DEVELOPING REGIONAL INPUT-OUTPUT MODELS FROM THE U.N. FORMAT ADOPTED BY THE U.S. IN THE NEW 1972 INPUT-OUTPUT MODEL

Dennis DiPietre, Rodney L. Walker, and David R. Martella

Since the mid-1940s, interest in the interrelationships of subnational economies has been growing. Part of this interest flows from a realization of the need to manage regional growth and mitigate the effects of economically unstable components of regional economies on the welfare of the people within the region. Aggregated macroeconomic models applied at the national level commonly provide insufficient information about their components, the regional economies. This lack has led to the development of state and regional macro models which can provide specific information relevant to state or local decision makers. Such information includes the availability of regional resources necessary to support and expand regional production and the impacts of changes in demand on the welfare of local inhabitants. These models are also useful in estimating the impacts of national policy on regional economies. The development of regional input-output models is an example of the trend toward fuller understanding of regional economies. State or regional input-output models can be constructed either by survey or by estimation from the national input-output model. Time and money constraints have increased the popularity of the latter approach among regional economists.

Typically, the three sequentially developed tables in the Leontief input-output analysis are the Transactions Table, the Direct Requirements Table, and the Total Requirements Table. The national Input-Output Tables for 1972, (released to the public in 1979) have been conceptually modified, according to the recommendations of the United Nations, and differ substantially from previously constructed tables.

The major differences include the introduction of a Make Table which describes the commodity output composition of each industry. From this table, a market share or production table can be constructed to reveal the percentage of each commodity produced by each industry. This accounting of secondary production relaxes the traditional assumption of one commodity produced per industry. The change will provide a more accurate estimation of output by eliminating overestimaton and the erroneous transaction links which result from the traditional method of treating secondary production. The differentiation of commodities and industries makes possible two Total Requirements Tables, one indicating the total commodity requirements of final demand and the other indicating the total industry requirements of final demand.

The purpose of our article is to present the conceptual method by which to develop a regional input-output model from the current national input-output model. A generalized model conforming to the new treatments introduced by the Bureau of Economic Analysis is described.

The national input-output model conforming to the new treatment of secondary products appears in *The Input Output Structure of the U.S. Economy, 1972* (Ritz). Our generalized model conforms to the changes appearing in that publication. For simplification, all elements of the national model are capitalized and all regional elements are in lower case letters in the equations and tables that follow. This simplification relaxes conventional mathematical notation. However, all vectors and matrices are identified by their dimensions in the descriptions following each equation. A hypothetical four-industry economy is used as an aid in exposition.

THE USE TABLE

The Use Table is a matrix which describes the component parts of the economy. In Table 1, the commodities appear as rows and the industries which produce these commodities as primary products appear along the corresponding column head. Down the rows of the intermediate part of the Use Table, each U_{ij} represents the amount of commodity i, both nationally produced and imported, that is used by industry j in the production of that industry's total output (G_j, primary and secondary products). U_{5j} describes industry j's use of imports for which there are no domestically pro-

Dennis DiPietre is former Graduate Research Assistant, Rodney L. Walker is former Assistant Professor, and David R. Martella is Research Associate, Department of Agricultural Economics and Rural Sociology, University of Arkansas.

TABLE 1. THE USE TABLE

		Industries 1	2	3	4	Final Demand	Exports	Imports	Total Final Demand	Total Commodity Output
Commodities	1	U_{11}	U_{12}	U_{13}	U_{14}	F_1	X_1	I_1	E_1	Q_1
	2	U_{21}	U_{22}	U_{23}	U_{24}	F_2	X_2	I_2	F_2	Q_2
	3	U_{31}	U_{32}	U_{33}	U_{34}	F_3	X_3	I_3	E_3	Q_3
	4	U_{41}	U_{42}	U_{43}	U_{44}	F_4	X_4	I_4	E_4	Q_4
Non-comparable Imports		U_{51}	U_{52}	U_{53}	U_{54}	F_5	X_5	I_5	E_5	0
Scrap		U_{61}	U_{62}	U_{63}	U_{64}	F_6	X_5	I_6	F_6	SC_t
Value Added		VA_1	VA_2	VA_3	VA_4					VA_t
Total Industry Output		G_1	G_2	G_3	G_4	F_t	X_t	I_t	E_t	

U_{ij} = purchases of commodity i by industry j from all sources
F_i = final demand for commodity i
X_i = exports of commodity i
I_i = imports of commodity i
E_i = total final demand for commodity i
Q_i = total commodity i output from all national industries
VA_j = value added by industry j
G_j = total industry j output

The part of the Use Table enclosed by dashed lines represents intermediate demand.

duced substitutes. U_{6j} describes industry j's use of scrap products. VA_j describes the value added (compensation to employees, indirect business taxes, and property-type income[1]) in the production of industry j's total output. The total output equation for each industry is:

$$G_j = \sum_{i=1}^{m} U_{ij} + VA_j$$

where

G_j = total industry j output (primary and secondary products)
U_{ij} = the dollar value of all inputs, including imports of domestically produced commodities, scrap, and imports of noncomparable commodities
VA_j = value added by industry j
m = number of commodities including scrap and noncomparable imports.

Across the first four columns of the Use Table, each U_{ij} denotes the dollar value of commodity i purchased by each industry j for use as an intermediate input. F_i is the value of commodity i purchased by final demanders. X_i is the value of commodity i exported, and I_i is a negative value that deletes the value of commodity i imported into the nation. E_i is total final demand for domestically produced commodity i. The sum of each U_{ij} in row_i (intermediate demand) plus final demand and exports, less imports of commodity i, yields Q_i. Q_i is the total domestic production of commodity i, no matter which domestic industry produces it. The fundamental equation of total commodity output is:

$$Q = Ui + E \tag{1}$$

where

Q = (m × 1) vector of total commodity outputs plus scrap and noncomparable imports
U = (m × n) intermediate part of the Use Table
i = (n × 1) a summation vector of 1's
E = (m × 1) vector of final demand including exports less imports
m = the number of commodities plus scrap and noncomparable imports
n = the number of industries.

THE MAKE TABLE

The Make Table (Table 2) is an industry by

TABLE 2. THE MAKE TABLE AND VECTOR H

		Commodities 1	2	3	4	Non-Comparable Imports	Scrap		Total Industry Output
Industries	1	V_{11}	V_{12}	V_{13}	V_{14}	0	0	H_1	G_1
	2	V_{21}	V_{22}	V_{23}	V_{24}	0	0	H_2	G_2
	3	V_{31}	V_{32}	V_{33}	V_{34}	0	0	H_3	G_3
	4	V_{41}	V_{42}	V_{43}	V_{44}	0	0	H_4	G_4
Total Commodity Output		Q_1	Q_2	Q_3	Q_4	0	0	H_t	

*The portion of the Make Table enclosed by the dashed lines is the estimate of V.

commodity matrix (reverse of the Use Table) which describes the dollar value of primary and secondary products produced by each industry. Across the columns, each V_{ij} is the value of commodity j produced by the industry at the left of the row. H_i is the dollar value of scrap produced by industry i.[2] The summation of each row element plus the vector H yields each industry's total output (G_i). The fundamental equation of total industry output is:

$$G = Vi + H \tag{2}$$

where

G = (n × 1) column vector of total industry output
V = (n × m) Make Table having zero values for noncomparable imports and scrap
i = (m × 1) summation vector of 1's
H = (n × 1) vector of scrap output.

The last two columns in V contain only zeros, which reflect the fact that there are no national production functions for imports or scrap. These columns must be present to maintain the conformability necessary for later

[1]Property-type income includes proprietor's income, rental income, corporate profits, and business transfer payments less subsidies.

[2]Note that within the estimate of V (the Make Table), scrap is entered as a column vector of zeros. The scrap vector H is included in Table 1 for the convenience of the reader.

manipulations of this matrix.

Down the columns, each V_{ij} is the value of commodity j produced by each industry i, so that the summation of each column yields the total commodity outputs (Q_j).

THE MARKET SHARE TABLE

The Market Share Table or D Table (Table 3)

TABLE 3. THE MARKET SHARE TABLE

		COMMODITY 1	2	3	4	Non-comparable Imports	Scrap
INDUSTRY	1	D_{11}	D_{12}	D_{13}	D_{14}	0	0
	2	D_{21}	D_{22}	D_{23}	D_{24}	0	0
	3	D_{31}	D_{32}	D_{33}	D_{34}	0	0
	4	D_{41}	D_{42}	D_{43}	D_{44}	0	0
Total		1.0	1.0	1.0	1.0	0	0

is constructed from the Make Table by dividing each element (V_{ij}) in the first (m − 2) columns by its corresponding column sum (Q_j). Each element in the D Table (D_{ij}) reveals in decimal percentages the share of commodity j produced by industry i. The sum of each column yields 1.0 or 100 percent of commodity j. The fundamental equation relating the Market Share Table to the Make Table is:

$$V = D\hat{Q} \tag{3}$$

where

V = (n × m) Make Table with zeros in the columns for noncomparable imports and scrap
D = (n × m) Market Share Table in decimal percentages
$\hat{Q}$ = (m × m) matrix of zeros except for the main diagonal which contains the vector Q.

THE DIRECT REQUIREMENTS TABLE

The Direct Requirements Table or B Table (Table 4) is developed from the intermediate part of the Use Table and expresses the commodity requirements of each industry necessary to produce one dollar of output. Each element (U_{ij}) in the Use Table is divided by its corresponding column sum (G_j) to accomplish the translation to a per-dollar-of-output basis. The amount of value added per dollar of output can then be solved as a residual.

TABLE 4. DIRECT REQUIREMENTS TABLE

		INDUSTRY 1	2	3	4
COMMODITY	1	B_{11}	B_{12}	B_{13}	B_{14}
	2	B_{21}	B_{22}	B_{23}	B_{24}
	3	B_{31}	B_{32}	B_{33}	B_{34}
	4	B_{41}	B_{42}	B_{43}	B_{44}
Non-comparable Imports		B_{51}	B_{52}	B_{53}	B_{54}
Scrap		B_{61}	B_{62}	B_{63}	B_{64}
Value Added		VA_{11}	VA_{12}	VA_{13}	VA_{14}
Total		1.0	1.0	1.0	1.0

$$VA_j = 1 - \sum_{i=1}^{m} \frac{U_{ij}}{G_j}$$

The resulting figures for value added are customarily appended to the B Table as the last row; thus the column sums equal 1.0.

Each B_{ij} expresses the value of commodity i necessary to produce one dollar's worth of industry j output. The fundamental equation relating the Use Table to the Direct Requirements Table is:

$$U = B\hat{G} \tag{4}$$

where

U = (m × n) portion of the Use Table
B = (m × n) Direct Requirements Table (excluding value added)
$\hat{G}$ = (n × n) matrix of zeros except for the main diagonal which is the vector G
m = number of commodities including noncomparable imports and scrap
n = number of industries.

THE TREATMENT OF SCRAP

Scrap is the unplanned output of an industry. The value of scrap is composed of the margins on the sale of used goods and some actual commodities. For example, leftover rails in the railroad industry may be sold to steel industries which melt them down and produce products. If scrap is produced, it is assumed to be produced in a fixed proportion to the industry's output. However, scrap is treated in such a way as to prevent its requirement as an input from generating output in the industries from which it is produced. The fundamental equation relating scrap to industry output is:

(5) $$H = \hat{P}G$$

where

- H = $(n \times 1)$ vector of scrap production
- $\hat{P}$ = $(n \times n)$ matrix of zeros except the main diagonal which contains the vector P, the fixed percentage of the industry's output that is scrap
- G = $(n \times 1)$ vector of total industry outputs.

THE TOTAL REQUIREMENTS TABLES

The development of the first Total Requirements Table (Table 5) involves estimating some Matrix M which describes the relationship of total final demand to total commodity output.

TABLE 5. COMMODITY BY COMMODITY TOTAL REQUIREMENTS TABLE

	COMMODITIES 1	2	3	4	NCI	SCRAP
COMMODITY 1	CC_{11}	CC_{12}	CC_{13}	CC_{14}	0	0
2	CC_{21}	CC_{22}	CC_{23}	CC_{24}	0	0
3	CC_{31}	CC_{32}	CC_{33}	CC_{34}	0	0
4	CC_{41}	CC_{42}	CC_{43}	CC_{44}	0	0
NCI	CC_{51}	CC_{52}	CC_{53}	CC_{54}	1.0	0
SCRAP	CC_{61}	CC_{62}	CC_{63}	CC_{64}	0	1.0
COMMODITY OUTPUT MULTPLIERS	CM_1	CM_2	CM_3	CM_4	1.0	1.0

CC_{ij} = total value of commodity i necessary to supply one dollar's worth of commodity j output to final demand.

CM_j = total value of all commodities necessary to supply one dollar's worth of commodity j output to final demand.

$$Q = ME$$

The estimation of matrix M follows.

Substituting equation 4 into equation 1 gives:

(6) $$Q = BG + E.$$

Substituting equation 3 into equation 2 gives:

(7) $$G - H = D\hat{Q}.$$

Substituting equation 5 into equation 7 and solving for G gives:

$$G - \hat{P}G = D\hat{Q}$$
$$(I - \hat{P})G = D\hat{Q}$$

(8) $$G = (I - \hat{P})^{-1}D\hat{Q}.$$

Letting $W = (I-P)^{-1}D$ gives:

(9) $$G = W\hat{Q}.$$

The W matrix transforms commodity outputs to industry outputs by inflating commodity outputs with scrap. The components of W reveal how this step is accomplished. $(I - \hat{P})^{-1}$ contains, along the main diagonal, the amount of industry output necessary to produce one dollar's worth of commodity. If an industry produces scrap, the corresponding entry in $(I - \hat{P})^{-1}$ will be greater than one. This is so because the value of scrap is a component of total industry output, but is not included in the value of commodity output.

The premultiplication of D, the Market Share Table, by $(I - \hat{P})^{-1}$ weights the industry output requirements to produce a dollar's worth of commodity by the share of total commodity output produced by each industry.

Substituting equation 9 into equation 6 and solving for Q gives:

$$Q = BWQ + E$$
$$(I-BW)Q = E$$
(10) $$Q = (I-BW)^{-1}E.$$

Notice that the $(I-BW)^{-1}$ matrix is the matrix M relating commodity output to total final demand. Each element in $(I-BW)^{-1}$ is the total requirement of the commodity named at the beginning of the row needed to supply one dollar of total final demand for the commodity named at the head of the column. The sums of each column are the total requirements of all commodities necessary to produce one dollar of total final demand for the commodity named at the head of each column. These are the commodity output multipliers.

The second Total Requirements Table (Table 6) is calculated by solving for some Matrix N

TABLE 6. THE INDUSTRY BY COMMODITY TOTAL REQUIREMENTS TABLE

	COMMODITIES 1	2	3	4	NCI	SCRAP
INDUSTRIES 1	IC_{11}	IC_{12}	IC_{13}	IC_{14}	0	0
2	IC_{21}	IC_{22}	IC_{23}	IC_{24}	0	0
3	IC_{31}	IC_{32}	IC_{33}	IC_{34}	0	0
4	IC_{41}	IC_{42}	IC_{43}	IC_{44}	0	0
INDUSTRY OUTPUT MULTIPLIERS	IM_1	IM_2	IM_3	IM_4	0	0

which describes the relationship between total industry output (G) and total final demand.

$$G = NE$$

Because W translates commodity output to industry output, matrix N is solved for directly in the following calculation.

Let $N = W(I-BW)^{-1}$

so that

$$G = W(I-BW)^{-1}E.$$

$W(I-BW)^{-1}$ is the matrix N relating total industry output to total final demand. Each element in $W(I-BW)^{-1}$ is the total requirement of the industry named at the beginning of the row necessary to supply one dollar of total final demand for the commodity named at the head of the column. The sums of each column are the total requirements of all industries necessary to supply one dollar of total final demand for the commodity named at the head of each column. These are the industry output multipliers.

From these two sets of multipliers, the model can demonstrate the overall impact on commodity output and industry output resulting from increases or decreases in total final demand.

THE REGIONAL MAKE TABLE

The first step in constructing a regional input-output model is the development of the regional Make Table. The regional Make Table is constructed with the assumption that regional industries produce the same primary and secondary products as the national industries and that each regional industry is proportionally identical to its corresponding national industry in the production of primary and secondary products. Given these assumptions and estimates of total regional output for each industry, the regional v_{ij}'s are computed as:

$$v_{ij} = \frac{V_{ij}}{G_i} g_i$$

v_{ij} = the regional Make Table elements
V_{ij} = the national Make Table elements
G_i = the national total industry i output
g_i = the regional total industry i output.

Regional total commodity outputs necessary for the estimation of the regional Use Table can be computed from the regional Make Table as:

$$q_i = \sum_{i=1}^{m} v_{ij} \qquad j = 1, 2, \ldots, n$$

where
q_i = total commodity i output
m = the number of commodities
n = the number of industries.

THE REGIONAL USE TABLE

After the total commodity outputs are obtained, the regional Use Table (Table 7) can be estimated. Given the national Use Table and estimates of total regional commodity outputs, the problem is to estimate regional purchases of each commodity i, scrap and noncomparable imports by each industry j (u_{ij}), regional final demand (f_i), regional imports (m_{ij}),[3] regional exports (x_i), and regional value added (va_{ij}) for each industry. These estimations can be accomplished by a variety of location quotient or pool techniques. Schaffer and Chu and Czamanski and Malizia summarize commonly used methods. The only impact on these techniques from the new treatment of secondary products is that commodity outputs should be used in calculating the location quotients (for instance) because industry outputs include scrap and secondary products. Once the final estimates of the regional Use Table are complete, the calculations to create the Direct Requirements Tables and the Total Requirements Tables are identical to those used for the generalized national model.

TABLE 7. REGIONAL USE TABLE

		INDUSTRIES 1	2	3	4	Final Demand	Exports	Total Commodity Output
COMMODITIES	1	u_{11}	u_{12}	u_{13}	u_{14}	f_1	x_1	q_1
	2	u_{21}	u_{22}	u_{23}	u_{24}	f_2	x_2	q_2
	3	u_{31}	u_{32}	u_{33}	u_{34}	f_3	x_3	q_3
	4	u_{41}	u_{42}	u_{43}	u_{44}	f_4	x_4	q_4
Non-comparable Imports		u_{51}	u_{52}	u_{53}	u_{54}	f_5	x_5	q_5
Scrap		u_{61}	u_{62}	u_{63}	u_{64}	f_6	x_6	q_6
Imports		m_1	m_2	m_3	m_4	f_7		
Value Added		va_1	va_2	va_3	va_4			
Total Industry Output		g_1	g_2	g_3	g_4			

UNIQUE FEATURES OF THE REGIONAL MODEL

First, unlike the national Use Table, the regional Use Table contains an import row for commodities not produced in the nation (non-

[3]For an explanation of the additional import row at the regional level, see the next section.

comparable imports) and for regionally produced commodities (m_{ij}) which must be imported. This second import row is necessary at the regional level because the region imports both from the nation and from international suppliers whereas the nation imports only from international sources. The extra import row accounts for those products which come into the region from extraregional suppliers.

Second, the national model contains four dummy industries which have zero values in every cell of the Use Table except value added and final demand. From these dummy sectors, the estimates of value added for the exogenous sectors of the regional Use Table can be computed. After this computation has been made, these dummy industries can be eliminated from the regional model. Every other element in the regional Use Table and all subsequent tables is unaffected by their omission.

After the estimation of the regional Total Requirements Tables, the income and employment multipliers can be developed by the standard formulation (Yan) utilizing the industry by commodity Total Requirements Table.

Generally, two types of each multiplier are developed. The Type I multipliers are those based on the assumption that personal consumption is exogenous to the model. The Type II multipliers are based on the assumption that personal consumption will change as the model is affected by an increase or decrease in total final demand. These multipliers are calculated by disaggregating a personal consumption vector from final demand and making it endogenous to the model, so that wage earners become another "industry" in the Use Table. The value added row is also brought up into the endogenous portion of the Use Table. Because the indirect business taxes component of value added cannot be disaggregated with available data, the Type II multipliers tend to be biased upward.

MAJOR ASSUMPTIONS AND LIMITATIONS

One major assumption of the current model and of input-output analysis is the fixed proportion of inputs. This restriction establishes the linear relationship between inputs and outputs. A second assumption of the model is a fixed relationship between primary and secondary products produced by an industry. If the industry is called on to double its output of primary product to meet a new final demand, its production of secondary products will also double. Third, there must be only one true cost of factors, so that input mix or substitution will not occur as increases or decreases in demand force new output levels and new demands for inputs on related industries. In other words, the relative prices for substitutable inputs remain constant and cannot change. This assumption fixes the technical coefficients for any level of output. Fourth, if an industry produces scrap, it is assumed to be produced in a fixed relationship to that industry's output.

A further assumption, which is related to estimating a regional model from the national model, is the assumption of identical product mix. In aggregating industries into a single production, one assumes that the components of the aggregated industry are proportionally identical in both the national and regional aggregations. This assumption suggests that a relatively fine level of disaggregation of all industries will produce the most accurate estimates of production technology and output. However, a fine level of disaggregation may be difficult to accomplish in models relying heavily on secondary data because of problems of disclosure. A tradeoff results between the accuracy of the model and the cost in terms of both time and money. If these product mix problems occur, they should be noted and the resulting analysis should be reflective of the error that is introduced.

FURTHER STUDY

An interesting and useful indication for further study is derived from the new treatment of secondary products adopted by the BEA (DiPietre). The creation of the Make Table provides an opportunity to estimate empirically the effects of both vertical integration and diversification by an industry. These effects were completely hidden in the one-industry, one-commodity method. With the aid of the industry by commodity Total Requirements Table, the impact on each industry producing a given commodity is captured in the industry output multipliers.

REFERENCES

Czamanski, Stanislav and Emil E. Malizia. "Applicability and Limitations in the Use of National Input-Output Tables for Regional Studies." *Reg. Sci. Assoc. Pap.* 23(1969):65-73.

DiPietre, Dennis D. "An Interindustry Analysis of the Structure of the Arkansas Economy with Special Investigations of the Poultry Industry." Masters thesis, University of Arkansas, 1979.

Leontief, Wassily. *Input-Output Economics.* New York: Oxford University Press, 1966.

Ritz, Phillip. *The Input-Output Structure of the U.S. Economy, 1972.* Bureau of Economic Analysis, U. S. Department of Commerce, 1979.

Schaffer, William and Kong Chu. "Non-survey Techniques for Constructing Regional Interindustry Models." *Reg. Sci. Assoc. Pap.* 23(1969):83-101.

U.S. Department of Commerce, Bureau of Economic Analysis. "Mathematical Derivation of the Total Requirements Tables for the 1972 Input-Output Study." Washington, D.C.: Government Document Publication Service, 1979.

Yan, Chiou-Shuang. *Introduction to Input-Output Economics.* New York: Holt, Rinehart, and Winston, 1969.