THE SHADOW WAGE RATE IN DEVELOPING COUNTRIES:

LITTLE AND MIRRLEES' FORMULATION RECONSIDERED

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Economic Development Report No. 206

February 1972

DEVELOPMENT RESEARCH GROUP
Center for International Affairs
Harvard University
Cambridge, Massachusetts

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Abstract

What is the socially optimal shadow wage rate for use in planning and evaluating government investment projects when, as is generally the case in developing countries, industrial wage rates are several times greater than the marginal product of labor in agriculture? Based on an analysis which stresses the social importance of savings in such countries, Little and Mirrlees conclude that the shadow wage should be high, not much below the industrial wage rate, and present a formula for use as a rough approximation. This paper derives the shadow wage from an explicit optimization model and shows that the Little-Mirrlees formula is based on very restrictive assumptions about the propensities to save of different social groups. When these assumptions are relaxed their formula represents not a rough approximation but an upper bound on the values that the shadow wage may reasonably take. Some hypothetical computations suggest that the resultant bias may be considerable, sufficient to completely reverse their general conclusion.

Portions of this research were supported by the Development Research Group through funds provided by the Agency for International Development, the National Science Foundation, and the Ford Foundation.

However, the views expressed in this report are those of the author and do not necessarily reflect those of the sponsoring agencies.

I

Fundamental to any technique of project appraisal is the calculation of accounting prices or shadow prices for factors, which are intended to reflect more accurately than market prices the social costs involved in factor use. Clearly such calculations imply the existence of a social welfare function whose value is maximized, subject to a specific set of constraints, when public investment decisions are based on those shadow prices -- whether that social welfare function is explicitly formulated or not. Preferably, then, one would construct an economy-wide optimization model, would find that solution which maximizes an explicitly formulated objective function, and would use the values of factors in the dual solution as accounting prices for project appraisal. But since the information required to construct such a model is generally not available, except at considerable cost, this procedure is seldom used and in practice various approximations are employed. The OECD Manual by Little and Mirrlees aims, among other things, to privide a consistent basis for these approximations.

The most crucial of these accounting prices is that for labor -- the shadow wage rate. The accounting price used in project planning and project appraisal will determine the capital-intensity of the investment and the amount of employment generated. Thus, depending on the size of the government investment budget, the shadow wage rate has direct implications for income distribution and (since different income groups have different propensities to save) for economic growth. In less-developed countries so few projects reach the stage where they can be formally appraised that very few are in fact rejected. The shadow wage rate affects the capital-intensity of investment not via selection among projects which are already formulated but in the formulation process itself, since we can assume that project planners will try to make their projects rate as highly as possible in terms of whatever technique of appraisal is in use. But the extent to which this will be true (and therefore the extent to which shadow pricing is really relevant) depends on the extent to which project planners are <u>aware</u> of the techniques of appraisal being used.

Assume that the amount of capital to be invested in government projects is given. In Figure 1 Q/K represents output per unit of capital, L/K represents labor used per unit of capital in government projects, and the slope of the curve represents the marginal product of labor. Suppose the market wage in the industrial sector (c) -- the wage rate that must actually be paid to labor in government projects -- is used in project planning. Then labor will be used up to the point where the value of its marginal product equals that wage rate, generating a level of employment corresponding to, say, n_1. If, on the other hand, project planners estimate the marginal product of labor (m) in the rural sector -- from which new labor employed in government projects presumably comes -- on the assumption that this is the real social opportunity cost of using that labor, and use this (much lower) value in their calculations a level of employment corresponding to n_2 may result.

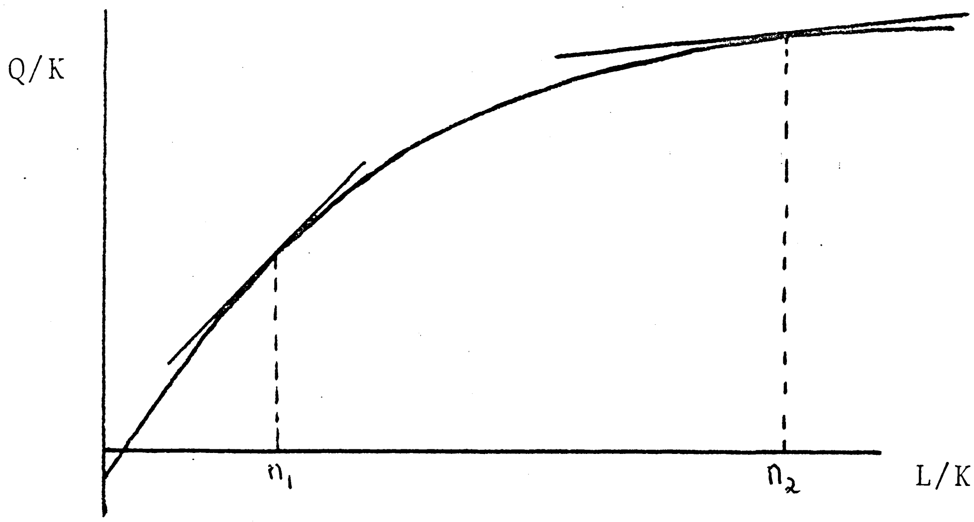

FIGURE 1

Little and Mirrlees' formula for the shadow wage rate puts it at some value between these two. Their reasoning is, very briefly, as follows. Withdrawing one man from agriculture reduces production by the average marginal product of labor in agriculture, m; but total consumption is increased by c-m. The total amount of savings available for further investment is reduced by c, the increase in consumption plus the loss in agricultural production (p. 160).[†] The shadow wage rate (SWR) is then set at the social cost of increasing employment by one unit.

$$SWR = c - \frac{1}{s_o} (c - m) \qquad (1)$$

-- where s_o is the shadow price of savings or the marginal utility of one dollar's worth of investment relative to the marginal utility of one dollar's worth of consumption. The "marginal utility of investment" is the marginal utility of the stream of consumption goods resulting from a unit of investment, discounted to the present (pp. 160-162). Although the shadow price of savings and the shadow wage are determined simultaneously -- since the shadow price of savings is a function of the shadow wage rate (via the rate of return on capital) as well as vice versa -- in this formulation the shadow price of savings is estimated prior to the shadow wage rate. Logically, then, the Little and Mirrlees formulae for estimating the shadow price of savings and the shadow wage rate should be viewed as an iterative procedure for obtaining consistent estimates; but the "one-shot" approach probably causes little error in practice.

Little and Mirrlees do not pretend that this formulation is entirely adequate, calling it "... a fairly simple formula..., which, though based on crude assumptions takes account of the main

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† Numbers in parentheses refer to page numbers in the Little and Mirrlees Manual cited at the end of the paper.

relevant considerations in a quantifiable manner." (p. 177). But they claim that their analysis shows that "...the developing countries can safely assume that the shadow wage rate is quite high, not far below the consumption level of workers on the project." (p. 176).

II

Little and Mirrlees cannot be faulted for their efforts to make their manual as simple and operational as possible. It is the task of this paper, however, to show that the conclusion that the shadow wage rate is "... quite high, not far below the consumption level of workers on the project." is a direct result of some of the "crude assumptions" and that when these assumptions are relaxed, the stated formula represents not a rough approximation, but an <u>upper bound</u> on the values that the shadow wage rate may reasonably take. This is done by deriving the shadow wage rate from an explicit optimization model.

Let: Sector A be traditional agriculture, and Sector B be the modern sector:

Y^A and Y^B be real income in sectors A and B;

L^A and L^B be labor used in sectors A and B;

$\bar{K}^A$ and $\bar{K}^B$ be capital used in sectors A and B (both assumed fixed);

F^A and F^B be functions;

C^A and C^B be real consumption in sectors A and B;

α be the perpensity to consume out of income in sector A (assumed equal for labor's share and capital's share)

β be the propensity to consume out of wages in sector B;

γ be the propensity to consume out of residual income in sector B (paid to the owners of capital used);

and w be the wage rate in sector B (equal to c in the Little-Mirrlees notation).

The following structural equations can now be formulated

$$Y^A = F^A(L^A, \bar{K}^A) \tag{2}$$

$$C^A = \alpha Y^A \tag{3}$$

$$Y^B = F^B(L^B, \bar{K}^B) \tag{4}$$

$$C^B = \beta w L^B + \gamma(Y^B - wL^B). \tag{5}$$

And the following definitional equations, where C, I, and L are total consumption, investment, and labor used, respectively

$$C = C^A + C^B \tag{6}$$

$$I = Y^A - C^A + Y^B - C^B \tag{7}$$

$$L = L^A + L^B. \tag{8}$$

Let the social welfare function be given by

$$U = U(C,I). \tag{9}$$

Since total capital is assumed fixed in each sector in this model, it is via the allocation of labor between the two sectors that optimization can occur. The problem is to find the shadow wage rate which results in a maximization of (9) subject to the constraints represented by equations (6) to (8). Since project planners will (hopefully) employ labor in their projects up to the point where its marginal product is equal to its accounting price (which is treated <u>as if</u> it were the market price), the desired shadow wage rate is equal to the marginal product of labor at the point on the production surface where the social welfare function (equation (9)) is a maximum.

To find this we set up the Lagrangian

$$\Phi = U(C, I) - \lambda_1 \{C - \alpha Y^A - \beta w L^B - \gamma(Y^B - wL^B)\}$$
$$- \lambda_2 \{I - (1-\alpha) Y^A - Y^B + \beta w L^B + \gamma(Y^B - wL^B)\}$$
$$- \lambda_3 \{L - L^A - L^B\}. \qquad (10)$$

By setting the derivatives whti respect to λ_1, λ_2 and λ_3 equal to zero we have the constraints corresponding to equations (6), (7) and (8) respectively. By setting the derivatives with respect to C, I, L^A and L^B equal to zero we obtain the following

$$\partial\Phi/\partial C = U_c - \lambda_1 = 0 \qquad (11)$$

$$\partial\Phi/\partial I = U_I - \lambda_2 = 0 \qquad (12)$$

$$\partial\Phi/\partial L^A = \lambda_1 \alpha F_L^A - \lambda_2 (1 - \alpha) F_L^A + \lambda_3 = 0 \qquad (13)$$

$$\partial\Phi/\partial L^B = \lambda_1 \{\beta w + \gamma(F_L^B - w)\} + \lambda_2\{F_L^B - \beta w - \gamma(F_L^B - w)\} + \lambda_3 = 0 \qquad (14)$$

Where $U_I = \partial U/\partial I$;

$U_c = \partial U/\partial C$;

$F_L^A = \partial F^A/\partial L^A$ (= m in Little-Mirrlees' notation);

$F_L^B = \partial F^B/\partial L^B$.

From equations (11) and (12) we have

$$\frac{\lambda_2}{\lambda_1} = \frac{U_I}{U_c} = S, \text{ the shadow price of savings.} \qquad (15)$$

From equations (13) and (14) we have

$$\lambda_1 \alpha F_L^A + \lambda_2(1 - \alpha) F_L^A = \lambda_1 \{\beta w + \gamma(F_L^B - w)\} + \lambda_2 \{F_L^B - \beta w - \alpha(F_L^B - w)\} \qquad (16)$$

Dividing both sides by λ_1 and using the result given in equation (15) we obtain

$$\alpha F_L^A + S(1 - \alpha) F_L^A = \beta w + \gamma F_L^B - \gamma w + SF_L^B - S\beta w - S\gamma F_L^B + S\gamma w, \qquad (17)$$

which, by rearranging terms gives the desired shadow wage rate:

$$F_L^B = \frac{w\{S(\beta - \gamma) + \gamma - \beta\} + F_L^A \{S(1 - \alpha) + \alpha\}}{S(1 - \gamma) + \gamma} . \qquad (18)$$

Now set $\alpha = 1$,

$\beta = 1$,

and $\gamma = 0$.

We then have $F_L^B = w - \frac{1}{S} (w - F_L^A)$, (19)

which exactly corresponds to the Little-Mirlees formula for the shadow wage rate (equation(1)). Thus Little and Mirrlees' formula is the special case of (18) where it is assumed that:

agricultural workers consume all their income ($\alpha = 1$);

industrial workers consume all their income ($\beta = 1$);

and capital-owners save all their income ($\gamma = 0$).

The first of these assumptions seems plausible enough. Since capital in traditional agriculture is (according to the Schultz theory) of low productivity, the savings rate can be expected to be low. But the second and third seem much less plausible. It is therefore of interest to see how relaxing these assumptions affects the result.

Differentiating F_L^B with respect to α and β yields

$$\partial F_L^B/\partial\alpha = \frac{-F_L^A\ (S - 1)}{S(1 - \gamma) + \gamma} < 0 \tag{20}$$

$$\partial F_L^B/\partial\beta = \frac{w(S - 1)}{S(1 - \gamma) + \gamma} > 0 \ . \tag{21}$$

Since S will be always greater than unity† (20) is always negative and (21) is always positive. Thus relaxing the assumption that agricultural workers consume all their income raises the estimate of the shadow wage, and relaxing the corresponding assumption for industrial workers lowers it. But since w is, in practice always considerably greater than F_L^A the absolute value of (21) is always considerably greater than that of (20). Thus since α is never likely to deviate from unity more than β, relaxing

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† Clearly S cannot be less than unity since the lower bound on the trade-off between consumption and investment is given for the current year by the national accounting identity $Y \equiv C + I$. Furthermore $S = 1 \Rightarrow F_L^B = F_L^A$ and there is no premium on the generation of an investible surplus. In this case the shadow pricing problem does not arise; but it is precisely because there is a premium on savings that shadow pricing of factors is of interest.

these two assumptions will always lead to a <u>lowering</u> of the estimate of the shadow wage rate. Differentiating F_L^B with respect to γ yields a very cumbersome expression but it can be shown to be always negative. Thus relaxing the assumption that all surplus income from the project is saved <u>further lowers</u> the estimate. These three assumptions combined yield an <u>upper bound</u> on the estimates of the shadow wage rate that can be reasonably obtained by relaxing them. The estimates given in Table 1 suggest that this may be of considerable quantitative significance. Table 1 gives alternative values of equation (18) assuming that the industrial wage rate (w) = 3, and the marginal product of labor in agriculture (F_L^A) = 1.

TABLE 1
Estimates of Shadow Wage Rate with Alternative Parametric Assumptions

	$\alpha = 1$, $\beta = 1$, $\gamma = 0$	$\alpha = 1$, $\beta = 0.8$, $\gamma = 0.2$	$\alpha = 0.9$, $\beta = 0.7$, $\gamma = 0.3$	$\alpha = 0.9$, $\beta = 0.7$, $\gamma = 0.5$
S = 1.5	1.667	1.357	1.222	1.080
S = 5.0	2.600	1.952	1.630	1.267
S = 9.0	2.788	2.081	1.728	1.320

The values for S of 1.5 and 9.0 are those used in examples given by Little and Mirrlees. The value of 5.0 has been added.

III

The values that α, β, and γ will take in a particular situation is an empirical question. But Little and Mirlees' assumptions yield an estimate of the shadow wage rate that is always biased upwards and Table 1 suggests that this bias may be considerable. For given values of α, β, and γ the bias increases with the difference between the industrial wage rate and the marginal product of labor in agriculture, and with the shadow price of savings.

The degree to which the upward bias in the shadow wage rate affects industrial employment and the capital intensity of projects will depend on the elasticity of the derived demand for labor in government investment projects. But the Little and Mirlees assumptions have been shown to effectively minimise the employment generated by government investment and to result in projects that are overly capital intensive.

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ACKNOWLEDGEMENTS

This paper owes much to the classroom teaching of Albert Fishlow and the helpful comments of Lance Taylor. All defects are of the author's making.