effect in raising the degree of substitution between taxed and untaxed goods. Income taxes which offset the incompleteness of the indirect tax system lower this elasticity and permit higher rates of redistribution, while higher levels of required government expenditure, R , raise taxes before redistribution and thereby raise the elasticity of substitution, limiting further tax increases.

This finding provides some reassurance about the generality of the results. Quantitative work on optimal taxation is seriously limited by the small number of tractable analytical models of consumer demand and labour supply, and the present model, though tractable, has a number of unsatisfactory limitations, of which the most important is the critical role played by δ , conflating both the labour supply elasticity with the marginal propensity to spend on leisure. The sensitivity analysis reported in fig. 6 suggests that the correct way to think of the budgetary implications of transition is on their effect on the elasticity of substitution between taxed and untaxed goods, ϵ , bearing in mind that this elasticity will depend not only on tax rates but also on the position that consumers find themselves on their labour supply schedule, itself affected by income opportunities, income levels and the structure of transfers.

6. Conclusions

Optimal indirect tax rates were found to be quite sensitive to elasticities of labour supply, inequality aversion, and the degree of inequality in the skill distribution, though indirect tax rates tend to exaggerate this sensitivity: the direct tax equivalent rate of $\tau = t/(1+t)$ (for complete indirect tax coverage) varies less, as does the degree of redistribution measured by g . Stern's (1976) conclusions that optimal tax rates tend to be rather high for plausible (ie low) labour supply elasticities and elasticities of substitution between consumption and leisure, as well as modest levels of required government expenditure, are confirmed for the case of complete tax coverage, but the main finding is that if the indirect tax system is incomplete and fails to cover all goods, then optimal tax rates fall quite sharply with coverage, and the degree of redistribution roughly halves if the indirect tax coverage falls from 100 per cent to 80

per cent in the absence of any direct taxation. Introducing direct taxes considerably ameliorates this effect, provided the degree of tax evasion is not too high.

Although changing attitudes to inequality and changes in the inequality of skills may lead to roughly offsetting changes in the optimal tax rate, these effects seem small compared to changes in the efficiency of tax collection as measured by tax coverage. In less technical language, the present political leaders may be less concerned with inequality as their support comes from votes explicitly expressed in elections and reflecting the interests of their supporters, rather than requiring the tacit acceptance of a wider but politically silent fraction of the population as before, and they may therefore wish to reduce the degree of redistribution. On the other hand, the recent rise in before-tax earnings inequality in countries like Hungary argues for more redistribution to offset the deteriorating income distribution. The claim here is that these two factors roughly balance, unless attitudes to inequality have become so insensitive that calls for safety nets can be ignored. A decrease in the efficiency of tax collection, as measured by the degree of tax evasion on personal income tax, or incompleteness of coverage on the indirect tax side, does provide a powerful argument for reducing not only the tax *share* in GDP, but also tax *rates*.

The practical question is whether the tax reforms undertaken with system reform were able to sufficiently reduce inefficiencies in subsidies and other distortions in pricing energy, housing, and other favoured goods, that the fall in the efficiency of tax administration could be offset by better tax design. Finally, of course, if the previous tax system were not only poorly designed but excessive given its own objectives, one might expect a more rationally designed system to reduce the tax share.

The other striking finding is that increases in non-redistributive expenditure (net of enterprise revenue) crowd out redistributive transfers to a considerable extent, so that for example a (permanent) fall in profits tax revenue of 1 per cent of GNP should lead to a cut in transfers of between one-third and two-thirds of 1 per cent of GNP, and only a small rise in the total tax share in GNP. The main determinant of the degree of redistribution was found to be the elasticity of substitution between consumption and leisure, ie between taxed and untaxed goods - a result that is probably fairly robust. The main effect of changes in parameters appears to be to change this elasticity and hence change the desirable degree of redistribution, and in most model specifications, changes in tax coverage would have a direct effect on this elasticity, suggesting that tax coverage is a critical determinant of redistribution.

The old Soviet-type tax system had little reason for personal income taxation, and the main practical reason for introducing a personal income tax was to offset the decline in profits transfers to the government when the old expropriatory system was replaced by a standard corporate income tax. It would have been possible to have recouped this increasing the rates of value added taxation above the former turnover tax rates, but this would not only have been more inflationary, but might have encouraged greater tax evasion. The argument given here is that direct personal income taxation is a valuable complement to indirect taxation, as it can assist in increasing the overall extent of tax coverage and hence lower the distortionary costs of tax collection. Income and commodity taxes are complementary in that income taxes where paid fall on all expenditure, while commodity taxes fall on incomes even where income tax is evaded.

The main conclusion is that the case for reducing taxes and expenditure in transforming economies is quite strong because of the extreme difficulty in ensuring an efficient and broad based tax system with high compliance. Given the difficulty of reducing expenditures, except in socially divisive or ethically unappealing ways, and given the inevitable erosion of the direct tax base through increased private activity and the fall in domestic demand, this strengthens the case for increasing the breadth of the indirect tax base, and tightening up tax administration to ensure compliance. It also follows that widening the indirect tax base while retaining direct taxes with compensating reductions in tax rates and adjustments in transfers is likely to be desirable in market economies like the US and the UK.

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Appendix A: Algebraic specification of the model

As Deaton (1983) explains, optimal tax calculations are bedeviled by the large number of potentially different first order conditions (one set for each consumer) that have to be simultaneously satisfied. The standard strategy is to look for a specification of consumer behaviour that allows perfect aggregation, for which the essential requirement is that the income supply functions be linear in the wage. The most general functional forms for which this holds are slightly more general than that considered here, in that the parameters δ and γ can be functions of prices. This dependence is suppressed in the form chosen here, ensuring that the optimal indirect tax system is uniform (and hence readily parameterised).

The chosen form of the indirect utility function of a worker whose skill level is w , facing consumer prices q , and receiving lump sum grant g is

$$V(g, q, w) = \frac{g + w(1 - \gamma_0) - q' \gamma}{w^\delta \prod_i q_i^{\beta_i(1 - \delta)}}, \quad \sum_i \beta_i = 1, \quad \beta_i > 0. \quad (A1)$$

Skills are distributed according to the distribution function $F(w)$, $\underline{w} \leq w \leq \bar{w}$, $F(\bar{w}) = 1$. Labour supply ℓ (as a fraction of total time) is found from Roy's identity $\ell = V_w/V_g$ (subscripts in this case indicating partial derivatives), so that

$$\ell = (1 - \delta)(1 - \gamma_0) - \frac{\delta}{w} (g - q' \gamma), \quad (A2)$$

provided that the wage is above two critical levels:

$$w \geq w_0 \equiv \left(\frac{\delta}{1 - \delta} \right) \left(\frac{g - q' \gamma}{1 - \gamma_0} \right), \quad w \geq - \left(\frac{g - q' \gamma}{1 - \gamma_0} \right). \quad (A3)$$

Here w_0 is the wage below which the worker will prefer to be unemployed, and will thus set $\ell = 0$, while the second inequality is required for concavity of the cost function (and to ensure that leisure is at least as high as the subsistence minimum, γ_0). The parameter δ is the marginal propensity to spend on leisure, or the fraction by which earned income, $w\ell$, is reduced if unearned income, g , is raised by one unit, from (A2). As Deaton (1983) points out, δ not only measures the disincentive effects of unearned income on labour supply, but $\delta - 1$ is the compensated wage elasticity of over-subsistence leisure, $1 - \ell - \gamma_0$. Merging these two effects is convenient but restrictive.

The demand for good i by someone of wage w is also found from Roy's identity: $x_i = -\partial V/\partial x_i / \partial V/\partial g$ and is linear in (above-subsistence) income $y(w)$ (as in the Linear Expenditure System):

$$x_i = \gamma_i + \frac{\beta_i y(w)}{q_i}, \quad (A4)$$

where

$$\begin{aligned} y(w) &= (1-\delta)[g + w(1-\gamma_0) - q' \gamma], & w \geq w_0, \\ y(w) &= g - q' \gamma, & w < w_0. \end{aligned}$$

and γ is the vector of subsistence requirements for goods.

When calibrating the model, two elasticities are critical - that between leisure and consumption, ϵ , and that of the labour supply to the wage, η . Consumption expenditure in this model is $c = g + w\ell$, and the ratio of leisure to consumption is $\phi \equiv (1-\ell)/c$:

$$\phi \equiv \frac{1-\ell}{c} = \frac{1-(1-\delta)(1-\gamma_0) + \delta\mu/w}{g - \delta\mu + w(1-\delta)(1-\gamma_0)}, \quad \mu \equiv g - q' \gamma.$$

$$\epsilon \equiv -\frac{d \ln \phi}{d \ln w} = \frac{\delta(g - q' \gamma)}{w(1-\ell)} + \frac{(1-\delta)(1-\gamma_0)w}{c}. \quad (A5)$$

The elasticity of labour supply is

$$\eta \equiv \frac{d \ln \ell}{d \ln w} = \frac{\delta\mu}{w\ell} = \frac{\delta(g - q' \gamma)}{w(1-\delta)(1-\gamma_0) - \delta(g - q' \gamma)}. \quad (A6)$$

If a subset of goods $i \in \Gamma$ is subject to a VAT at rate t , total indirect tax revenue will be

$$T = r\gamma_i + \frac{t\beta_i}{1+t} \int_{\underline{w}}^{\bar{w}} y(w) dF(w), \quad \gamma_i = \sum_{i \in \Gamma} \gamma_i, \quad \beta_i = \sum_{i \in \Gamma} \beta_i. \quad (A7)$$

In practice, the integral is evaluated in two parts, the first over $[\underline{w}, w_0]$, the second over $[w_0, \bar{w}]$, to allow for the two different functional definitions of $y(w)$ in (A4). Tax

revenue depends linearly on transfers, g , and hence g can be computed once R is specified (though in the presence of endogenous unemployment the computation requires a sequence of iterations).

The (constrained) optimal indirect tax problem is now to choose a rate of VAT, t , and a feasible (balanced budget) level of lump-sum grants, g , given a specification of the set of taxable goods, Γ , and required exhaustive public expenditure, R , to maximise social welfare, W :

$$W = \int_{\underline{w}}^{\bar{w}} U(V(g, t, w)) dF(w) = u U(V(g, t, w_0)) + \int_{w_0}^{\bar{w}} U(V(g, t, w)) dF(w), \quad u = \int_{\underline{w}}^{w_0} dF, \quad (A8)$$

where u is the fraction of the population unemployed (i.e. choosing zero hours work), and $U(V(g, t, w))$ is the social value of the income enjoyed by a worker of skill w when the VAT tax rate is t and grant is g . It will be convenient to normalise before-tax prices so that $p_i = 1$, and

$$V(g, t, w) = \frac{g + w(1 - \gamma_0) - (1 + t)\gamma_t - \gamma_n}{w^\delta (1 + t)^{\beta(1 - \delta)}}, \quad \gamma_n \equiv \sum_{i \in \Gamma} \gamma_i. \quad (A9)$$

Society's attitudes to inequality will be reflected in the choice of the function U , and it is convenient to have a simple parameterisation for inequality aversion, and hence to take $U(c) = c^{1-\nu}/(1-\nu)$, $\nu \neq 1$ (and $\ln c$ for $\nu = 1$), where c is consumption. Some care is required in normalising utility, for if we take the function $U(x) = x^{1-\nu}/(1-\nu)$, $\nu \neq 1$, this has the consequence that the social weight of extra consumption by a worker of wage w is not $c^{-\nu}$ but $V^{-\nu}$, and V increases as $w^{1-\delta}$. It seems desirable to follow tradition and ask that our parameterisation have the property that $\nu = 0$ corresponds to indifference to distribution (and hence an optimal redistributive tax of zero, ie $g = 0$), which would not be the case if we took $U(x) = x^{1-\nu}/(1-\nu)$.

The solution adopted here is to measure the standard of living by the level of consumption that the worker would choose if he faced no taxes, that yields the same level of utility that his actual choices faced with distortionary taxes yield. The value of c is:

$$c = (1 - \delta)g' + w(1 - \delta)(1 - \gamma_0) + \delta p' \gamma,$$

where g' is the lump-sum grant required to equalise the utility level when facing undistorted prices with those actually achieved:

$$\frac{g' + w(1-\gamma_0) - (\gamma_t + \gamma_n)}{w^\delta} = \frac{g + w(1-\gamma_0) - (1+t)\gamma_t - \gamma_n}{w^\delta (1+t)^{\beta(1-\delta)}}.$$

Substituting for g' the value for $c(g, t, w)$ is

$$c(g, t, w) = (1-\delta) \left(\frac{g + w(1-\gamma_0) - (1+t)\gamma_t - \gamma_n}{(1+t)^{\beta(1-\delta)}} \right) + \gamma_t + \gamma_n. \quad (\text{A10})$$

Social welfare is then

$$W = u \frac{c(g, t, w_0)^{1-\nu}}{1-\nu} + \frac{1}{1-\nu} \int_{w_0}^{\infty} c(g, t, w)^{1-\nu} dF(w), \quad \nu \neq 1. \quad (\text{A11})$$

This has the required property that $dU/dg = kc^{-\nu}$, where $k = (1-\delta)(1+t)^{-\beta(1-\delta)}$, the same for all individuals.

Adjusting for the distortionary effects of taxation suggests that consumption c in the formula for the elasticity of substitution between consumption and leisure, ϵ , in (A5) should be replaced by its non-distorted equivalent from (A10).

Model validation

Kaiser, Wiegard and Zimmerman (1989) argue persuasively that optimal tax studies are computationally complex and often hard to replicate. They list various pitfalls facing the computational analyst (not peculiar to optimal tax calculations) and suggest the importance of attempting to replicate known results (in order to provide some check on the computations). They also argue that numerical findings need to be defended by economic arguments and explanations as a further check. Anyone who has attempted to write computer programs to perform these calculations will be aware of the potential for errors and hence the need for checks. The ideal situation is one in which the model in question is identical to a model for which solutions have already been derived for a specific set of parameters.

Stern's (1976) model assumes an isoelastic direct utility function $u(c, \ell) = [\alpha(1-\ell)^\mu + (1-\alpha)c^{-\mu}]^{-1/\mu}$. Mirrlees' (1971) model assumes a unit elasticity of substitution between consumption and leisure, ϵ , and a direct utility function $u(c, \ell) = \ln c + \ln(1-\ell)$. Stern's model corresponds to Mirrlees' (1971) model for $\alpha = 0.5$, $\mu = 0$, and an inequality aversion (our ν) of 1 (the log case). The model of this paper coincides with these two if $\delta = 0.5$, $\gamma = 0$, $\gamma_0 = 0$, with complete uniform indirect

taxation, though as mentioned above, one must be careful to transform the indirect utility function appropriately.

Mirrlees (1971) computed optimal (non-linear) income taxes assuming that skills were log-normally distributed in the population with parameters (μ, σ) (the mean and standard deviation of the associated normal distribution) taken as $(-1, 0.39)$ to approximate the UK skill distribution. The log-normal density function is:

$$f(w) = dF(w) = \frac{1}{w\sigma\sqrt{2\pi}} \exp\left(\frac{-(\ln w - \mu)^2}{2\sigma^2}\right) \quad (\text{A12})$$

One of the advantages of the log-normal distribution is that it allows explicit solutions for the reference case $\delta = 0.5$, $\gamma = 0$, $\gamma_0 = 0$, for which the indirect utility function reduces to $(g + w)/\sqrt{w(1+t)}$. In that case, if there is no unemployment, the equation for the lump-sum transfer g reduces to

$$g = \frac{\bar{w}t}{2+t}, \quad \bar{w} = \int_{\underline{w}}^{\bar{w}} w dF(w).$$

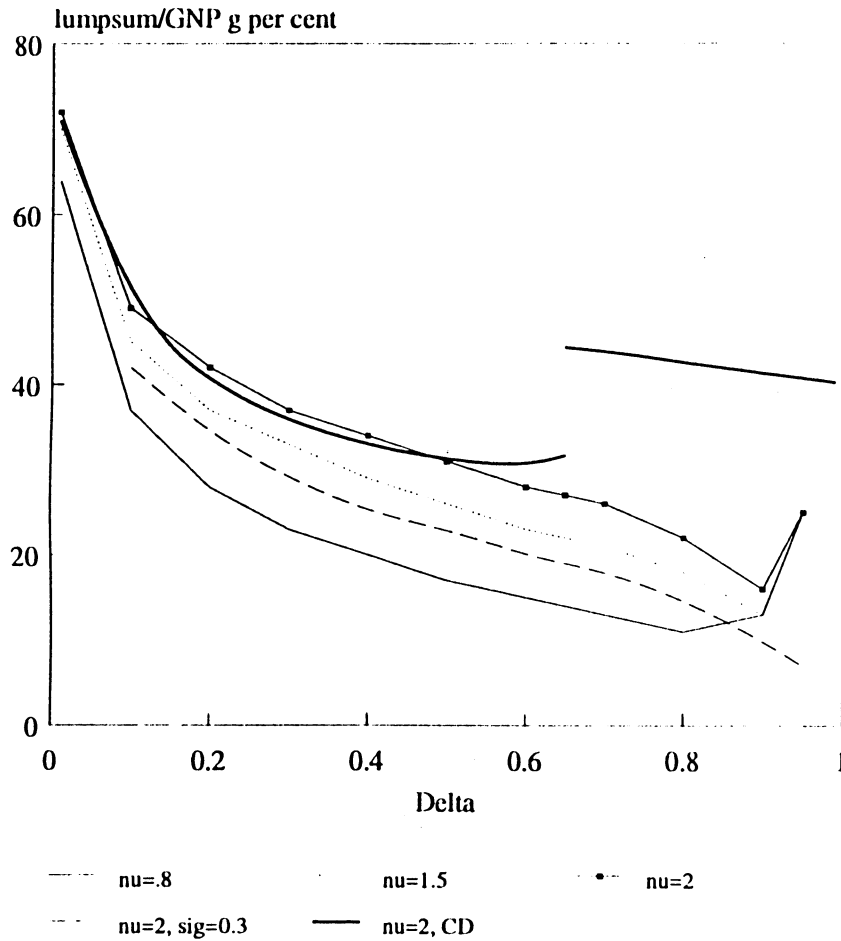
Social welfare is then

$$W = \frac{1}{\sqrt{1+t}} \left(\frac{\bar{w}t}{2+t} \int \frac{1}{\sqrt{w}} dF + \int \sqrt{w} dF \right), \quad \text{where } \int w^k dF = \exp(k\mu + \sigma^2/2).$$

This can be numerically computed and checked against the computer results, and again they confirm the computer calculations.

The next standard problem with optimal tax calculations is that of potential nonconcavity in the relation between social welfare and the tax rate - a problem that bedevils the search for a globally optimal tax rate. Fig. A1 graphs the equivalent income tax rate, τ , against δ for a variety of parameter specifications. If, as in these computations, required expenditure R is zero, then the ratio of the lump-sum grant, g , to GDP at market prices, $Y = g + \int w \ell dF$, is equal to the equivalent PIT rate τ , otherwise it will be less (but remains the most useful measure of the extent of redistribution, as it is equal to the share of redistributive transfers in GDP). The data for fig. A1 are given in table 1, and all plots have $\gamma_0 = 0.5$, $R = 0$. The four lowest plots (at $\delta = 0.8$) have $\gamma_t = 0.01$, $\gamma_n = 0.01$. All plots have $\sigma = 0.39$ except for the dashed line where, as noted, $\sigma = 0.3$. (This case is chosen as a possible reference case

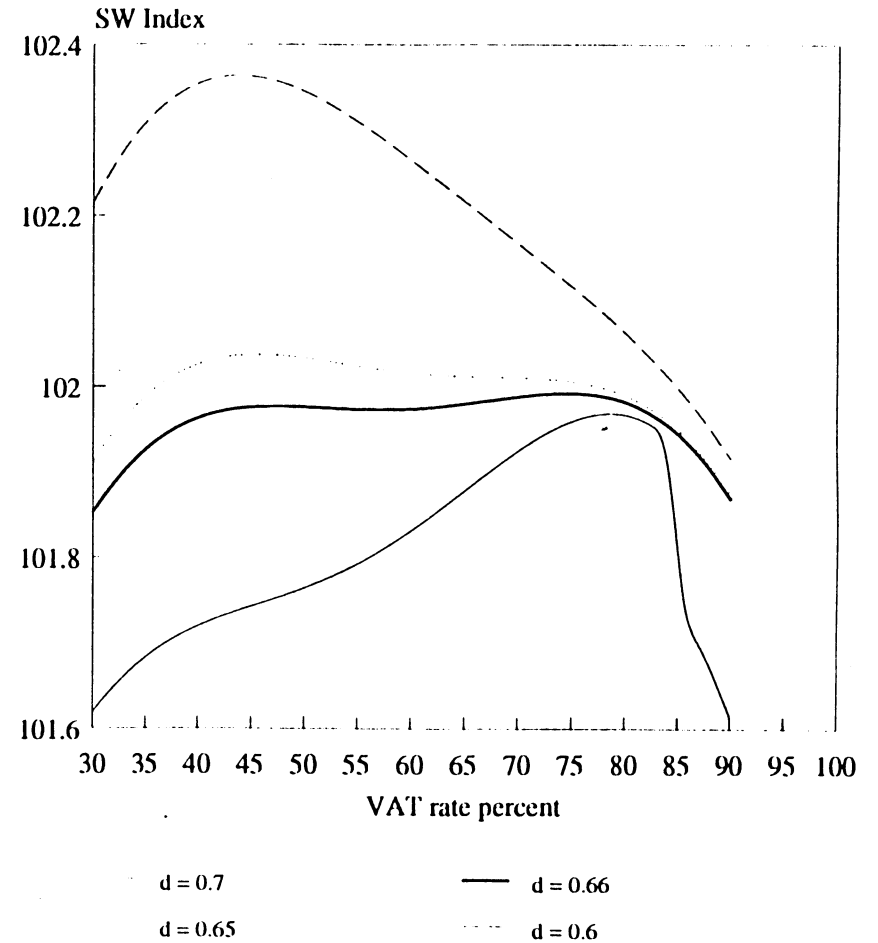
Optimal tax rates varying inequality aversion



G0=.5;GT=.01,GN=.01,R=0; CD: GT=GN=0

Fig. A1

Social Welfare as a function of VAT rate



G0=0.5, GN=GT=0.0, Nu=2.0,R=0,Beta=1

Fig. A2

for the more egalitarian CEE countries, as discussed below.) The bold discontinuous plot, labelled CD for Cobb-Douglas, has $\gamma = 0$.

The first three plots are monotonically decreasing until the tax falls to the point at which the poorest workers would be driven below subsistence, and at that point the tax system is set to generate at least subsistence income. This ceases to be a problem if the skill distribution is less unequal, as shown in the dashed line. Finally, the upper bold plot is discontinuous at about $\delta = 0.66$. For a range of values of δ near 0.66 the graph of W against t has two local maxima, as fig. A2 shows, and their relative height changes, leading to a discontinuous change in the global maximum (compare $\delta = 0.65$ and $\delta = 0.66$). At the point of discontinuity, social preferences are finely balanced between a larger degree of redistribution of a smaller total GDP (as deadweight losses are larger) and a smaller degree of redistribution of a larger GDP. The distributional consequences of the two alternatives are quite different, and are concealed by the aggregations of socially weighted outcomes.

Extensions to allow for direct taxation

Income tax revenue is $\alpha\tau\int w\ell_n dF$, and so total direct and indirect tax revenue is

$$T = \alpha\tau(1-\delta)(1-\gamma_0)\bar{w} - \frac{\alpha\tau\delta}{1-\tau}(g - q'\gamma) + r\gamma_i + \frac{t\beta_i}{1+t} \int_{\underline{w}}^{\bar{w}} [\alpha y_n(w) + (1-\alpha)y(w)] dF(w), \quad (A13)$$

The revenue available for redistribution as a uniform lump-sum grant, g , is

$$g = \alpha\tau(1-\delta)(1-\gamma_0)\bar{w} - \frac{\alpha\tau\delta}{1-\tau}(g - q'\gamma) + r\gamma_i + \frac{t\beta_i(1-\delta)}{1+t}(g - q'\gamma + (1-\alpha\tau)(1-\gamma_0)\bar{w}) - R, \quad (A14)$$

which requires further manipulation to collect all terms in g on the left-hand side.

Equivalent consumption for a worker paying income tax (the counterpart to (A10)) is:

$$c_n = (1-\delta) \left(\frac{g + w(1-\tau)(1-\gamma_0) - (1+t)\gamma_i - \gamma_n}{(1-\tau)^\delta(1+t)^{\beta_i(1-\delta)}} \right) + \gamma_i + \gamma_n. \quad (A15)$$

This is also the value of consumption to use in the calculation of the elasticity of substitution between leisure and consumption, ϵ , together with $w_n = (1-\tau)w$. Social welfare is:

$$W = \frac{\alpha}{1-\nu} \int_{\underline{w}}^{\bar{w}} c_n^{1-\nu} dF + \frac{1-\alpha}{1-\nu} \int_{\underline{w}}^{\bar{w}} c^{1-\nu} dF(w), \quad \nu \neq 1, \quad w_0 < \underline{w}. \quad (A16)$$

The desirability of introducing income tax

If we can show that $dW/d\tau > 0$ at $\tau = 0$, then social welfare would be increased by introducing a small income tax starting from zero. From (A16), provided $\nu \neq 1$, and $\underline{w} < w_0$:

$$\frac{\partial W}{\partial \tau} \Big|_{\tau=0} = \int_{\underline{w}}^{\bar{w}} \left(\alpha c_n^{-\nu} \frac{dc_n}{d\tau} + (1-\alpha) c^{-\nu} \frac{dc}{d\tau} \right) dF(w) = \int_{\underline{w}}^{\bar{w}} \left(\alpha \frac{\partial c_n}{\partial \tau} + \frac{\partial c}{\partial g} \frac{dg}{d\tau} \right) c^{-\nu} dF(w), \quad (A17)$$

where the second equality follows from the fact that $c_n = c$ at $\tau = 0$. The derivatives may be evaluated from (A10), (A14) and (A15):

$$\frac{\partial c}{\partial g} = \frac{1-\delta}{(1+t)^{\beta_t(1-\delta)}},$$

$$\frac{\partial c_n}{\partial \tau} = \frac{-(1-\delta)(1-\gamma_0)w}{(1+t)^{\beta_t(1-\delta)}} + \frac{(1-\delta)\delta[g+w(1-\gamma_0)-q'\gamma]}{(1+t)^{\beta_t(1-\delta)}} = \frac{-(1-\delta)[(1-\delta)w(1-\gamma_0)-\delta(g-q'\gamma)]}{(1+t)^{\beta_t(1-\delta)}}.$$

$$\frac{dg}{d\tau} = \alpha \frac{(1-\delta)(1-\frac{t\beta_t}{1+t})(1-\gamma_0)\bar{w}-\delta(g-q'\gamma)}{1-\frac{t\beta_t(1-\delta)}{1+t}} = \alpha \left((1-\gamma_0)\bar{w}-\delta \frac{(1-\gamma_0)\bar{w}+g-q'\gamma}{1-\frac{t\beta_t(1-\delta)}{1+t}} \right). \quad (A18)$$

Therefore

$$\frac{\partial W}{\partial \tau} = \frac{1-\delta}{(1+t)^{\beta_t(1-\delta)}} \int_{\underline{w}}^{\bar{w}} c^{-\nu} \left(\frac{dg}{d\tau} + \alpha[\delta(g-q'\gamma)-(1-\delta)(1-\gamma_0)w] \right) dF(w), \quad (A19)$$

and from (A18)

$$\frac{\partial W}{\partial \tau} = \frac{\alpha(1-\delta)}{(1+t)^{\beta_i(1-\delta)}(1+t-\beta_i(1-\delta))} \int_{\underline{w}}^{\bar{w}} c^{-\gamma} dF(w) \left([(1+t-\delta\beta_i(1-\delta))\bar{w} - (1-\delta)(1+t-\beta_i(1-\delta))w](1-\gamma_0) - \delta t\beta_i(1-\delta)(g-q'\gamma) \right). \quad (A20)$$

This is hard to sign, though if $\delta = 0$ and $\beta_i = 1$, it is negative, as the integrand is proportional to $c^{-\gamma}((1+t)\bar{w} - w)$ and $c^{-\gamma}$ and w are negatively correlated.

The present model may exaggerate the effect of changes in environmental parameters, as they may lead to larger changes in ϵ holding δ constant than might be the case with a different specification of preferences. Some sense of the strength of the structural constraints placed on the various elasticities is provided by fig. A3, showing the variation in ϵ and η with the log of the wage. The distribution function is shown plotted on the LH y-axis as the nearly straight dashed line, so the percentiles of the wage distribution can be readily located - and the median is indicated by the horizontal line.¹³ It is striking that ϵ increases with w (and is constrained from falling too low by the transfers which dominate at low wage rates), while η falls very sharply with the wage. Note also that correcting consumption for the indirect tax rate substantially increases the elasticity (shown as the continuous line labelled *epsilon (Cnet)*). Stern's (1976) model assumed a constant value for ϵ and might therefore lead to rather less dramatic responses of transfers to tax coverage, holding the parameter ϵ in the underlying utility function constant.

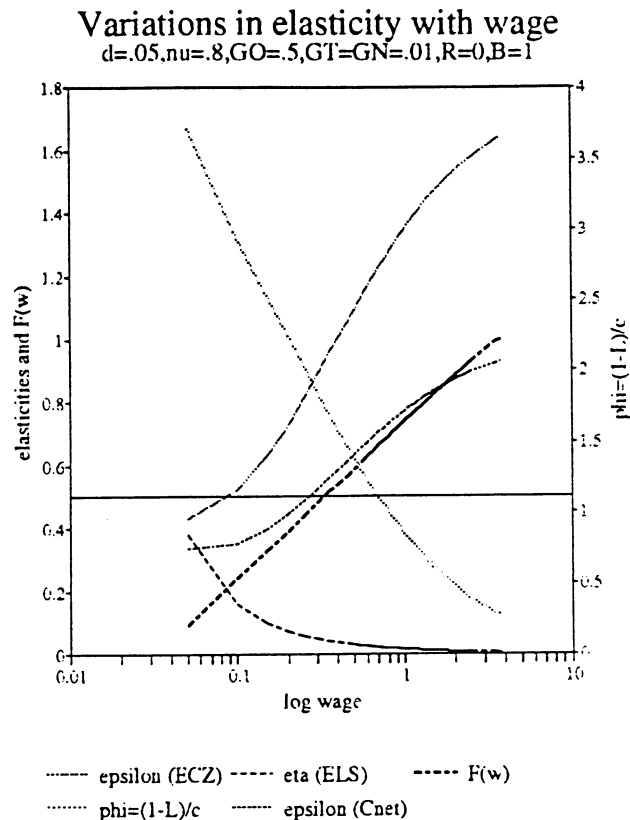


Fig. A3

¹³ The underlying parameters are $\delta = 0.05$, $\gamma_0 = 0.5$, $\gamma_n = .01$, $\gamma_t = .01$, $\beta_i = 1$, $t = .84$, $R = 0$, $g = 46\%$ of GNP.

Appendix B: Computation

The computation was programmed in FORTRAN and proceeded as follows. The skill distribution was truncated at a lower and upper level, to avoid excessively small values which create computational difficulties or inefficiencies. The distribution was then integrated over these limits and re-scaled. The initial value of GDP and the average wage were calculated for zero taxes, and required government expenditure was specified as a fraction of this initial GDP. The tax rate was then varied, and for each tax rate the level of lump sum grant, g , satisfying (8) was determined by a simple iterative procedure. Given this value of g , the cut-off level of wages, w_0 , and hence unemployment were computed. If unemployment was positive, a new estimate for g was computed until convergence was achieved. Social welfare was then computed by integration, using the NAG Fortran routine DO1AJF. A standard golden section search for the maximum of social welfare as a function of the tax rate was used to find local maxima, and periodic plots of the social welfare index against the tax rate were computed to check for multiple local maxima. Values of the lump sum grant as a percentage of GDP at market prices, and required government expenditure as a per cent of GDP at market prices were calculated. The social welfare index used was the level of uniformly distributed consumption equal in social welfare to that achieved, expressed as a per cent of the zero tax level, provided zero taxes were feasible, and otherwise expressed as a per cent of the minimum feasible tax rate. Positive required public expenditure or high subsistence requirements relative to minimum wages both required positive tax rates for feasibility. The unemployment rate was computed, but remained below one per cent for all calculations.

Income tax was introduced by nesting the optimal VAT calculations within a DO loop varying the rate of income tax, after suitably modifying the equations for g , V , W , etc, and tested by setting $\tau = 0$ to ensure that the earlier results were replicated.

The output was read into a spreadsheet for analysis and graphing, using Quattro Pro 4.0 and Harvard Graphics 2.0.

Table 1 Relationship between tax rates and delta

percentages

σ	0.39		0.39		0.39		0.3					
v	0.8		1.5		2		2					
δ	t	g/Y	ϵ	t	g/Y	ϵ	t	g/Y	ϵ	t	g/Y	ϵ
0.01	178	64		231	70	33	260	72	31			
0.05	84	46		115	54	52	132	57	50	103	51	55
0.1	59	37	69	83	45	62	97	49	60	74	42	65
0.2	39	28	77	59	37	72	71	42	69	52	34	74
0.3	31	23	81	48	33	76	59	37	74	41	29	78
0.4	25	20	82	41	29	78	51	34	76	34	25	79
0.5	21	17	81	36	26	78	45	31	76	30	23	78
0.6	18	15	78	31	23	75	40	28	74	25	20	76
0.7	15	13	73	27	21	71	35	26	69	21	18	71
0.8	12	11	64	22	18	61	29	22	60	17	15	62
0.9	14	13	44	14	13	44	19	16	43	11	10	44
0.95	34	25	23	34	25	23	34	25	23	7	7	27

Table 2 Effect of incomplete tax coverage

σ	0.39	0.39	0.39	0.39	0.3	0.39	0.39	0.39
δ	0.05	0.05	0.05	0.03	0.05	0.05	0.03	0.05
v	1.5	1.2	0.8	0.8	0.8	0.5	0.5	0.5
R %	0	0	0	0	0	0	0	3.7-4.8

β	Value added tax rates percent							
1	115	104	84	108	63	65	85	69
0.95	85	75	59	67	43	45	51	47
0.9	70	62	48	51	34	35	38	38
0.8	55	48	36	38	24	25	27	27
0.7	47	40	30	30	19	20	21	22
0.6	42	35	25	26	16	17	17	18

β	Ratio of lump sum transfers to market GDP %							
1	54	51	46	52	39	39	46	37
0.95	44	41	35	38	29	29	32	27
0.9	37	35	29	31	23	24	25	21
0.8	29	26	21	22	16	16	17	13
0.7	23	20	16	16	11	12	12	8
0.6	18	16	12	12	8	9	9	5

β	ϵ (elasticity between leisure and consumption) %							
1	52	54	59	52	65	64	57	66
0.95	60	63	67	63	73	73	70	75
0.9	66	68	73	71	74	78	76	80
0.8	73	76	80	79	85	84	84	87

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